HW4. 선형대수학

1.

ⓐ 참

ⓑ 참

ⓒ 참

ⓓ 참

4.

ⓐ 1, 3행 바꾸는 기본행렬이므로

$$\begin{bmatrix} 0&0&1&0\\0&1&0&0\\1&0&0&0\\0&0&0&1 \end{bmatrix}^{-1} = \begin{bmatrix} 0&0&1&0\\0&1&0&0\\1&0&0&0\\0&0&0&1 \end{bmatrix}$$

ⓑ 역행렬은 $\begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&\frac{1}{4} \end{bmatrix}$

ⓒ $\begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&-3&0&1 \end{bmatrix}$

d. $\begin{bmatrix} 1&0&2\\0&1&0\\0&0&1 \end{bmatrix}$

5.

ⓐ $\begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&5&0\\0&0&0&1 \end{bmatrix}$ ⓑ $\begin{bmatrix} 1&0&0&0&0\\0&0&1&0&0\\0&1&0&0&0\\0&0&0&1&0\\0&0&0&0&1 \end{bmatrix}$

ⓒ $\begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&4&1&0\\0&0&0&1 \end{bmatrix}$

10

(a) $A = \begin{bmatrix} 1&-3\\4&-9 \end{bmatrix}$

$[A|I] \to [I|A^{-1}]$

$$\left[\begin{array}{cc|cc} 1&-3&1&0\\4&-9&0&1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1&-3&1&0\\0&3&-4&1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1&0&-3&1\\0&3&-4&1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1&0&-3&1\\0&1&-\frac{4}{3}&\frac{1}{3} \end{array}\right]$$

∴ 역행렬은 $\begin{bmatrix} -3&1\\-4/3&1/3 \end{bmatrix}$

(b) $\left[\begin{array}{cc|cc} 3&6&1&0\\4&7&0&1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 3&6&1&0\\1&1&-1&1 \end{array}\right]$

$$\sim \left[\begin{array}{cc|cc} 1&2&1/3&0\\1&1&-1&1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1&2&1/3&0\\0&-1&-4/3&1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1&0&-7/3&2\\0&-1&-4/3&1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1&0&-7/3&2\\0&1&4/3&-1 \end{array}\right]$$

∴ 역행렬은 $\begin{bmatrix} -7/3&2\\4/3&-1 \end{bmatrix}$

(c) $\left[\begin{array}{ccc|ccc} 1&0&-2&1&0&0\\-3&1&4&0&1&0\\2&-3&4&0&0&1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1&0&-2&1&0&0\\0&1&-2&3&1&0\\0&-3&8&-2&0&1 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|ccc} 1&0&-2&1&0&0\\0&1&-2&3&1&0\\0&0&2&7&3&1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1&0&0&8&3&1\\0&1&0&10&4&1\\0&0&2&7&3&1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1&0&0&8&3&1\\0&1&0&10&4&1\\0&0&1&7/2&3/2&1/2 \end{array}\right]$$

∴ 역행렬은 $\begin{bmatrix} 8&3&1\\10&4&1\\7/2&3/2&1/2 \end{bmatrix}$

(d)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -4 & -7 & 3 & 0 & 1 & 0 \\ -2 & -6 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 10 & 2 & 1 \end{array}\right]$$

∴ 역행렬이 존재하지 않는다.

$\boxed{17}$

$$\begin{cases} (a-\lambda)x_1 + bx_2 = 0 \\ cx_1 + (d-\lambda)x_2 = 0 \end{cases}$$

$$\cancel{c}x_1 = \frac{(\lambda-d)x_2}{c}$$

$$\Rightarrow \frac{(a-\lambda)(\lambda-d)x_2}{c} + bx_2 = 0$$

$$bx_2 = \frac{(\lambda-a)(\lambda-d)x_2}{c}$$

$$(cb - (\lambda-a)(\lambda-d))x_2 = 0$$

$$(cb - \lambda^2 + a\lambda + d\lambda - ad)x_2 = 0$$

이때 $x_2 \neq 0$이려면

$\boxed{\therefore \lambda^2 - (a+d)\lambda + ad - bc = 0 \text{ 이어야 한다}}$

$\boxed{19.}$

ⓐ $A = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

$$E_1A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} = u$$

$$A = E^{-1}u = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$

ⓑ $A = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$ $E_1 = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_1A = \begin{bmatrix} 1 & 3 & 1 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_2E_1A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 4 & 9 & 2 \end{bmatrix}$ $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

