Pattern Analysis and Applications
https://doi.org/10.1007/s10044-019-00862-8

THEORETICAL ADVANCES

Adaptive detection of FOREX repetitive chart patterns

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Received: 11 March 2019 / Accepted: 26 November 2019

Abstract
The global financial ecosystem has evolved and matured along with the ever-changing world economy that grew increasingly complicated due to globalisation. As traders are often inundated with information from various sources when formulating trading strategies, numerous analysis methods have been developed to ease the decision-making process. However, factors such as prior experience and knowledge of the trader as well as various psychological factors often influence the final trading decision. Focusing on charting-based analysis, it still suffers from drawbacks due to the time-warping properties of the chart patterns and the reliance on a large number of pre-defined chart patterns. Hence, in order to address the gaps within the FOREX research, the paper endeavours to propose a novel chart detection algorithm. The auto-segmentation implementation within the algorithm utilises piecewise linear regression to detect chart patterns within the FOREX historical data. By successfully extracting the repetitive chart patterns and subsequently establishing its similarities using Agglomerative Hierarchical Clustering, the information provided could potentially be used to assist traders in solidifying their investment decisions. The experimental results obtained show that repetitive chart patterns can indeed be successfully detected and extracted from the FOREX historical data.

Keywords Repetitive trends · FOREX market analysis · Technical analysis · Chart pattern detection

1 Introduction

The FOREX market is undoubtedly the largest and most liquid financial market in the world [1, 2] with an average daily trading volume that surpassed the trillion-dollar threshold since the year 1995 [3]. As traders often strive to maximise profitability when trading, the ability to analyse and forecast currency exchange price is essential in determining the subsequent course of action when trading. Therefore, an understanding of how the various external factors affect the currency exchange price is required when predicting its future values. Extensive knowledge of the complex financial ecosystem requires a thorough study of the underlying characteristics and mechanics of FOREX trading, namely:

- How FOREX trading works (from basic trading rules to the various trading and prediction platform).
- Sufficient knowledge of the different types of trading data provided/available.
- The utilisation of various analysis methods for FOREX market forecasting.

As the topic of FOREX trading is too broad to discuss in its entirety within the paper, additional information relating to the general characteristics of the FOREX market and trading technicalities is available in the following literature by Baiynd [4], Coulling [5], and Gallo [6]. Even though various analysis and forecasting methods have been implemented throughout the years, numerous problems and challenges which affect the forecasting accuracy and the overall return on investment (ROI) still exist.

Although traders are traditionally assumed to approach the currency exchange in an entirely rational and efficient manner, the reality is significantly different. The idea that market traders are boundedly rational *satisficers* [7, 8] comes into play whereby the trading decisions made are typically not ideal and heavily influenced by their behaviour and information available at the point of time. Apart from that, traders

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Published online: 04 December 2019

also tend to develop their own rule of thumb/heuristics based on their prior experience and knowledge. Therefore, the trading style (intraday vs interday) and rules developed are based on the trader's personal preference. Thus, it introduces an element of psychology into FOREX trading.

The choice of trading strategies used will inadvertently affect the selection of trading data used for analysis as it is released at different time intervals. Traders have the option to choose among some of the financial data commonly used such as historical currency exchange price, interest rates, gross domestic products (GDP), quarterly trade balance numbers, and newspaper and Twitter newsfeeds. Although traders participating in intraday trading often focus on the use of high-frequency information such as tick data for analysis, lower-frequency information such as daily closing price and various newsfeeds is sufficient for interday traders. Researchers such as Aloud et al. [9] and Neely and Weller [10] have noted that research based on high-frequency trading data is scarce.

With the advent of computational advancements, the FOREX state-of-the-art research has expanded into an interdisciplinary study encompassing methods such as econometrics and computational finance to ease the analysis and trading process. The widespread acceptance of integrating computational methods into the trading world has enabled the development of the proposed algorithm based on the charting-based analysis. In this study, the focus is on the detection of repetitive chart patterns within the FOREX historical data. It aims to address the gaps within the FOREX analysis and forecasting research by simplifying the detection of repetitive chart patterns within the currency exchange data.

The proposed algorithm utilises piecewise linear regression (PLR) [11, 12] to isolate the various trends occurring within the nonlinear currency exchange data using breakpoints. Utilising the PLR results obtained as a guideline, the proposed up-trend and down-trend representation within the FOREX historical data can be extracted. Subsequently, similarities among all the chart patterns extracted are determined using Agglomerative Hierarchical Clustering [13, 14].

Nevertheless, in order to fully comprehend the rationale behind the proposed algorithm, additional details are provided in the following sections. Section 2 provides a more detailed overview of the various state-of-the-art research on FOREX analysis and forecasting. From the findings obtained, Sect. 3 focuses on the implementation of the proposed algorithm to identify and extract the chart patterns within the FOREX historical data. Subsequently, the chart patterns obtained are further analysed to establish its repetitive nature. As the algorithm consists of multiple connected modules, Sect. 4 provides a more in-depth discussion on the results obtained at each stage. Finally, Sect. 5 summarises all the relevant results of the proposed algorithms as well as the possible future enhancements.

2 FOREX market analysis

Although market analysis remains a crucial component in trading, the fact of the matter is that FOREX market is continuously affected by various external factors such as political stability, economic events, terms of trade, and economic performance. Thus, it presents an additional layer of complexity to market analysis due to the subjectivity involved in the decision-making process. Classical theories such as the Random Walk Analysis [15] and Efficient Market Hypothesis [16] promote the belief that historical FOREX prices hold no value in predicting future currency exchange prices.

Proponents of the Efficient Market Hypothesis initially posit that the FOREX market is inherently efficient and follows a random walk pattern. This is because efficient markets fully integrate all the available information and that the prices adjust almost immediately as the information is disseminated. It is also the core belief of the Efficient Market Hypothesis that the more efficient the market, the more random the price fluctuation making it harder to predict [8, 17]. However, evidence of excess returns while using technical analysis in the foreign exchange market goes against the theory of the Efficient Market Hypothesis [8]. Hence, researchers such as Lo [17] proposed the adaptive market hypothesis model which incorporates behavioural biases when trading.

There are currently three popular techniques used by researchers for FOREX market analysis and forecasting, namely Fundamental Analysis, Sentiment Analysis, and Technical Analysis. However, following discussion centres upon the use of technical analysis, specifically charting-based analysis for FOREX market analysis and forecasting based on several reasons. First, it is widely accepted and used within the trading community. Secondly, fundamental analysis and sentiment analysis are highly subjective. As the FOREX market is affected by numerous external factors, traders often compile information from various sources to make an informed trading decision. New development often happens at a rapid speed in the trading world. Therefore, the information might have already been reflected in the price depending on the efficiency of the market. The chart patterns and technical indicators obtained often provide additional information to the traders for decision-making. Trend indicators such as simple moving average (SMA), exponential moving average (EMA), and moving average convergence divergence (MACD) not only show the direction and strength of the current trend but incorporated with additional trading rules to obtain an entry/exit guideline.

