

## MEAN-FIELD THEORY

The mean-field Nambu-Gor'kov Hamiltonian is

$$H = \frac{1}{2} \iint d^3\mathbf{r} d^3\mathbf{r}' \hat{\Psi}^\dagger(\mathbf{r}') \begin{pmatrix} \xi(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\hat{1} & \hat{\Delta}(\mathbf{r},\mathbf{r}') \\ \hat{\Delta}^\dagger(\mathbf{r},\mathbf{r}') & -\xi(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\hat{1} \end{pmatrix} \hat{\Psi}(\mathbf{r}) \quad (1)$$

$$\equiv \frac{1}{2} \iint d^3\mathbf{r} d^3\mathbf{r}' \hat{\Psi}^\dagger(\mathbf{r}') \hat{\tau}_3 \left( \xi(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \hat{\Delta}(\mathbf{r},\mathbf{r}') \right) \hat{\Psi}(\mathbf{r}). \quad (2)$$

where

$$\hat{\Delta}(\mathbf{r},\mathbf{r}') = \begin{pmatrix} 0 & \hat{\Delta}(\mathbf{r},\mathbf{r}') \\ -\hat{\Delta}^\dagger(\mathbf{r},\mathbf{r}') & 0 \end{pmatrix} \quad (3)$$

When the system is homogeneous, the two-point gap function only depends on the relative coordinate, i.e.,  $\hat{\Delta}(\mathbf{r},\mathbf{r}') = \hat{\Delta}(\mathbf{r}-\mathbf{r}')$ . In this case, we can transform to momentum space, and the Hamiltonian becomes

$$H = \frac{1}{2} \sum_{\mathbf{p}} \hat{\Psi}_{\mathbf{p}}^\dagger \hat{\tau}_3 (\xi_{\mathbf{p}} + \hat{\Delta}(\mathbf{p})) \hat{\Psi}_{\mathbf{p}}. \quad (4)$$

where  $\hat{\Psi}_{\mathbf{p}}^\dagger = (c_{\mathbf{p}\uparrow}^\dagger, c_{\mathbf{p}\downarrow}^\dagger, c_{-\mathbf{p}\uparrow}, c_{-\mathbf{p}\downarrow})$ .

## FINITE TEMPERATURE

The imaginary time Green's function is

$$\hat{G}(\mathbf{r}\tau, \mathbf{r}'\tau') = \hat{G}(\mathbf{r}, \mathbf{r}'; \tau - \tau') = - \left\langle T_\tau \hat{\Psi}(\mathbf{r}, \tau) \hat{\Psi}^\dagger(\mathbf{r}', \tau') \right\rangle \equiv \begin{pmatrix} \hat{G} & \hat{F} \\ \hat{F} & \hat{G} \end{pmatrix} \quad (5)$$

The unperturbed Matsubara Green's function is

$$\hat{G}_0(\mathbf{p}, i\epsilon_n) = (i\epsilon_n - \hat{\tau}_3 \xi_{\mathbf{p}})^{-1} = - \frac{i\epsilon_n + \hat{\tau}_3 \xi_{\mathbf{p}}}{\epsilon_n^2 + \xi_{\mathbf{p}}^2}. \quad (6)$$

In bulk, the Gorkov Green's function satisfies

$$\hat{G}^{-1}(\mathbf{p}, i\epsilon_n) = \hat{G}_0^{-1}(\mathbf{p}, i\epsilon_n) - \hat{\tau}_3 \hat{\Delta}(\mathbf{p}) = i\epsilon_n - \hat{\tau}_3 \xi_{\mathbf{p}} - \hat{\tau}_3 \hat{\Delta}(\mathbf{p}). \quad (7)$$

which gives

$$\hat{G}(\mathbf{p}, i\epsilon_n) = - \frac{i\epsilon_n + \hat{\tau}_3 \xi_{\mathbf{p}} + \hat{\tau}_3 \hat{\Delta}(\mathbf{p})}{\epsilon_n^2 + \xi_{\mathbf{p}}^2 + |\Delta|^2} \quad (8)$$

