# Social Choice and Irreducible Values: A Political Economy Approach of Ideology[1]

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## Abstract

*In the present paper we prove that any social choice function satisfies Arrow's principle of Independence of Irrelevant Alternatives (IIA), if individual behavior is menu-dependent. Therefore, Arrow's 'General Possibility Theorem' is not valid when individual preferences are determined by irreducible values. In this context, any aggregation device that fulfills Non-dictatorship and Paretian Unanimity principles (simple majority, for example) also satisfies IIA. This could be an important result for social choice theory, inasmuch as individual behavior determined by irreducible values (self-interest, ideology, ethics, social norms, for example) can validate representative democracy. The relative importance of such values, and the possibility of preference reversals, determine the dynamic of social choice according to democratic principles.*

## 1. Introduction

Arrow's 'General Possibility Theorem' establishes that there is no democratic election design that produces transitive and complete social preference relations and also satisfies Non-dictatorship (ND), Paretian Unanimity (PU), and Independence of Irrelevant Alternatives (IIA) principles. Since then, several alternative approaches had been trying to solve this paradox. Buchanan [1954] and Tullock [1967] had made important critical comments on Arrow's paradox. They argue that majority voting has some beneficial aspects and that Arrow's theorem does not have much to say about democratic elections schemes. Majority rule is an acceptable one just because it allows logrolling and discussion from which relative unanimity emerges (see, for example da Silva, 1996). In democracy, competing alternatives could be tested and, if so, replaced by new or old competitive alternatives.

The analysis made by economists about the behavior of people in market place is logically consistent and it can be tested. Throughout empirical testing one can explain how market and even firms operates when some decision are made. In public sphere, however, the explanation of the link between private action and collective choice is more difficult because the existence of some logical problems, as illustrated by Arrow(1963). The theory of collective action is in this sense less developed than the individual one. Nevertheless, we ought to explain collective choice and by explanation we mean (i) the

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construction of a logically consistent theory and, (ii) its empirical testing. Despite the conclusions of Arrow(1963), democracy really works and it is a strong empirical evidence that introduces the necessity of alternative approaches to public choice decisions. This is the context of the paper.

The democratic process naturally leads to bargaining, vote trade, collusion, and minimal consensus in society. Inside democratic process, self-interest, ideology, ethics, and social norms play their role. However, traditional binary choice theory assumes that those values can be reduced to a simple criterion usually associated to the notion of utility. In the present paper we apply the model of decision where choice is determined by an ordered set of irreducible criteria developed by Moldau (1988, 1992, and 1994) in order to discuss individual behavior induced by ideology, ethics, and social norms. This approach we call choice by irreducible values model (CIVM).

Contrary to traditional binary choice theory, in which preference relation $R$ is the primitive notion, CIVM departs from the primitive notion of criterion's relative importance. Nonetheless, from its axioms it is possible to derive a weak preference relation $R$ (complete and transitive) — Moldau (1994). Garcia (1994) had proved an important property of CIVM (Theorem 1): for finite opportunity sets, individual behavior is menu-dependent in the sense of Sen (1993, 1994, and 1995), that is, changes in opportunity set can reverse individual preference over any two options. This is the key issue to analyze the basic problem of social choice posed by Arrow (1963).

Firstly, we are going to prove (Theorem 2) that any social choice function satisfies the principle of Independence of Irrelevant Alternatives, if individual behavior is menu-dependent. Therefore, Arrow's 'General Possibility Theorem' is not valid when individual preferences are determined by irreducible values. In this context, any aggregation device that fulfills Non-dictatorship and Paretian Unanimity principles (simple majority, for example) also satisfies IIA.

Secondly, we are going to show that this is an important result for social choice theory, inasmuch as individual behavior determined by irreducible values (self-interest, ideology, ethics, social norms, for example) can validate representative democracy. The relative importance of such values, and the possibility of dynamic preference reversals, determine the inner nature of social choice according to democratic principles. As it was noticed by political scientists and thinkers, from Locke to Tocqueville and, passing throughout Hanna Arent and Antonio Gramsci, democracy implies non-radicalism, persuasion and tolerance, values and ideologies changes and so on.

