1

Lecture 10 a : Epidemics on networks & Advanced spreading processes

Mar 19.

Epi. on networks

- not every node can infect every other node.
- infections only go across edges.

nodes ? edges ?

exposure network $G = (V, E)$

transmission network $G_T = (V, E_T)$

(i) people (i) → (j)

depends on pathogen

$E_T \subseteq E$

G is weighted, undirected, but G_T is directed
(always a tree for SIR)

* See the figure on Page 12 in Lecture 9 notes, for "transmission networks in the wild".

Note : these data are hard to get ! (need "contact tracing")

The meaning of R_0 in a network

- avg # of secondary infection.
- not an intrinsic property of pathogen
- function of $Pr(k)$ in G and β.

Dynamics on the network

» β, γ are now probabilities

» each time step, loop over infected nodes: (1) apply edge rules; (2) apply node rules.

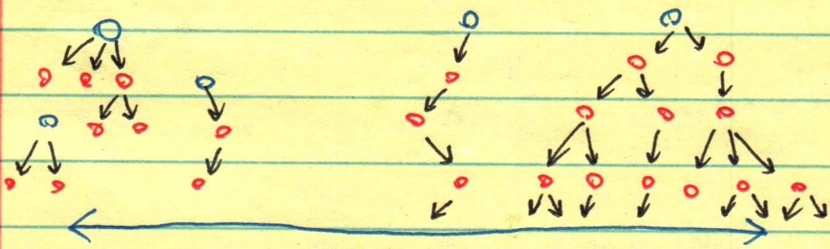
$\{ (I) \text{---} (S) \} \xrightarrow{\beta} \{ (I) \text{---} (I) \}$ edge rule

$\{ (I) \} \xrightarrow{\gamma} \{ (R) \}$ node rule (SIR)

$\xrightarrow{\gamma} \{ (S) \}$ node rule (SIS)

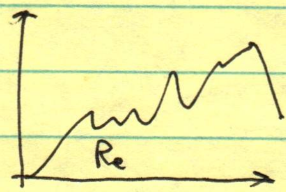
* See figure on Page 14 for an example.
* Use the Jupyter notebook on Canvas to explore.

The rule of R_0.



$R_0 < 1$ ("sub-critical" small outbreaks) | $R_0 = 1$ ("critical" outbreaks of all sizes) | $R_0 > 1$ ("super-critical" global epidemics)

R_e : effective # 2^{nd}ary infections.



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* See Table on Page 16 for examples of real R_0 values for real diseases, along w/ estimates of the required vax. rate to reach herd immunity.

$$\left(1 - \frac{1}{R_0}\right)$$

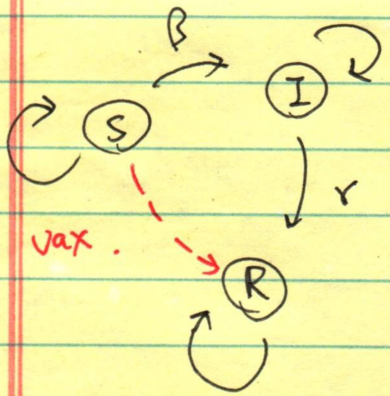
* To stop an epidemic on a network.

» "herd immunity" → when S is very small ($R < 1$)

naturally acquired $\quad I \rightarrow R$

vs.

vaccinations $\quad S \rightarrow R$



SIR-V model

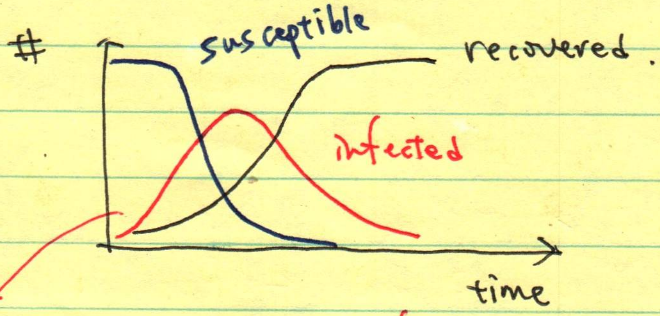
network strategies

1) high deg. nodes vax first
2) cut bridges between modules
3) vax. essential workers.
4) vax. vulnerable.

④

We have learned ......

the compartmental model ... on a network.



simple, idealized epidemics (unimodal, smooth)

assumptions:

closed (no new nodes join)
dynamics are fixed
well mixed (either clique or G(n,p))

But, usually ...

* See Figure 1 of Lecture 10's notes.
* See Fall 2022 scribbles: COVID-19 pandemic (yr 1, fully immune-naive population)

What do you see? - periodicity
- seasonality (hemisphere)
- increasing trend

All not reproduced by SIR

multimodal, highly irregular ...

§ What drives this heterogeneity ?

>> G varying over time (social distancing ; waxing / waning)
("community structure" may play a role)

>> Re-infection is possible (SIS)
or, duration in R is short

$$
\begin{tikzcd}
S \arrow[r] & I \arrow[d] \\
& R \arrow[ul]
\end{tikzcd}
$$

>> Population density → more exposures.

>> pathogen evolution

>> bad data, etc.

①

Lecture 10b. Social "contagions" Mar 21, 2024.

Still the key idea: edges are the mechanism of transmission

Two standard models of social distancing: $\Gamma = \{ \underset{\text{inactive}}{I}, \underset{\text{active}}{A} \}$

» Independent cascade model (IC)

Kempe et al (2003)

(gossip model) — cost of activation is <u>low</u>.

P_{ij} = information $i \rightarrow j$

"Influence maximization"

transmission is independent of each other.

Granovetter — Schelling (1978)

» Linear threshold model (LT)

(peer pressure model, aka, social reinforcement)

threshold $\theta_u = U(0,1)$

activation function for u

$$\sum_{(u,v) \in E} I[x_v = A] \times \frac{1}{k_u} \geq \theta_u$$

Two examples (see Lecture Notes)

IC : transmission network is always a tree.

end if no new activate nodes

» When over a cascade will high degree nodes tend to activate? (early in cascade)

» what process does IC reduce to if $p = 1$? (BFS tree)

LT: » when over a cascade will ________? (activate "later" in cascade)

» what process does LT reduce to if $\theta_u = \frac{1}{k_u}$? (IC w/ $p=1$)

(2)

Simple vs. Complex contagion
(IC) (LT)

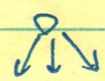
>> see Guibeault & Centola (2021)

Gist: locally-tree structure is hard to mediate complex contagion
(no ladder)

What do real social contagions look like (on Twitter)

>> see Goel et al. (2015)

key ingr. 10^9 cascades

- $\langle S \rangle = 1.3$
- 99% only friends retweet 
- 0.025% go to ≥ 100 nodes.

"punctuated equilibrium"

"viral evolution"

See Canvas (and next page) for the Papers that we discussed!

Final question for the class:

What kind of things in your real life have felt like network influence?

Venmo.

③ Papers that we discussed on Mar 19.

Paper 1 The hidden geometry of complex, network-driven contagion phenomena (Science, 2013)

Link 1 www.nextstrain.org / "punctuated equilibrium"

Paper 2 The structural virality of online diffusion (Management Science, 2015)

Paper 3 Topological measures for identifying and predicting the spread of complex contagions (nature communications, 2021)

Paper 4 The spread of behavior in an online social network experiment (Science, 2010)

Paper 5. The role of social networks in information diffusion (WWW, 2012)

Paper 6. The diffusion of microfinance (Science, 2013)