수치해석 과제#5

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8.3

편집기 - C:₩Users₩JunGi_Kim₩Desktop₩2020학년 3학년 2학기₩수치해석₩과제

matlin.m | circuitcal.m | circuitcal2.m | hardproblem.m

```
1   function matlin()
2 -   equation_1 = [0 -7 5];
3 -   equation_2 = [0 4 7];
4 -   equation_3 = [-4 3 -7];
5 -   b = [50; -30; 40];
6
7 -   A = [equation_1 ; equation_2; equation_3];
8
9 -   solution = A₩b;
10
11 -  disp("solution is below");
12 -  disp(solution);
13 -  disp("Transpose matrix is below");
14 -  disp(A');
15 -  disp("Inverse of the coefficient matrix is below");
16 -  disp(inv(A));
17
18 -  end
```

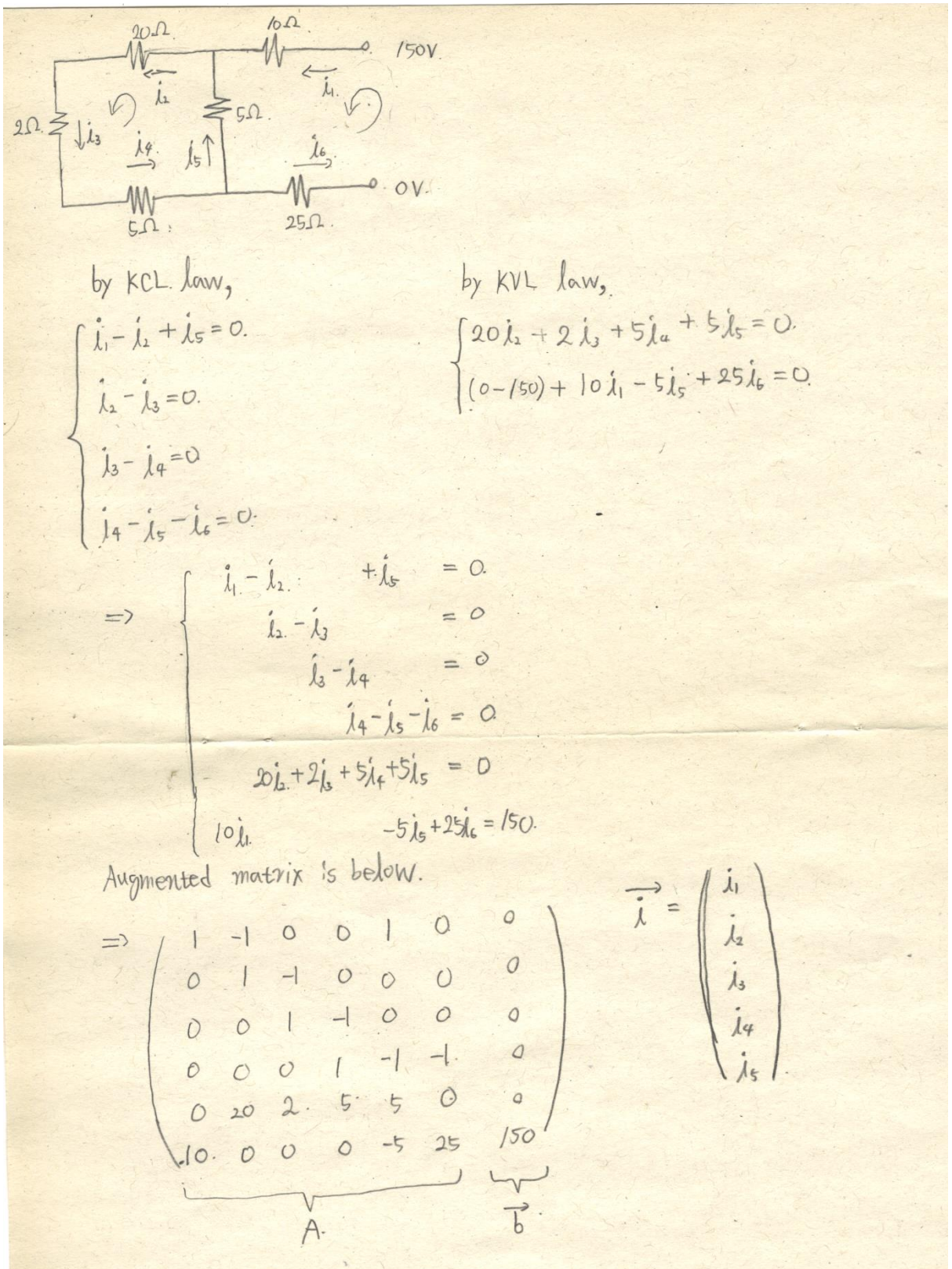
각 방정식에서 coefficient는 row vector로 표현하였습니다.