$E_3E_2E_1A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{bmatrix}$ $E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

$$E_4E_3E_2E_1A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix} = u$$

$$A = \begin{bmatrix} 2 & \cancel{0} & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} u$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

200
-3

(c)

$$A=\begin{bmatrix}1&1&-1\\6&2&2\\-3&4&1\end{bmatrix} \quad E_1=\begin{bmatrix}1&0&0\\-6&1&0\\0&0&1\end{bmatrix}$$

$$E_1A=\begin{bmatrix}1&1&-1\\0&-4&8\\-3&4&1\end{bmatrix} \quad E_2\begin{bmatrix}1&0&0\\0&1&0\\3&0&1\end{bmatrix}$$

$$E_2E_1A=\begin{bmatrix}1&1&-1\\0&-4&8\\0&7&-2\end{bmatrix} \quad E_3=\begin{bmatrix}1&0&0\\0&1&0\\0&\frac{7}{4}&1\end{bmatrix}$$

$$E_3E_2E_1A=\begin{bmatrix}1&1&-1\\0&-4&8\\0&0&12\end{bmatrix}=U$$

$$A=\begin{bmatrix}1&0&0\\6&1&0\\-3&-\frac{7}{4}&1\end{bmatrix}\begin{bmatrix}1&1&-1\\0&-4&8\\0&0&12\end{bmatrix}$$

(d)

$$A=\begin{bmatrix}2&-1&3\\4&2&1\\-6&-1&2\end{bmatrix} \quad E_1=\begin{bmatrix}1&0&0\\-2&1&0\\0&0&1\end{bmatrix}$$

$$E_1A=\begin{bmatrix}2&-1&3\\0&4&-5\\-6&-1&2\end{bmatrix} \quad E_2=\begin{bmatrix}1&0&0\\0&1&0\\3&0&1\end{bmatrix}$$

$$E_2E_1A=\begin{bmatrix}2&-1&3\\0&4&-5\\0&-4&11\end{bmatrix} \quad E_3=\begin{bmatrix}1&0&0\\0&1&0\\0&1&1\end{bmatrix}$$

$$E_3E_2E_1A=\begin{bmatrix}2&-1&3\\0&4&-5\\0&0&6\end{bmatrix}$$

$$A=\begin{bmatrix}1&0&0\\2&1&0\\-3&-1&1\end{bmatrix}\begin{bmatrix}2&-1&3\\0&4&-5\\0&0&6\end{bmatrix}$$

$\boxed{20}$

(a)

$$\begin{bmatrix}1&-1&0\\-1&2&-1\\0&-1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}2\\6\\1\end{bmatrix}$$

$$A=\begin{bmatrix}1&-1&0\\-1&2&-1\\0&-1&2\end{bmatrix} \quad E_1=\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}$$

$$E_1A=\begin{bmatrix}1&-1&0\\0&1&-1\\0&-1&2\end{bmatrix} \quad E_2=\begin{bmatrix}1&0&0\\0&1&0\\0&1&1\end{bmatrix}$$

$$E_2E_1A=\begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}$$

$$A=\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}$$

$=Lu$

$Ax=b$

$Ly=b \quad y=ux$

$$\left[\begin{array}{ccc|c}1&0&0&2\\-1&1&0&0\\0&-1&1&1\end{array}\right]\sim\left[\begin{array}{ccc|c}1&0&0&2\\0&1&0&2\\0&0&1&3\end{array}\right]$$

$$y=\begin{bmatrix}2\\2\\3\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&-1&0&2\\0&1&-1&2\\0&0&1&3\end{array}\right]\sim\left[\begin{array}{ccc|c}1&0&0&7\\0&1&0&5\\0&0&1&3\end{array}\right]$$

$x_1=7,\ x_2=5,\ x_3=3$

(b)

$$A=\begin{bmatrix}1&2&1\\2&3&3\\-3&-10&2\end{bmatrix}\quad E_1=\begin{bmatrix}1&0&0\\-2&1&0\\3&0&1\end{bmatrix}$$

$$E_1A=\begin{bmatrix}1&2&1\\0&-1&1\\0&-4&5\end{bmatrix}\quad E_2=\begin{bmatrix}1&0&0\\0&1&0\\0&-4&1\end{bmatrix}$$

$$E_2E_1A=\begin{bmatrix}1&2&1\\0&-1&1\\0&0&1\end{bmatrix}$$

$$A=\begin{bmatrix}1&0&0\\+2&1&0\\-3&4&1\end{bmatrix}\begin{bmatrix}1&2&1\\0&-1&1\\0&0&1\end{bmatrix}$$

