

선형대수학

HW2

p18.

[7.]

(a) 참

(b) 거짓, 기약행사대리꼴 조건이다.

(c) 거짓, 0으로만 구성된 행이 있을 수 있다

(d) 거짓, 이유 ① 모든 행이 맨 오른쪽 성분만 0이 아닌 경우, 불가능하다

이유 ② 3로만 구성된 행을

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ 행렬은 역행렬 존재}$$

이 경우 3행은 맨 오른쪽 성분만

0이 아닌 값으로 나머지는 0인 행이다.

하지만 위의 행렬은

unique solution이다

(e) 참

[11]

②번, ① \Rightarrow 4개의 외적 값이
이상이② \Rightarrow 맞다③ \Rightarrow 2행이 3행과 마주라면
true④ \Rightarrow 2행 pivot이 1행의
pivot보다 작을 수 있다

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[12]

(a)

$$\begin{bmatrix} 1 & -5 & 4 & | & -3 \\ 2 & -7 & 3 & | & -2 \\ -2 & 1 & 7 & | & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -5 & 4 & | & -3 \\ 0 & -6 & 10 & | & -3 \\ -2 & 1 & 7 & | & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & | & -3 \\ 0 & -6 & 10 & | & -3 \\ 0 & -9 & 15 & | & -7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -5 & 4 & | & -3 \\ 0 & -6 & 10 & | & -3 \\ 0 & -3 & 5 & | & -\frac{7}{2} \end{bmatrix} \sim \begin{bmatrix} 1 & -5 & 4 & | & -3 \\ 0 & -3 & 5 & | & -\frac{7}{2} \\ 0 & 0 & 0 & | & -\frac{1}{2} + \frac{3}{2} \end{bmatrix}$$

 \Rightarrow 3행때문에 불능

(c)

$$\begin{bmatrix} 0 & 1 & 5 & | & -4 \\ 1 & 4 & 3 & | & -2 \\ 2 & 7 & 1 & | & -2 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 5 & | & -4 \\ 1 & 4 & 3 & | & -2 \\ 2 & 7 & 1 & | & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & 3 & | & -2 \\ 0 & 1 & 5 & | & -4 \\ 0 & -1 & -5 & | & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 3 & | & -2 \\ 0 & 1 & 5 & | & -4 \\ 0 & 0 & 0 & | & -2 \end{bmatrix}$$

 \Rightarrow 3행때문에 불능

(e)

$$\begin{bmatrix} 1 & -1 & 0 & 0 & | & 5 \\ 0 & 1 & -4 & 1 & | & 0 \\ -1 & 6 & 1 & 5 & | & 3 \\ 0 & -1 & 5 & 4 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -6 & 0 & 0 & | & 5 \\ 0 & 0 & 1 & 5 & | & 0 \\ -1 & 6 & 1 & 5 & | & 3 \\ 0 & -1 & 5 & 4 & | & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -6 & 0 & 0 & | & 5 \\ 0 & 0 & 1 & 5 & | & 0 \\ 0 & 0 & 1 & 5 & | & 3 \\ 0 & -1 & 5 & 4 & | & 0 \end{bmatrix}$$

 \Rightarrow 2행, 3행 보충 \therefore 불능

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$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 0 & b \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2-b \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -5 & -b \end{array} \right]$$

a) 불가능하면 $a \neq 5, b = 0$

b) 가능하면 $a = 5, b = 0$

$$\sim \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 1 & -\frac{17}{24} \\ 0 & 0 & 2 & 0 & 2 & 7 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 6 & 12 & -4 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & 0 & 1 & \frac{7}{2} \\ 0 & 0 & 0 & 1 & 2 & -\frac{2}{3} \\ 0 & 0 & 0 & 0 & 1 & -\frac{17}{24} \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & 0 & 0 & \frac{7}{2} + \frac{17}{24} \\ 0 & 0 & 0 & 1 & 0 & -\frac{2}{3} + \frac{17}{12} \\ 0 & 0 & 0 & 0 & 1 & -\frac{17}{24} \end{array} \right]$$

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$$\left[\begin{array}{ccccc|c} 1 & -2 & 4 & -8 & 16 & 3 \\ 1 & -1 & 1 & -1 & 1 & 5 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 4 \\ 1 & 2 & 4 & 8 & 16 & 10 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc|c} 0 & -2 & 4 & -8 & 16 & 2 \\ 0 & -1 & 1 & -1 & 1 & 4 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 3 \\ 0 & 2 & 4 & 8 & 16 & 9 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc|c} 0 & 0 & 8 & 0 & 24 & 11 \\ 0 & 0 & 2 & 0 & 2 & 7 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 3 \\ 0 & 2 & 4 & 8 & 16 & 9 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 24 & -17 \\ 0 & 0 & 2 & 0 & 2 & 7 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 2 & 6 & 14 & 3 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & 0 & 0 & \frac{7}{2} \\ 0 & 0 & 0 & 1 & 0 & -\frac{2}{3} \\ 0 & 0 & 0 & 0 & 1 & -\frac{17}{24} \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 3 - \frac{10}{24} - \frac{3}{2} + \frac{17}{24} \\ 0 & 0 & 1 & 0 & 0 & \frac{10}{24} \\ 0 & 0 & 0 & 1 & 0 & \frac{3}{4} \\ 0 & 0 & 0 & 0 & 1 & -\frac{17}{24} \end{array} \right]$$