## 2. Irreducible Values and Menu-dependence

The standard idea of choice determined by utility maximization presupposes the reducibility of all choice criteria to a unique measure of value, usually called utility function. This means that agent's values can be reduced to a single measure of comparison, which enables the establishment of preference relations. Nonetheless, this approach hides some important questions about the structure and the role of individual values on the comparisons establishment, that is, about mechanics of motivational system. In fact, there is no substantive reason to believe that multiple criteria reflecting a spread range of needs, wants, and objectives, could be reduced to a unique criterion[4]. The neglect in treating irreducible criteria is widely criticized in literature[5].

Moldau (1988,1993) presents an alternative method to treat the problem of choice by irreducible criteria, which we call Choice by Irreducible Values Model (CIVM). According his approach, individual choice is determined by agent's preference ordering over the opportunity set, as it is in most of binary choice theories. The basic difference is that, in his model, agent's preference orderings are determined by the ***relative importance*** of his set of irreducible criteria. In this section we present CIVM and we deduce their properties.

## 2.1 The Model

The problem of choice involves two basic sets: the set of irreducible criteria and the opportunity set. The set of irreducible criteria is denoted by $J$ and it is supposed to be formed by $m$ elements, $m \geq 1$. The opportunity set is denoted by $X$ and it is formed by $n$ elements, $n \geq 1$. Both sets are supposed to be finite.

As we said before, the primitive notion in CIVM is the relative importance of the irreducible criteria. Comparisons of any two options from $X$ are established by the binary relation $\underline{\phi}$ over the product space $JxX$ of pairs $(j,x)$, where $j$ and $x$ are variables from $J$ and $X$, respectively. The proposition $(j',x')\,\underline{\phi}\,(j'',x'')$ means that criterion $j'$ at option $x'$ is "at least as important as" criterion $j''$ at option $x''$. On the basis of the relative importance relation we can define relations "more important than" and "as important as".

$$\forall j',j'' \in J \text{ and } \forall x',x'' \in X: \quad (j',x')\,\phi\,(j'',x'') \Leftrightarrow (j',x')\,\underline{\phi}\,(j'',x'') \wedge \neg (j'',x'')\,\underline{\phi}\,(j',x')$$

$$(j',x')\,\&\,(j'',x'') \Leftrightarrow (j',x')\,\underline{\phi}\,(j'',x'') \wedge (j'',x'')\,\underline{\phi}\,(j',x') .$$

Based on $\underline{\phi}$ we can define a non-preference relation on $X$ according criterion $j$ as follows: for any option and for any criterion, $x'$ is "at least as good as" $x''$ according criterion $j$, if criterion $j$ at $x'$ is as important as it is at $x''$. From this notion follow the definitions of preference and indifference relations according $j$.

$$\forall j \in J \text{ and } \forall x',x'' \in X: \quad x' Q_j x'' \Leftrightarrow (j,x'')\,\underline{\phi}\,(j,x')$$

$$x' P_j x'' \Leftrightarrow (j,x'')\,\phi\,(j,x')$$

$$x' I_j x'' \Leftrightarrow (j,x'')\,\&\,(j,x') .$$

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[4] Ainslie (1985), for example, suggests a basic opposition between *"visceral satisfactions, closely associated with the consumption of a concrete object ... and more subtle satisfactions, such as Knowledge of 'the ideal'..., pursuit of wisdom..."*.

[5] Moldau (1993) summarizes the controversy around the problem of multiple and irreducible criteria.

It is convenient to note that any option is preferred to another if, and only if, criterion $j$ at $x'$ is less important than it is at $x''$. This means that the relative importance of any criterion raises when the necessity behind it is satisfied. In this sense, Moldau (1993, p.358, fn.) says: "*the preference relation according to a given criterion is defined in terms of an attempted reduction of that criterion's importance*." So we can read the proposition "criterion $j$ at option $x'$ is more important than criterion $j$ at option $x''$" as follows: at option $x'$, criterion $j$ is more satisfied than it is at option $x''$.