$LUx=b$

$Ly=b \quad y=Ux$

$$\left[\begin{array}{ccc|c}1&0&0&2\\2&1&0&1\\-3&4&1&0\end{array}\right]\sim\left[\begin{array}{ccc|c}1&0&0&2\\0&1&0&-3\\0&4&1&6\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&0&0&2\\0&1&0&-3\\0&0&1&+18\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1&2&1&2\\0&-1&1&-3\\0&0&1&+18\end{array}\right]\sim\left[\begin{array}{ccc|c}1&2&0&-16\\0&-1&0&-21\\0&0&1&+18\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&0&0&-58\\0&-1&0&-21\\0&0&1&18\end{array}\right]\sim\left[\begin{array}{ccc|c}1&0&0&-58\\0&1&0&21\\0&0&1&18\end{array}\right]$$

$x_1=-58,\ x_2=21,\ x_3=18$

23

(c)

$$M=\left[\begin{array}{cc|cc}0&5&2&0\\-9&-10&-4&4\\\hline 1&2&2&4\\3&5&4&6\end{array}\right]=\begin{bmatrix}A&B\\C&D\end{bmatrix}$$

$$D^{-1}=\frac{1}{-4}\begin{bmatrix}6&-4\\-4&2\end{bmatrix}=\begin{bmatrix}-\frac{3}{2}&1\\1&-\frac{1}{2}\end{bmatrix}$$

$$BD^{-1}=\begin{bmatrix}-3&2\\10&-6\end{bmatrix}$$

$$A-BD^{-1}C=\begin{bmatrix}0&5\\-9&-10\end{bmatrix}-\begin{bmatrix}3&4\\-8&-10\end{bmatrix}$$

$$=\begin{bmatrix}-3&1\\-1&0\end{bmatrix}$$

$$(A-BD^{-1}C)^{-1}=+\begin{bmatrix}0&-1\\1&-3\end{bmatrix}$$

$=$

$$D^{-1}C=\begin{bmatrix}-\frac{3}{2}&1\\1&-\frac{1}{2}\end{bmatrix}\begin{bmatrix}1&2\\3&5\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{3}{2}+3&-3+5\\1-\frac{3}{2}&2-\frac{5}{2}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{3}{2}&2\\-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} (A-BD^{-1}C)^{-1} & -(A-BD^{-1}C)^{-1}BD^{-1} \\ -D^{-1}C(A-BD^{-1}C)^{-1} & D^{-1}+D^{-1}C(A-BD^{-1}C)^{-1}BD^{-1} \end{bmatrix}$$

$$(A-BD^{-1}C)^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & -3 \end{bmatrix}$$

$$-(A-BD^{-1}C)^{-1}BD^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 10 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -6 \\ 33 & -20 \end{bmatrix}$$

$$-D^{-1}C(A-BD^{-1}C)^{-1} = \begin{bmatrix} -\frac{3}{2} & -2 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & \frac{3}{2}+6 \\ \frac{1}{2} & -2 \end{bmatrix} = \begin{bmatrix} -2 & \frac{15}{2} \\ \frac{1}{2} & -2 \end{bmatrix}$$

$$D^{-1}+D^{-1}C(A-BD^{-1}C)^{-1}BD^{-1}$$

$$= \begin{bmatrix} -\frac{3}{2} & 1 \\ 1 & -\frac{1}{2} \end{bmatrix} + \begin{bmatrix} +2 & -\frac{15}{2} \\ -\frac{1}{2} & +2 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 10 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{2} & 1 \\ 1 & -\frac{1}{2} \end{bmatrix} - \begin{bmatrix} 6+75 & -4-45 \\ -\frac{3}{2}-20 & 1+12 \end{bmatrix}$$