The bulk quasiclassical Green's function is

$$\hat{g}(\mathbf{p}, i\epsilon_n) = \int d\xi_p \hat{G}(\mathbf{p}, i\epsilon_n) \hat{\tau}_3 = -\pi \frac{i\epsilon_n \hat{\tau}_3 - \hat{\Delta}(\mathbf{p})}{\sqrt{|\Delta|^2 + \epsilon_n^2}} = \frac{\pi}{\sqrt{|\Delta|^2 + \epsilon_n^2}} \begin{pmatrix} -i\epsilon_n & \hat{\Delta} \\ -\hat{\Delta}^\dagger & i\epsilon_n \end{pmatrix} \equiv \begin{pmatrix} \hat{g} & \hat{f} \\ \hat{f} & \hat{g} \end{pmatrix} \quad (9)$$

and it satisfies the normalization condition

$$\hat{g}(\mathbf{p}, i\epsilon_n)^2 = -\pi^2 \hat{1}. \quad (10)$$

The bulk quasiclassical Green's function satisfies

$$[i\epsilon_n \hat{\tau}_3 - \hat{\Delta}, \hat{g}] = 0 \quad (11)$$

But when near the boundary or impurities, the quasiclassical Green's function  $\hat{g}(\mathbf{R}, \mathbf{p}; \epsilon)$  and the gap function  $\hat{\Delta}(\mathbf{R}, \mathbf{p})$  are inhomogeneous, and are described by the Eilenberger equation and gap equation.

$$[i\epsilon_n \hat{\tau}_3 - \hat{\Delta}(\mathbf{R}, \mathbf{p}), \hat{g}(\mathbf{R}, \mathbf{p}; \epsilon)] + i\hbar \mathbf{v}_p \cdot \nabla \hat{g}(\mathbf{R}, \mathbf{p}; \epsilon) = 0 \quad (12)$$

and

$$\hat{\Delta}(\mathbf{R}, \mathbf{p}) = T \sum_{\epsilon_n = -\omega_c}^{\omega_c} \left\langle v(\mathbf{p}, \mathbf{p}') \hat{f}(\mathbf{R}, \mathbf{p}', \epsilon_n) \right\rangle_{p'} \quad (13)$$

Here  $\mathbf{R} = \frac{1}{2}(\mathbf{r} + \mathbf{r}')$  is the center of the pair. Now we neglect the spin structure, i.e. discard the little hats. In bulk, we have

$$\Delta(\mathbf{p}) = T \sum_{\epsilon_n = -\omega_c}^{\omega_c} \left\langle v(\mathbf{p}, \mathbf{p}') \frac{\pi \Delta(\mathbf{p}')}{\sqrt{|\Delta|^2 + \epsilon_n^2}} \right\rangle_{p'} \quad (14)$$

Move everything to the right hand side and subtract Eq. (13) from it, we get

$$0 = T \sum_{\epsilon_n = -\infty}^{\infty} \left[ \left\langle v(\mathbf{p}, \mathbf{p}') \frac{f(\mathbf{R}, \mathbf{p}', \epsilon_n)}{\Delta(\mathbf{R}, \mathbf{p})} \right\rangle_{p'} - \left\langle v(\mathbf{p}'', \mathbf{p}') \frac{\pi}{\sqrt{|\Delta|^2 + \epsilon_n^2}} \frac{\Delta(\mathbf{p}')}{\Delta(\mathbf{p}'')} \right\rangle_{p'} \right] \quad (15)$$

which gives

$$\Delta(\mathbf{R}, \mathbf{p}) = \frac{T \sum_{\epsilon_n}^{\infty} \left\langle v(\mathbf{p}, \mathbf{p}') \hat{f}(\mathbf{R}, \mathbf{p}', \epsilon_n) \right\rangle_{p'}}{T \sum_{\epsilon_n}^{\infty} \left\langle v(\mathbf{p}'', \mathbf{p}') \frac{\pi}{\sqrt{|\Delta|^2 + \epsilon_n^2}} \frac{\Delta(\mathbf{p}')}{\Delta(\mathbf{p}'')} \right\rangle_{p'}} \quad (16)$$