명령 창

```
>> matlin()
solution is below
  -15.1812
   -7.2464
   -0.1449

Transpose matrix is below
     0     0    -4
    -7     4     3
     5     7    -7

Inverse of the coefficient matrix is below
   -0.1775   -0.1232   -0.2500
   -0.1014    0.0725         0
    0.0580    0.1014         0

fx >>
```

8.14

문제 해석 및 방정식 세우기



by KCL law,

$$\begin{cases} i_1 - i_2 + i_5 = 0 \\ i_2 - i_3 = 0 \\ i_3 - i_4 = 0 \\ i_4 - i_5 - i_6 = 0 \end{cases}$$

by KVL law,

$$\begin{cases} 20i_2 + 2i_3 + 5i_4 + 5i_5 = 0 \\ (0-150) + 10i_1 - 5i_5 + 25i_6 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} i_1 - i_2 + i_5 = 0 \\ i_2 - i_3 = 0 \\ i_3 - i_4 = 0 \\ i_4 - i_5 - i_6 = 0 \\ 20i_2 + 2i_3 + 5i_4 + 5i_5 = 0 \\ 10i_1 - 5i_5 + 25i_6 = 150 \end{cases}$$

Augmented matrix is below.

$$\Rightarrow \left(\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 20 & 2 & 5 & 5 & 0 & 0 \\ 10 & 0 & 0 & 0 & -5 & 25 & 150 \end{array}\right)$$

A $\quad$ $\vec{b}$

$$\vec{i} = \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{pmatrix}$$

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hardproblem.m | circuitcal.m | +

```
1   function circuitcal()
2 –   A = [1 -1 0 0 1 0;...
3         0 1 -1 0 0 0;...
4         0 0 1 -1 0 0;...
5         0 0 0 1 -1 -1;...
6         0 20 2 5 5 0;...
7         10 0 0 0 -5 25;]
8
9 –   disp("coefficient matrix is below");
10 –  disp(A);
11
12 –  b = [0; 0; 0; 0; 0; 150];
13
14 –  solution = A₩b;
15
16 –  disp("solution is below")
17 –  disp(solution);
18
19
20 –  end
```

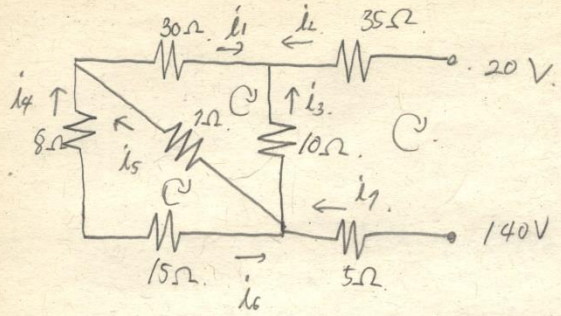
명령 창

```
    10     0     0

solution is below
    3.8247
    0.5976
    0.5976
    0.5976
   -3.2271
    3.8247

fx >>
```

순서대로 i1, i2, i3, i4, i5, i6에 대한 전류 값입니다.

8.15

문제 해석 및 방정식 세우기



by KCL law,

$$\begin{cases} i_1 + i_2 + i_3 = 0 \\ -i_1 + i_4 + i_5 = 0 \\ -i_4 - i_6 = 0 \\ -i_3 - i_5 + i_6 + i_7 = 0 \end{cases}$$

by KVL law,

$$\begin{cases} 8i_4 - 7i_5 - 15i_6 = 0 \\ 30i_1 - 10i_3 + 7i_5 = 0 \\ -35i_2 + 10i_3 + 5i_7 + (20 - 140) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} i_1 + i_2 + i_3 = 0 \\ -i_1 + i_4 + i_5 = 0 \\ -i_4 - i_6 = 0 \\ -i_3 - i_5 + i_6 + i_7 = 0 \\ 8i_4 - 7i_5 - 15i_6 = 0 \\ 30i_1 - 10i_3 + 7i_5 = 0 \\ -35i_2 + 10i_3 + 5i_7 = 120 \end{cases}$$

