수치해석 과제#3

2015111113 김준기

4.22

centerdiffapp.m | macforcos.m

```matlab
function centerdiffapp(true_value, func, x, step_size)
    displaybytable()
    %보기 좋게 측정값들을 도표 형식으로 표현하기 위해 도표의 머리부분을 출력합니다.
    h = step_size;
    for i = 0:10
        tmp_result = (func(x + h) - func(x - h))/(2*h);
        tmp_true_error = true_value - tmp_result;
        displaybytable(i, h, tmp_result, tmp_true_error)
        h = h/10;
    end

end

function displaybytable(count, step_size, approximation, true_error)
    if(nargin == 0)
        disp("|--------|--------------------|-------------------------|--------------------|")
        disp("| count  |     step size      |      approximation      |     true error     |")
        disp("|--------|--------------------|-------------------------|--------------------|")
    else
        fprintf("|  %3d   |    %13.10f   |        %12.7f       | %17.12f  |\n", count, step_size, approximation, true_error)
        disp("|--------|--------------------|-------------------------|--------------------|")
    end
end
```

처음에는 argument를 아무것도 전달해주지 않아 도표의 머리 부분만 출력 되게 합니다.

실질적으로 centered approximation difference를 계산하는 부분입니다.

명령 창

```
>> centerdiffapp(-0.5, @(x) cos(x), pi/6, 1)
|--------|--------------------|-------------------------|--------------------|
| count  |     step size      |      approximation      |     true error     |
|--------|--------------------|-------------------------|--------------------|
|   0    |    1.0000000000    |        -0.4207355       |  -0.079264507596   |
|--------|--------------------|-------------------------|--------------------|
|   1    |    0.1000000000    |        -0.4991671       |  -0.000832916766   |
|--------|--------------------|-------------------------|--------------------|
|   2    |    0.0100000000    |        -0.4999917       |  -0.000008333292   |
|--------|--------------------|-------------------------|--------------------|
|   3    |    0.0010000000    |        -0.4999999       |  -0.000000083333   |
|--------|--------------------|-------------------------|--------------------|
|   4    |    0.0001000000    |        -0.5000000       |  -0.000000000833   |
|--------|--------------------|-------------------------|--------------------|
|   5    |    0.0000100000    |        -0.5000000       |  -0.000000000013   |
|--------|--------------------|-------------------------|--------------------|
|   6    |    0.0000010000    |        -0.5000000       |   0.000000000014   |
|--------|--------------------|-------------------------|--------------------|
|   7    |    0.0000001000    |        -0.5000000       |  -0.000000000263   |
|--------|--------------------|-------------------------|--------------------|
|   8    |    0.0000000100    |        -0.5000000       |   0.000000002512   |
|--------|--------------------|-------------------------|--------------------|
|   9    |    0.0000000010    |        -0.5000000       |  -0.000000014141   |
|--------|--------------------|-------------------------|--------------------|
|  10    |    0.0000000001    |        -0.5000000       |   0.000000041370   |
|--------|--------------------|-------------------------|--------------------|
>>
```

4.25

centerdiffapp.m | macforcos.m

```matlab
function macforcos(x, es, maxnumofiteration)
    % es is pre-specified tolerance
    % calculate cos(x) value using by Maclaurin-series expansion

    ea = 100;
    % ea is approximation relative error

    i = 0;
    % i is number of iteration for now

    tmp_result = 0;
    % temporary result of calculation
    i = 0;
    disp("|-------|--------------------|--------------------------------|")
    disp("| count |    approximation   |   approximation error relative |")
    disp("|-------|--------------------|--------------------------------|")
    while(1)
       tmp_result = tmp_result + ( (-1)^i * (x)^(2*i)/factorial(2*i) );
       if i ~= 0
        ea = abs((tmp_result - previous_approximation)/tmp_result)*100;
       end
       fprintf("| %3d  |  %15.10f   |        %15.10f         |₩n", i+1, tmp_result, ea)
       disp("|-------|--------------------|--------------------------------|")
       i = i+1;
       if (ea < es | i > maxnumofiteration),break,end
       % pre-specified tolerance 값보다 작다면 반복 중지

       previous_approximation = tmp_result;
    end
end
```

반복횟수가 증가할수록 이전 맥클로린 급수의 결과 값에 현재 항만큼 추가로 계산하여 더해줍니다.

