ROB 422 HW1 Junhao TU.

1. a.

$$p + 2q = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ -2 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 \\ -5 \\ 15 \end{bmatrix}$$

b.

$$p \cdot r = -5 + 2 + 3 = 0 \qquad r \cdot p = -5 + 2 + 3 = 0$$

c.

$$q \times r = \begin{bmatrix} q_2 r_3 - q_3 r_2 \\ q_3 r_1 - q_1 r_3 \\ q_1 r_2 - q_2 r_1 \end{bmatrix} = \begin{bmatrix} -6 + 14 \\ -7 + 12 \\ 8 - 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 6 \end{bmatrix}$$

$$r \times q = \begin{bmatrix} r_2 q_3 - r_3 q_2 \\ r_3 q_1 - r_1 q_3 \\ r_1 q_2 - r_2 q_1 \end{bmatrix} = \begin{bmatrix} -14 + 6 \\ -12 + 7 \\ 2 - 8 \end{bmatrix} = \begin{bmatrix} -8 \\ -5 \\ -6 \end{bmatrix}$$

d.

$$\|p\| = \sqrt{5^2 + (-1)^2 + 1^2} = \sqrt{27} = 3\sqrt{3}$$

$$\|q\| = \sqrt{(-4)^2 + (-2)^2 + 7^2} = \sqrt{69}$$

e.

$$\|p - q\| = \sqrt{(5+4)^2 + (-1+2)^2 + (1-7)^2} = \sqrt{118}$$

2. $p$ and $q$ are orthogonal thus $p \cdot q = 0$

$$p \cdot q = -4 - 3k + k^2 = 0 \Rightarrow k_1 = 4 \ \& \ k_2 = -1.$$

3. Given the dimensions of w and z, we can determine the dimensions of X and Y as well. X will have the same number of rows as W and Y will have the same number of columns as W.

$$W \in R^{2\times1},\ X \in R^{2\times3},\ Y \in R^{2\times1},\ Z \in R^{2\times3}$$

$$W = \begin{bmatrix} 1 \\ 5 \end{bmatrix} \quad X = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 8 \end{bmatrix} \quad Y = \begin{bmatrix} 9 \\ 13 \end{bmatrix} \quad Z = \begin{bmatrix} 10 & 11 & 12 \\ 14 & 15 & 16 \end{bmatrix}$$

4.

$$\begin{bmatrix} 3 & -1 & -3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 & 9 & 10 \\ 0 & -5 & -4 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -9 \\ 3 \end{bmatrix}$$

5.

Code refers to the 'HW1_q5.py' script in gradescope.

Here, I will present the output of the code.

a) $X_a = \begin{bmatrix} 0.4 \\ 2.4 \\ -3 \end{bmatrix}$

b) X has no solution or infintely many solutions.

c) $X_c = \begin{bmatrix} -2.5 \\ -4 \end{bmatrix}$

6. Code refers to the 'HW1_q6.py' script in gradescope.

Here, I will present the output of the code.

a. $A+2B = \begin{bmatrix} -3 & -2 \\ 11 & -7 \end{bmatrix}$ b. $AB = \begin{bmatrix} 6 & -8 \\ -10 & -3 \end{bmatrix}$ $BA = \begin{bmatrix} -8 & -2 \\ -5 & 11 \end{bmatrix}$ c. $A^T = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

d. $B^2 = \begin{bmatrix} -4 & 10 \\ -20 & 1 \end{bmatrix}$ e. $A^TB^T = \begin{bmatrix} -8 & -5 \\ -2 & 11 \end{bmatrix}$ $(AB)^T = \begin{bmatrix} 6 & -10 \\ -8 & -3 \end{bmatrix}$ f. $\det(A) = -7$

g. $B^{-1} = \begin{bmatrix} -0.214 & 0.143 \\ -0.286 & -0.143 \end{bmatrix}$

7.

$$R = Rotz(\pi)\cdot Roty(\pi/4)\cdot Rotz(\pi/3) \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\pi) & -\sin(\pi) & 0 \\ \sin(\pi) & \cos(\pi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\frac{\pi}{4}) & 0 & \sin(\frac{\pi}{4}) \\ 0 & 1 & 0 \\ -\sin(\frac{\pi}{4}) & 0 & \cos(\frac{\pi}{4}) \end{bmatrix} \times \begin{bmatrix} \cos(\frac{\pi}{3}) & -\sin(\frac{\pi}{3}) & 0 \\ \sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \times \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & -1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \times \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ -\frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

8.

