Contextual Typing

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Type Inference and what we believe ...

- Having reasonable and meaningful annotations is good;
- Local information is good;
- Having guidelines for language designers and programmers is good;
- Scalability is necessary;
- Implementation can be easily derived.

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- Types are propogated to neighbouring expressions;

Bidirectional Typing: Problems Statement

- Trade-off between expressive power and backtracking;
  - more expressive, less syntax-directness;
  - all-or-nothing inference strategy;
- Unclear annotatability and rule duplication;
- Inexpressive subsumption.

Our Proposal: Contextual Typing

- Quantitative Type Assignment Systems;
  - as a specification for programmers;
  - tells you where the annotations are needed;
  - parametrised with a counter: $\Gamma \vdash_n e : A$
- Syntax-directed algorithmic Type Systems
  - is decidable;
  - parametrised with a context: $\Gamma \vdash \Sigma \Rightarrow e \Rightarrow A$

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Recap

- Contextual typing is a lightweight approach to type inference
  - that exploits partially known contextual information;
- It enables several improvements over bidirectional typing