Solutions of chapter 9

("Gauge theories in particle physics", 3rd, Aitchison and Hey)

Chapter 9, Deep inelastic electron-nucleon scattering

Problems 9.1, P_{272}.

The various normalization factors in equation (9.3) and (9.11) may be checked in the following way. The cross section for inclusive electron-proton scattering may be written (equation(9.11)):

$$d\sigma = (\frac{4\pi\alpha}{q^2})^2 \frac{1}{4[(k\cdot p)^2 - m^2M^2]^{1/2}} 4\pi M L_{\mu\nu} W^{\mu\nu} \frac{d^3k'}{2\omega'(2\pi)^3} \tag{9.104}$$

in the usual one-photon exchange approximation, and the tensor $W^{\mu\nu}$ is related to hadronic matrix elements of the electormagnetic current operator by equation (9.3) :

$$e^2 W^{\mu\nu}(q,p) = \frac{1}{4\pi M}\frac{1}{2}\sum_s \sum_X \langle p; p, s|\hat{j}^\mu_{em}(0)|X; p'\rangle \langle X; p'|\hat{j}^\nu_{em}(0)|p; p, s\rangle (2\pi)^4 \delta^4(p+q-p')$$

where the sum X is over all possible hadronic final states. If we consider the special case of elastic scattering, the sum over X is only over the final protons's degrees of freedom:

$$e^2 W^{\mu\nu}_{el}(q,p) = \frac{1}{4\pi M}\frac{1}{2}\sum_s \sum_{s'} \langle p; p, s|\hat{j}^\mu_{em}(0)|p; p', s'\rangle \langle p; p', s'|\hat{j}^\nu_{em}(0)|p; p, s\rangle (2\pi)^4 \delta^4(p+q-p') \frac{1}{(2\pi)^3}\frac{d^3p'}{2E'}$$

Now use equation (8.206) with $\mathcal{F}_1 = 1$ and $k = 0$(i.e. the electromagnetic current matrix element for a 'point' proton) to show that the resulting cross section identical to that for elastic $e\mu$ scattering.

Solution of problem 9.1

Using (8.206) with $\mathcal{F}_1 = 1$ and $k = 0$ for the current matrix elements, and canceling a factor of e^2, we obtain

$$\begin{aligned} W^{\mu\nu}_{el} &= \frac{1}{8\pi M}\sum_{s,s'} \bar{u}(p,s)\gamma^\mu u(p',s')\bar{u}(p',s')\gamma^\nu u(p,s)(2\pi)^4\delta^4(p+q-p')\frac{1}{(2\pi)^3}\frac{d^3\boldsymbol{p}'}{2E'} \\ &= \frac{1}{8\pi M} Tr\{\gamma^\mu(\not{p}'+M)\gamma^\nu(\not{p}+M)\}(2\pi)^4\delta^4(p+q-p')\frac{1}{(2\pi)^3}\frac{d^3\boldsymbol{p}'}{2E'} \end{aligned} \tag{8.171}$$

where the Trace is $2M^{\mu\nu}$(8.186). Equation (9.104) then becomes

$$d\sigma = (\frac{4\pi\alpha}{q^2})^2 \frac{1}{4[(k\cdot p)^2 - m^2M^2]^{1/2}} L_{\mu\nu} M^{\mu\nu} (2\pi)^4\delta^4(p+q-p')\frac{1}{(2\pi)^3}\frac{d^3\boldsymbol{p}'}{2E'}\frac{d^3k'}{2\omega'(2\pi)^3}$$

This is precisely the formula which yields the cross section for elastic eμ scattering(8.184).

Problems 9.2, P_{273}.

(a)Perform the contraction $L_{\mu\nu}W^{\mu\nu}$ for inclusive inelastic electron-proton scattering (remember $q^{\mu}L_{\mu\nu} = q^{\nu}L_{\mu\nu} = 0$). Hence verify that the inclusive differential cross section in terms of "laboratory" variables, and neglecting the electron mass, has the form

$$\frac{d^2\sigma}{d\Omega dk'} = \frac{\alpha^2}{4k^2 sin^4(\theta/2)}[W_2 cos^2(\theta/2) + W_1 sin^2(\theta/2)]$$

(b) By calculating the Jacobian

$$J = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix}$$

for a change of variables $(x, y) \rightarrow (u, v)$

$$dudv = |J|dxdy$$

find expressions for $d^2\sigma/dQ^2dv$ and $d^2\sigma/dxdy$, where Q^2 and v have their usual significance, and x is the scaling variable $Q^2/2Mv$ and $y = v/k$.