2.1 Technical analysis

The technical analysis, which dates back to the 1700s, primarily focused on the use of historical data for FOREX market forecasting. However, the effectiveness of technical analysis has often been viewed with a sceptical eye. As discussed by Schulmeister [18] as well as Neely and Weller [8], studies on its profitability have frequently led to mixed results when taken into consideration. Therefore, it is a conundrum as to the reason traders would keep using technical analysis when trading. In fact, surveys conducted within the trading community have shown that up to 90% of the market participants [8, 18, 19] and 30–40% of the market professionals regard technical analysis as their primary trading technique [8, 18].

There are several plausible explanations which contribute to the overall acceptance of technical analysis. Firstly, the fact that market traders are boundedly rational *satisficers* means technical analysis signals are often not followed blindly. The traders are often influenced by their personal biases, experience, and heuristics developed over time. Secondly, traders often do not rely on a single analysis method alone and additional information available is often sought to make an informed decision when trading.

2.1.1 Assumptions

Technical analysis was developed based on three fundamental guiding principles, viz: (1) the market price *discounts* everything, (2) asset price moves in trend, and (3) history repeats itself [8]. While seemingly straightforward, further investigation on each of the principle would provide an invaluable insight into the possibility of integrating it into market analysis. The first principle stems from the belief that market fundamentals are assimilated into the price fluctuation. Therefore, it eliminates the need to analyse fundamental data with the focus shifted to the use of historical trading data.

Subsequently, the two remaining principles emphasise heavily on the fact that technical analysts believe in the existence of various underlying patterns within the FOREX historical data. While proponents of Random Walk Analysis [15] and Efficient Market Analysis [16, 20] often view price fluctuation to be random and unpredictable, the opposite is true for technical analysis whereby price move in trends and underlying patterns such as discussed in Sect. 2.2 could potentially be extracted. The identification of chart patterns eventually leads to the third assumption on the repeatability of extracted patterns over time [8, 21]. The chart patterns identified within the FOREX historical data often have recurring potential regardless of the duration taken for development. Therefore, the chart patterns identified and subsequently extracted are worth monitoring.

2.1.2 Types of technical analysis

In general, technical analysis encompasses two main methods, namely: mechanical analysis and charting-based analysis. While mechanical analysis focuses on the use of statistically based methods, charting-based analysis relies on the use of pre-defined chart patterns. Therefore, in order to gain a better understanding of the different approaches towards technical analysis, further information on both the analysis methods along with the issues commonly faced by technical analysts is provided in the following discussions.

2.1.2.1 Charting-based analysis The charting-based analysis revolves around the identification of a set of 53 pre-defined chart patterns as documented in the Encyclopedia of Chart Patterns [22] to forecast the price fluctuations based on commonly accepted inference rules adopted by the traders. Each of the documented chart patterns could be associated with one of the following signals: (1) reversal, (2) continuation, or (3) bilateral movement of the trend. Successful identification and matching of the chart patterns as it develops over time inform the traders to pay attention to the corresponding buy or sell signal related to the chart pattern.

Nevertheless, thorough scrutiny of all the state-of-the-art implementation reveals that there is only a limited amount of research focusing on charting-based analysis. Focusing on the use of both synthetic and stock data, Fu et al. [23] explored the use of perceptually important point (PIP) to detect five chart patterns selected, namely head-and-shoulder, double tops, triple tops, rounded top, and spike top. The outlines of the currency price fluctuations obtained are matched using both template and rule-based technique. Wan and Si [24, 25] later expanded the idea of using PIP for chart pattern matching by including all 53 of the pre-defined chart patterns. By classifying the chart patterns into five major groups, an ANFIS-based chart pattern matching [24] and rule-based approach [25] were developed.

Also implementing a rule-based chart pattern matching technique, Bandara et al. [26] utilised the local extrema detected using kernel regression smoother of ten different chart patterns achieving a pattern identification accuracy of 96%. Deviating from the previous state-of-the-art implementation discussed, Liu and Kwong [27] employed the use of wavelet decomposition feature of 14 different chart patterns for pattern recognition. The proposed implementation manages to achieve up to 81% accuracy in detecting chart patterns.

Canelas et al. [28–30] outlined another approach towards detection of repeating chart pattern. By using the SAX representation of the financial time series, pattern recognition is used to find similar pattern occurring within the historical data to generate a buy or sell signal using a genetic algorithm (GA) optimisation process [28]. The implementation is further enhanced by improving the GA optimisation process [29] as well as extending the pattern discovery process to cater to multidimensional financial data [30].

Only a few researchers have directly combined the use of currency exchange trends and chart patterns. Albeit using stock data as input, Parracho et al. [31] employed a template matching technique with GA optimisation to explore the use of the overall uptrend, downtrend, along with three different breakout patterns to forecast the buy and sell signals. Wu et al. [32] approach charting-based analysis in a slightly different manner as a sliding window is used to extract the chart from the TAIEX stock data. The raw data obtained are subsequently transformed using discrete wavelet transform (DWT). To aggregate similar chart patterns, *K*-means clustering is subsequently used for clustering. The modified AprioriAll algorithm is subsequently used to find the frequently repeating chart patterns that are most similar to the query pattern for trend prediction.

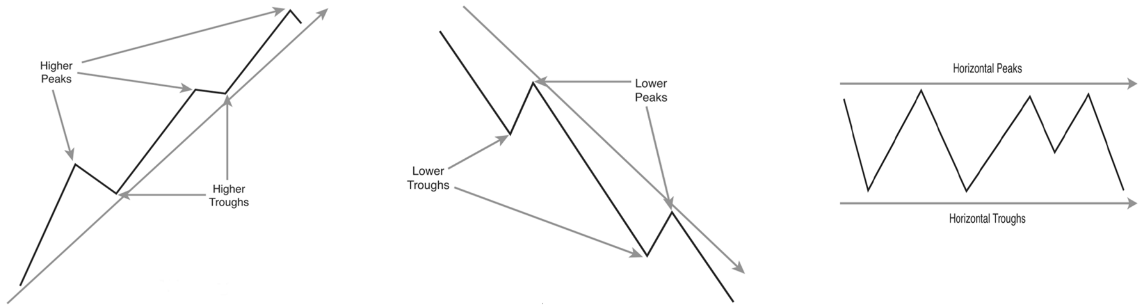
By using both the chart patterns and the original historical data, a hybrid WT-DTW method is proposed by Bagheri et al. [21] to match the chart patterns while combining it with the forecasting result obtained using a Wavelet-ANFIS-QPSO system in a decision support system to obtain trading signals of buy, sell, or neutral based on the predicted market trends. Subsequent testing using FOREX data reveals a hit rate of 68.98%. Lee et al. [33], as well as Tiong et al. [34, 35], proposed the use of up-trend and down-trend chart patterns for detection using different machine learning techniques. Lee et al. [33] combined a total of six different features extracted from the chart pattern for forecasting using Hidden Markov Model (HMM) model with high accuracy of 95.31%. On the other hand, research by Tiong et al. [34, 35] incorporated an additional feature while utilising a combination of ANN machine learning for chart pattern matching for dynamic time-warping (DTW)-based forecasting.

