# “Random or not so random?”

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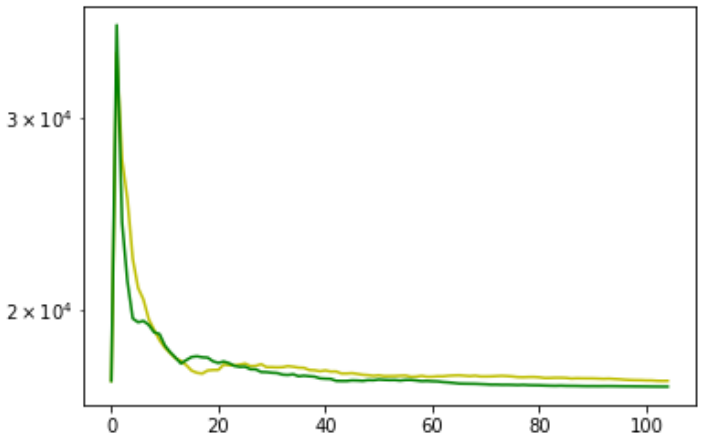
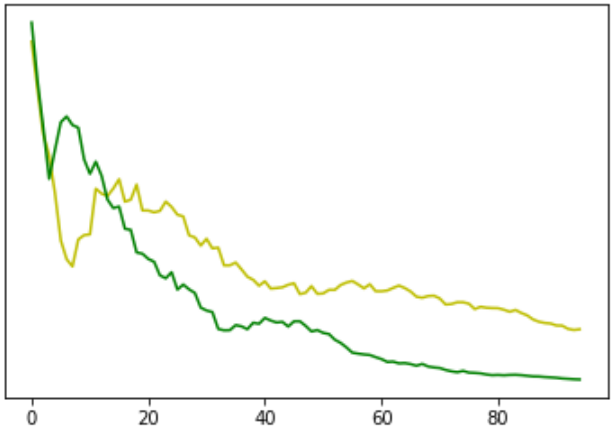
**Task 1:**

**Histories of objective function View:**

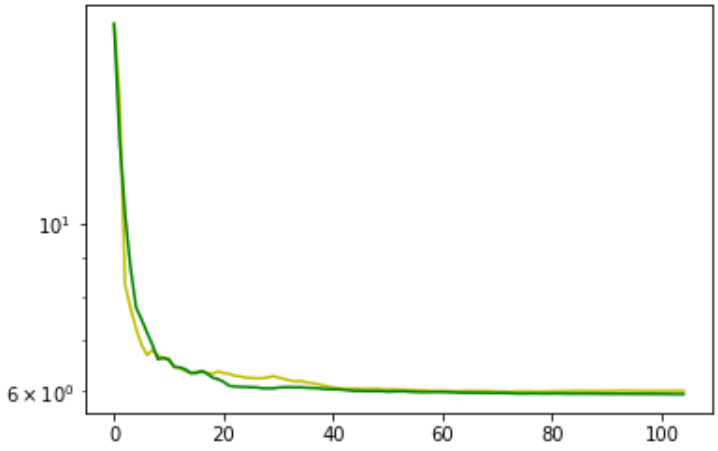
Set 1 View: using “generate_random_numbers(105, 0.5, 20.0, "normal")”

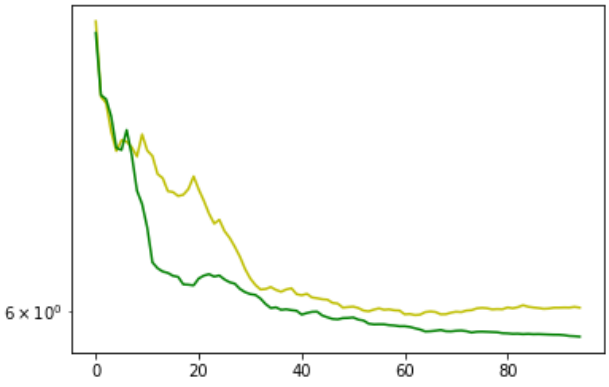



(closer look: last 90 points)



Set 2 View: using “generate_random_numbers(105, 0.5, 1.0, "uniform")”




(closer look: last 90 points)

**Conclusion:** The strategy without replacement have better performance in both cases.

**Proof of "IDG_wo_task1 must converge to the true solution"**

(Note: result using n+1 because we start from x0 = 0)

$k=0$

$x_1 = 0 - \frac{1}{1+0} \cdot (-y_{i0}) = y_{i0}.$

$k=1$

$x_2 = y_{i0} - \frac{1}{2}(y_{i0} - y_{i1})$

$= \frac{1}{2} y_{i0} + \frac{1}{2} y_{i1}$

$k=2$

$x_3 = \frac{1}{2} y_{i0} + \frac{1}{2} y_{i1} - \frac{1}{3}(\frac{1}{2} y_{i0} + \frac{1}{2} y_{i1} - y_{i2})$

$= \frac{1}{3} y_{i0} + \frac{1}{3} y_{i1} + \frac{1}{3} y_{i2}$

⋮

$k=n+1$

$x_n = \frac{1}{n}(y_{i0} + y_{i1} + \ldots + y_{in})$

$\Rightarrow x_{opt} = \frac{1}{n} \sum_{k=0}^{n} y_{ik}$ , since without replacement,

$y_{i0}, y_{i1} \ldots y_{in}$ include all value in $y$

Claim $x_n = \frac{1}{n} \sum_{k=0}^{n-1} y_{ik}$

Base case $n=1$.

