Solutions of detector physics

Problem 5.1.

(1). A Geiger-Müler counter (dead time 500 μs) measures in a strong radiation field a count rate of 1 kHz. What is the dead-time corrected true rate ?

(2). A point-like radioactive $\gamma-$ray source leads to a count rate of $R1 = 90000$ per second in a GM counter at a distance of $d_1 = 10cm$. At $d_1 = 30cm$ one gets $R2 = 50000$ per second. What is the dead time of the GM counter, if absorption effects in the air can be neglected ?

Solution of problem 5.1.

(1). For, Geiger-Müler counter and any other detectors all have a *dead time* τ_D where no particles after an event can be recorded. The dead time can be as short as 1 ns in Cherenkov counters, but in Geiger-Müler tubes it can account for 1 ms.

If the count rate is N, the counter is dead for the fraction $N \times \tau_D$ of the time, i.e., it is only sensitive for the fraction $1 - N \times \tau_D$ of the measurement time. The *true count rate* – in the absence of dead-time effects – would then be

$$N_{true} = \frac{N}{1 - N \times \tau_D}$$

In part (1) of this exercise, the 1kHz should be understood as N in above formula. As a result, the N_{true} can be figured out as,

$$\begin{aligned} N_{true} &= \frac{N}{1 - N \times \tau_D} \\ &= \frac{1kHz}{1 - 1kHz \times 500\mu s} \\ &= 2kHz \end{aligned}$$

So, the true rate is 2 kHz.

(2). For this part of exercise, one should assume the surface of GM counter is not big : in terms of its surface approximately can be considered to be equivalent to the spheral surface at both places. Besides, one has to recognize that an element of sphere surface can be described as $ds = dl_\theta dl_\phi \hat{\boldsymbol{r}} = r^2 sin\theta d\theta d\phi \hat{\boldsymbol{r}}$. As a result, the measured γ of GM counter in $d_1 = 10cm$ should be 9 times than the one in $d_2 = 30cm$ place. Moreover and obviously, the true count rate should be same no matter where the real place it is. So,

$$\begin{aligned} & N_{true} = \frac{N}{1 - N \times \tau_D} \\ & \Rightarrow \frac{90kHz}{1 - 90kHz \times \tau} = 9 \times \frac{50kHz}{1 - 50kHz \times \tau} \\ & \Rightarrow \tau = 10\mu s \end{aligned}$$

So, the GM counter has a dead-time of $10\mu s$.

Problem 8.1.

In an experiment an η meson with total energy $E_0 = 2000MeV$ is produced in the laboratory frame. Estimate the width of the η mass peak measured in a calorimeter which has an energy and angular resolution of $\sigma_E/E = 5\%, \sigma_\theta = 0.05$ radian, respectively ($m_\eta = 547.51MeV$).

Solution of problems 8.1

A typical energy spectrum measured in a calorimeter with high-resolution is usually asymmetric, with a rather long 'tail' to lower energies, and the energy resolution is conventionally parameterized as, $\sigma_E = FWHM/2.35$, where FWHM is the usual "Full Width Half Maximum" of a (Gaussian) spectrum.

Since the energy resolution of calorimeter is $\sigma_E/E = 5\%$ and the invariant mass peak of η meson is $m_\eta = 547.41MeV$, so,

$$\sigma_{m_\eta} = m_\eta \times 5\% = 547.41MeV \times 5\% = 27.3MeV$$

Therefore, the FWHM of η mass peak is equal to,

$$FWHM = 2.35 \times 27.37MeV = 64.3MeV.$$

comment 0, energy resolution of EMC.

In principle, the energy resolution is obtained from fit according to experiments individually. Therefore the energy resolution has different formula under variant context. That's to say, there is no universal expression of EMC's energy resolution. For instance, the CMS electromagnetic calorimeter can be approximated as

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{\sqrt{E}} \oplus c = \sqrt{(\frac{a}{\sqrt{E}})^2 + (\frac{b}{\sqrt{E}})^2 + c^2}$$

where " a " stands for photoelectron statistics, " b " for the electronics noise, and " c " appears due to the calibration uncertainty and crystal non-univormity.

The NA48 experiment approximates the energy resolution of its LKr calorimeter by above formula with a set of the following parameters of $a = 3.2\%, b = 9\%, c = 0.42\%$.

The OPAL experiment at CERN reported an energy resolution dominated by the stochastic term, or the term of " $a/\sqrt{E}$ " above.

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E[GeV]}}$$

comment 1, angular resolution of EMC.

In calorimeters without longitudinal segmentation the photon angles (or coordinates) are measured usually as corrected "centre of gravity " of the energy deposition.

$$\theta_\gamma = \frac{\sum \theta_i E_i}{\sum E_i} F_\theta(\varphi, \theta, E), \varphi_\gamma = \frac{\sum \varphi_i E_i}{\sum_i} F_\varphi(\varphi, \theta, E)$$

where E_i, θ_i, φ_i are the energy deposited in the ith calorimeter element with the angular coordinates θ_i, φ_i, F is correction function(like Fano factor).

BaBar experiment presented its angular resolution as,

$$\sigma(\theta) = \frac{4.2mrad}{\sqrt{E[GeV]}}$$

Problem 8.2.

Photons of $1GeV(1000MeV)$ energy are detected in a NaI(Tl) calorimeter which has an energy resolution $\sigma_E/E = 1.5\%/(E[GeV])^{1/4}$. Determine how the pulse-height distribution would change if an aluminum sheet of $L = 0.5X_0$ thickness would be placed in front of the calorimeter. Estimate the resulting decrease of the energy resolution.

Solution of problems 8.2

The number and the energy of show particles at depth $t = n \cdot X_0(X_0$ is the radiation length)in EMC could be expressed as,

$$N(t) = 2^t = 2^{n \cdot X_0}, E(t) = E_0 \cdot 2^{-t} = E_0 \cdot 2^{-n \cdot X_0}$$

So, if there is an aluminum sheet of $L = 0.5X_0$ thickness would be placed in front of the calorimeter, then the energy deposited E(t) in EMC will decrease since part of the E_0 lost in the aluminum layer. And the number of produced particle also decrease accordingly. As a result, the pulse-height distribution will change like this : the peak will move down to lower energy position, while the FWTH of histogram will become wider which means resolution will be worse than before.

Under the context of this exercise, it's reasonable to assume EMC has an origin depth of $6.5X_0$, and for NaI(Tl), its radiation length is 2.588 cm. And

$$\frac{\sigma_E}{E} = \frac{\sqrt{E}}{E} = \frac{1}{\sqrt{E}}$$

And the times of energy variance of two conditions are ,

$$2^{0.5}X_0 = \sqrt{2} \times 2.588 = 3.6$$

So, their energy resolution is

$$(\frac{\sigma_E}{E})_{aluminum} = \sqrt{3.6}(\frac{\sigma_E}{E}) = 1.9(\frac{\sigma_E}{E}) = 2.86\%/(E[GeV])^{1/4}$$