One of the main complications faced by chartist is the time-warping properties of the chart pattern identified within the currency exchange data. As the chart patterns do not develop over a specific time duration, the extracted chart patterns have different lengths. Apart from that, 53 chart patterns are a huge number to take into consideration when matching. Most importantly, the documented chart patterns only represent chart patterns that have been discovered over the years without taking into consideration those that have not been discovered and formally documented. From the state-of-the-art review, there are also only a small number of charting-based analysis conducted in the FOREX market. Although the lack of study could be directly related to trader's initial reservation, Park and Irwin [36] posit that certain chart patterns might indeed be profitable although there is limited evidence in establishing its profitability due to the varying results obtained from different chart patterns, currency exchange pair, and sample period tested.

2.1.2.2 Mechanical-based analysis The mechanical analysis revolves around the use of indicators derived from mathematical functions using the past and present exchange rate as input to analyse underlying trends which provide a more credible mathematical justification and theoretical background. There are currently six well-known indicators in use, viz: trend, momentum, volume, volatility, cycle, and Bill Williams' indicator [6, 37]. Based on the mechanical indicators computed, trading rules are often formed to assist traders in making trading decisions on buying/selling a particular currency pair. Due to the proliferation of interdisciplinary approach in trading, statistical methods such as ARIMA and GARCH have been used to forecast the currency exchange price.

Albeit achieving a high forecasting accuracy and ROI value, there are still issues associated with the conventional FOREX forecasting methods. The use of methods such as ARIMA and Box–Jenkins assumes that the currency exchange data are linear, while the opposite is true [38]. Thus, traders began incorporating more nonlinear methods such as TAR, ARCH, and GARCH to that adapts better to the data fluctuation. As it is a parametric model, certain values need to be pre-specified [38]. Similar to the computation of the numerous technical indicators, it is therefore not as adaptable to the different currency exchange prices and its fluctuations. Although commonly used values have been established within the trading community, there are no clear guidelines on the final selection. The few non-parametric models investigated did not outperform the random walk model [38].

Therefore, traders have been looking into using machine learning models alongside technical indicators to deal with nonlinear data. It has contributed to the development of numerous trading portfolio/automated forecasting systems. The machine learning techniques commonly used are artificial neural network (ANN), support vector machine (SVM), genetic algorithm (GA), neuro-fuzzy computing, and support vector machine (SVM). The ANN models used by researchers such as ridge polynomial neural network (RPNN), basic multi-layer neural network, and various hybrid combinations have consistently shown positive forecasting [39–43]. In fact, Rehman et al. [42] were able to achieve an average accuracy of 98.5%. By using daily, weekly, and monthly data, Emam [39] have found that the mean squared error (MSE) obtained during both the testing and real market simulation is very similar to the prediction result obtained with daily data being the best, followed by weekly and monthly data. Yao et al. [44] choose to develop a neuro-fuzzy-based portfolio management system which optimises the trading schedule by incorporating a fuzzy cerebellar model articulation controller (FCMAC) forecasting module. It manages to achieve a level of near human expert.



(a) Uptrend **(b)** Downtrend **(c)** Sideway trend

Fig. 1 Trends within the FOREX historical data as taken from Kirkpatrick and Dahlquist [53]

For algorithms developed based on GA, such as implemented by Kato et al. [45], it strives to learn the various buying and selling strategy produced using technical indicators. Slany [46] have also utilised a variant of genetic programming (GP) for trend prediction that produces 12% - 31% correct predictions. Support vector regression (SVR) has been used in conjunction with GHSOM in a two-stage architecture proposed by De Brito and Oliveira [47–49]. The architecture was initially used by Hsu et al. [50] for stock market forecasting. In another study, Bahramy and Crone [51] have found that the information provided by Bollinger Band indicator significantly increases the forecasting accuracy when used in conjunction with SVR. On the other hand, Baasher and Fakhr [52] have explored the use of SVM for feature selection and classification algorithm with the best performance exceeding 77%.

2.2 FOREX repetitive trends and chart patterns

It is relatively easy for the human brain to analyse the formation of chart patterns and its repetition throughout the entire duration of the FOREX historical data. However, the identification of the series of peaks and troughs that within the data is often subjective and depends heavily on the trader's perspective. The FOREX trends detected by connecting the adjacent peaks and troughs detected within the historical data can be generalised in a broader perspective to encompass the following: (1) uptrend, (2) downtrend, and (3) sideway trend as shown in Fig. 1.

The hidden patterns are observed to occur when analysed under two conditions: (1) seasonality and time of day and (2) matched to pre-defined chart patterns. While the FOREX historical data might appear to fluctuate randomly, a thorough examination of the data reveals that peaks and troughs are often detected at certain hours of the day and shift according to the season due to daylight saving time (DST). On the other hand, chart patterns can also be identified by comparing it against the pre-defined patterns documented in the Encyclopedia of Chart Patterns [22].

Ito and Hashimoto [54], as well as Aloud et al. [9], have conducted an investigation on the existence of intraday and seasonal patterns within the FOREX historical data. Focusing on the study by Ito and Hashimoto [54], it was conducted using the number of deals, price change, return volatility, and bid–ask spread for both the USD/JPY and EUR/USD currency exchange pairs. While the original EBS dataset used encompasses data from 1 January 1999 to 31 December 2001 with a 1-second interval, it is essential to confirm that the patterns detected occur consistently over time using a different dataset. Hence, the price fluctuation of the tick dataset as provided by HistData [55] during the winter (January) and summer (June) season for the year 2015 is also thoroughly examined. The focus is to compare the number of deals with the original study using the tick data as a market activity indicator. Figures 2 and 3 show that the AUD/USD, EUR/USD, GBP/USD, and JPY/USD exchange pairs fluctuate similarly throughout the day with the peaks and troughs forming at different times depending on the season. Apart from that, it also has the same peaks and troughs as previously identified in the original work. Therefore, it substantiates the fact that there are repetitive patterns within the FOREX time series data.