Solution of problem 9.2, (a)

Omitting the terms involving q_μ and q_ν from the expression (9.10) fro $W^{\mu\nu}$, we need to evaluate

$$\begin{aligned} L_{\mu\nu}W^{\mu\nu} = L^{\mu\nu}W_{\mu\nu} &= 2[k'^{\mu}k^{\nu} + k'^{\nu}k^{\mu} + (q^2/2)g^{\mu\nu}][-g_{\mu\nu}W_1 + (p_\mu p_\nu/M^2)W_2] \\ &= 2[(-2k'\cdot k - 2q^2)W_1 + (2p\cdot k' p\cdot k + q^2p^2/2)W_2/M^2] \end{aligned}$$

In the "Laboratory" system, and neglecting the electron mass (compare (8.217) and (8.128)),

$$p\cdot k' = \omega' M, p\cdot k = \omega M, q^2 = -2k\cdot k'$$

So,

$$(2p\cdot k' p\cdot k + q^2p^2/2)/M^2 = 2\omega\omega' + q^2/2 = 2\omega\omega' - k\cdot k'$$

Writing as usual $k = \omega = |\boldsymbol{k}|, k' = \omega' = |\boldsymbol{k}'|$, we have

$$k\cdot k' = kk'(1 - cos\theta) = 2kk' sin^2\theta/2$$

and

$$2\omega\omega' - k\cdot k' = kk'(1 + cos\theta) = 2kk' cos^2/2$$

Hence

$$L_{\mu\nu}W^{\mu\nu} = 4kk'[2W_1 sin^2\theta/2 + W_2 cos^2\theta/2]$$

And from (9.104),

$$d\sigma = (\frac{4\pi\alpha}{q^2})^2 \frac{1}{4[(k\cdot p)^2 - m^2M^2]^{1/2}} 4\pi M 4kk'[2W_1 sin^2\theta/2 + W_2 cos^2\theta/2]\frac{d^3\boldsymbol{k}'}{2\omega'(2\pi)^3}$$

Now $(q^2)^2 = 16k^2k'^2 sin^4\theta/2$, and $[(k\cdot p)^2 - m^2M^2]^{1/2}$ = k M (neglecting the electron mass). And $d^3\boldsymbol{k}' = k'^2dk'd\Omega$. Hence,

$$d\sigma = \frac{\alpha^2}{4k^2 sin^4\theta/2}[2W_1 sin^2\theta/2 + W_2 cos^2\theta/2]dk'd\Omega$$

Proof is done.

Problems 9.3, P_{273}.

Consider the description of inelastic electron-proton scattering in terms of virtual photon cross sections:

(a) In the 'laboratory' frame with $p^\mu = (M, 0, 0, 0)$ and $q^\mu = (q^0, 0, 0, q^3)$, evaluate the transverse spin sum

$$\frac{1}{2}\sum_{\lambda=\pm 1} \epsilon_\mu(\lambda)\epsilon_\nu^*(\lambda)W^{\mu\nu}$$

Hence show that the 'Hand' cross section for transverse virtual photons is

$$\sigma_T = (4\pi^2\alpha/K)W_1$$

(b) Using the definition

$$\epsilon_s^\mu = (1/\sqrt{Q^2})(q^3, 0, 0, q^0)$$

and rewriting this in terms of the 'laboratory' 4-vectors p^μ and q^μ, evaluate the longitudinal / scalar virtual photons cross section. Hence show that

$$W_2 = \frac{K}{4\pi^2\alpha}\frac{Q^2}{Q^2+\nu^2}(\sigma_S + \sigma_T).$$

Solution of problem 9.3, (a)