$x_1 = x_0 - \frac{1}{1+0} \cdot (-y_{i0}) = y_{i0}.$

Assume $x_n = \frac{1}{n} \sum_{k=0}^{n-1} y_{ik}$

$x_{n+1} = \frac{1}{n} \sum_{k=0}^{n-1} y_{ik} - \frac{1}{n+1}(\frac{1}{n} \sum_{k=0}^{n-1} y_{ik} - y_{in})$

$= (\frac{1}{n} - \frac{1}{n+1} \cdot \frac{1}{n}) \sum_{k=0}^{n-1} y_{ik} + \frac{1}{n+1} y_{in}$ $\frac{n+1-1}{n(n+1)} = \frac{1}{n+1}$

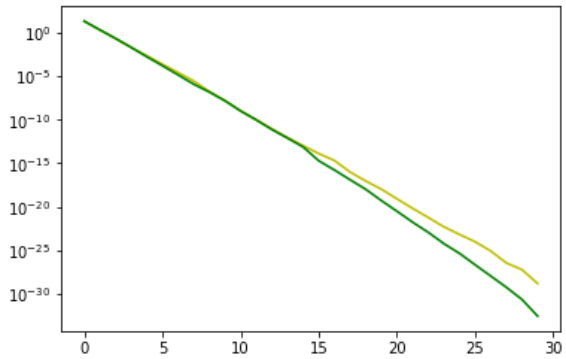
$= \frac{1}{n+1} (\sum_{k=0}^{n-1} y_{ik} + y_{in})$

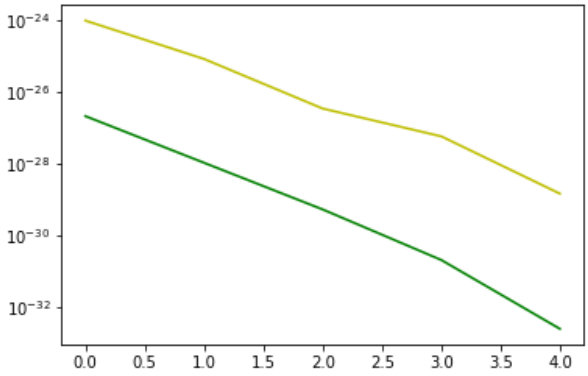
$= \frac{1}{n+1} \sum_{k=0}^{n} y_{ik}$ ☑

Note: In both task 2 and task 3, we run both functions (wr and wo) 200 times and generate the mean value for each point to make the result more accurate.

**Task 2**

**Histories of objective function View:**



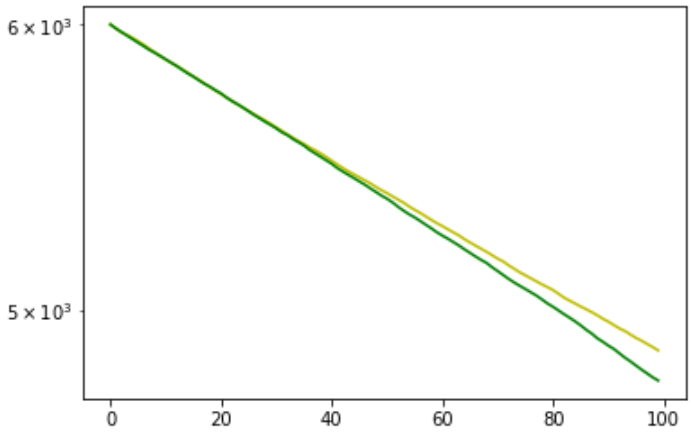




(closer look: last 5 points)

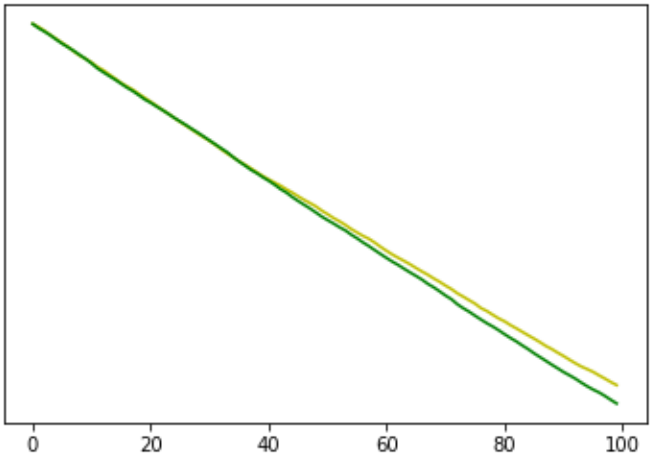
**Conclusion:** The strategy without replacement have better performance in this case.

**Task 3:**

**Histories of objective function View:**




**Convergence:**




**Conclusion:** The strategy without replacement have better performance in this case.