3 Repetitive chart pattern detection algorithm

The current state-of-the-art research has shown that there is no shortage of research within the FOREX analysis and forecasting domain. Nonetheless, there are pros and cons associated with each analysis method. By shifting the research focus towards technical analysis, it has been found there is a relatively big gap within the area of charting-based analysis. Albeit its widespread acceptance among the trading community, the charting-based analysis received significantly less

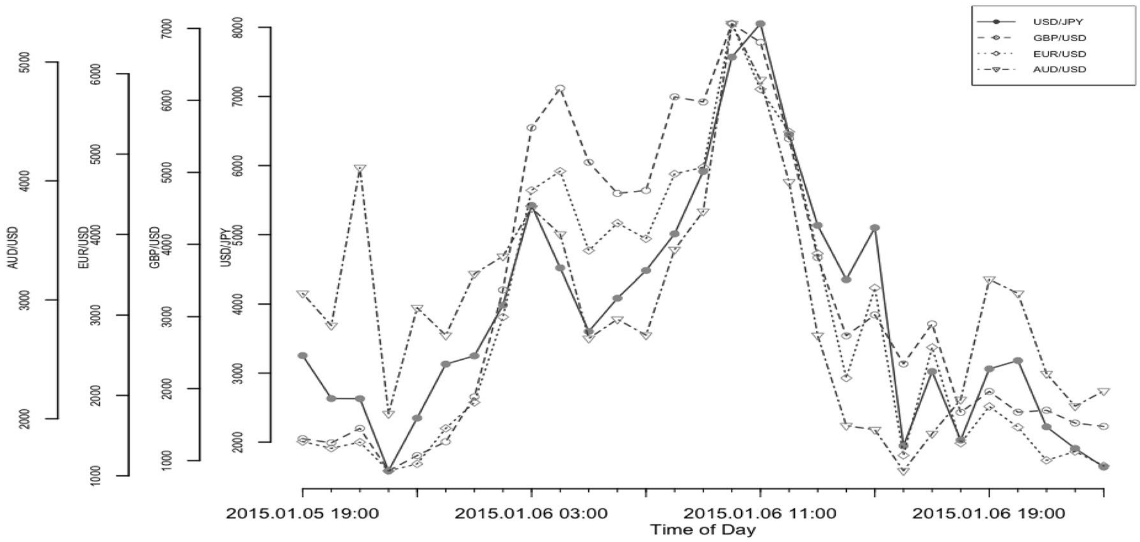


Fig. 2 FOREX intraday activities (January), winter 2015

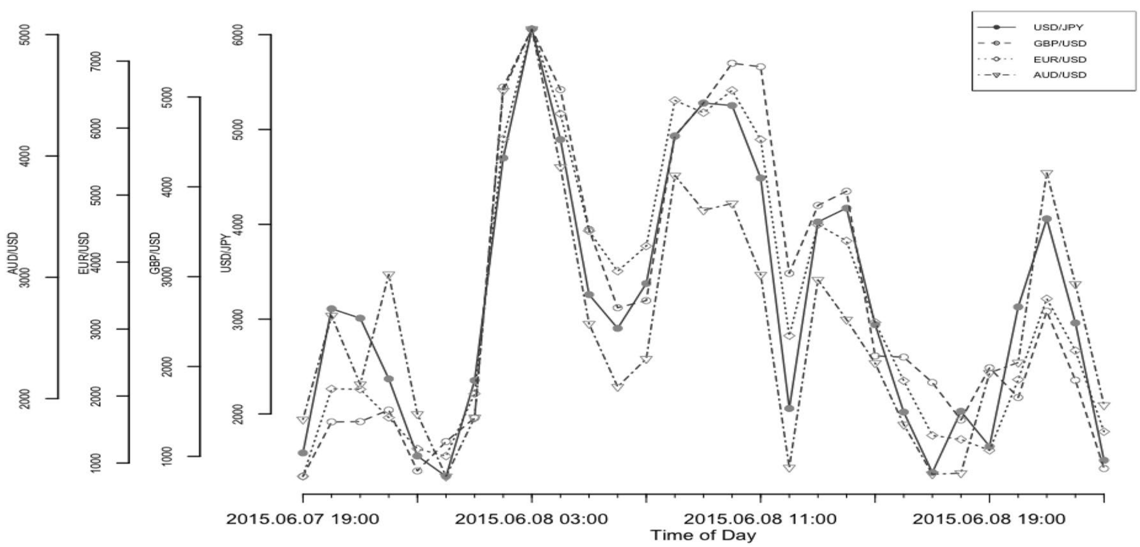
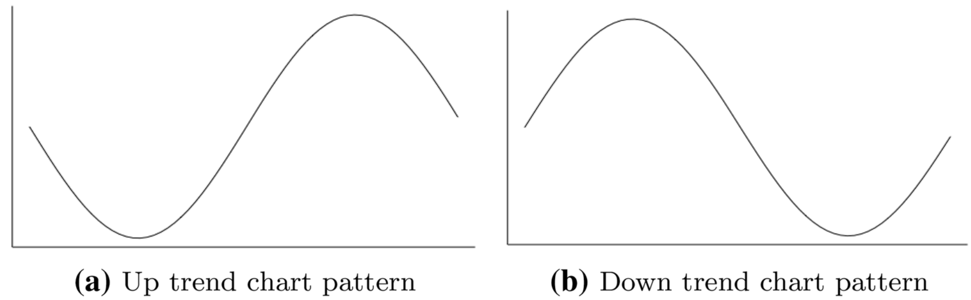


Fig. 3 FOREX intraday activities (June), summer 2015

attention compared to the mechanical-based analysis. The identification of chart patterns within the FOREX historical data also proves to be a difficult task because of the number of chart patterns documented in the Encyclopedia of Chart Patterns [22]. Apart from that, it is also hard to make a direct comparison of all the chart patterns extracted as it is affected by time-warping issues whereby the chart patterns do not develop over a specific period.

Hence, the proposed algorithm endeavours to address the problems previously mentioned based on the initial assumptions held by technical analysts, as discussed in Sect. 2.1.1. It aims to simplify the analysis and forecasting process using an auto-segmentation process. The auto-segmentation process is extremely crucial as it works to identify trends within the FOREX historical data according to the fluctuation of each currency exchange pair. Not only does the regression of each segment represent a specific trend detected, but it is also subsequently used for the identification and extraction of chart patterns. Another approach towards simplification is implemented by reducing the number of chart patterns used for identification to only the up-trend and down-trend patterns. The up-trend and down-trend chart patterns are identified from the FOREX historical data based on the fluctuation of the currency exchange data moving above and below the regression line at each segment. Finally, the repetitive nature of the chart patterns is

Fig. 4 FOREX chart patterns used for detection



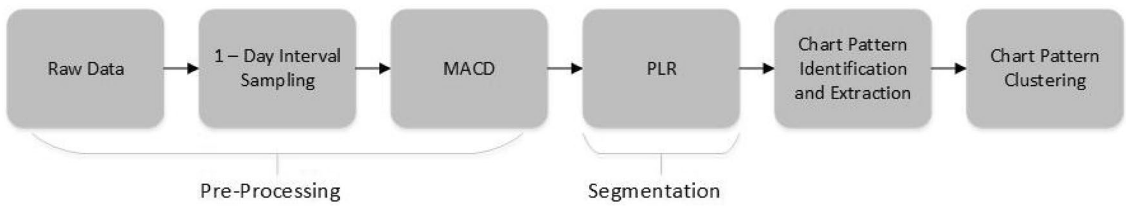


Fig. 5 Proposed algorithm

established by determining the similarities among the chart patterns extracted using Agglomerative Hierarchical Clustering [13, 14].

3.1 Algorithm implementation

The proposed research relies heavily on the existence of chart patterns and its repetition within the FOREX historical data for analysis. Therefore, it is essential to establish what constitutes a successful detection of chart patterns before delving deeper into the algorithm implementation. It is explicitly defined to encompass both the identification, extraction, and clustering of similar chart patterns to determine the repeatability of the chart patterns.

