Family name, comma, personal names:_____

Your student ID:_____

Your signature (as on UTorID): _____

UNIVERSITY OF TORONTO
Faculty of Arts and Science

ACT230H1F 20129 TERM TEST 1

Duration – 50 Minutes

Aids:
All non-programmable calculators allowed.
Scrap paper provided by proctors if needed.

Instructor: Keith Sharp PhD FCIA FSA CFA

NOTES:
1. This question book:
    a. Term test has 11 questions including an ungraded privacy version identifier question.
    b. It's OK to write on the question book even if 'don't write on book' gets stamped on it
2. This is a closed book exam.
3. Scantrons and bubbling time:
    a. make sure you've indicated your letter answers on the Scantron before time's up
    b. for finals, exam president probably has Registrar's instructions: no extra time for bubbles
    c. pencil preferred-you can erase it-but ink also works.
4. Blanks:
    a. 10 points correct, two if blank, zero points if wrong
    b. So expectation if you guess is the same as leaving a blank.
5. Privacy enhancement:
    a. Please stay in your seats and don't talk till all materials have been collected.
    b. Photo ID on desk during exam-we'll be making a map of where you sit
6. Name and student ID and signature please on
    a. this question paper
    b. Scantron, with, for ID and name, bubbles and letters, for eyes and computers.
7. Good luck!

1.*(Assignment)* Eric receives 12,000 from a life insurance policy. He uses the fund to purchase two different annuities, each costing 6,000. The first annuity is a 24-year annuity-immediate paying 4Y per year. The second annuity is an 8-year annuity-immediate paying 9Y per year. Both annuities are based on an annual effective interest rate of i, i>0. Determine i.

(A) Less than 4.000%
(B) 4.000% but less than 4.300%
(C) 4.300% but less than 4.600%
(D) 4.600% but less than 4.900%
(E) 4.900% or more

(B) Solution

$6000 = 4Y \, a_{\overline{24}|}$

$6000 = 9Y \, a_{\overline{8}|}$

Take quotient

$(1-v^{24})/(1-v^8) = 2.25$

$1 + v^8 + v^{16} = 2.25$

$v^8 = [-1 + root(1+5)]/2 = 0.7247449$

$i = 0.0410626$

2. *(Assignment)* You are given the following information about a guaranteed investment contract.

Initial deposit to guaranteed investment contract: $100,000

Purchase date: 1/1/88

Maturity date: 1/1/98

Interest credited on initial deposit: 9% per year, reinvested at the end of each year

Interest credited on additions: 6% per year, reinvested at the end of each year

In what range is the accumulated value of the contract as of 1/1/98?

(A) Less than $180,000

(B) $180,000 but less than $195,000

(C) $195,000 but less than $210,000

(D) $210,000 but less than $225,000

(E) $225,000 or more

(D)Solution

$100,000 + 9,000 \, s_{\overline{10}|} \, (at \, 6\%)$

$=100,000 + 9,000 * [(1.06^{10} - 1)/0.06]$

$=100,000 + 9,000 * 13.18079$

$=\$218,627$

3. *(Assignment)* Jennifer deposits 1000 into a bank account. The bank credits interest at a nominal annual rate of i convertible semi-annually for the first 7 years and a nominal annual rate of 2i convertible quarterly for all years thereafter. The accumulated amount in the account at the end of 5 years is X. The accumulated amount in the account at the end of 9.5 years is 1980. Calculate X to the nearest dollar.

(A) 1201
(B) 1226
(C) 1251
(D) 1329
(E) The correct answer is not given by (A), (B), (C) or (D)

(D) Solution:

At t=9.5,

$$1,000 * (1+i/2)^{14} * (1+2i/4)^{(9.5-7)*4} = 1,980$$

$$1,000 * (1+i/2)^{14} * (1+i/2)^{10} = 1,980$$

Hence $(1+i/2) = 1.980^{1/24}$

At t=5:

$$X=1,000 * (1+i/2)^{10} =1,000 * 1.980^{10/24} = 1,329.26$$

4. *(Assignment)* At an annual effective interest rate of i, i>0, the following are all equal:

(i) the present value of 10,000 at the end of 6 years:
(ii) the sum of the present values of 6,000 at the end of year t and 46,000 at the of year 2t; and
(iii) 5,000 immediately.

Calculate the present value of a payment of 4,000 at the end of year t+3 using the same annual effective interest rate.