Augmented matrix is,

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 8 & -7 & -15 & 0 & 0 \\ 30 & 0 & -10 & 0 & 7 & 0 & 0 & 0 \\ 0 & -35 & 10 & 0 & 0 & 0 & 5 & 120 \end{pmatrix}$$

circuitcal2.m | hardproblem.m

```
function circuitcal2()
A = [1 1 1 0 0 0 0; -1 0 0 1 1 0 0; 0 0 0 -1 0 -1 0; ...
    0 0 -1 0 -1 1 1; 0 0 0 8 -7 -15 0; 30 0 -10 0 7 0 0; ...
    0 -35 10 0 0 0 5];
disp("coefficient matrix is below");
disp(A);
b = [0; 0; 0; 0; 0; 0; 120];

solution = A\b; % solution is x when the equation form is like to be the form Ax = b
disp("solution is below");
disp(solution)
end
```

명령 창

```
>> circuitcal2()
coefficient matrix is below
     1     1     1     0     0     0     0
    -1     0     0     1     1     0     0
     0     0     0    -1     0    -1     0
     0     0    -1     0    -1     1     1
     0     0     0     8    -7   -15     0
    30     0   -10     0     7     0     0
     0   -35    10     0     0     0     5

solution is below
    0.5534
   -2.5107
    1.9573
    0.1291
    0.4243
   -0.1291
    2.5107
```

순서대로 i1, i2, i3, i4, i5, i6, i7에 대한 전류 값입니다.

fx >>

9.1. Given the equations

$$2x_1 - 6x_2 - x_3 = -38.$$
$$-3x_1 - x_2 + 1x_3 = -34.$$
$$-8x_1 + x_2 - 2x_3 = -20.$$

(a). Solve by Gauss elimination

above set of linear equation is converted augmented matrix,

$$\begin{pmatrix} 2 & -6 & -1 \\ -3 & -1 & 1 \\ -8 & 1 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -38 \\ -34 \\ -20 \end{pmatrix}$$

$$\times \frac{-3}{2} \left(\begin{pmatrix} 2 & -6 & -1 & -38 \\ -3 & -1 & 1 & -34 \\ -8 & 1 & -2 & -20 \end{pmatrix} \right) \times \frac{-8}{2}$$

$$\begin{pmatrix} 2 & -6 & -1 & -38 \\ 0 & -10 & 5.5 & -91 \\ 0 & -23 & -6 & -172 \end{pmatrix} \times \frac{-23}{-10}$$

$$\begin{pmatrix} 2 & -6 & -1 & -38 \\ 0 & -10 & 5.5 & -121 \\ 0 & 0 & ① & ② \end{pmatrix}$$

$$① : -6 - 5.5 \times \frac{23}{10}$$
$$-6 - \frac{11}{2} \times \frac{23}{10}$$
$$-6 - \frac{253}{20} = -\frac{313}{20} = -18.65$$

$$② : -172 - (-91.) \times \frac{23}{10} = \frac{313}{10} = 31.3$$

$$\begin{pmatrix} 2 & -6 & -1 & -38 \\ 0 & -10 & 5.5 & -91. \\ 0 & 0 & -18.65 & 31.3 \end{pmatrix}$$

back elimination

$$-18.65 x_3 = 31.3.$$
$$x_3 = \frac{31.3}{-18.65} = -2.$$
$$-10x_2 + 5.5 \times (-2) = -91.$$
$$x_2 = -\frac{1}{10}(-91 - 5.5 \times (-2)) = 8.$$
$$2x_1 - 6 \times 8 + 2 = -38$$
$$x_1 = \frac{1}{2}(48 - 2 - 38)$$
$$= 4$$

$\therefore$) $x_1 = 4, \quad x_2 = 8, \quad x_3 = -2$

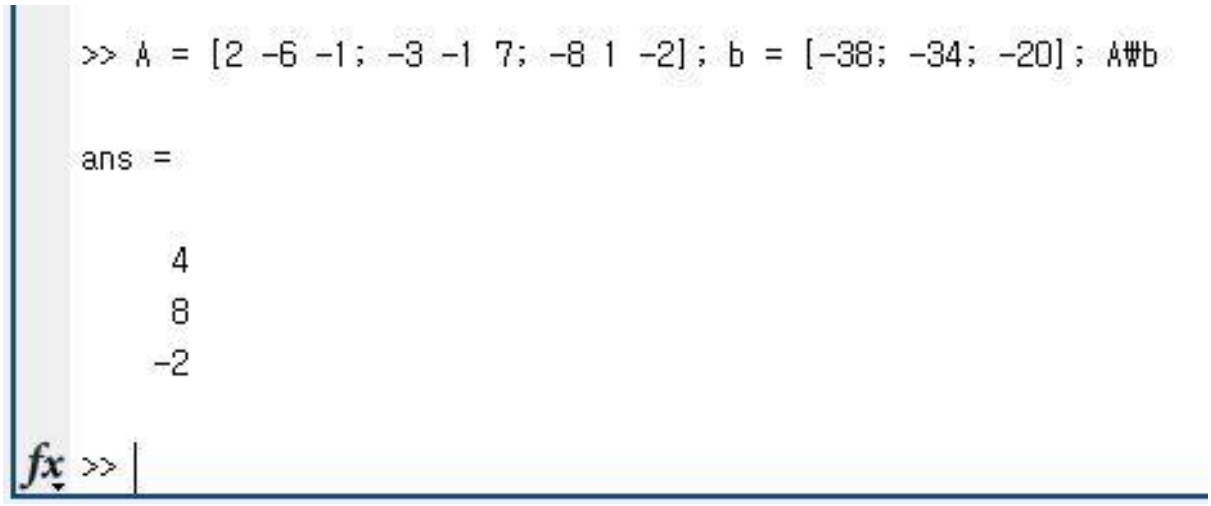
(b). substitute your results into the original equations to check your answers.