The basic sine and displaced sine trigonometric chart patterns depicted in Fig. 4 are used for identification purposes. As the proposed algorithm is based heavily on the existence of currency exchange trends and patterns, the chart patterns proposed have two major significances. Not only does it work to simplify the pattern matching process, but it also indirectly represents the current trend of the currency exchange price. It also ensures that unknown repeating patterns are taken into consideration as it is based on the currency exchange trends.

As shown in Fig. 5, the proposed algorithm consists of six major implementation steps.[1] The first three steps are essentially data pre-processing steps crucial in preparing the FOREX historical data for the following processes. First and foremost, the FOREX historical data are selected and obtained from HistData [55]. The closing price of the FOREX market was used for analysis as technical analysis relies on using the past currency exchange price for analysis. It is also readily available from various sources for free from numerous reputable sources online such as Oanda [56], HistData [55], and various central bank Web sites.

Four of the major currency pairs in trading are utilised in the research, namely AUD/USD, EUR/USD, GBP/USD, and USD/JPY, as it is actively traded and monitored within the trading community. Information obtained from the state-of-the-art research has shown that most of the research utilises at least one of the major currency exchange pairs. Once the FOREX historical data are acquired, the sampling and normalisation process begins. The FOREX historical data obtained are subsequently sampled at a 1-day interval (daily) frequency as the data provided initially by HistData [55] are at a 1-min interval. Alternatively, the sampling process could be omitted if the original dataset has a 1-day interval. The rationale behind the sampling of the data can be linked back to the prior literature review as 11 out of 13 the research previously reviewed are conducted using daily data.

Subsequently, the MACD technical indicator shown in Eq. 1, which is derived using the EMA indicator in Eq. 2, is applied for normalisation purposes. Referring to Eq. 2, P_q denotes the current exchange price, q is the q_{th} data of the exchange price, and EMA_q is the EMA value of the q_{th} price index. δ refers to the decay factor that can be calculated as $2/(m+1)$, whereby m is the time period window defined. From the initial clustering results presented in Fig. 6, it can be seen that the clustering result is affected by the currency exchange price range over time. By using the original

[1] R version 3.4.0 was utilised for the development of the entire project [14].

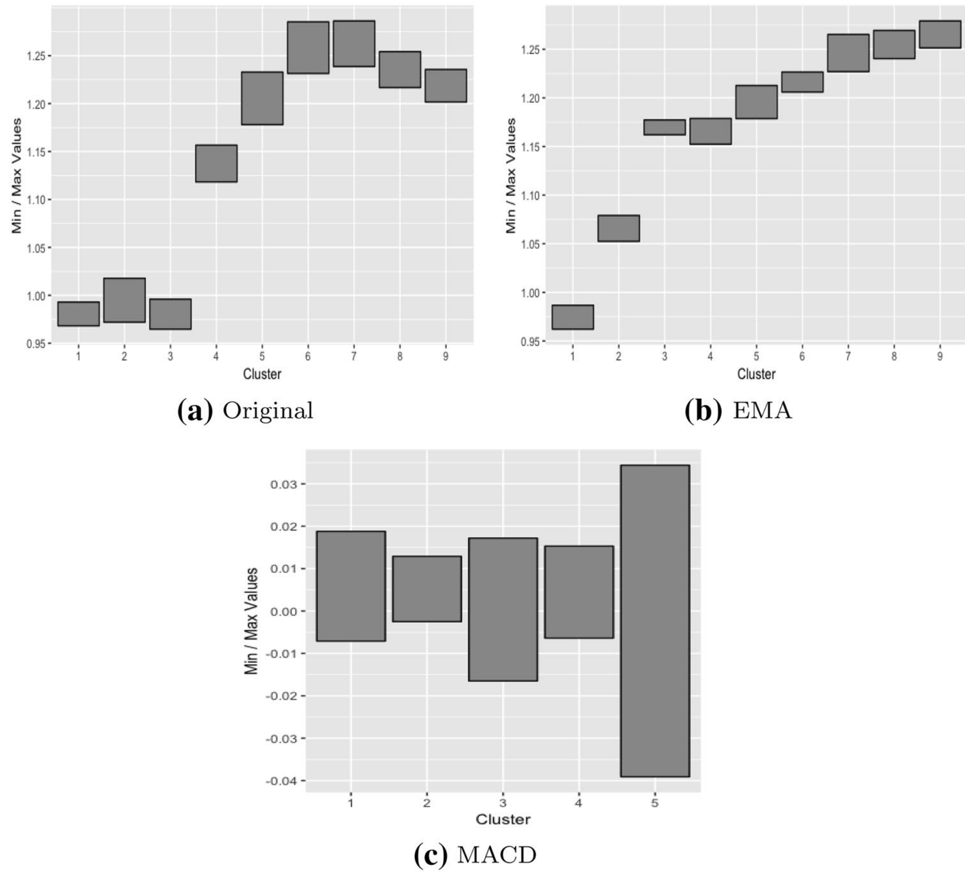


Fig. 6 EUR/USD currency exchange clustering range

currency exchange price as well as the EMA technical indicator, the chart patterns obtained are often clustered within a specific currency exchange price range. However, the use of MACD technical indicator can be seen to eliminate the issue. Therefore, the MACD technical indicator with the EMA value of 12 and 26 is used for normalisation purposes. Both the EMA values were chosen based on the fact that not only is it widely used, but it also ensures that the results obtained are consistent and do not stray from the values that traders will typically implement for analysis.

$$\text{MACD} = (\text{12-day EMA}) - (\text{26-day EMA}) \tag{1a}$$

$$\text{Signal} = \text{9-day EMA of the MACD line} \tag{1b}$$

$$\text{EMA}_q(P, M) = \left(P_q - \text{EMA}_{q-1}\right) \times \delta + \text{EMA}_{q-1} \tag{2a}$$

$$= (1 - \delta) \times \text{EMA}_{q-1} + \delta \times P_q \tag{2b}$$

The work of Tiong et al. [34] utilising the linear regression line (LRL) to look for repetitive chart patterns has inspired the development of the segmentation module of the proposed algorithm. The work was later adapted by Yong et al. [57], whereby technical indicators were used as input to determine whether similar chart patterns still exist. However, the linear regression is only able to identify a single trend across the FOREX historical data. As FOREX historical data have been proven to be nonlinear, the proposed algorithm implements PLR [11, 12] for segmentation purposes.[2] The PLR was chosen for implementation as it allows for more flexibility in terms of isolating the various trends occurring within the dataset using break-points. However, the number of break-points varies depending on the number of trends existing within the data. Thus, the optimal number of break-points has to be determined.

The PLR break-point exploration phase outlined in Algorithm 1 helps to identify the minimum and maximum number of break-points across all four currency pairs. It is dependent on two main factors discussed further below:

– *Stopping Condition* The two main stopping conditions established for the algorithm are: (1) length of the dataset / 2 and (2) PLR computation error.

[2] The R *segmented* package version 0.5–2.0 was utilised to compute the PLR break-points [11, 12].

