NO.
year month day ( )

선형대수학

HW3

$\boxed{10}$

ⓐ 참

ⓑ 거짓 반례를 들면 $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \times \begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}$

$AB = \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix} = \begin{bmatrix}3 & 8\\ 4 & 18\end{bmatrix}$

$BA \quad \begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} = \begin{bmatrix}7 & 10\\ 10 & 14\end{bmatrix}$

$AB \neq BA$.

ⓒ 거짓 A가 0 행렬이고 $B \neq C$인 행렬이면

$AB = 0 = BA$지만 $B \neq C$다.

ⓓ 참.

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$AB = \begin{bmatrix}+2 & 3\\ -1 & 2\end{bmatrix}\begin{bmatrix}1 & 9\\ -3 & k\end{bmatrix} = \begin{bmatrix}-7 & 18+3k\\ -7 & -9+2k\end{bmatrix}$

$BA = \begin{bmatrix}1 & 9\\ -3 & k\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix} = \begin{bmatrix}-7 & 21\\ -6-k & -9+2k\end{bmatrix}$

$18 + (k \times 3) = 21$
$-6 - k = -7$
$-9 + 2k = -9 + 2k$

를 만족하는 k는 1이다. $\boxed{k=1}$

$\boxed{15}$ → 답지답에 오류가 있음.

$B = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$AB = \begin{bmatrix}3 & -6\\ -2 & 4\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \begin{bmatrix}3a-6c & 3b-6d\\ -2a+4c & -2b+4d\end{bmatrix}$

$3a - 6c = 0 \Rightarrow a = 2c$
$3b - 6d = 0 \Rightarrow b = 2d$
$-2a + 4c = 0 \Rightarrow a = 2c$
$-2b + 4d = 0 \Rightarrow b = 2d$

$BA = \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}3 & -6\\ -2 & 4\end{bmatrix} = \begin{bmatrix}3a-2b & -6a+4b\\ 3c-2d & -6c+4d\end{bmatrix}$

$3a = 2b \qquad b = \frac{3}{2}a, \ d = \frac{3}{4}a$
$3c = 2d \qquad c = \frac{2}{3}d = \frac{2}{3}\cdot\frac{3}{4}a = \frac{1}{2}a$

$\therefore B = \begin{bmatrix}a & \frac{3}{2}a\\ \frac{1}{2}a & \frac{3}{4}a\end{bmatrix}$의 형태면

$AB = BA = 0$ 를 만족한다

$(a \in \mathbb{R})$

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ⓐ 참.

ⓑ 참

ⓒ 참

ⓓ 거짓 - 역행렬의 유일성

A가 $n \times n$이고 가역적일때

$AB_1 = I_n \quad AB_2 = I_n$을

만족하는 B_1, B_2가 있다고 하자

$A^{-1}AB_1 = A^{-1}I_n, \ B_1 = A^{-1}$

$A^{-1}AB_2 = A^{-1}I_n, \ B_2 = A^{-1}$

$\therefore A^{-1} = B_1 = B_2$

Keun Young Co.

ⓔ 증명 $(AB)^{-1} = B^{-1}A^{-1}$ 이다

$AB(AB)^{-1} = I_n$

$A^{-1}AB(AB)^{-1} = A^{-1}I_n$

$B^{-1}A^{-1}AB(AB)^{-1} = B^{-1}A^{-1}I_n$

$(AB)^{-1} = B^{-1}A^{-1}$

17 풀이①

$A \times B = \begin{bmatrix} -3 & -11 \\ 1 & 17 \end{bmatrix}$

$A^{-1}AB = A^{-1}\begin{bmatrix} -3 & -11 \\ 1 & 17 \end{bmatrix}$

$B = A^{-1}\begin{bmatrix} -3 & -11 \\ 1 & 17 \end{bmatrix}$

이때 $A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} 5 & 3 \\ 3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-4}\begin{bmatrix} 5 & 3 \\ 3 & 1 \end{bmatrix}$

$B = -\frac{1}{4}\begin{bmatrix} 5 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -3 & -11 \\ 1 & 17 \end{bmatrix}$

$= -\frac{1}{4}\begin{bmatrix} -12 & -55+51 \\ -9+1 & -33+17 \end{bmatrix}$

$= -\frac{1}{4}\begin{bmatrix} -12 & -4 \\ -8 & -16 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