The transverse polarization vectors are given in (9.38): $\epsilon^\mu(\lambda = \pm 1) = \mp 2^{-1/2}(0, 1, \pm i, 0)$. These satisfy $\epsilon(\lambda = \pm 1) \cdot p = 0$ in the laboratory frame, and also (9.40) : $q \cdot \epsilon = \epsilon \cdot q = 0$. Hence in the produce $\epsilon_\mu \epsilon_\nu^* W^{\mu\nu}$, with $W^{\mu\nu}$ given by (9.10), only the contraction with $-g^{\mu\nu}W_1$ survives, leading to

$$\begin{aligned}
\frac{1}{2}\sum_{\lambda=\pm 1} \epsilon_\mu(\lambda)\epsilon_\nu^*(\lambda)W^{\mu\nu} &= \frac{1}{2}[\epsilon^\mu(\lambda = 1)\epsilon_\mu^*(\lambda = 1) + \epsilon^\mu(\lambda = -1)\epsilon_\mu^*(\lambda = -1)]W_1 \\
&= \frac{1}{2}[(-\frac{1}{\sqrt{2}})^2(0, 1, i, 0)(0, 1, -i, 0)^T + (\frac{1}{\sqrt{2}})^2(0, 1, -i, 0)(0, 1, i, 0)^T]W_1 \\
&= \frac{1}{2}(1 + 1)W_1 \\
&= W_1
\end{aligned}$$

Therefore, (9.46), $\sigma_T = (4\pi^2\alpha/K)W_1$, follows from (9.45).

Solution of problem 9.3, (b)

From (9.47) the longitudinal / scalar virtual photon cross section is, $\sigma_S = (4\pi^2\alpha/K)\epsilon_\mu^*(\lambda = 0)\epsilon_\nu(\lambda = 0)W^{\mu\nu}$, where $W^{\mu\nu}$ given by (9.10), and where $\epsilon^\mu(\lambda = 0)$ is real and given by (9.41), and satisfies $q \cdot \epsilon = 0$(see (9.40)). Thus in the contractions with $W^{\mu\nu}$, terms involving q^μ and q^ν can be dropped. The W_1 term in '$\epsilon \cdot \epsilon W$ ' is then simply $(-\epsilon \cdot \epsilon W_1 = -W_1)$ (note (9.42)), while the W_2 term is

$$\begin{aligned}
&\frac{1}{M_2}\epsilon(\lambda = 0) \cdot p\epsilon(\lambda = 0) \cdot pW_2 \\
&= \frac{1}{M^2Q^2}(q^3M)(q^3M)W_2 \qquad \text{(for longitudinal/scalar photon, } \epsilon_s^\mu(\lambda = 0) = (1/\sqrt{Q^2})(q^3, 0, 0, q^0)) \\
&= \frac{(q^3)^2}{Q^2}W_2 = \frac{Q^2 + (q^0)^2}{Q^2}W_2 = (1 + \frac{\nu^2}{Q^2})W_2
\end{aligned}$$

and (9.48) follows from these results.

From (9.46) we obtain $W_1 = [K/4\pi^2\alpha]\sigma_T$, and substituting this into (9.48) gives the required result for W_2.

Problems 9.4, P_{274}.

In this question, we consider a representation of the 4×4 Dirac matrices in which(see section 4.4)

$$\alpha = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix} \qquad \beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Define also the 4×4 matrix

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and the Dirac four-component spinor $\begin{pmatrix} \phi \\ \chi \end{pmatrix}$. Then the two-component spinors ψ, χ satisfy

$$\boldsymbol{\sigma} \cdot \boldsymbol{p}\phi = E\phi - m\chi$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{p}\chi = -E\chi + m\phi$$

(a) Show that for a massless Dirac particle, ϕ and χ become helicity eigenstates(see section 4.3) with positive and negative helicity respectively.

(b) Defining

$$P_R = \frac{1+\gamma_5}{2} \qquad P_L = \frac{1-\gamma_5}{2}$$

show that $P_R^2 = P_L^2 = 1, P_R P_L = 0 = P_L P_R$, and that $P_R + P_L = 1$. Show also that

$$P_R \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} \phi \\ 0 \end{pmatrix} \qquad P_L \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} 0 \\ \chi \end{pmatrix}$$

and hence that P_R and P_L are projection operators for massless Dirac particles, onto states of definite helicity. Discuss what happens when $m \neq 0$.

(c)The general massless spinor u can be written

$$u = (P_L + P_R)u \equiv u_L + u_R$$

where u_L, u_R have the indicated helicity. Show that

$$\bar{u}\gamma^\mu u = \bar{u}_L \gamma^\mu u_L + \bar{u}_R \gamma^\mu u_R$$

where $\bar{u}_L = u_L^\dagger \gamma^0$, $\bar{u}_R = u_R^\dagger \gamma^0$; and deduce that in electromagnetic interactions of massless fermions helicity is conserved.