Relation $\underline{\phi}$ is supposed to satisfy the following two axioms:

**Axiom 1** (comparability): $\forall j, j' \in J$ and $\forall x', x'' \in X$, $(j, x')\,\underline{\phi}\,(j', x'') \vee (j', x'')\,\underline{\phi}\,(j, x')$.

**Axiom 2** (transitivity): $\forall j, j', j'' \in J$ and $\forall x', x'', x''' \in X$,

$$(j, x')\,\underline{\phi}\,(j', x'') \wedge (j', x'')\,\underline{\phi}\,(j'', x''') \Rightarrow (j, x')\,\underline{\phi}\,(j'', x''').$$

Given Axioms 1 and 2, we can say that $\underline{\phi}$ establishes a weak relative importance ordering and a weak-preference ordering according $j$ on $X$. But it is also possible to establish an weak-importance ordering at $x$ on $J$. This is the basic issue to introduce the rule which determines the overall preference relation on $X$. Let $k(j,x)$ be an integer between 1 and $m$ which ranks the criteria in order of importance.

$\forall j, j' \in J$ and $\forall x' \in X$: $\quad (j, x')\,\phi\,(j', x') \Rightarrow k(j, x') < k(j', x')$.[6]

This means that if criterion $j$ is more important than criterion $j'$ at option $x'$, then the rank number of $(j, x')$ is smaller than the rank number of $(j', x')$; that is, the greater the relative importance of criterion $j$ at option $x'$, smaller its rank number. For the $k$th ranked criterion, we can define the relations of non-preference, preference and indifference, as follows:

$$\forall k \leq m \text{ and } \forall x', x'' \in X: x' Q_k x'' \Leftrightarrow (j(k, x'), x')\,\underline{\phi}\,(j(k, x''), x'')$$

$$x' P_k x'' \Leftrightarrow (j(k, x''), x'')\,\phi\,(j(k, x'), x')$$

$$x' I_k x'' \Leftrightarrow (j(k, x'), x')\,\tilde{\phi}\,(j(k, x''), x'').$$

Finally, we can define the overall preference relation $P$ on $X$ as follows:

$$\forall x', x'' \in X: x' P x'' \Leftrightarrow \exists g\,(g \geq 1 \wedge x' P_k x'') \wedge \forall k\,(k < g \Rightarrow x' I_k x'').$$

This definition says that any option $x'$ is preferred to $x''$ if, and only if, there is some criterion $g$th ranked for which $x'$ is preferred to $x''$ and, for any other criterion ranked above $g$ — that is, any other criterion more important than the gth —, $x'$ is indifferent to

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[6] Ties between importance are supposed to be broken arbitrarily. For this issue see Moldau (1993, sec.IV).

$x''$. In other words, the overall preference $P$ is determined by the least important criterion for which there is no tie. The least satisfied criterion overcome those which are more satisfied than it. The overall indifference relation $I$ and the weak preference relation $R$ have the following definitions:

$$\forall x', x'' \in X: \quad x'Ix'' \Leftrightarrow \forall k\,(k = 1K\ ,m \Rightarrow x'I_k x''\ .$$
$$x'Rx'' \Leftrightarrow (x'Ix'' \vee x'Px''$$

Moldau (1993, p.359-60) had proven that $Q$ , $Q_k$, and $R$ are complete and transitive relations on $X$. Therefore, from a set of irreducible criteria the individual can order his opportunity set. Although basic properties of preference ordering are the same of any binary choice model, CIVM has a special feature which we discuss next.

