MEAN-FIELD THEORY

The mean-field Nambu-Gor'kov Hamiltonian is

$$H = \frac{1}{2}\iint \mathrm{d}^3\mathbf{r}\mathrm{d}^3\mathbf{r}'\widehat{\Psi}^\dagger(\mathbf{r}') \begin{pmatrix} \xi(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\hat{1} & \hat{\Delta}(\mathbf{r},\mathbf{r}') \\ \hat{\Delta}^\dagger(\mathbf{r},\mathbf{r}') & -\xi(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\hat{1} \end{pmatrix} \widehat{\Psi}(\mathbf{r}) \tag{1}$$

$$\equiv \frac{1}{2}\iint \mathrm{d}^3\mathbf{r}\mathrm{d}^3\mathbf{r}'\widehat{\Psi}^\dagger(\mathbf{r}')\widehat{\tau}_3 \left(\xi(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + \widehat{\Delta}(\mathbf{r},\mathbf{r}')\right) \widehat{\Psi}(\mathbf{r}). \tag{2}$$

where

$$\widehat{\Delta}(\mathbf{r},\mathbf{r}') = \begin{pmatrix} 0 & \hat{\Delta}(\mathbf{r},\mathbf{r}') \\ -\hat{\Delta}^\dagger(\mathbf{r},\mathbf{r}') & 0 \end{pmatrix} \tag{3}$$

When the system is homogeneous, the two-point gap function only depends on the relative coordinate, i.e., $\hat{\Delta}(\mathbf{r},\mathbf{r}') = \hat{\Delta}(\mathbf{r}-\mathbf{r}')$. In this case, we can transform to momentum space, and the Hamiltonian becomes

$$H = \frac{1}{2}\sum_{\mathbf{p}} \widehat{\Psi}^\dagger_{\mathbf{p}}\widehat{\tau}_3(\xi_{\mathbf{p}} + \widehat{\Delta}(\mathbf{p}))\widehat{\Psi}_{\mathbf{p}}. \tag{4}$$

where $\widehat{\Psi}^\dagger_{\mathbf{p}} = (c^\dagger_{\mathbf{p}\uparrow}, c^\dagger_{\mathbf{p}\downarrow}, c_{-\mathbf{p}\uparrow}, c_{-\mathbf{p}\downarrow})$.

FINITE TEMPERATURE

The imaginary time Green's function is

$$\widehat{G}(x,x') = -\left\langle T_\tau \widehat{\Psi}(x)\widehat{\Psi}^\dagger(x')\right\rangle. \tag{5}$$

where $x = (\mathbf{r},\tau)$. The unperturbed Matsubara Green's function is

$$\widehat{G}_0(\mathbf{p}, i\epsilon_n) = (i\epsilon_n - \widehat{\tau}_3\xi_{\mathbf{p}})^{-1} = -\frac{i\epsilon_n + \widehat{\tau}_3\xi_{\mathbf{p}}}{\epsilon_n^2 + \xi_{\mathbf{p}}^2}. \tag{6}$$

The Gorkov Green's function satisfies

$$\widehat{G}^{-1}(\mathbf{p}, i\epsilon_n) = \widehat{G}_0^{-1}(\mathbf{p}, i\epsilon_n) - \widehat{\tau}_3\widehat{\Delta}(\mathbf{p}) = i\epsilon_n - \widehat{\tau}_3\xi_{\mathbf{p}} - \widehat{\tau}_3\widehat{\Delta}(\mathbf{p}). \tag{7}$$

which gives

$$\widehat{G}(\mathbf{p}, i\epsilon_n) = -\frac{i\epsilon_n + \widehat{\tau}_3\xi_{\mathbf{p}} + \widehat{\tau}_3\widehat{\Delta}(\mathbf{p})}{\epsilon_n^2 + \xi_{\mathbf{p}}^2 + |\Delta|^2} \tag{8}$$

The quasiclassical Green's function is

$$\widehat{g}(\mathbf{p}, i\epsilon_n) = \int \mathrm{d}\xi_p \widehat{G}(\mathbf{p}, i\epsilon_n)\widehat{\tau}_3 = -\pi \frac{i\epsilon_n \widehat{\tau}_3 - \widehat{\Delta}(\mathbf{p})}{\sqrt{|\Delta|^2 + \epsilon_n^2}} = \frac{\pi}{\sqrt{|\Delta|^2 + \epsilon_n^2}} \begin{pmatrix} -i\epsilon_n & \hat{\Delta} \\ -\hat{\Delta}^\dagger & i\epsilon_n \end{pmatrix} \equiv \begin{pmatrix} \hat{g} & \hat{f} \\ -\hat{f}^\dagger & -\hat{g} \end{pmatrix} \tag{9}$$

and it satisfies the normalization condition

$$\widehat{g}(\mathbf{p}, i\epsilon_n)^2 = -\pi^2 \widehat{1}. \tag{10}$$

The bulk quasiclassical Green's function satisfies

$$\left[i\epsilon_n \widehat{\tau}_3 - \widehat{\Delta}, \widehat{g} \right] = 0 \tag{11}$$