(d) In weak interactions an axial vector current $\bar{u}\gamma^\mu \gamma_5 u$ also enters. Is helicity still conserved ?

(e) Show that the 'Dirac' mass term $m\bar{\psi}\psi$ may be written as $\hat{\bar{\psi}}_L^\dagger \bar{\psi}_R + \hat{\bar{\psi}}_R^\dagger \bar{\psi}_L$.

Solution of problem 9.4, (a)

In the limit $m \to 0$ the spinors ϕ and χ satisfy $\boldsymbol{\sigma} \cdot \boldsymbol{p}\phi = E\phi$ and $\boldsymbol{\sigma} \cdot \boldsymbol{p}\chi = -E\chi$ respectively, where in both cases $E = |\boldsymbol{p}|$(massless). Hence ϕ satisfies

$$\frac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{|\boldsymbol{p}|}\phi = \frac{E}{|\boldsymbol{p}|}\phi = \phi$$

which shows it has positive helicity (compare (4.67)); similarly χ has negative helicity.

(b)

For example,

$$P_R P_L = (\frac{1+\gamma_5}{2})(\frac{1-\gamma_5}{2}) = \frac{1}{4}[1-\gamma_5^2] = 0$$

When $m \neq 0$, the operators P_R and P_L still project out the ϕ and χ components of the 4-component spinor, but these 2-component objects are no longer (with $m \neq 0$) helicity eigenstates (since, for example, $(\boldsymbol{\sigma} \cdot \boldsymbol{p}/|\boldsymbol{p}|\phi)$ is no longer equal to ϕ or -ϕ).

(c)

We may write

$$\bar{u}\gamma^\mu u = u^\dagger (P_R + P_L)\gamma^0\gamma^\mu(P_R + P_L)u$$

We exploit the fundamental relation $\gamma^\mu\gamma_5 = -\gamma_5\gamma^\mu$(see (J. 11)). Consider one 'cross' term :

$$\begin{aligned} u^\dagger P_R\gamma^0\gamma^\mu P_L u &= u^\dagger\gamma^0 P_L\gamma^\mu P_L u \\ &= u^\dagger\gamma^0\gamma^\mu P_R P_L u && (\text{due to } \gamma^\mu\gamma_5 = -\gamma_5\gamma^\mu,\ P_L\gamma^\mu = \gamma^\mu P_R) \\ &= 0 && (P_R P_L = 0, \text{ c.f. (b)}) \end{aligned}$$

Similarly for the term $u^\dagger P_L\gamma^0\gamma^\mu P_R u$. The only surviving terms are

$$u^\dagger P_R\gamma^0\gamma^\mu P_R u + u^\dagger P_L\gamma^0\gamma^\mu P_L u$$

which is just $\bar{u}_R\gamma^\mu u + \bar{u}_L\gamma^\mu u$. Hence 'R' states connect only to 'R' states, and similarly for 'L' states, and so helicity(in the massless limit) is conserved.

Note that '$\bar{u}_R$' could perhaps more clearly be written as $\overline{u_R}$ since we form it by taking the dagger of u_R and then multiplying by γ^0 - i.e. we take the Dirac 'bar' of u_R. $\bar{u}_R$ is however the conventional notation.

(d)

In this case a typical cross term is

$$\begin{aligned} u^\dagger P_R\gamma^0\gamma^\mu\gamma_5 P_L u &= u^\dagger\gamma^0 P_L\gamma^\mu\gamma_5 P_L u \\ &= u^\dagger\gamma^0\gamma^\mu P_R\gamma_5 P_L u && (P_L\gamma^\mu = \gamma^\mu P_R) \\ &= u^\dagger\gamma^0\gamma^\mu\gamma_5 P_R P_L u && (P_R\gamma^5 = \gamma^5 P_R) \\ &= 0 \end{aligned}$$

and again helicity is conserved.