## 2.2 Menu-dependence

Menu-dependence behavior can be defined as the occurrence of preference reversals[7] when there is some change on agent's opportunity set. According to Sen (1994), the basic condition for internal consistency of choice fails in a situation like this. Suppose that individual $i$ prefers option $x'$ to $x''$, $x', x'' \in X$. Now, assume that we reduce his opportunity set picking up alternative $x'''$ from $X$ and then he says that option $x''$ is preferred to $x'$. So, we can say that his preference is menu-dependent, inasmuch changes on menu imply preference reversals.[8]

In this section, we will argue that CIVM doesn't exclude the possibility of menu-dependence behavior. This proposition was first presented in Garcia (1994) and here we only reproduce the general argument of the original proof. If we pay attention on the mathematical structure of the function which determines overall preferences, we can see that it is quite the same of that proposed by Arrow (1963) to social decision functions (also called aggregation devices). First, consider that each criterion is an individual of Arrow's system. Therefore, the overall preference would be a kind of social preference determined by individual values.

In order to study inter-menu problems we introduce another variable on our analysis: $t$. Menu $t$ denotes a specific "situation" in which the individual must establish, based on his values $J$ , their preferences $P$ on $X$ . Note that the set of criteria and the opportunity set are fixed on menu $t$. In this sense, any two situations can differ to each order, either because they have different opportunity sets or as a result of differences on criteria sets (changes on individual values, for example). $T$ denotes the set of all possible menus;

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[7] This term is usually associated to the failure of independence axiom of expected utility theory — Karni (1987). Notwithstanding, we decide to use this terminology since problems are quite the same.

[8] This behavior is also known as dependence of irrelevant alternatives in social choice theory. According Arrow (1963), it is a basic requisite for both individual and social rationality. For this issue see also Mackay (1980) and Sen (1993).

since $X$ and $J$ are finite, $T$ must be finite too. $X_T$ denotes the set of all $X_t$, such that $t \in T$.

Given axioms 1 and 2, we know that the weak preference relation according $j$ is a complete and transitive non preference relation on $X$. Let restate this proposition including menu specification as follows:

$$\forall t \in T,\ \forall j \in J, \text{ and } \forall x', x'', x''' \in X:\ x'Q_{jt}x'' \vee x''Q_{jt}x'$$
$$x'Q_{jt}x'' \wedge x''Q_{jt}x''' \Rightarrow x'Q_{jt}x'''$$

Denoting the weak preference according $j$ in situation $t$ by $R_j$ and the weak overall preference in situation $t$ by $R$ , we can deduce Lemma 1.

***Lemma 1*** - *Given Axioms 1 and 2, for any situation* $t \in T$, *and for any two options* $x', x'' \in X$ , *if* $\forall j(j \in J_t \Rightarrow x'R_{jt}x''$ , *then* $x'R_tx''$.

*Proof: Suppose that is was not the case. Then, there would be a situation* $t \in T$ *and two options* $x', x'' \in X$ , *such that* $\forall j(j \in J_t \Rightarrow x'R_{jt}x''$ , *but* $x''P_tx'$. *We know that if* $\forall j(j \in J_t \Rightarrow x'R_{jt}x''$ , *then for any two criteria* $\jmath, f \in J$ , $(f, x'') \succ (f, x'$ . *Let* $\jmath$ *be the decisive criterion on* $x'$. *In this case, we know that* $(f, x') \succ (\jmath, x'$ *and, by Axiom 2, that* $(f, x'') \succ (\jmath, x'$ *Nonetheless, if* $x''$ *was preferred to* $x'$, *then there would be two decisive criteria* $\jmath, f \in J$ *of rank number k such that* $(\jmath, x') \succ (f, x''$ , *which constitutes a contradiction. Therefore, we can conclude that for any situation* $t \in T$, *and for any two options* $x', x'' \in X$ , *if* $\forall j(j \in J_t \Rightarrow x'R_{jt}x''$ , *then* $x'R_tx''$. □

Lemma 1 says that CIVM satisfies the well-known *weak Paretian Unanimity* condition imposed on Arrow's system. Now, let presuppose that there is no criterion which, for any situation, is decisive in determining the preference relation between any two alternatives. This is to say: let assume the condition of *Non-dictatorship* (Arrow, 1963). In our approach, this assumption implies that we explicitly exclude the possibility of lexicographic criteria in determining individual's preference. Therefore, any criterion can be satiated and, this being the case, it has its relative importance diminished. The possibility of lexicographic criteria is not excluded in CIVM, so we postulate Axiom 3, called non-dominance condition.