$\boxed{\therefore B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}}$

풀이②

$A^{-1} = ?$

$[A \mid I_n]$

$= \left[\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ -3 & 5 & 0 & 1 \end{array}\right]$

$= \left[\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ 0 & -4 & 3 & 1 \end{array}\right]$

$= \left[\begin{array}{cc|cc} 1 & -3 & 1 & 0 \\ 0 & 1 & -\frac{3}{4} & -\frac{1}{4} \end{array}\right]$

$= \left[\begin{array}{cc|cc} 1 & 0 & 1-\frac{9}{4} & -\frac{3}{4} \\ 0 & 1 & -\frac{3}{4} & -\frac{1}{4} \end{array}\right]$

$A^{-1} = \begin{bmatrix} -\frac{5}{4} & -\frac{3}{4} \\ -\frac{3}{4} & -\frac{1}{4} \end{bmatrix}$

$B = A^{-1} \times \begin{bmatrix} -3 & -11 \\ 1 & 17 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ ---> 풀이①과 같음

$\therefore B = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

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A는 2x2 행렬

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3a+2b \\ 3c+2d \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a+b \\ c+d \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$\begin{cases} 3a+2b=1 \\ a+b=3 \end{cases}$ $\begin{cases} 3c+2d=-1 \\ c+d=7 \end{cases}$

$\begin{cases} 3a+2b=1 \\ 2a+2b=6 \end{cases}$ $\begin{cases} 3c+2d=-1 \\ 2c+2d=14 \end{cases}$

$a=-5$
$b=8$

$c=-15$
$d=22$

$\begin{bmatrix} -5 & 8 \\ -15 & 22 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -9 \end{bmatrix}$ $\therefore A\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -9 \end{bmatrix}$

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ⓐ 참

ⓑ 거짓 행렬 A가 skewed 되면 $A = -A^T$ 이다

ⓒ 참 $A = \frac{1}{2}\underbrace{(A+A^T)}_{sym} + \frac{1}{2}\underbrace{(A-A^T)}_{skew}$

ⓓ 참 linear combination

ⓔ 참

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ⓐ $BA = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ -1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 12 & 4 & 0 \\ -3 & 6 & 20 \end{bmatrix}$

ⓑ $BC = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ -1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 10 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 42 & 0 & 0 \\ 1 & 4 & 20 \end{bmatrix}$

ⓒ $DE = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} 9 & 6 & 3 \\ 0 & -2 & 4 \\ 0 & 0 & 11 \end{bmatrix} = \begin{bmatrix} 18 & 4 & 33 \\ 0 & -6 & 56 \\ 0 & 0 & 77 \end{bmatrix}$

ⓓ $AB = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ -1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ -5 & 15 & 20 \end{bmatrix}$

$ABC = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ -5 & 15 & 20 \end{bmatrix}\begin{bmatrix} 10 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 30 & 0 & 0 \\ 84 & 0 & 0 \\ 5 & 20 & 100 \end{bmatrix}$

ⓔ $E^T = \begin{bmatrix} 9 & 0 & 0 \\ 6 & -2 & 0 \\ 3 & 4 & 11 \end{bmatrix}$

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$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$

$= \frac{1}{2}\left(\begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 2 \\ 6 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 5 & 3 \\ 3 & 2 & 4 \end{bmatrix}\right) + \frac{1}{2}\left(\begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 2 \\ 6 & 3 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 1 & 5 & 3 \\ 3 & 2 & 4 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 4 & 5 & 9 \\ 5 & 10 & 5 \\ 9 & 5 & 8 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -3 & -3 \\ 3 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 4 & 2 & 6 \\ 8 & 10 & 4 \\ 12 & 6 & 8 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 2 \\ 6 & 3 & 4 \end{bmatrix}$

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ⓐ If A is invertible & skew, Is A^{-1} a skew mtx?