(e)

The Dirac mass term is

$$\bar{\hat{\psi}}\hat{\psi} = \hat{\psi}^\dagger(P_R + P_L)\gamma^0(P_R + P_L)\hat{\psi}$$

Consider a 'diagonal' term :

$$\hat{\psi}^\dagger P_R\gamma^0 P_R\hat{\psi} = \hat{\psi}^\dagger\gamma^0 P_L P_R\hat{\psi} = 0$$

and similarly for the other diagonal term. Only the 'L - R' and 'R - L' terms survive.

Problems 9.5, P_{274}.

In the HERA colliding beam machine, positrons of total energy $27.5GeV$ collide head on with protons of total energy $820GeV$. Neglecting both the positron and protons rest masses, calculate the center-of-mass energy in such a collision process.

Some theories have predicted the existence of 'leptoquarks', which could be produced at HEAR as a resonance state formed from the incident positron and the struck quark. How could a distribution of such events look, if plotted versus the variables x ?

Solution of problem 9.5

Neglecting the positron and proton masses, their 4-momenta are $p_{e^+} = (k, 0, 0, -k)$, say, and $p_p = (p, 0, 0, p)$. Then,

$$W_{CM}^2 = (p_{e^+} + p_p)^2 = (k + p)^2 - (k - p)^2 = 4kp.$$

So $W_{CM} = 2\sqrt{kp} = 300.3GeV$.

A leptoguark of mass M_{lq} formed as a resonance state of the e^+ and the struck quark would appear as a peak in the effective mass of the e^+ and the quark, at an effective mass equal to M_{lq}. In a simple parton model picture, this effective mass is $\sqrt{(p_{e^+} + xp_p)^2}$. So we expect a peak when

$$p_{e^+}^2 + 2xp_{e^+} \cdot p_p + x^2 p_p^2 = M_{lq}^2$$

or, neglecting he positron and proton masses, $2xp_{e^+} \cdot p_p = M_{lq}^2 \Rightarrow x = M_{lq}^2 / W_{CM}^2$, which is the peak of x.

Problems 9.6, P_{275}.

(a)By the expedient of inserting a $\delta-$ function, the differential cross section for Drell-Yan production of a lepton pair of mass $\sqrt{q^2}$ may be written as

$$\frac{d\sigma}{dq^2} = \int dx_1 dx_2 \frac{d^2\sigma}{dx_1 dx_2} \delta(q^2 - sx_1 x_2)$$

Show that this is equivalent to the form

$$\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{9q^4} \int dx_1 dx_2 x_1 x_2 \delta(x_1 x_2 - \tau) \times \sum_a e_a^2 [q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1) q_a(x_2)]$$

which, since $q^2 = s\tau$, exhibits a scaling law of the form

$$s^2 d\sigma / dq^2 = F(\tau)$$

(b) Introduce the Feynman scaling variable

$$x_F = x_1 - x_2$$

with

$$q^2 = sx_1 x_2$$

and show that

$$dq^2 dx_F = (x_1 + x_2) s dx_1 dx_2$$

Hence show that the Drell-yan formula can be rewritten as

$$\frac{d^2\sigma}{dq^2 dx_F} = \frac{4\pi\alpha^2}{9q^4}\frac{\tau}{(x_F^2+4\tau)^{1/2}}\sum_a e_a^2[q_a(x_1)\bar{q}_a(x_2)+\bar{q}_a(x_1)q_a(x_2)]$$

Solution of problem 9.6

(a)

Using (9.92) for $d^2\sigma/dx_1dx_2$, we have

$$\begin{aligned}\frac{d\sigma}{dq^2} &= \int dx_1 dx_2 \frac{4\pi\alpha^2}{9q^2}\sum_a e_a^2[q_a(x_1)\bar{q}_a(x_2)+\bar{q}_a(x_1)q_a(x_2)]\delta(q^2-sx_1x_2)\\ &= \frac{4\pi\alpha^2}{9q^2}\int dx_1dx_2\sum_a e_a^2[q_a(x_1)\bar{q}_a(x_2)+\bar{q}_a(x_1)q_a(x_2)]\frac{1}{s}\delta(q^2-sx_1x_2)\end{aligned}$$

with the help of $(E.29)$, and then writing $1/s = x_1x_2/q^2$(equalizing the two terms in the δ function) and $\tau = q^2/s$, we obtain the desired formula.