**Axiom 3** : $\neg\exists j \in J$, such that $\forall t \in T$ and $\forall x', x'' \in X$ , $x'R_{jt}x'' \Rightarrow x'R_tx''$.

Now, we can define menu-dependence condition: any function which determines overall preference based on $J$ is said to be menu-dependent if, and only if, there are two situations $t, t' \in T$ and there two options $x', x'' \in X_T$ such that criteria sets are the same in both situation $(J_t = J_{t'}$ , but $x'R_tx'' \wedge \neg(x'R_{t'}x''$ . Therefore, menu-dependence is the negation of *independence of irrelevant alternatives*.

Finally, we can postulate Theorem 1:

***Theorem 1*** *- Given Axioms 1, 2, and 3, individual behavior is menu-dependent.*
*Proof: The same argument used to prove Arrow's 'General Possibility Theorem'.*[9]

This is a very intuitive result: since comparisons are established according to the relative importance of the $k$th criteria at $x'$ and at $x''$, the preference relation between any two options doesn't depend solely on $x'$ and $x''$. Therefore, menu-dependence behavior emerges as a result of choice determined by a set of irreducible values, none of them prevailing over all other criteria in any possible situation.

## 3. Social Choice and Irreducible Values

The basic property of menu-dependence of individual preference has important consequences on social choice. In this section we prove that, if for all individual Axioms 1 to 3 hold, then for any social decision function $r$, $r$ satisfies the principle of *Independence of Irrelevant Alternatives* (Arrow, 1963). After that, we analyze simple majority rule under conditions of irreducible and non-dominant individual values.

### 3.1 Independence of Irrelevant Alternatives

Let $H$ be the set of all individuals $i$, such that it has $h \geq 1$ elements. Now, we can say that, if Axioms 1, 2 and 3 hold for all individuals $i$, then their behavior are menu-dependent. That is to say, if for any individual, its choice is determined by a set of irreducible and non-dominant criteria, then menu-dependence holds for all individuals of society.

**Axiom 4** : $\forall i \in H$ individual behavior satisfies Axioms 1, 2, and 3.

Given Axiom 4, we can say that $\forall i \in H$ there are two situations $t', t'' \in T$ and there two options $x', x'' \in X_{t'}, X_{t''}$ such that $x' R_{t'} x'' \wedge \neg (x' R_{t''} x''$ . In the same way, we can say that if Axioms 1 to 4 hold, then we have that the following proposition also holds:

**Proposition 1** : $\forall$ , if $\forall t', t'' \in T$ and $\forall x', x'' \in X_T$, $x' R_{it'} x'' \Rightarrow x' R_{it''} x''$, then $i \notin H$ .

In social choice theory, Independence of Irrelevant Alternatives is defined as follows: $\forall t', t'' \in T$ and $\forall x', x'' \in X_T$, if $\forall i \in H$ $x' R_{it'} x'' \Rightarrow x' R_{it''} x''$, then $x' S_{t'} x'' \Rightarrow x' S_{t''} x''$, where $S$ denotes social weak-preference relation over any two options and it is supposed to be complete and transitive. To our purpose, we define IIA by its negation form:

[9] For this issue see Arrow (1963), Mackay (1980) and Garcia (1994).

$\forall t',t'' \in T$, $\forall x',x'' \in X_T$, if $x' S_{t'} x'' \wedge \neg (x' S_{t''} x''$, then $\exists i \in H \quad x' R_{it'} x'' \wedge \neg (x' R_{it''} x''$.

Now, we can state the sufficient condition for IIA. In order to do this, it is useful to note that, if for any two situation and any two options, there is an individual whose preference is menu-dependent, then we can say[10] that if $x' S_{t'} x'' \wedge \neg (x' S_{t''} x''$, then $\exists i \in H$ $x' R_{it'} x'' \wedge \neg (x' R_{it''} x''$ , $\forall t',t'' \in T$, $\forall x',x'' \in X_T$. Therefore, we only need to prove that $\forall t',t'' \in T$, $\forall x',x'' \in X_T$, $\exists i \in H \quad x' R_{it'} x'' \wedge \neg (x' R_{it''} x''$ , in order to prove that IIA is satisfied. In order to simplify our analysis, let proposition 2 describe the negation of the necessary condition for IIA.