Table 1 PLR break-point looping conditions

Algorithm	Looping conditions
A1	Single iteration process – Main iteration K value: $\min_g - \max_g$
A2	Fine-level segmentation for both iterations – Main iteration K value: $\min_g - \max_g$ – Secondary iteration K value: $\min_g$ – Length of main iteration segment
A3	Coarse-level segmentation for the first iteration; fine-level segmentation for the second iteration – Main iteration K value: $\min_g$ – 11 (Number of years) – Secondary iteration K value: $\min_g$ – Length of main iteration segment
A4	Coarse-level segmentation for both first and second iterations – Main iteration K value: $\min_g$ – 11 (Number of years) – Secondary iteration K value: $\min_g$ – 12 (Number of months)
A5	Mid-level segmentation for both first and second iterations – Main iteration K value: $\min_g$ – (Length of data / 11 (Number of years)) – Secondary iteration K value: $\min_g$ – (Length of main iteration segment / 12 (Number of months))

– *Number of Break-points* The number of break-points (quantiles) chosen to represent the initial break-points (denoted as K) is adjusted each iteration.

Algorithm 1 Break-point Detection and Segmentation Exploration Phase

Input: FOREX Historical Data: Normalised Data
Output: PLR Result

```
for K = 1 to Stopping Condition do
    Perform PLR calculation and obtain break-points.
    Calculate Mean Squared Error (MSE) value for each segment.
end for
```

The MSE value calculated at each iteration is used to determine the number of break-points. The number of break-points which produces the smallest MSE value is taken as the minimum value, while the maximum value is determined by the largest break-point value taken into consideration before stopping. The *global minimum* and *global maximum* values ($\min_g$ and $\max_g$) are determined by taking the smallest minimum value and the largest maximum value across all currency exchange pair.

Once the $\min_g$ and the $\max_g$ values have been determined, the iterative PLR segmentation is conducted according to the pseudocode in Algorithm 2. A total of five different algorithms are used to segment the currency exchange data as shown in Table 1. While Algorithm A1 adheres to a single iteration process, Algorithms A2–A5 explore a two-level segmentation method with different settings to segment the data. The degree of segmentation is determined by the number of break-points considered at each iteration. Algorithm A2 takes into consideration fine-level segmentation for both iterations. However, the number of break-points considered for Algorithms A3 and A4 is at a coarse-level segmentation for the main iteration, while the second iteration is tested using both fine- and coarse-level segmentation. Algorithm A5 focuses on mid-level segmentation for both iterations. The MSE value calculated at each iteration aids in choosing the PLR break-points that produces the best fit with the normalised data.

Algorithm 2 Break-point Detection and Segmentation

Input: FOREX Historical Data: Normalised Data
Output: Segmented FOREX historical data

```
for K = min_g to Stopping Condition do
    Perform PLR calculation and obtain break-points
    Calculate Mean Squared Error (MSE) value for each segment
end for
Select PLR with the minimum Mean Squared Error (MSE) value for segmentation
if Algorithm selected: A2 A3 A4 A5 then
    for K = min_g to Stopping Condition do
        Perform PLR calculation and obtain break-points
        Calculate Mean Squared Error (MSE) value for each segment
    end for
end if
Select PLR for the secondary iteration with the minimum Mean Squared Error (MSE) value for segmentation
```

The PLR break-points obtained during the segmentation step are crucial as it represents the overall trend of the FOREX market. The lines formed by connecting the neighbouring break-points are used to identify and extract the proposed chart patterns based on the crossover between the normalised data and PLR lines. Nevertheless, the crossovers between the normalised data and PLR lines have to be monitored in order to remove noisy chart patterns identified by requiring either side of chart patterns are required to have at least 1% of the segment length. Referring back to Algorithm 2, it can be seen that the number of segments and final chart patterns extracted is unpredictable depending on the rate of currency exchange price fluctuation along with the regression estimated for each segment. Once successfully extracted, the chart patterns are subjected to an Agglomerative Hierarchical Clustering process using the DTW distance matrix as input to establish the similarities among chart patterns with different lengths.[3,4] Similar to the complications faced when choosing the optimal number of PLR break-points for segmentation and the chart patterns extracted, there are no clear guidelines for the final number of clusters obtained for each of the currency exchange pair. Therefore, a repetitive process examining the clustering result with adjustments made to the number of clusters is executed as outlined in Algorithm 3. It can be seen from the algorithm that the defined starting condition and ending condition of the loop range from 2 to (number of trends/2).

The rationale behind the choice of starting and stopping conditions is as follows:

- The minimum number of clusters has to be more than one cluster.
- The minimum number of trends within a cluster has to be at least two in a single cluster. This is because isolated chart patterns are not useful in establishing similarities between the patterns. Hence, trends clustered in isolation are rejected.

Table 2 Dataset information

Currency dataset	AUD/USD	EUR/USD	GBP/USD	USD/JPY
Number of data points	3354	3353	3355	3357
Duration of data	2 January 2002 to 31 December 2012			
Time interval	1-Day interval			
Technical indicator	MACD(12,26,9)			

Table 3 Minimum and maximum PLR break-point values

Currency pair	Minimum PLR break-point	Maximum PLR break-point
AUD/USD	5	96
EUR/USD	1	99
GBP/USD	1	99
USD/JPY	3	100

[3] The R *dtw* package version 1.18-1 was utilised to compute the DTW distance matrix [58, 59].

[4] The R *stats* package from R version 3.4.0 and *dendextend* package version 1.5.2 were utilised for clustering [13, 14].

