Dirac Monopole

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2.2 Relativistic Lagrangian mechanics

$$L = -\frac{m_0 c^2}{\gamma}$$

$$L = -\frac{m_0 c^2}{\gamma} + q\dot{\vec{r}} \cdot \vec{A}$$

$$\vec{p} = \gamma m\dot{\vec{r}} + q\vec{A}$$

$$\dot{\vec{r}} = \frac{c(\vec{p} - q\vec{A})}{\sqrt{(\vec{p} - q\vec{A})^2 + m^2 c^2}}$$

$$\gamma = \frac{\sqrt{(\vec{p} - q\vec{A})^2 + m^2 c^2}}{mc}$$

$$H = \dot{\vec{r}} \cdot \vec{p} - L$$

$$H = \sqrt{[c(\vec{p} - q\vec{A})]^2 + m^2 c^4}$$

2.3 Orbit and symmetry breaking

$$H = \sqrt{[c(\vec{p} - q\vec{A})]^2 + m^2 c^4}$$

3 Laplace-Runge-Lenz vector

3.1 Coulomb potential

$$H = \frac{1}{2} m\dot{\vec{r}}^2 - \frac{k}{r}$$

$$\vec{M} = \vec{p} \times \vec{L} - mk\hat{r}$$

3.2 Harmonic potential

$$H = \frac{1}{2} m\dot{\vec{r}}^2 + \frac{1}{2} m\omega^2 r^2$$

$$\vec{M} = \vec{p} \times \vec{L} - \vec{p}^2\vec{r} - \vec{p}(\vec{r} \cdot \vec{p})$$

3.3 Dirac monopole

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