

Homework 1

<One more Puzzle with SMT Solver>

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Description:

- Constraints:** I found about 4 constraints to solve this puzzle.
 - Every cell in the puzzle should have a value between 1 and 9.
 - The sum of Black-colored cells in a single column should be value of the label at its column.
 - The sum of White-colored cells in a single row should be value of the label at its row.
 - Every cell should have its value at its given puzzle.

2. Constraints as logical formula:

I uploaded the picture below at my github repository for better reading. In, this write-up, I will explain about Q1~5, The conditions that are need to be satisfied.

- Q1 is corresponding to the constraint "a."
- Q2 is for making $B(i,j)$ to have either value of 1 or 0
- Q3 and Q4 are corresponding to the constraint "b." and "c."
- Q5 is corresponding to the constraint "d."
- Constraints "a", "b", "c" and "d" are stated at Description - 1.

3. Demonstration

20	23	30	29	34	6	9	21	19	
8	2	1	8	1	3	5	7	6	18
9	1	4	2	5	6	3	1	7	28
3	5	1	4	9	1	3	9	1	8
8	6	6	3	5	1	1	4	1	4
8	6	6	2	6	8	3	3	9	31
8	7	8	8	4	5	2	1	1	18
4	8	3	5	5	2	1	2	8	24
1	2	8	8	8	3	7	2	7	22
8	3	9	5	9	2	1	4	9	35

<Test 1. Example given in the "homework+1.pdf">

1	6	4	7	0	
1	3	5	6	9	20
2	3	4	7	1	3

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1 1 0 0 0

0 1 1 1 0

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<Test2. test case made by myself to check whether the program can solve rectangular puzzle>

Discussion:

- While solving this puzzle, I found that there is a puzzle called "pixel puzzle" which is to fill pixels with the information give on the top side and left-most side. I think I can solve this puzzle by applying this puzzle's solution.
- I learnt that using multiplication in the formula may result Z3 Solver cannot find the solution as multiplication makes the solution non-linear. I had an issue when I use multiply sign in the formula.

The Github Repository:

<https://github.com/junhyung9985/NumberCros/blob/master/README.md>

"(a)"

$$Q_1: \bigwedge_{z=2}^M \bigwedge_{j=1}^{n-1} (1 \leq A(z,j) \leq 9)$$

→ Specifying the range of available values that $A(z,j)$ and $B(z,j)$ can hold.

"(b)"

$$Q_2: \bigwedge_{z=2}^M \bigwedge_{j=1}^{n-1} ((B(z,j)=0) \vee (B(z,j)=1))$$

→ "the label of each column is the same as the sum of the numbers in the Black squares of the column"

$$Q_3: \bigwedge_{j=1}^{n-1} \left(\sum_{z=2}^M (A(z,j) \times B(z,j)) \right) = A(1,j)$$

→ "the label of each row is the same as the sum of the numbers in the White squares of the row."

"(c)"

$$Q_4: \bigwedge_{z=2}^M \left(\sum_{j=1}^{n-1} (A(z,j) \times (1 - B(z,j))) \right) = A(z,n)$$

→ "Assigning $A(z,j)$ (except $A(1,n)$) with a value at corresponding location of puzzle plate (array)."

"(d)"

$$Q_5: \bigwedge_{z=1}^M \bigwedge_{j=1}^n ((z \neq 1) \vee (j \neq n)) \rightarrow (A(z,j) = \text{puzzle}[z][j])$$

→ "Assigning $A(z,j)$ (except $A(1,n)$) with a value at corresponding location of puzzle plate (array)."

∴ $Q_1 \wedge Q_2 \wedge Q_3 \wedge Q_4 \wedge Q_5$

intb puzzle [n+1][n+1];

padding → For easier representation.

labels

"Puzzle to color"

→ To read (M-1)X(N-1) puzzle...

<How I read my puzzle>

labels ⇒ (1,1), (1,2), ..., (1,n-1)
are located at... (2,n), (3,n), ..., (M,n)

$A(z,j)$ ⇒ Value located at i-th row and j-th column
(1 ≤ z ≤ M, 1 ≤ j ≤ n-1)
c.f. $A(1,n)$ doesn't exist.

$B(z,j)$ ⇒ color at the location of i-th row and j-th column.
(2 ≤ z ≤ M, 1 ≤ j ≤ n-1)

$\{ B(z,j)=1 \rightarrow \text{black color}$
 $B(z,j)=0 \rightarrow \text{white color}$