$$= \begin{bmatrix} -82.5 & +50 \\ 22.5 & -13.5 \end{bmatrix}$$

$$\therefore M^{-1} = \begin{bmatrix} 0 & -1 & 10 & -6 \\ 1 & -3 & 33 & -20 \\ -2 & 7.5 & -82.5 & 50 \\ 0.5 & -2 & 22.5 & -13.5 \end{bmatrix}$$

```python
import numpy as np

# 행렬 A를 출력하는 함수
def pprint(msg, A):
    print("---", msg, "---")
    (n,m) = A.shape
    for i in range(0, n):
        line = ""
        for j in range(0, m):
            line += "{0:.2f}".format(A[i,j]) + "₩t"
        print(line)
    print("")

A = np.array([[1., 2.], [3., 4.]])
pprint("A", A)

Ainv1 = np.linalg.matrix_power(A, -1) # matrix_power( )를 사용한 역행렬 A-1 계산
pprint("linalg.matrix_power(A, -1) => Ainv1", Ainv1)

Ainv2 = np.linalg.inv(A) # inv( )를 사용한 역행렬 A-1 계산
pprint("np.linalg.inv(A) => Ainv2", Ainv2)

pprint("A*Ainv1", np.matmul(A, Ainv1)) # 행렬 A와 역행렬 A-1의 곱
pprint("A*Ainv2", np.matmul(A, Ainv2)) # 행렬 A와 역행렬 A-1의 곱

B = np.random.rand(3,3) # 난수를 이용한 3x3 행렬 B 생성
pprint("B =", B)
Binv = np.linalg.inv(B) # 역행렬 B-1 계산
pprint("Binv =", Binv)
pprint("B*Binv =", np.matmul(B, Binv)) # 행렬 B와 역행렬 B-1의 곱

# CX = D의 해 계산
C = np.array([[5, 3, 2, 1], [6, 2, 4, 5], [7, 4, 1, 3], [4, 3, 5, 2]])
D = np.array([[4], [2], [5], [1]])
x = np.matmul(np.linalg.inv(C), D)
pprint("x", x) # 해 x 출력
pprint("C*x", np.matmul(C, x)) # C*x의 결과가 D와 같은지 확인
```

```
--- A ---
1.00    2.00
3.00    4.00

--- linalg.matrix_power(A, -1) => Ainv1 ---
-2.00   1.00
1.50    -0.50

--- np.linalg.inv(A) => Ainv2 ---
-2.00   1.00
1.50    -0.50

--- A*Ainv1 ---
1.00    0.00
0.00    1.00

--- A*Ainv2 ---
1.00    0.00
0.00    1.00

--- B = ---
0.08    0.40    0.40
0.14    0.15    0.63
0.81    0.83    0.94

--- Binv = ---
-3.32   -0.31   1.63
3.29    -2.16   0.03
-0.07   2.18    -0.36

--- B*Binv = ---
1.00    0.00    0.00
-0.00   1.00    0.00
0.00    -0.00   1.00

--- x ---
1.31
-0.38
-0.31
-0.77

--- C*x ---
4.00
2.00
5.00
1.00
```

```python
import numpy as np

def pprint(msg, A):
    print("---", msg, "---")
    (n,m) = A.shape
    for i in range(0, n):
        line = ""
        for j in range(0, m):
            line += "{0:.2f}".format(A[i,j]) + "₩t"
        print(line)
    print("")

#LU 분해 함수
def LU(A):
    (n,m) = A.shape
    L = np.zeros((n,n)) # 행렬 L 초기화
    U = np.zeros((n,n)) # 행렬 U 초기화

    # 행렬 L과 U 계산
    for i in range(0, n):
        for j in range(i, n):
            U[i, j] = A[i, j]
            for k in range(0, i):
                U[i, j] = U[i, j] - L[i, k]*U[k, j]
        L[i,i] = 1
        if i < n-1:
            p = i + 1
            for j in range(0,p):
                L[p, j] = A[p, j]
                for k in range(0, j):
                    L[p, j] = L[p, j] - L[p, k]*U[k, j]
                L[p,j] = L[p,j]/U[j,j]
    return L, U
```

```python
A = np.array([[5, 3, 2, 1], [6, 2, 4, 5], [7, 4, 1, 3], [4, 3, 5, 2]])
b = np.array([[4], [2], [5], [1]])

# 행렬 A의 LU 분해
L, U = LU(A)
pprint("A", A)
pprint("L", L)
pprint("U", U)

# LU 분해를 이용한 Ax=b의 해 구하기
x = LUSolver(A,b)
pprint("x", x)
```

```
--- A ---
5.00    3.00    2.00    1.00
6.00    2.00    4.00    5.00
7.00    4.00    1.00    3.00
4.00    3.00    5.00    2.00

--- L ---
1.00    0.00    0.00    0.00
6.00    1.00    0.00    0.00
7.00    1.06    1.00    0.00
4.00    0.56    -1.15   1.00

--- U ---
5.00    3.00    2.00    1.00
0.00    -16.00  -8.00   -1.00
0.00    0.00    -4.50   -2.94
0.00    0.00    0.00    -4.81

--- x ---
-0.06
1.54
-0.38
0.46
```