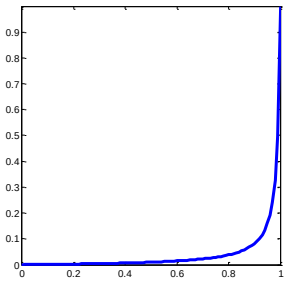
Normalized Sigmoid Function

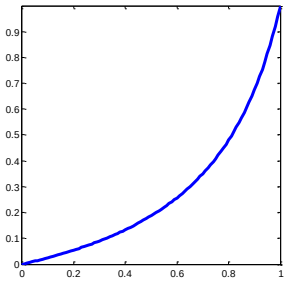
Mar 26 2018
Jungwon Kang

I referred to the site:
https://dinodini.wordpress.com/2010/04/05/normalized-tunable-sigmoid-functions/

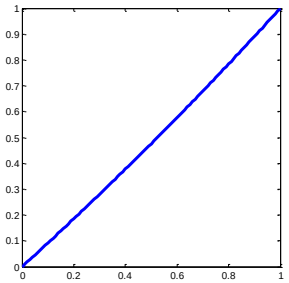
The default curve is $y = \frac{kx}{k-x+1}$, where k is a constant. The following curves are the curves according to k, where x varies 0.0 to 1.0, and accordingly y also moves 0.0 to 1.0.



$k = 0.01$



$k = 0.3$



$k = 10.0$

We wanted to make a normalized sigmoid function (NSF) that is bounded on $x \in [0.0, 1.0]$ and $y \in [0.0, 1.0]$ as well.

In order to make the NSF, we utilized the above default curve. We split $x \in [0.0, 1.0]$ into two split case: 1) $x \in [0.0, 0.5)$ and 2) $x \in [0.5, 1.0]$.

1) $x \in [0.0, 0.5)$

$y = \frac{kx}{k-x+1}$ (default) → $y = \frac{1}{2}\frac{k(2x)}{k-(2x)+1}$ (Making $x \in [0.0, 0.5)$ and $y \in [0.0, 0.5)$)

The final $y = \frac{kx}{k-2x+1}$, and it is $x \in [0.0, 0.5)$ and $y \in [0.0, 0.5)$.

2) $x \in [0.5, 1.0]$

To make this case, we need three steps: (i), (ii), and (iii) applying offset.

(i) inversion of y (new $y = 0.5 - y$)

$y = 0.5 - \frac{kx}{k-2x+1}$

(ii) inversion of x (new $x = 0.5 - x$)

$y = 0.5 - \frac{k(0.5-x)}{k-2(0.5-x)+1} = 0.5 - \frac{0.5k-kx}{k+2x}$

(iii) applying offset (new $x = x - 0.5$, new $y = y + 0.5$)

$y = \frac{kx+2x-1}{k+2x-1}$

In summary,

$$y = \begin{cases} \frac{kx}{k-2x+1} & x \in [0.0, 0.5) \\ \frac{kx+2x-1}{k+2x-1} & x \in [0.5, 1.0] \end{cases}$$

Here, $y \in [0.0, 1.0]$. The curve is as follows: