Problem 7.11

(a) Show that the Fourier transform of the free-field equation for A_μ(i.e. the one in the previous question with j_μ^{em} set to zero) is given by (7.87).

Solutions of problem 7.11 (a):

The equation (7.87) reads as,

$$(-k^2 g^{\nu\mu} + k^\nu k^\mu)\tilde{A}_\mu(k) \equiv M^{\nu\mu}\tilde{A}_\mu(k) = 0 \tag{7.87}$$

And the Fourier transform of free-field equation for A_μ is $\tilde{A}_\mu(k) = \Box A^\mu - \partial^\mu(\partial_\nu A^\nu)$.

$$\begin{aligned}
&(-k^2 g^{\nu\mu} + k^\nu k^\mu)\tilde{A}_\mu(k) \\
&= (-k^2 g^{\nu\mu} + k^\nu k^\mu)[\Box A^\mu - \partial^\mu(\partial_\nu A^\nu)] \\
&= (-k^\nu k_\nu g^{\nu\mu} + k^\nu g^{\mu\nu} k_\nu)[\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu)] \\
&= \partial_\mu k^\nu(-k_\nu g^{\nu\mu} + g^{\mu\nu} k_\nu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\
&= \partial_\mu k^\nu(-k_\nu g^{\nu\mu}\partial^\mu A^\nu + k_\nu g^{\nu\mu}\partial^\nu A^\mu + g^{\mu\nu} k_\nu \partial^\mu A^\nu - g^{\mu\nu} k_\nu \partial^\nu A^\mu) \\
&= \partial_\mu k^\nu(-k_\nu \partial_\nu A^\nu + k_\nu g^{\nu\mu}\partial^\nu A^\mu + k^\nu \partial^\mu A^\nu - k^\mu \partial^\nu A^\mu) \qquad (g_{\nu\mu} = g^{\nu\mu})
\end{aligned}$$