**Proposition 2** : $\exists t',t'' \in T$, $\exists x',x'' \in X_T$, such that $\forall i \in H \quad x' R_{it'} x'' \Rightarrow x' R_{it''} x''$.

Next, Lemma 2 proves that proposition 1 is inconsistent to the negation of the necessary condition for IIA (proposition 2), which is the same thing to say that proposition 1 is a necessary condition for IIA.

***Lemma 2** - Proposition 1 implies the necessary condition for IIA.*

*Proof: Suppose that was not the case. In this case, we have that propositions 1 and 2 hold simultaneously. Therefore:*

$\forall$ , *if* $\forall t',t'' \in T$ *and* $\forall x',x'' \in X_T$, $x' R_{it'} x'' \Rightarrow x' R_{it''} x''$, *then* $i \notin H$ *(proposition 1) and*

$\exists t',t'' \in T$, $\exists x',x'' \in X_T$, *such that* $\forall i \in H \quad x' R_{it'} x'' \Rightarrow x' R_{it''} x''$ *(proposition 2).*

*From proposition 2, we have that* $\forall i \quad i \in H \Rightarrow (x' R_{it'} x'' \Rightarrow x' R_{it''} x''$ *and, from proposition 1, we have that* $\forall i \ (x' R_{it'} x'' \Rightarrow x' R_{it''} x'') \Rightarrow i \notin H$. *Therefore, we have that* $\forall i \quad i \in H \Rightarrow i \notin H$. *From this proposition we can conclude that* $\neg \exists i, i \in H$, *otherwise there would be a contradiction such that* $i \in H \wedge i \notin H$. *So, H is necessarily an empty set. Nonetheless, this proposition contradicts the premise that the number of elements from H is greater than 1, h≥1. Therefore, we can conclude that proposition 1 implies the necessary condition for IIA.* □

***Theorem 2** - Given Axioms 1 to 4, for any social decision function r, r satisfies the principle of Independence of Irrelevant Alternatives.*

*Proof: It is sufficient to consider that Axioms 1 to 4 imply proposition 1, which implies the necessary condition for IIA, according to Lemma 2.* □

### 3.2 Simple Majority

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[10] This logical argument can be described as follows: if A is true, then B⇒A is also true. This is so, because any proposition B, either true or false, can imply any true proposition A.

Given Theorem 2, we can ask whether Arrow's 'General Possibility Theorem' remains valid in a context of individual choice guided by irreducible values. In this section we analyze a democratic election design based on simple majority which satisfies principles of Unrestricted Scope (US), Paretian Unanimity (PU), and Non-dictatorship (ND). Before that, we proceed to introduce some definitions and premises about voter behavior and majority rules.

Assume that any voter *i* satisfies Axioms 1 to 3. For any election *t*, we assume that, for any voter $i$, his vote on any candidate $x \in X$ is a function of his individual preferences over $X$. $V_{i,t}(x$ denotes the value *i*'s vote on *x* in election *t*, such that $V_{i,t}(x) =$ if *i* votes on candidate *x*, and $V_{i,t}(x) = 0$ if *i* doesn't vote on *x*. Now, we introduce the basic relation between voter preferences and his vote: $\forall i \in H$ $\forall t \in T$, and $\forall x', x'' \in X$, $V_{i,t}(x') = 1 \Leftrightarrow x' P_{it} x''$. As a consequence we have that, if any voter has more than one candidate in his most preferred equivalence class, then for any candidate $V_{i,t}(x) = 0$.