$A = -A^T$

$A^{-1} = (-A^T)^{-1}$

$A^{-1} = (-A^{-1})^T$

$A^{-1} = -(A^{-1})^T$

$\therefore$ A^{-1}역시 반대칭 행렬이다

ⓑ $A = -A^T,\ B = -B^T$

$A+B = -A^T - B^T = -(A+B)^T$

$A-B = -A^T + B^T = -(A-B)^T$

$A = -A^T$; $A^T = -A^T$

$kA = k(-A^T) = -(kA)^T$

```python
import numpy as np

# 행렬 A를 출력하는 함수
def pprint(msg, A):
    print("---", msg, "---")
    (n,m) = A.shape
    for i in range(0, n):
        line = ""
        for j in range(0, m):
            line += "{0:.2f}".format(A[i,j]) + "\t"
        print(line)
    print("")

A = np.array([[3.,0.,0.], [0.,2.,0.],[0.,0.,5.,]])
B = np.array([[1.,0.,0.], [4.,2.,0.],[-1.,3.,4.,]])
C = np.array([[10.,0.,0.], [1.,0.,0.],[2.,1.,5.,]])
v = np.array([[10.], [20.],[30.]])

pprint("A+B", A+B) # 행렬의 합 A+B
pprint("A-B", A-B) # 행렬의 차 A-B

pprint("3*A ", 3*A) # 행렬의 스칼라배 3A
pprint("2*v ", 2*v) # 벡터의 스칼라배 2v

pprint("matmul(A,B)", np.matmul(A,B)) # 행렬의 곱 AB
pprint("matmul(A,C)", np.matmul(A,C)) # 행렬의 곱 AC
pprint("A*v", A*v) # 행렬과 벡터의 곱 Av

pprint("matrix_power(A, 2)", np.linalg.matrix_power(A, 2)) # 행렬의 거듭제곱 A2
pprint("matrix_power(A, 3)", np.linalg.matrix_power(A, 3)) # 행렬의 거듭제곱 A3

pprint("A*B", A*B) # 행렬의 성분별 곱셈 A*B
pprint("A/B", A/B) # 행렬의 성분별 나눗셈 A/B
pprint("A**2 == A*A", A**2) # 행렬의 성분별 거듭제곱 A**2

pprint("A.T", A.T) # 행렬의 전치 AT
pprint("v.T", v.T) # 벡터의 전치 vT

M = np.diag([1, 2, 3]) # 대각행렬 diag(1,2,3) 생성
pprint("diag(1,2,3) =", M)

D11 = np.array([[1, 2], [3, 4]])
D12 = np.array([[5], [6]])
D21 = np.array([[7, 7]])
D22 = np.array([[8]])
D = np.block([[D11, D12], [D21, D22]]) # 블록행렬 D 생성
pprint("block matrix", D)
```

```
--- A+B ---
4.00    0.00    0.00
4.00    4.00    0.00
-1.00   3.00    9.00

--- A-B ---
2.00    0.00    0.00
-4.00   0.00    0.00
1.00    -3.00   1.00

--- 3*A  ---
9.00    0.00    0.00
0.00    6.00    0.00
0.00    0.00    15.00

--- 2*v  ---
20.00
40.00
60.00

--- matmul(A,B) ---
3.00    0.00    0.00
8.00    4.00    0.00
-5.00   15.00   20.00

--- matmul(A,C) ---
30.00   0.00    0.00
2.00    0.00    0.00
10.00   5.00    25.00

--- A*v ---
30.00   0.00    0.00
0.00    40.00   0.00
0.00    0.00    150.00

--- matrix_power(A, 2) ---
9.00    0.00    0.00
0.00    4.00    0.00
0.00    0.00    25.00
```

```
--- matrix_power(A, 3) ---
27.00   0.00    0.00
0.00    8.00    0.00
0.00    0.00    125.00

--- A*B ---
3.00    0.00    0.00
0.00    4.00    0.00
-0.00   0.00    20.00

--- A/B ---
3.00    nan     nan
0.00    1.00    nan
-0.00   0.00    1.25

--- A**2 == A*A ---
9.00    0.00    0.00
0.00    4.00    0.00
0.00    0.00    25.00

--- A.T ---
3.00    0.00    0.00
0.00    2.00    0.00
0.00    0.00    5.00

--- v.T ---
10.00   20.00   30.00

--- diag(1,2,3) = ---
1.00    0.00    0.00
0.00    2.00    0.00
0.00    0.00    3.00

--- block matrix ---
1.00    2.00    5.00
3.00    4.00    6.00
7.00    7.00    8.00
```