$$2 \times 4 - 6 \times 8 - (-2) = 8 - 48 + 2$$
$$= -38.$$

$$-3 \times 4 - 8 + 1 \times (-2) = -12 - 8 - 14$$
$$= -34.$$

$$-8 \times 4 + 8 - 2 \times (-2) = -32 + 8 + 4$$
$$= -20.$$

Matlab 커맨드 창을 이용해 결과를 검증하였습니다.



9.12

문제 해석 및 방정식 세우기

$C(x)$. C = concentration, t = time, x = distance, D = diffusion coefficient, U = fluid velocity, k = a first-order decay rate.

$D = 2$, $U = 1$, $k = 0.2$, $C(0) = 80$, $C(10) = 20$, $x = [0, 10]$

$$D\frac{d^2C}{dx^2} - U\cdot\frac{dC}{dx} - kC = 0$$

At $\frac{dC}{dx} = \frac{C(x_{i+1}) - C(x_i)}{\Delta x}$

At $\frac{d^2C}{dx^2} = \frac{1}{\Delta x}\left\{\frac{C(x_{i+2}) - C(x_{i+1})}{\Delta x} - \frac{C(x_{i+1}) - C(x_i)}{\Delta x}\right\}$

$$= \frac{1}{\Delta x}\left\{\frac{C(x_{i+2}) - 2C(x_{i+1}) + C(x_i)}{\Delta x}\right\}$$

$i+1 \longrightarrow i$.

$$= \frac{C(x_{i+1}) - 2C(x_i) + C(x_{i-1})}{\Delta x^2}$$

$$D\cdot\frac{C(x_{i+1}) - 2C(x_i) + C(x_{i-1})}{\Delta x^2} - U\cdot\frac{C(x_{i+1}) - C(x_i)}{\Delta x} - kC = 0$$

$(\Delta x = 1,\ D = 2,\ U = 1)$

$0.8C(x_i)$

$$2(C(x_{i+1}) - 2C(x_i) + C(x_{i-1})) - C(x_{i+1}) + C(x_i) - 0.2C(x_i) = 0$$

$$C(x_{i+1}) - 3.2C(x_i) + 2C(x_{i-1}) = 0$$

$$2C(x_{i-1}) - 3.2C(x_i) + C(x_{i+1}) = 0$$

$$\begin{pmatrix} C_1 \\ \vdots \\ C_9 \end{pmatrix}$$

9개. $\begin{cases} 2C(x_0) - 3.2C(x_1) + C(x_2) = -160 \\ 2C(x_1) - 3.2C(x_2) + C(x_3) = 0 \\ \vdots \\ 2C(x_8) - 3.2C(x_9) + C(x_{10}) = -20 \end{cases}$ $\Rightarrow$ $\begin{pmatrix} -3.2 & 1 & 0 & \cdots & 0 & -160 \\ 2 & -3.2 & 1 & \cdots & 0 & 0 \\ & & \vdots & & & \\ 0 & 0 & \cdots & 2 & -3.2 & -20 \end{pmatrix}$

편집기 - C:₩Users₩JunGi_Kim₩Desktop₩2020학년 3학년 2학기₩수

hardproblem.m

```
1   function hardproblem()
2 –   A = zeros(9);
3 –   disp(A);
4
5 –   A(1, 1) = -3.2;
6 –   A(1, 2) = 1;
7
8 –   disp(A);
9 –   j = 1;
10 –  for i = 2:1:8
11 –      A(i, j) = 2;
12 –      A(i, j + 1) = -3.2;
13 –      A(i, j + 2) = 1;
14 –      j = j + 1;
15 –  end
16 –  A(9, 8) = 2;
17 –  A(9, 9) = -3.2;
18
19 –  disp(j);
20 –  disp(A);
21 –  b = [-160; 0; 0; 0; 0; 0; 0; 0; -20];
22 –  solution = A₩b;
23 –  disp(solution);
24
25 –  x = 0:1:10;
26 –  y = [80 solution' 20];
27
28 –  plot(x, y, 'or', 'MarkerFaceColor', 'red');
29 –  end
```

1번째 행과 9번째 행을 제외하고는 반복문을 이용해 방정식을 구성했습니다.

