

Numerical Analysis

<Home Assignment #3>

(Deadline: June 13, 2025)

Problem 1. The following data were gathered to determine the relationship between pressure and temperature of nitrogen.

| | | | | | | |
|---------------|------|------|------|-------|-------|-------|
| $T, ^\circ C$ | -40 | 0 | 40 | 80 | 120 | 160 |
| $p, N/m^2$ | 6990 | 8100 | 9350 | 10500 | 11700 | 12800 |

Use **least-squares regression** to fit a straight line to the data in the table above. Employ the equation $p = \alpha T + \beta$ to determine α and β on the basis of these data.

Problem 2. The force on a sailboat mast can be represented by the following function:

$$F = \int_0^H 200 \left(\frac{z}{5+z} \right) e^{-z^2/H} dz \quad (1)$$

where z = the elevation above the deck and H = the height of the mast. Compute F for the case where $H = 30$ using the methods below.

- a) The composite trapezoidal rule ($n = 2$).
- b) The composite Simpson's 1/3 rule ($n = 2$).
- c) The two-point Gauss-Legendre formula.

Problem 3. Solve the following initial value problem over the interval from $t = 0$ where $y(0) = 1$. Display all your results on the same graph.

$$\frac{dy}{dt} = yt^2 - 1.1y$$

- a) Analytically.
- b) Using Euler's method with $h = 0.5$.
- c) Using the midpoint method with $h = 0.5$.
- d) Using the fourth-order RK method with $h = 0.5$.