The overall value of any candidate *x*, denoted by $V_t(x$ is the summation of $V_{i,t}(x$, $i = 1, \ldots, h$. That is: $V_t(x) = \sum_{i=1}^{h} V_{it}(x$. For any candidate *x*, $h \geq V_t(x) \geq 0$. Given individual votes on any election we can define relations of social weak-preference (*S*), social preference (*SP*), and social indifference (*SI*). $\forall i \in H$ $\forall t \in T$, and $\forall x', x'' \in X$ :

$x' S_t x'' \Leftrightarrow V_t(x') \geq V_t(x''$ ,

$x' SP_t x'' \Leftrightarrow V_t(x') > V_t(x''$ , and

$x' SI_t x'' \Leftrightarrow V_t(x') = V_t(x''$ .

Given this democratic election design, we can prove that $S$ is a complete and transitive relation on $X$. In the other hand, this social decision function satisfies Paretian Unanimity and Non-dictatorship. The proofs for Lemmas 3 to 6 are presented in Appendix 1. Inasmuch any social decision function which satisfies Axioms 1 to 4 also satisfies Independence of Irrelevant Alternatives, we can say that this particular election design is an Arrovian social choice.

***Theorem 3*** *- Given Axioms 1 to 4, the simple majority rule described above satisfies conditions IIA, US, UP, and ND.*

*Proof: Directly from Theorem 2 and Lemmas 3 to 6.* □

Let see a very simple example of how this election design can be applied. Suppose an election among three candidates $x'$, $x''$, and $x'''$ based on individuals' preferences of $i'$, $i''$, and $i'''$. Figure 1 shows voters' preferences. In this situation, each candidate receives one vote: $i'$ votes on $x'$, $i''$ votes on $x'''$, and $i'''$ votes on $x''$. Therefore, we can say that $x'$ is socially indifferent to $x''$, which is also indifferent to $x'''$.

**Figure 1 - Example of an election**

| | | | |
|---|---|---|---|
| | $i'$ | $x'Px''$ | $x''Px'''$ |
| *voters* | $i''$ | $x'''Px'$ | $x'Px''$ |
| | $i'''$ | $x''Px'''$ | $x'''Px'$ |

This example is also useful to illustrate the intuitive idea behind the choice by irreducible values and the menu-dependence behavior. Suppose that for any reason candidate $x'''$ is excluded from the opportunity set. In this case, we have a new menu such that, if individual preferences were not changed, we would have the election of $x'$, that is, $x'SPx''$. This example would illustrate the possibility of social menu-dependence. Nonetheless, this argument cannot be applied, because when we exclude $x'''$ from the opportunity set, the menu-dependent behavior of voters can promote changes on individual preferences and votes.[11]

Finally, we can imagine situations in which ties are not accepted — elections for public offices, for example. In fact, the democratic election design described above admits the possibility of two candidates receive the same number of votes. If they are the most preferred candidates, but election needs only one winner, there would be indetermination. In cases like this, we could think of elections in two steps: the two most voted candidates run in another election. This procedure could generate only one winner.[12]

## 4. Concluding Remarks

The first important conclusion of this paper is that tolerance and persuasion are the basis of democracy normative framework. From Axiom 3 we can perceive the nature of democracy as a non-dominant individual value schema or an Institutional set that demands tolerance.

This conclusion naturally leads to another important remark derived from the results presented here. The formation of a coalition can promote the emergence of one single winner. A coalition corresponds to an agreement on the part of two or more players to

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[11] Arrow (1963) argues that this social behavior would be not rational. Here, we propose an alternative approach to this problem. In our analysis, this behavior cannot occur since the exclusion of any candidate changes voters' preferences on the remaining candidates.

[12] This procedure assures the occurrence of a Condorcet winner.

coordinate their actions so as to bring about an outcome that is more advantageous to members of the coalition than the outcome that prevails from an non-cooperative action. Despite free riding, as proposed initially by Olson (1965), the logic of collective action implies that, if transaction costs involved in a coalition formation are low or nil, individual agents improve private well-being implementing collective action. In this sense, if a coalition prefers $x'$ to $x$ it means that every member prefers $x'$ to $x$. So, a coalition can be modeled as an individual. In society, where there are many coalitions, a majority can be formed. This result can emerge from a dynamic ideological bargaining. If the transaction involved in the process is costless, the process could, and we here are just suggesting, select Condorcet winners as final outcomes.