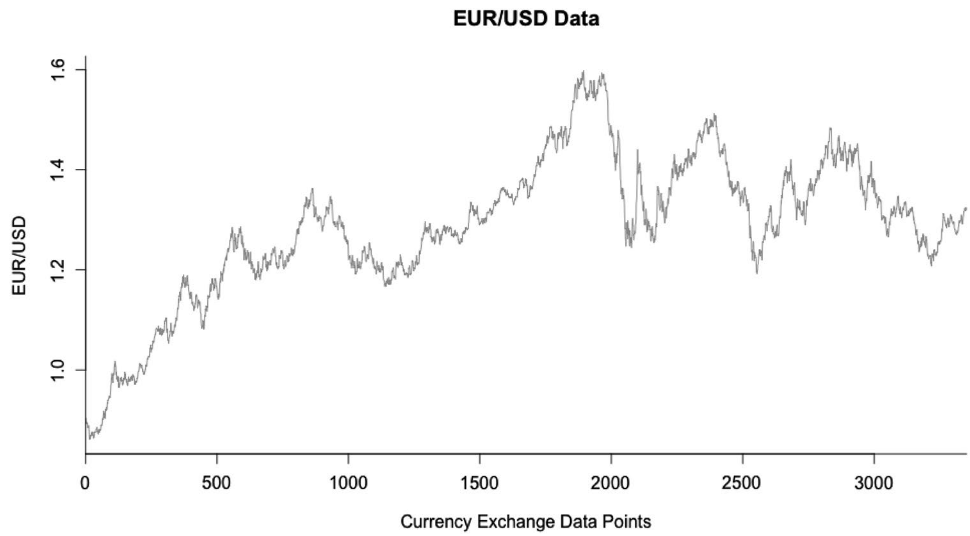


Fig. 7 EUR/USD dataset representation

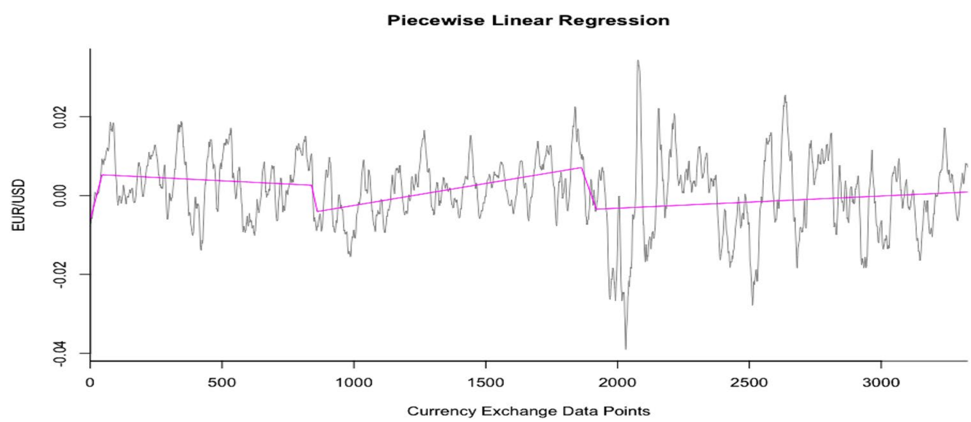


Fig. 8 PLR segmentation representation

Algorithm 3 Iterative Clustering Algorithm.

Input: DTW Distance Matrix of Extracted Chart Patterns for a particular currency exchange pair

Output: Chart Pattern Clustering Result

for l `= 2 to` $(number\ of\ trends/2)$ **do**
    Perform agglomerative hierarchical clustering to obtain l number of cluster.
    Calculate the mean value for each cluster.
    Calculate the overall mean value for l number of cluster using the results obtained in the previous step.
end for
The optimal number of cluster chosen is the number of cluster with the smallest mean value.
The rejected trends are combined into an additional cluster.

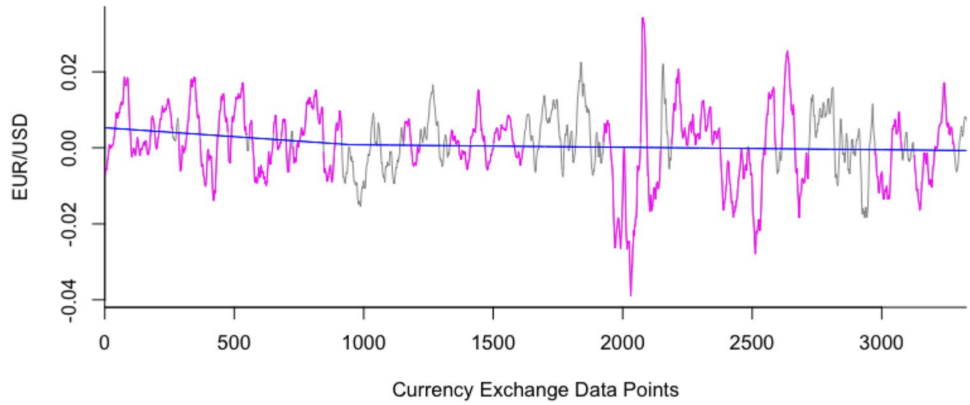


Fig. 9 EUR/USD chart pattern identification and extraction

Table 4 FOREX historical data segmentation results

Currency pair	Algorithm	Number of segments extracted	Number of chart patterns extracted	Minimum length of chart patterns extracted	Maximum length of chart patterns extracted
AUD/USD	A1	3	20	30	147
	A2	95	123	4	148
	A3	95	123	4	148
	A4	14	33	7	148
	A5	90	126	4	148
EUR/USD	A1	2	22	31	294
	A2	83	103	4	294
	A3	83	103	4	294
	A4	6	30	4	294
	A5	68	92	4	294
GBP/USD	A1	2	14	53	164
	A2	39	59	4	163
	A3	39	59	4	163
	A4	14	39	5	163
	A5	36	60	4	163
USD/JPY	A1	3	34	28	163
	A2	155	214	4	113
	A3	155	214	4	113
	A4	8	44	4	168
	A5	143	209	4	113

4 Results and discussion

The results obtained at every major step of the proposed algorithm are further scrutinised in three separate sections. Section 4.1 discusses the pre-processing step of the currency exchange data, which includes the data selection, acquisition, and normalisation results of the dataset. The segmentation results, along with the extraction of chart patterns from each of the currency exchange pair tested, are available in Sect. 4.2. Finally, the clustering results of the chart patterns extracted indicating its repetitive nature are further investigated in Sect. 4.3.

4.1 Dataset

The experimental results for the proposed algorithm are obtained using the AUD/ USD, EUR/USD, GBP/USD, and USD/JPY currency exchange pairs provided by HistData [55]. Table 2 shows a summary of the raw data before and after the pre-processing stages. An example of the EUR/USD dataset before normalisation is shown in Fig. 7.

Table 5 Chart pattern clustering results

Currency pair	Algorithm	Total number of clusters	Number of clustered trends	Number of rejected trends
AUD/USD	A1	7	17	3
	A2	34	96	27
	A3	34	96	27
	A4	8	25	8
	A5	36	99	27
EUR/USD	A1	4	15	7
	A2	27	80	23
	A3	27	80	23
	A4	9	24	6
	A5	21	68	24
GBP/USD	A1	3	10	4
	A2	15	45	14
	A3	15	45	14
	A4	14	34	5
	A5	17	47	13
USD/JPY	A1	9	26	8
	A2	62	169	45
	A3	62	169	45
	A4	12	34	10
	A5	55	160	49

4.2 Segmentation and chart pattern identification and extraction

The PLR exploration phase is performed according to Algorithm 1 previously outlined to determine the number of break-points across all four currency exchange pairs. Analysing the MSE value at each iteration of Algorithm 1, the number of minimum and maximum break-points for each currency exchange pair is documented in Table 3. Once the $\min_g$ and $\max_g$ values have been determined, PLR segmentation is performed using Algorithms A1–A5. Figure 8 shows an example of the segmentation using Algorithm A1 on the normalised EUR/USD currency exchange pair. The proposed chart patterns identified within the FOREX historical data using the best fit PLR as a guide, as shown in Fig. 9, are subsequently extracted for clustering.

The number of segments and chart patterns extracted along with the minimum and maximum length of the chart patterns obtained during the segmentation as well as the chart pattern identification and extraction phase is listed in Table 4. Albeit producing segments and chart patterns with varying lengths, it can be seen that Algorithms A1–A5 were able to segment the FOREX historical data successfully.