(b)

$$\begin{aligned}dq^2dx_F &= \begin{vmatrix}\frac{\partial q^2}{\partial x_1} & \frac{\partial q^2}{\partial x_2}\\ \frac{\partial x_F}{\partial x_1} & \frac{\partial x_F}{\partial x_2}\end{vmatrix} dx_1dx_2\\ &= \begin{vmatrix} sx_2 & sx_1\\ 1 & -1\end{vmatrix} dx_1dx_2\\ &= -s(x_1+x_2)dx_1dx_2\end{aligned}$$

The minus sign can be absorbed by appropriate choice of limits in the $q^2\!-\!x_F$ integration. In the variables(x_1,x_2), the integration is over the square $0\le x_1\le 1, 0\le x_2\le 1$. Consider performing the integration holding x_1 fixed and integrating over x_2, and then integrating over x_1. Take $x_1 = 1/2$ as an example, this line maps into the line $2q^2/s + x_F = 1/2$, and it is traversed in the sense of q^2/s increasing (from 0 to 1/2) but x_F decreasing (from 1/2 to $-1/2$). We can reverse the sense in which x_F is covered by invoking the minus sign from the determinant.

The variables x_1 and x_2 are given in terms of x_F and τ by $x_1 - x_2 = x_F$ and $x_2 = \tau/x_1$. So we have

$$x_1 - \tau/x_1 = x_F$$

Solving for x_1 (which is greater than 0) we find

$$x_1 = \frac{1}{2}[x_F + x_F^2 + 4\tau^{1/2}]$$

and hence

$$x_2 = \frac{1}{2}[-x_F + x_F^2 + 4\tau^{1/2}]$$

so that $x_1 + x_2 = (x_F^2+4\tau)^{1/2}$. Hence

$$\begin{aligned}d^2\sigma &= \frac{4\pi\alpha^2}{9q^2}\sum_a e_a^2[q_a(x_1)\bar{q}_a(x_2)+\bar{q}_a(x_1)q_a(x_2)]dx_1dx_2 \qquad \text{(from (9.92))}\\ &= \frac{1}{x_1+x_2)s}\{\ldots\}dq^2dx_F\\ &= \frac{1}{(x_F^2+4\tau)^{1/2}}\frac{4\pi\alpha^2\tau}{9q^4}\sum_a e_a^2[q_a(x_1)\bar{q}_a(x_2)+\bar{q}_a(x_1)q_a(x_2)]dq^2dx_F\end{aligned}$$

which leads to the desired expression.

Problems 9.7, P_{275}.

Verify that if the quarks participating in the Drell-Yan subprocess $q\bar{q} \to \gamma \to \mu\bar{\mu}$ had spin-0, the CM angular distribution of the final $\mu^+\mu^-$ pair would be proportional to $(1 - cos^2\theta)$.

Solution of problem 9.7

Let the 4-momenta of the incoming q and $\bar{a}$ be k and k_1 respectively, and let those of the outgoing μ^- and μ^+ be p' and p_1. Then the $q - \bar{q} - \gamma$ vertex, for scalar quarks, is proportional to $(k - k_1)^\mu$, while the $\gamma - \mu^- - \mu^+$ vertex is same as in problem 8.19(b). Thus in evaluating the unpolarized cross section we need the contraction

$$\begin{aligned} T &= (k-k_1)^\mu (k-k_1)^\nu (p'_\mu p_{1\nu} + p'_\nu p_{1\mu} - (Q^2/2) g_{\mu\nu}) \\ &= 2p' \cdot (k-k_1) p_1 \cdot (k-k_1) - (Q^2/2)(k-k_1)^2 \end{aligned}$$

Introduce the Mandelstam variables

$$s = (k+k_1)^2 = Q^2, t = (k-p')^2 = (k_1-p_1)^2, u = (k-p_1)^2 = (k_1-p')^2$$

Then, neglecting lepton masses,

$$\begin{aligned} T &= 2[-\frac{t}{2} + \frac{u}{2}][-\frac{u}{2} + \frac{t}{2}] - (Q^2/2)(-Q^2) \\ &= -\frac{1}{2}(t-u)^2 + \frac{1}{2}(Q^2)^2 \\ &= -\frac{1}{2}(4k^2 cos\theta)^2 + \frac{1}{2}(4k^2)^2 \\ &\propto (1 - cos^2\theta) \end{aligned}$$

where k is the CM momentum.