(A) Less than 700.000
(B) 700.000 but less than 730.000
(C) 730.000 but less than 760.000
(D) 760.000 but less than 790.000
(E) 790.000 or more

(D, and E also accepted which results from rounding t) Solution

$10,000*v^6 = 6,000*v^t + 46,000*v^{2t} = 5,000$

Hence $v^6 = 0.5$

$56v^{2t} + 6v^t - 5 = 0$

$v^t = [-6 \quad +- \quad (36+4*46*5)^{0.5}]/(2*46)$

$= [-6 \quad +- \quad (36+4*46*5)^{0.5}]/(2*46)$

Taking positive answer, $v^t = 0.2708641$

Hence $4,000 \, v^{t+3} = 4,000 * 0.2708641 * 0.5^{0.5} = 766.11$

5. You borrow $20,000 to buy a car and agree to repay the loan in 48 equal monthly installments. The first is paid a month after the loan is made. The interest rate is 6% per annum compounded monthly. Calculate the amount of each installment.

(A) Less than $465.000
(B) $465.000 but less than $470.000
(C) $470.000 but less than $475.000
(D) $475.000 but less than $480.000
(E) $480.000 or more

(B) Solution:

$j^{(12)} = 0.06$

$j^{(12)}/12 = 0.005$

$L = K\, a_{\overline{48}|}\, (at\ 0.5\%)$

$20,000 = K * 42.580318$

$K = \$469.70\ per\ month$

6.On January 1, 2015 you deposit $10,000 in Blue Bank, which pays 5% interest effective. Each January 1, 2016 through 2025, you remove the interest earned in the previous year and deposit it with Red Bank at 6% per annum interest effective. Calculate your total wealth on January 1, 2025, giving the answer to the nearest $10

(A) $16,560
(B) $16,570
(C) $16,580
(D) $16,590
(E) The correct answer is not given by (A), (B), (C) or (D)

(D) Solution

$10,000 +\ 0.05*10,000*s_{\overline{10}|}\, (at\ 6\%)= 10,000 + 500*13.18079 = \$16,590.397$

7.On a day 350 years ago, the Dutch bought Manhattan Island for $30. The land is now worth $3,000 billion. Inflation has averaged 4 percent effective per year. Calculate the real rate of return effective per year. (Hint: if your calculator has trouble with big numbers, try splitting a big number into two factors).

(A)   Less than 3.000%
(B)   3.000% but less than 3.500%
(C)   3.500% but less than 4.000%
(D)   4.000% but less than 4.500%
(E)   4.500% or more

(B)Solution
$1+i = (A(350)/A(0))^{1/350} = 1.075049$

$i^{real} = (1+i)/(1+r)-1 = 1.075049/1.04 -1 = 0.0337$

Surprisingly, not such a great investment, though maybe inflation averaged less that 4% And tax?

8. You want to have a big birthday celebration some year. The current price per person at your chosen hall is $40.00 but it inflates 4% per year. Currently your bank account is big enough to invite only 450 people. You pay 40% income tax on interest income, every year, and your bank account pays 9% per annum effective. You had your 21st birthday today. Which birthday will be the first at which you can afford to invite 500 people and still have a little money to spare?

(A) 28 th birthday
(B) 29 th birthday
(C) 30 th birthday
(D) 31 st birthday
(E) The correct answer is not given by (A), (B), (C) or (D)

(B) Solution:

(1+Real rate of interest, after tax)=

$(1 + 0.09*(1-0.4))/(1.04) = 1.054/1.04 = 1.0134615$

$500/450 = (1.054/1.04)^t$

$t = \ln(500/450)/(\ln(1.054/1.04)) = 7.879$

So, to have money left over, $21+8=29^{th}$ birthday

9. You deposit A(0)=$5,000 into a bank. For the first n=3 years it pays interest of 7% per annum effective compound. For the last ten months it pays interest of 2% per annum simple. Assume, as usual, that the interest stays in the account to accumulate.

Calculate the accumulated amount after 3 years 10 months.

(A) Less than $6,200.000
(B) $6,200.000 but less than $6,300.000
(C) $6,300.000 but less than $6,400.000
(D) $6,400.000 but less than $6,500.000
(E) $6,500.000 or more

(B)Solution

$A(3\ 4/12) = 5,000 * 1.07^3 * (1+ 0.02*10/12) = 6,227.30$

10. At an effective annual compound interest rate of i>0, it is found that an investment doubles in a years, triples in b years, and $1 grows to $5 in c years.

To what amount to the nearest dollar does $10 grow in 3a+4b years?.

(A) $5,680
(B) $6,080
(C) $6,480
(D) $6,880
(E) The correct answer is not given by (A), (B), (C) or (D)

(C) Solution:

$10*2^3*3^4 = 6480$

11. IMPORTANT Please mark this question on your Scantron with G. This is the version-identifier question. If you fail to bubble this question with G, correct reporting of your grade will be delayed since the software won't know the question order of your test.