a. Given $T^0_r$, the position of mobile robot can be expressed as

$$r^0 = \begin{bmatrix} 1.7 \\ 2.1 \\ 0 \end{bmatrix}.$$

$$\therefore V = P - r^0 = \begin{bmatrix} 0.1 \\ -1.4 \\ 0 \end{bmatrix} - \begin{bmatrix} 1.7 \\ 2.1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.6 \\ -3.5 \\ 0 \end{bmatrix}$$

b. To align the robot's x-axis with the vector $V$, the x-axis is the normalized $V$:

$$X_{axis} = \frac{V}{\|V\|}, \quad X_{axis} = \frac{\begin{bmatrix} -1.6 \\ -3.5 \\ 0 \end{bmatrix}}{\sqrt{1.6^2 + 3.5^2}} = \begin{bmatrix} -0.416 \\ -0.909 \\ 0 \end{bmatrix}$$

Given that the robot's z axis must not change: $Z_{axis} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$Y_{axis} = Z_{axis} \times X_{axis} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -0.416 \\ -0.909 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.909 \\ -0.416 \\ 0 \end{bmatrix}$$

$$\therefore T_{pose} = \begin{bmatrix} -0.416 & 0.909 & 0 & 1.7 \\ -0.909 & -0.416 & 0 & 2.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C. A valid rotation matrix R should staisfy the following conditions

1. $R \times R^T = I$ 2. $\det(R) = 1$

for (1),
$$\begin{bmatrix} -0.416 & 0.909 & 0 \\ -0.909 & -0.416 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -0.416 & -0.909 & 0 \\ 0.909 & -0.416 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

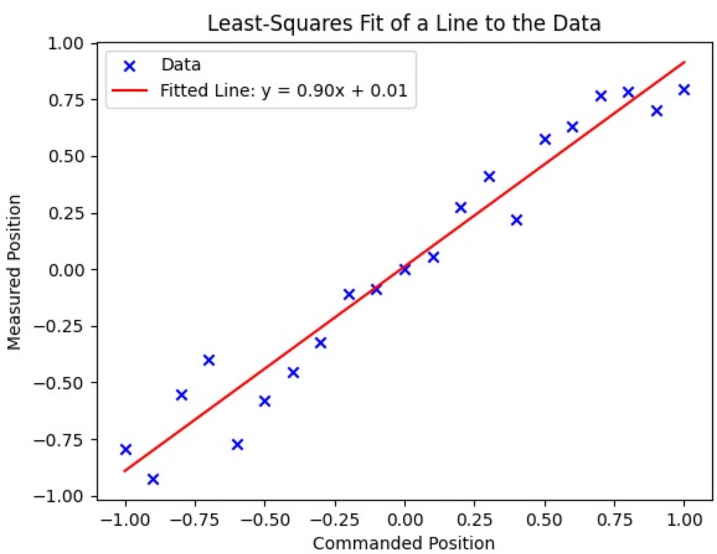
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for (2), $\det \begin{bmatrix} -0.416 & 0.909 & 0 \\ -0.909 & -0.416 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -0.416(-0.416) - 0.909 \times (-0.909) + 0 = 1$

Therefore, both conditions are satisfied, so R is a valid rotation matrix.

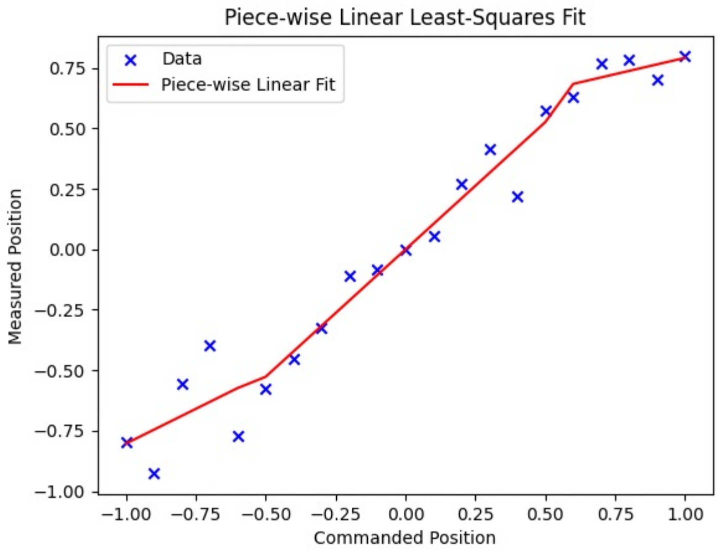
9.

a.




Slope: 0.9026 Intercept: 0.0111 Sum of squared errors: 0.3106

b. the least-squares problem is overdetermined. The number of equations is larger than variables, thus the problem has no exact solutions. In this case, it can be regarded as overdetermined c.




Parameters:

Slope of first line: 0.5749

Intercept of first line: -0.2291

Slope of second line: 1.0531

Intercept of second line: -0.001

Slope of third line: 0.2698

Intercept of third line: 0.5219

Sum of squared errors: 0.2308

prediction for 0.68 : 0.7053