We have the Digamma function

$$K(T) = T \sum_{\epsilon_n = -\omega_c}^{\omega_c} \frac{\pi}{|\epsilon_n|} \approx \ln \left( 1.13 \frac{\omega_c}{T} \right) \quad (17)$$

In bulk at  $T_c$ , we have

$$\Delta(\mathbf{p}) = K(T_c) \langle v(\mathbf{p}, \mathbf{p}') \Delta(\mathbf{p}') \rangle_{p'} \quad (18)$$

and we have

$$K(T_c) - K(T) = \ln(T/T_c) = \frac{\Delta(\mathbf{p}'')}{\langle v(\mathbf{p}'', \mathbf{p}') \Delta(\mathbf{p}') \rangle_{p'} \big|_{T_c}} - T \sum_{\epsilon_n = -\omega_c}^{\omega_c} \frac{\pi}{|\epsilon_n|} \quad (19)$$

Substitute in Eq. (13), we get another equation to determine the gap inhomogeneity:

$$\ln \frac{T}{T_c} = T \sum_{\epsilon_n = -\infty}^{\infty} \left[ \frac{\langle v(\mathbf{p}, \mathbf{p}') f(\mathbf{R}, \mathbf{p}', \epsilon_n) \rangle_{p'} / \Delta(\mathbf{R}, \mathbf{p})}{\langle v(\mathbf{p}'', \mathbf{p}') \Delta(\mathbf{p}') \rangle_{p'} / \Delta(\mathbf{p}'') \big|_{T_c}} - \frac{\pi}{|\epsilon_n|} \right] \quad (20)$$

If you want temperature dependence of bulk gap, you can substitute in Eq. (14) to get

$$\ln \frac{T}{T_c} = T \sum_{\epsilon_n = -\infty}^{\infty} \left[ \frac{\pi \langle v(\mathbf{p}, \mathbf{p}') \Delta(\mathbf{p}') / \Delta(\mathbf{p}) \sqrt{|\Delta|^2 + \epsilon_n^2} \rangle_{p'}}{\langle v(\mathbf{p}'', \mathbf{p}') \Delta(\mathbf{p}') \rangle_{p'} / \Delta(\mathbf{p}'') \big|_{T_c}} - \frac{\pi}{|\epsilon_n|} \right] \quad (21)$$

## RICCATI PARAMETERIZATIONS

The Riccati parameterization of the Green's function is

$$\hat{g} = -i\pi \hat{N} \begin{pmatrix} 1 + \hat{a}\hat{\underline{a}} & 2\hat{a} \\ -2\hat{\underline{a}} & -1 - \hat{\underline{a}}\hat{a} \end{pmatrix} \quad (22)$$

where  $\hat{N}$  is

$$\hat{N} = \begin{pmatrix} (1 - \hat{a}\hat{\underline{a}})^{-1} & 0 \\ 0 & (1 - \hat{\underline{a}}\hat{a})^{-1} \end{pmatrix} \quad (23)$$

The inverse is

$$\hat{a} = (\hat{g} - i\pi)^{-1} \hat{f} \quad (24)$$

$$\hat{\underline{a}} = (\hat{\underline{g}} + i\pi)^{-1} \hat{\underline{f}} \quad (25)$$

Substitute in the bulk Green's function to get the bulk Ricatti amplitude:

$$\hat{a} = \frac{i\hat{\Delta}}{\epsilon_n + \sqrt{|\Delta|^2 + \epsilon_n^2}} \quad (26)$$

$$\underline{\hat{a}} = \frac{i\hat{\Delta}^\dagger}{\epsilon_n + \sqrt{|\Delta|^2 + \epsilon_n^2}} \quad (27)$$

The matrix Ricatti equation is

$$i\hbar\mathbf{v}_p \cdot \nabla \hat{a} + 2\epsilon\hat{a} + \hat{a}\hat{\Delta}^\dagger\hat{a} + \hat{\Delta} = 0 \quad (28)$$

$$i\hbar\mathbf{v}_p \cdot \nabla \underline{\hat{a}} - 2\epsilon\underline{\hat{a}} - \underline{\hat{a}}\hat{\Delta}\underline{\hat{a}} - \hat{\Delta}^\dagger = 0 \quad (29)$$