But when near the boundary or impurities, the quasiclassical Green's function $\widehat{g}(\mathbf{r}, \mathbf{p}; \epsilon)$ and the gap function $\widehat{\Delta}(\mathbf{r}, \mathbf{p})$ are inhomogeneous, and are described by the Eilenberger equation and gap equation.

$$\left[\epsilon \widehat{\tau}_3 - \widehat{\Delta}(\mathbf{r}, \mathbf{p}), \widehat{g}(\mathbf{r}, \mathbf{p}; \epsilon) \right] + i\hbar \mathbf{v}_p \cdot \nabla \widehat{g}(\mathbf{r}, \mathbf{p}; \epsilon) = 0 \tag{12}$$

and

$$\hat{\Delta}(\mathbf{r}, \mathbf{p}) = \left\langle v(\mathbf{p}, \mathbf{p}')T \sum_{\epsilon_n = -\omega_c}^{\omega_c} \hat{f}(\mathbf{r}, \mathbf{p}', \epsilon_n) \right\rangle_{p'} \tag{13}$$

NUMERICAL SOLUTION

The $(1, 1)$ and $(1, 2)$ elements of the Eilenberger equation are

$$i\epsilon_n \hat{g} + \hat{\Delta}\hat{f}^\dagger + i\hbar \mathbf{v}_p \cdot \boldsymbol{\nabla} \hat{g} = 0 \tag{14}$$

$$i\epsilon_n \hat{f} + \hat{\Delta}\hat{g} + i\hbar \mathbf{v}_p \cdot \boldsymbol{\nabla} \hat{f} = 0 \tag{15}$$

Note that $\hat{\Delta}$ and $\hat{f}$ has the same spin structure, so we can substitute in $\hat{g}$ and $\hat{f}$, divide by T_c, and we get

$$i(2n+1)\pi t g(\mathbf{r}, \hat{\mathbf{p}}, n) + \hat{\delta}(\mathbf{r}, \hat{\mathbf{p}}) f^*(\mathbf{r}, \hat{\mathbf{p}}, n) + i(\hat{\mathbf{p}}_x \partial_x + \hat{\mathbf{p}}_y \partial_y) g(\mathbf{r}, \hat{\mathbf{p}}, n) = 0 \tag{16}$$

$$i(2n+1)\pi t f(\mathbf{r}, \hat{\mathbf{p}}, n) + \hat{\delta}(\mathbf{r}, \hat{\mathbf{p}}) g(\mathbf{r}, \hat{\mathbf{p}}, n) + i(\hat{\mathbf{p}}_x \partial_x + \hat{\mathbf{p}}_y \partial_y) f(\mathbf{r}, \hat{\mathbf{p}}, n) = 0 \tag{17}$$

where $t = T/T_c$, $\delta = \Delta/T_c$, $\xi = \hbar v_f/T_c$ and $\vec{\partial} = \xi \boldsymbol{\nabla}$.

For the gap equation, we ...

$$\delta = \tag{18}$$

We first assume an initial guess for the gap function $\delta(\mathbf{r}, \hat{\mathbf{p}})$. For each trajectory $\hat{\mathbf{p}}$ along which the quasiparticle propagates, we can solve the Eilenberger equation to get the Green's function $g(\mathbf{r}, \hat{\mathbf{p}}, n)$ and the anomalous Green's function $f(\mathbf{r}, \hat{\mathbf{p}}, n)$. Then we iterate to update the gap function $\delta(\mathbf{r}, \hat{\mathbf{p}})$ by solving the gap equation. Repeat until convergence.

^{3}HE-A

For ^{3}He-A, the spin structure is $\hat{\mathbf{d}} \cdot (i\vec{\sigma}\sigma_y) = \sigma_x$. In general, we have

$$\hat{\Delta}(\mathbf{r}, \mathbf{p}) = \hat{\sigma}_x(\Delta_1(\mathbf{r}, \mathbf{p}) + i\Delta_2(\mathbf{r}, \mathbf{p})) \tag{19}$$

in bulk we have $\Delta_1(\mathbf{p}) = \Delta p_x$ and $\Delta_2(\mathbf{p}) = \Delta p_y$. We can also write the anomalous Green's function as

$$\hat{f}(\mathbf{r}, \mathbf{p}, \epsilon_n) = \hat{\sigma}_x\Bigg(f_1(\mathbf{r}, \mathbf{p}, \epsilon_n) + if_2(\mathbf{r}, \mathbf{p}, \epsilon_n)\Bigg) \tag{20}$$

EDGE GAP PROFILE

We assume translational invariance along the edge, i.e. the y direction. We choose the gap to be real along the x direction. We assume the momentum dependence of the gap is given by

$$\hat{\Delta}(x, \mathbf{p}) = \hat{\sigma}_x(\Delta_1(x)p_x + i\Delta_2(x)p_y) \tag{21}$$