The last important conclusion leads to some reflection about the role of electoral researches made by news papers and independent institutions. The feasible outcomes set $X$ is altered when one candidate is said to be "out of the game". The electoral pools researches can alter the information set and the agents' preference orderings, without any change on individual values. This is an important intuitive result derived from the theory proposed above. The information set can alter agents' perceived menu and, if so, it becomes a very important instrument for the decisions involved in collective choice.

The main conclusion is that democracy requires a normative rule linked with tolerance, bargain and, translating to economics jargon, dynamic preference orderings. The existence of radicals guided by lexical values would fail to meet the condition of non-dominant values posed by Axiom 3. In this sense, democracy is incompatible with antiestablishmentarian behavior. This is the core of democratic collective choice process.

**Appendix**

***Lemma 3*** - *Given Axioms 1 to 4, the simple majority rule determines a complete social weak-preference relation on $X$ for any election $t$.*

*Proof: Suppose that it was not the case. Therefore, there would be an election $t'$ in which there were two options $x'$ and $x''$, such that $\neg(x' S_t x''$ and $\neg(x'' S_t x'$. In this case, the definition of social weak-preference would imply that: $\neg(V_t(x') \geq V_t(x''))$ and $\neg(V_t(x'') \geq V_t(x'))$. This would be the same thing to say that $V_t(x') > V_t(x''$ and $V_t(x'') > V_t(x'$, which would constitute a contradiction. Therefore, for any election $t$ and any two candidates $x'$ and $x''$, $x' S_t x''$ or $x'' S_t x'$. So we have that $S$ is a complete binary relation on $X$.* $\square$

***Lemma 4*** - *Given Axioms 1 to 4, the simple majority rule determines a transitive social weak-preference relation on $X$ for any election $t$.*

*Proof: Suppose that it was not the case. Therefore, there would be an election $t'$ in which there were three options $x'$, $x''$, and $x'''$ such that $x' S_t x''$ and $x'' S_t x'''$, but*

$\neg(x'S_t x'''$. *In this case, the definition of social weak-preference would imply that:* $V_t(x')\geq V_t(x''$ *and* $V_t(x'')\geq V_t(x'''$, *but* $\neg(V_t(x')\geq V_t(x'''))$. *Nonetheless, if* $V_t(x')\geq V_t(x''$ *and* $V_t(x'')\geq V_t(x'''$, *we would have that* $V_t(x')\geq V_t(x'''$, *which would constitute a contradiction. Therefore, for any election* $t$ *and any candidates* $x'$, $x''$, *and* $x'''$, *if* $x'S_t x''$ *and* $x''S_t x'''$, *then* $x'S_t x'''$. *So we have that* $S$ *is a transitive binary relation on* $X$. □

**Lemma 5** - *Given Axioms 1 to 4, we have that simple majority rule satisfies Paretian Unanimity principle.*

*Proof: Suppose that it was not the case. Therefore, there would be an election* $t$ *in which there were two options* $x'$ *and* $x''$, *such that* $\forall i\in H\ (x'R_{it}x''$, *but* $\neg(x'S_t x''$. *In this case, the definition of social weak-preference would imply that* $V_t(x'')>V_t(x'$, *because* $\neg(V_t(x')\geq V_t(x''))$. *Nonetheless, if it was the case, we would have that* $\exists i\in H\ (x''P_{it}x'$. *This would contradict the statement that* $\forall i\in H\ (x'R_{it}x''$. *Therefore, for any election* $t$ *and any two candidates* $x'$ *and* $x''$, *if* $\forall i\in H\ (x'R_{it}x''$, *then* $x'S_t x''$. *So, we have that simple majority rule satisfies Paretian Unanimity principle.* □

**Lemma 6** - *Given Axioms 1 to 4, we have that simple majority rule satisfies Non-dictatorship principle.*

*Proof: Directly from the definition of simple majority rule.* □

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