Although the maximum length of the chart patterns length is comparable for all the different algorithms, the two-level segmentation renders some of the chart patterns too short to train the machine learning model for recognition and forecasting with the minimum value recorded at only four data points.

It must be noted that the number of segments and chart patterns relies on the fluctuation of the FOREX historical data used. Based on the algorithm, the number of segments and chart patterns varies depending on the trend movements detected. The trends detected are unique to the duration of the historical data used and the currency exchange pair used. Therefore, there are no fixed number of segments or chart patterns to the final results obtained.

4.3 Clustering

Referring back to Algorithm 3 used for clustering, the mean values of the distance matrix for each cluster and the overall mean value are calculated at each iteration. By selecting the number of clusters for each of the currency pair that produces the lowest mean value overall yields the results shown in Table 5. The mean values of the distance matrix for each cluster along the overall mean value for each currency pair

Table 6 Cluster results analysis

Mean values	Currency pair			
	AUD/USD	EUR/USD	GBP/USD	USD/JPY
Overall mean value	0.0005700056	0.0008325156	0.001016741	0.05415509
Cluster 1	0.0005101067	0.0006263221	0.0009505495	0.04729401
Cluster 2	0.0006063254	0.001013301	0.001253475	0.05421295
Cluster 3	0.0005665245	0.0010315142	0.0008461988	0.06624497
Cluster 4	0.0005987533	0.0006589245	–	0.05068173
Cluster 5	0.0004532057	–	–	0.08310810
Cluster 6	0.0004397750	–	–	0.04163723
Cluster 7	0.0008153485	–	–	0.05079803
Cluster 8	–	–	–	0.05935532
Cluster 9	–	–	–	0.03406351
Rejected cluster	0.004895295	0.004811448	0.005661208	0.3667747

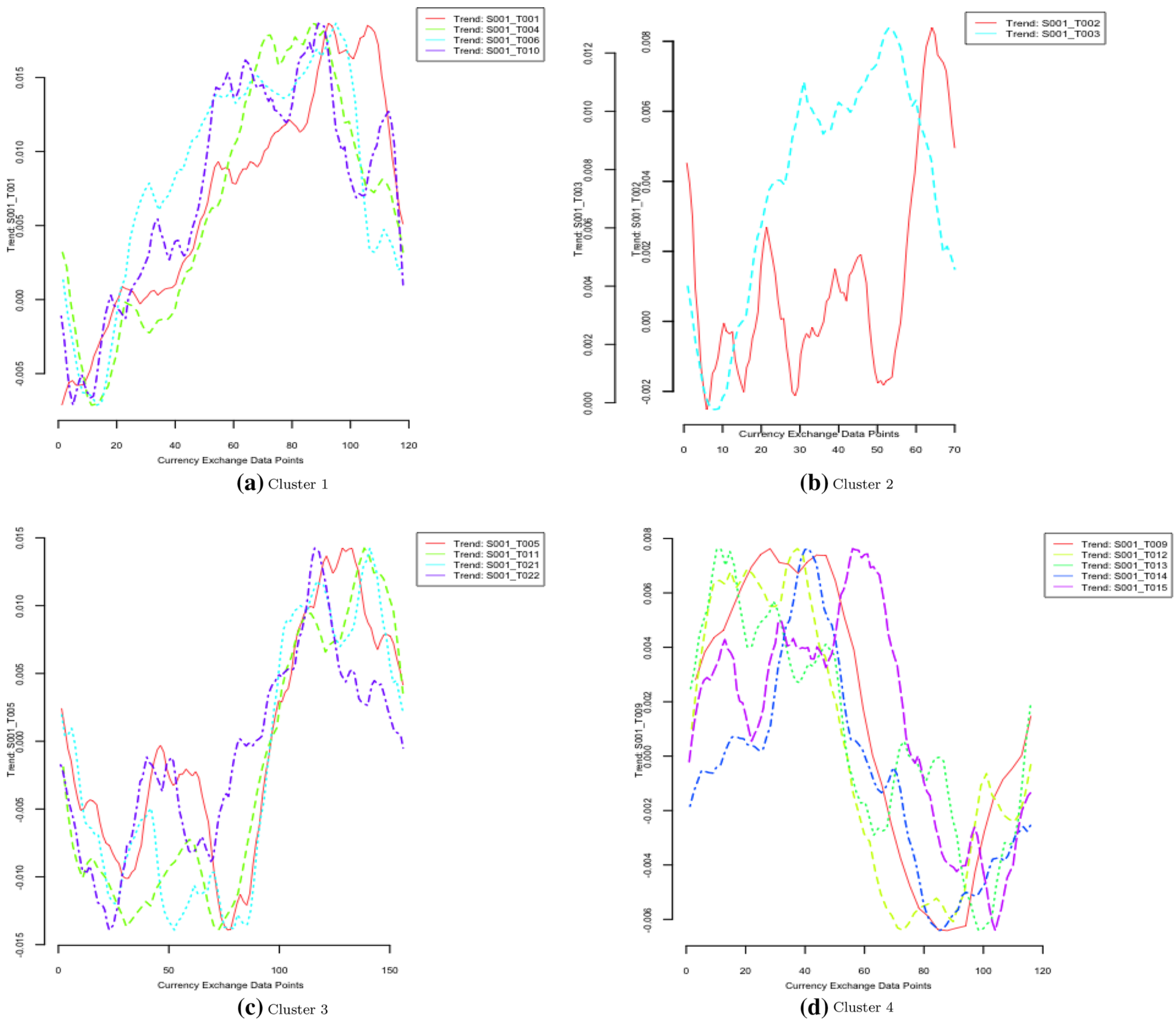


Fig. 10 EUR/USD clustering result using algorithm A1

are shown in Table 6. Visualisations of four of the clusters obtained using the EUR/USD currency exchange pair are subsequently provided in Fig. 10 for clarity. Although the clustering process outcome can be seen to segregate similar chart patterns into the same cluster successfully, Algorithms A2 and A3 produce the same clusters. The reasoning behind acquiring similar cluster could be traced back to the fact that the segmentation process yields the same chart patterns for both the algorithms. From the observed clustering results obtained combined with the longer trends extracted, Algorithm A1 would be a more suitable choice for implementation.

The experimental results reveal that the chart patterns extracted are unique and different across the various currency exchange pairs. The number of unique repeating chart patterns for each currency exchange pair found over the course study is as follows:

– AUD/USD: 7
– EUR/USD: 4
– GBP/USD: 3
– USD/JPY: 9

5 Conclusion

In conclusion, the extensive investigation into the state-of-the-art FOREX research has revealed that the FOREX price does indeed move in trends and that history repeats itself as posited by technical analysts. The chart patterns which possess crucial information for forecasting could be successfully extracted from the FOREX historical data using the proposed algorithm. Not only could the chart patterns be successfully extracted using all five algorithms, the clustering results obtained further justify the existence of repetitive chart patterns within the FOREX historical data. From the results obtained, further research opportunities involve optimising the segmentation process to perform more effectively and extract chart patterns with longer lengths.

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