### <sup>3</sup>HE-A

For <sup>3</sup>He-A, the spin structure is  $\hat{\mathbf{d}} \cdot (i\vec{\sigma}\hat{\sigma}_y) = \hat{\sigma}_x$ , and we have two components of the gap function,

$$\hat{\Delta}(\mathbf{R}, \mathbf{p}) = \hat{\sigma}_x(\Delta_1(\mathbf{R})p_x + \Delta_2(\mathbf{R})p_y) \quad (30)$$

In bulk we have  $\hat{\Delta}(\mathbf{R}, \mathbf{p}) = \hat{\sigma}_x\Delta(p_x + ip_y) = \hat{\sigma}_x\Delta e^{i\phi_p}$ . The interaction is  $v(\mathbf{p}, \mathbf{p}') = 3v_0\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}'$ , and the gap-equation reduces to

$$\ln \frac{T}{T_c} = T \sum_{\epsilon_n=-\infty}^{\infty} \left[ \frac{2 \langle \mathbf{p}_{x,y} \cdot \mathbf{p}' f(\mathbf{R}, \mathbf{p}', \epsilon_n) \rangle_{p'}}{\Delta_{1,2}(\mathbf{R})} - \frac{\pi}{|\epsilon_n|} \right] \quad (31)$$

The 2d bulk gap equation ( $|\Delta|^2$  does not depend on  $\mathbf{p}$ ) is

$$\ln \frac{T}{T_c} = T \sum_{\epsilon_n=-\infty}^{\infty} \left[ \frac{\pi}{\sqrt{|\Delta|^2 + \epsilon_n^2}} - \frac{\pi}{|\epsilon_n|} \right] \quad (32)$$

### S-WAVE PAIRING

For s-wave pairing, the spin structure is  $i\hat{\sigma}_y$ , and we have

$$\hat{\Delta}(\mathbf{R}, \mathbf{p}) = i\hat{\sigma}_y\Delta(\mathbf{R}) \quad (33)$$

In bulk we have  $\hat{\Delta}(\mathbf{R}, \mathbf{p}) = i\hat{\sigma}_y\Delta$ . The interaction is  $v(\mathbf{p}, \mathbf{p}') = v_s$ , and the gap-equation reduces to

$$\ln \frac{T}{T_c} = T \sum_{\epsilon_n=-\infty}^{\infty} \left[ \frac{f(\mathbf{R}, \epsilon_n)}{\Delta(\mathbf{R})} - \frac{\pi}{|\epsilon_n|} \right] \quad (34)$$

### <sup>3</sup>HE-A EDGE GAP PROFILE

We assume translational invariance along the edge, i.e. the  $y$  direction. We also assume the gap to be real(imaginary) along the  $p_x(p_y)$  direction.

$$\hat{\Delta}(x, \mathbf{p}) = \hat{\sigma}_x(\Delta_1(x)p_x + i\Delta_2(x)p_y) \quad (35)$$

Here  $\Delta_1(x)$  and  $\Delta_2(x)$  are real functions, and we have

$$\hat{\Delta}(x, \mathbf{p}) = \begin{pmatrix} 0 & \hat{\sigma}_x(\Delta_1(x)p_x + i\Delta_2(x)p_y) \\ -\hat{\sigma}_x(\Delta_1(x)p_x - i\Delta_2(x)p_y) & 0 \end{pmatrix} \quad (36)$$

$$= i\hat{\sigma}_x(\Delta_2p_y\hat{\tau}_1 + \Delta_1p_x\hat{\tau}_2) \quad (37)$$

We can always write the anomalous Green's function as

$$\hat{f}(\mathbf{R}, \mathbf{p}, \epsilon_n) = \hat{\sigma}_x \left( f_1(\mathbf{R}, \mathbf{p}, \epsilon_n) + if_2(\mathbf{R}, \mathbf{p}, \epsilon_n) \right) \quad (38)$$

where  $f_{1,2}$  are real functions, and we have

$$\hat{g} = \hat{g}\hat{\tau}_3 + i\hat{\sigma}_x(f_2\hat{\tau}_1 + f_1\hat{\tau}_2) \quad (39)$$