명령 창

```
68.1346
58.0308
49.4294
42.1124
35.9010
30.6583
26.3045
22.8579
20.5362

fx >>
```

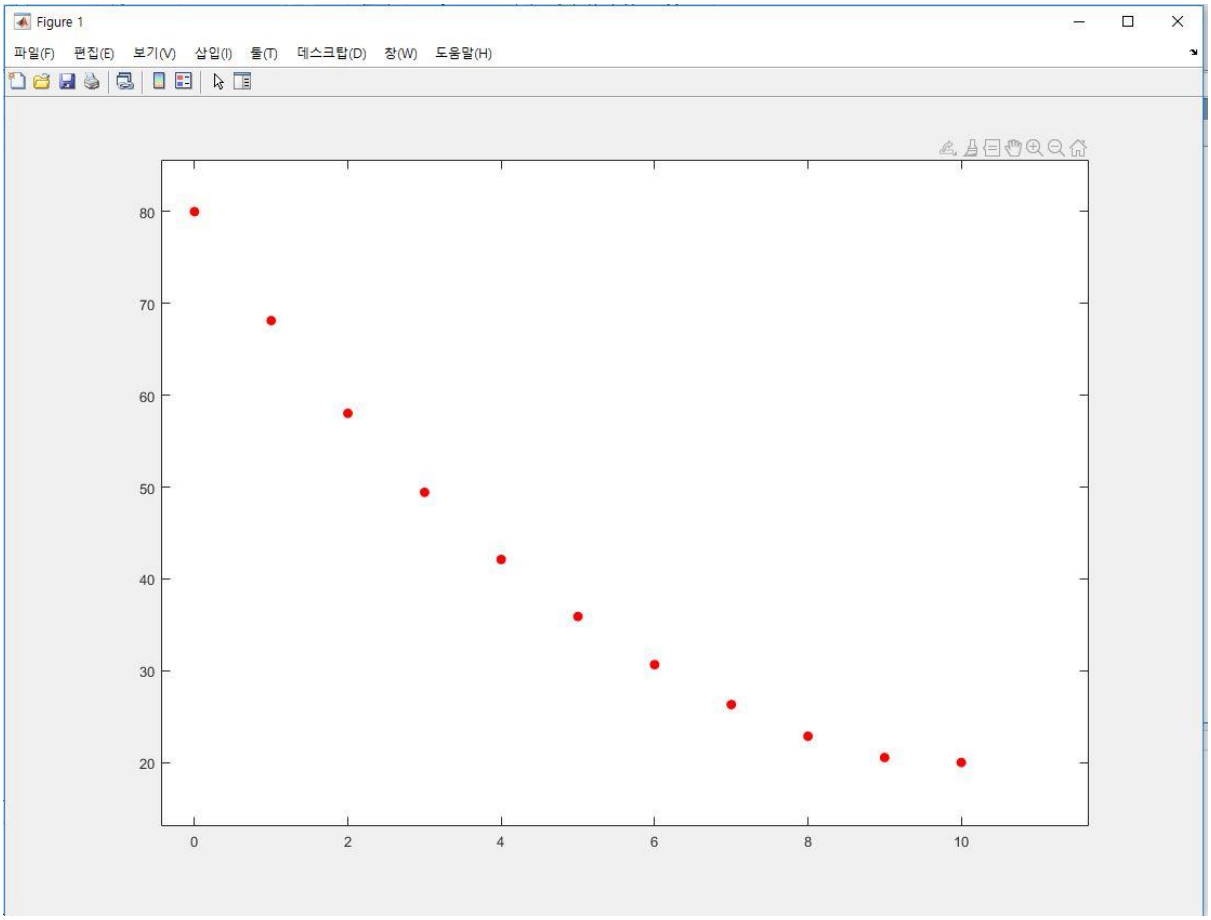


Figure 1
파일(F) 편집(E) 보기(V) 삽입(I) 툴(T) 데스크탑(D) 창(W) 도움말(H)
80
70
60
50
40
30
20
0
2
4
6
8
10