명령 창

```
>> macforcos(pi/3, 0.05, 100)
|-------|---------------------|---------------------------------|
| count |    approximation    |   approximation error relative  |
|-------|---------------------|---------------------------------|
|   1   |     1.0000000000    |         100.0000000000          |
|-------|---------------------|---------------------------------|
|   2   |     0.4516886444    |         121.3914413022          |
|-------|---------------------|---------------------------------|
|   3   |     0.5017962015    |           9.9856389838          |
|-------|---------------------|---------------------------------|
|   4   |     0.4999645653    |           0.3663531975          |
|-------|---------------------|---------------------------------|
|   5   |     0.5000004334    |           0.0071736146          |
|-------|---------------------|---------------------------------|
```

명령 창

```
>> macforcos((7*pi)/3, 0.05, 100)
```

count	approximation	approximation error relative
1	1.0000000000	100.0000000000
2	-25.8672564252	103.8658912393
3	94.4409882109	127.3898620876
4	-121.0491757026	178.0186958422
5	85.7233061178	241.2091777425
6	-37.7302336765	327.2005703778
7	12.5251892061	401.2348400937
8	-2.3124518196	641.6410884671
9	1.0096040652	329.0454148570
10	0.4262411416	136.8621812211
11	0.5087325166	16.2150781299
12	0.4991380711	1.9222027179
13	0.5000720436	0.1867676032
14	0.4999948335	0.0154421784

```
fx >>
```

수치 해석 과제 HW#3 　　　　　　　　　　　　　　2015111113 　김 준 기

4.11 true value $\cos(\frac{\pi}{3}) = 0.5$., $\varepsilon_s = 0.05$

term1. $\cos(\frac{\pi}{3}) = 1$. $E_t = -0.5$.

term2. $\cos(\frac{\pi}{3}) = 1 - \frac{(\frac{\pi}{3})^2}{2} = 0.4516886$.

$E_t = 0.0483113$

$\varepsilon_a = \left| \frac{0.4516868 - 1}{0.4516868} \times 100\% \right| = 121.391\%$.

term3. $\cos(\frac{\pi}{3}) = 1 - \frac{(\frac{\pi}{3})^2}{2} + \frac{(\frac{\pi}{3})^4}{4!} = 0.5017962$

$E_t = -0.0017962$.

$\varepsilon_a = \left| \frac{0.5017962 - 0.4516886}{0.5017962} \times 100\% \right|$

$= 9.98563\%$.

term4. $\cos(\frac{\pi}{3}) = 1 - \frac{(\frac{\pi}{3})^2}{2} + \frac{(\frac{\pi}{3})^4}{4!} - \frac{(\frac{\pi}{3})^6}{6!}$

$= 0.49996456$

$E_t = 0.0000354346$.

$\varepsilon_a = \left| \frac{0.49996456 - 0.5017962}{0.49996456} \times 100\% \right|$

$= 0.3663\%$.

term5. $\cos(\frac{\pi}{3}) = 1 - \frac{(\frac{\pi}{3})^2}{2} + \frac{(\frac{\pi}{3})^4}{4!} - \frac{(\frac{\pi}{3})^6}{6!} + \frac{(\frac{\pi}{3})^8}{8!}$

$= 0.50000043\cancel{34}$

$E_t = -0.0000004334$

$\varepsilon_a = \left| \frac{0.5000004334 - 0.49996456}{0.5000004334} \times 100\% \right|$

$= 0.00717\% < \varepsilon_s$ (0.05).

4.13. $f(x) = 25x^3 - 6x^2 + 7x - 88$.

$f(3) = 554$.

zero.