In this case, we can even write the gap equation as

$$\Delta_{1,2}(x) = T \sum_{\epsilon_n = -\omega_c}^{\omega_c} \langle v(\mathbf{p}_{x,y}, \mathbf{p}') f_{1,2}(x, \mathbf{p}', \epsilon_n) \rangle_{p'} \quad (40)$$

which becomes

$$\ln \frac{T}{T_c} = T \sum_{\epsilon_n = -\infty}^{\infty} \left[ \frac{2 \langle \mathbf{p}_{x,y} \cdot \mathbf{p}' f_{1,2}(x, \mathbf{p}', \epsilon_n) \rangle_{p'}}{\Delta_{1,2}(x)} - \frac{\pi}{|\epsilon_n|} \right] \quad (41)$$

#### Numerical solution

The iteration of the gap function is

$$\delta_{1,2}(x) = \frac{\sum_n \langle \mathbf{p}_{x,y} \cdot \mathbf{p}' f_{1,2}(x, \mathbf{p}', n) \rangle_{p'}}{\sum_n \frac{\pi}{\sqrt{1+e_n^2/\delta(t)^2}} \frac{\pi}{2}} \quad (42)$$

or equivalently

$$\delta_{1,2}(x)\delta(T) = \frac{t \sum_n 2 \langle \mathbf{p}_{x,y} \cdot \mathbf{p}' f_{1,2}(x, \mathbf{p}', n) \rangle_{p'}}{\ln t + \sum_n 1/|2n+1|} \quad (43)$$

where  $t = T/T_c$ ,  $\delta(\mathbf{p}, x) = \Delta(\mathbf{p}, x)/|\Delta(T)|$ , and  $\delta(T) = |\Delta(T)|/T_c$ . The Ricatti equation is

$$ip_x \partial_x a(\mathbf{p}, x) + 2ea(\mathbf{p}, x) + a(\mathbf{p}, x)\delta^*(\mathbf{p}, x)a(\mathbf{p}, x) + \delta(\mathbf{p}, x) = 0 \quad (44)$$

$$ip_x \partial_x \underline{a}(\mathbf{p}, x) - 2e\underline{a}(\mathbf{p}, x) - \underline{a}(\mathbf{p}, x)\delta(\mathbf{p}, x)\underline{a}(\mathbf{p}, x) - \delta^*(\mathbf{p}, x) = 0 \quad (45)$$

where  $e = \epsilon/|\Delta(T)|$ ,  $\xi = \hbar v_f/|\Delta(T)|$  and  $\vec{\partial} = \xi \nabla$ . For Matsubara frequency, we replace  $e \rightarrow i(2n+1)\pi t/\delta(T)$ . We can get  $g$  and  $f_{1,2}$  from

$$g = -i\pi \frac{1 + a\underline{a}}{1 - a\underline{a}} \quad (46)$$

$$f = f_1 + if_2 = -i\pi \frac{2a}{1 - a\underline{a}} \quad (47)$$

The local current density is

$$\mathbf{j}(\mathbf{R}) = 2N_f T \sum_n \langle \mathbf{v}_p g(\mathbf{R}, \mathbf{p}, \epsilon_n) \rangle_p \quad (48)$$

We first assume an initial guess for the gap function  $\delta(\mathbf{r}, \hat{\mathbf{p}})$ . For each trajectory  $\hat{\mathbf{p}}$  along which the quasiparticle propagates, we can solve the Eilenberger equation to get the Green's function  $g(\mathbf{r}, \hat{\mathbf{p}}, n)$  and the anomalous Green's function  $f(\mathbf{r}, \hat{\mathbf{p}}, n)$ . Then we iterate to update the gap function  $\delta(\mathbf{r}, \hat{\mathbf{p}})$  by solving the gap equation. Repeat until convergence.