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Mar 12, 2024

Lecture 9a : Spreading Processes on Networks.

- Project proposal / presentation / writeup reminder.
- PS 5, 6 smaller

- dynamics on a network (assume G is fixed relative to timescale of dynamics)

- what things spread across a network ?
  » (ask for examples)

- state variable $x_i \in \Gamma$

  (transportation "people navigating" (not spreading))

  x_i state variable on each node i
  Γ various states possible

update function $x_i^{t+1} = \cancel{f(G)}$ instead of .. $= f(x^t_{nb(i)})$

} Key idea: Network edges are the mechanism of transmission.
(representation choice !)

- what are the nodes ?
  » (ask for examples)
- what are edges ?
  » (ask for examples)
- Three kinds of spreading processes on networks. (non-exhaustive)
  - (this wk) » Epidemic models (bio pathogens)
  - (next wk) » Social adoption models (preferences)
  - » Biological computation models (neural cascades, boolean networks)

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Two dichotomies

- continuous v.s. discrete time

$$G_t \xrightarrow{f} G_{t+1} \xrightarrow{f} \cdots$$

- simple v.s. complex contagions

bio pathogen — costly social behavior. (peer pressure modeling)

— low-cost social behavior

Epidemic models of spreading

epidemiology : A study of all diseases.

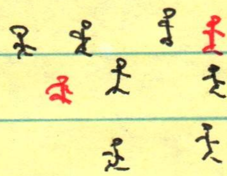
infectious disease ep: : diseases that spread between people

network epi : diseases that spread on a social network.

populations

- individuals
- internal systems
- tissues
- cells

molecules



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Kermack–McKendrick

ID ep: warm up : the compartmental model (from 1927 !)

- let's first ignore the network (assume all-to-all connections, also called "well-mixed")

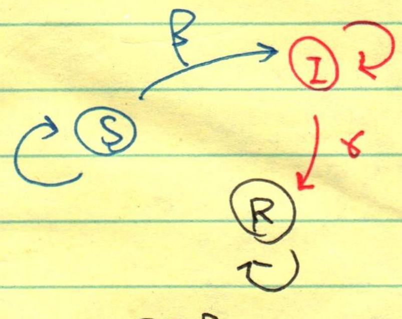
- each node has a state variable

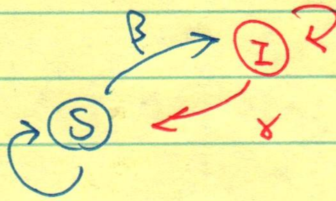
$$x_i \in \{S, I, R\}.$$
$$x_i \in \{S, I\}$$

S: susceptible — not infected + can be infected.
I: infected / infectious
R: recovered / removed — not infected + cannot be infected again.

- define state transition rules to update each x_i
  these rules are a model of the disease.

state diagrams $S \to I_1 \to I_2 \to I_3 \to R$





SIR

- chicken pox
- measles

* anything you can get a childhood vaccine for

SIS

influenza   common cold.
≈ covid

* anything for which infection does not confer long-standing immunity

§ Specifying SIR / SIS. (see figure on previous page)

β : transmission rate of infection

higher β : spreads faster / faster rate of $S \to I$.

lower β : slower spread / S stays larger for longer.

r : recovery rate from infection

higher r : faster recovery

lower r : more time in I.

§ SIS dynamics

- let N be total population (constant)
- an infected person transmits disease to a susceptible ($S \to I$) person w/ prob. $\bar{\beta}$ per unit time
- an infected person recovers ($I \to S$) w/ prob r per unit time
- let $S(t)$ and $I(t)$ denote # S and # I ;

  ≫ Note :

  $$N = S(t) + I(t)$$

  at $t=0$ $S(0) = N-1$, $I(0) = 1$.

- how $S(t)$ evolves

$$S(t+\Delta t) = S(t) - \underbrace{[S(t) + \bar{\beta}\, \Delta t\, I(t)]}_{S \to I} + \underbrace{[I(t) \times r\, \Delta t]}_{I \to S}$$

≫ Then we convert difference eq'n to a differential eq'n.

» variables

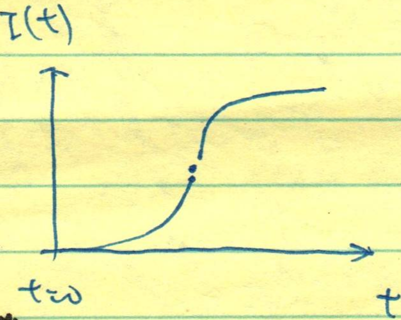
$$s = \frac{S}{N} \qquad i = \frac{I}{N} \qquad \beta = \bar{\beta} N$$

» rate equation

divide by Δt.

$$\frac{di}{dt} = \beta i (1-i) - \gamma i$$

solution



Question: Will an epidemic spread? (given small seed)

- let i be very small (relative to N), so, approx. $(1-i) \approx 1$.

$$\frac{di}{dt} \approx \beta i - \gamma i = i(\beta - \gamma)$$

- simple ODE w/ $\boxed{\text{logistic function}}$ as solution

$$\frac{\beta}{\gamma} : \begin{cases} >1 & \text{exponential spread} \\ =1 & \text{high var.} \\ <1 & \text{does not spread.} \end{cases}$$

Highlights:

$R_0 \equiv \beta / \gamma$ (exponential rate when $i \ll 1$)

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See Lecture Notes.

$$\frac{di}{dt} = 0$$
$$i(\infty) = 1 - \beta/r.$$

page 7 :
(2.2.2)

left → infected eq. at (steady state) $1 - \beta/r$
(SIS)

right
(SIR)

SIR in the wild :

ideal cond. for SIR are rare (what are they ?)

conditions.
- 100 % naive pop.
- closed pop.
- "homogeneous" mixing.

but, 1978 influenza outbreak @ English boarding school w/ $n = 763$ boys.

≈ 2.1 days infectious before isolating.

$\hat{\beta} = 0.00234$

$r = 0.476.$

character / models ! compartmental !

Covid edition : SEIR model w/ death and vaccinations.

See Pubar et al., Science (2021)
- vaccinated / unvaccinated / no vacc. (refused).
- vaccination rate / infected fatality rate (IFR), etc.

exponential growth : covid-19 $I(t) \approx R_0^{\,t}$.

See page 9

Lecture 16: March 14, 2024

" Snow day "

- See the Jupyter Notebook for basic epidemic simulations (on Canvas)