$f(3) = f(1) = -62$.

$\varepsilon_t = \frac{554 + 62}{554} \times 100\% = 111.19\%$.

first.

$f(3) = f(1) + f'(1) \times 2 = 78$.

$f'(x) = 75x^2 - 12x + 7 \Rightarrow f'(1) = 70$.

$\varepsilon_t = \frac{554 - 78}{554} \times 100\% = 85.92\%$.

second.

$f(3) = f(1) + f'(1) \times 2 + \frac{f''(1)}{2!} \times 2^2 = 354$

$f''(x) = 150x - 12 \Longrightarrow f''(1) = 138$.

$\varepsilon_t = \frac{554 - 354}{554} \times 100\% = 36.10\%$.

third.

$f(3) = f(1) + f'(1) \times 2 + \frac{f''(1)}{2!} \times 2^2 + \frac{f'''(1)}{3!} \times 2^3 = 554$

$f'''(x) = 150$.

$f'''(1) = 150$.

$\varepsilon_t = \frac{554 - 554}{554} \times 100\% = 0$.

4.16. true value $f'(2) = 283$.

$f(x_{i+1}) = f(2.25) = 25 \times (2.25)^3 - 6 \times (2.25)^2 + 7 \times (2.25) - 88 = 182.140625$

$f(x_i) = f(2) = 25 \times (2^3) - 6 \times (2^2) + 7 \times 2 - 88 = 102$

$f(x_{i-1}) = f(1.75) = 25 \times (1.75)^3 - 6 \times (1.75)^2 + 7 \times 1.75 - 88 = 39.859375$

• forward difference approximations,

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$

$$\left(\text{when } O(h) = -\frac{f''(x_i)}{2!}h - \frac{f'''(x_i)}{3!}h^2 - \cdots \right)$$

$$f'(2) \simeq \frac{f(x_{i+1}) - f(x_i)}{h} = \frac{182.140625 - 102}{0.25} = 320.5625$$

이는 truncation error인 $O(h)$만큼 true value와 차이가 남을 알려줍니다. 이를 확인하기 위해 $O(h)$를 계산하면,

$$O(h) = -\frac{f''(x_i)}{2!}h - \frac{f'''(x_i)}{3!}h^2$$

f(x)가 3차 polynomial이므로, 3차 도함수까지만 존재.

$$= -\frac{f''(2)}{2!} \times 0.25 - \frac{f'''(2)}{3!} \times 0.25^2$$

$$= -\frac{150 \times 2 - 12}{2!} \times 0.25 - \frac{150}{3!} \times 0.25^2$$

$$= -37.5625$$

이 값을 approximation값 320.5625에 더하면, $320.5625 - 37.5625 = 283$ true value가 됩니다.

따라서 backward difference approximation과 centered difference approximation 각 각에 대해서 approximation 값과 truncation error $O(h)$, $O(h^2)$은 아래와 같습니다.

• backward difference approximation.

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + O(h)$$

$$\left(\text{when } O(h) = \frac{f''(x_i)}{2!}h - \frac{f'''(x_i)}{3!}h^2 + \cdots \right)$$

$$f'(2) \simeq \frac{102 - 39.859375}{0.25} = 248.5625$$

$$O(h) = \frac{150 \times 2 - 12}{2} \times 0.25 - \frac{150}{3 \times 2} \times 0.25^2$$

$$= 34.4375$$

따라서,

$$f'(2) = 248.5625 + 34.4375 = 283$$

• centered difference approximation.

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$$

$$\left(\text{when } O(h^2) = -\frac{f^{(3)}(x_i)}{3!}h^2 + \frac{f^{(5)}(x_i)}{5!}h^4 - \cdots \right)$$

$$f'(2) \simeq \frac{182.140625 - 39.859375}{2 \times 0.25} = 284.5625$$

$$O(h^2) = -\frac{150}{6} \times 0.25^2 = -1.5625$$

따라서,

$$f'(2) = 284.5625 - 1.5625 = 283$$