$$\therefore a_0 = 1$$

$$a_1 = -\frac{5}{4}$$

$$a_2 = \frac{10}{24}$$

$$a_3 = \frac{3}{4}$$

$$a_4 = -\frac{17}{24}$$

$$p(x) = 1 - \frac{5}{4}x + \frac{10}{24}x^2 + \frac{3}{4}x^3 - \frac{17}{24}x^4$$

def Functions

```
import numpy as np

def pprint(msg, A):
    print("----", (msg), "----")
    (n,m) = A.shape
    for i in range(0,n):
        line = ""
        for j in range(0,m):
            line += "{0:.2f}".format(A[i,j])+"Wt"
            if j==n-1:
                line += " "
        print(line)
    print("")

def gauss(A):
    (n,m) = A.shape

    for i in range(0,n):
        maxE=abs(A[i,i]); maxR = i
        for k in range(i+1,n):
            if abs(A[k,i]) > maxE:
                maxE = abs(A[k,i])
                maxR = k

        for k in range(i,m):
            tmp = A[maxR,k]
            A[maxR,k] = A[i,k]
            A[i,k] = tmp

        piv = A[i,i]
        for k in range(i,m):
            A[i,k] = A[i,k]/piv

        for k in range(0,n):
            if k != i:
                o = A[k,i]/A[i,i]
                for j in range(i,m):
                    if i==j:
                        A[k,j] = 0
                    else:
                        A[k,j] = A[k,j] - o * A[i,j]

        pprint(str(i+1)+"번째 반복", A)

    x = np.zeros(n)
    for i in range(0,n):
        x[i] = A[i,n]
    return x
```

P79, (b)

```
A = np.array([[3.,6.,-3.,6.],[2.,7.,4.,28.],[2.,-6.,4.,2.]])

pprint("주어진 문제", A)
x = gauss(A)

(n,m) = A.shape; line = "해:₩t"
for i in range(0,n):
    line += "{0: .2f}".format(x[i]) + "₩t"
print(line)
```

--- 주어진 문제 ---

3.00	6.00	-3.00		6.00
2.00	7.00	4.00		28.00
2.00	-6.00	4.00		2.00

--- 1번째 반복 ---

1.00	2.00	-1.00		2.00
0.00	3.00	6.00		24.00
0.00	-10.00	6.00		-2.00

--- 2번째 반복 ---

1.00	0.00	0.20		1.60
0.00	1.00	-0.60		0.20
0.00	0.00	7.80		23.40

--- 3번째 반복 ---

1.00	0.00	0.00		1.00
0.00	1.00	0.00		2.00
0.00	0.00	1.00		3.00

해: 1.00 2.00 3.00

P79, (d)

```
A = np.array([[2.,0.,-6.,-8.],[0.,1.,2.,3.],[3.,6.,-2.,-4.]])

pprint("주어진 문제", A)
x = gauss(A)

(n,m) = A.shape; line = "해: "
for i in range(0,n):
    line += "{0: .2f}".format(x[i]) + " "
print(line)
```

--- 주어진 문제 ---

2.00	0.00	-6.00		-8.00
0.00	1.00	2.00		3.00
3.00	6.00	-2.00		-4.00

--- 1번째 반복 ---

1.00	2.00	-0.67		-1.33
0.00	1.00	2.00		3.00
0.00	-4.00	-4.67		-5.33

--- 2번째 반복 ---

1.00	0.00	-3.00		-4.00
0.00	1.00	1.17		1.33
0.00	0.00	0.83		1.67

--- 3번째 반복 ---

1.00	0.00	0.00		2.00
0.00	1.00	0.00		-1.00
0.00	0.00	1.00		2.00

해: 2.00 -1.00 2.00

P79, (f)

```
A = np.array([[2.,0.,0.,-4.,-10.],[0.,3.,3.,0.,0.],[0.,0.,1.,4.,-1.],[-3.,2.,3.,1.,5.]])

pprint("주어진 문제", A)
x = gauss(A)

(n,m) = A.shape; line = "해:₩t"
for i in range(0,n):
    line += "{0: .2f}".format(x[i]) + "₩t"
print(line)
```

--- 주어진 문제 ---

2.00	0.00	0.00	-4.00		-10.00
0.00	3.00	3.00	0.00		0.00
0.00	0.00	1.00	4.00		-1.00
-3.00	2.00	3.00	1.00		5.00

--- 1번째 반복 ---

1.00	-0.67	-1.00	-0.33		-1.67
0.00	3.00	3.00	0.00		0.00
0.00	0.00	1.00	4.00		-1.00
0.00	1.33	2.00	-3.33		-6.67

--- 2번째 반복 ---

1.00	0.00	-0.33	-0.33		-1.67
0.00	1.00	1.00	0.00		0.00
0.00	0.00	1.00	4.00		-1.00
0.00	0.00	0.67	-3.33		-6.67

--- 3번째 반복 ---

1.00	0.00	0.00	1.00		-2.00
0.00	1.00	0.00	-4.00		1.00
0.00	0.00	1.00	4.00		-1.00
0.00	0.00	0.00	-6.00		-6.00

--- 4번째 반복 ---

1.00	0.00	0.00	0.00		-3.00
0.00	1.00	0.00	0.00		5.00
0.00	0.00	1.00	0.00		-5.00
0.00	0.00	0.00	1.00		1.00

해: -3.00 5.00 -5.00 1.00