Linear Algebra

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May 2024

Inro

when we talk about systems of linear equations, if it is a simple equation like this,

$$2x + y = 3$$
$$x + 2y = 4$$

It is easy for you to solve it using your intuition, but if we have many equations, how can we know the answer of systems of equations? Also, I hope you can think of how we can determine how many solutions to those equations there are. These all lead us to explore.

Gaussian Elimination

If there is a problem, you will find that it already has the answers. We are not machines, and we can't solve complicated questions easily. Because when the problems become bigger, we don't know how to deal with their complexity. So, we can find a general and systematic way to solve it.

I won't introduce how we use Gaussian elimination; it's not a difficult thing, and you can find it in many books. I think the most important thing in your learning is to understand why we use it and to combine this knowledge with what you already know.

Why Gaussian Elimination Makes It Easy

It's a good question that needs to be answered. You can solve the two linear equations easily, but if there are 7 equations and 7 unknowns, I think it is not easy for you. So, when we meet a complex problem, how can we solve it. This question is just like asking you to solve the basic multiplication problem: $9999 * 9999$. You will think that is not a difficult question because you know the basic rules for solving it. But you will also think it is so complicated that you may make errors if you try to solve it. It's normal to think of something like this, so if we want to solve it, the first thing is to reduce the complexity, because we already know a feasible way to do it. How can we reduce the complexity? It's just like asking you a question—what is an easy way to solve this question, or what are you familiar with? We are good at solving something we have already seen, so we can try to transform it into the form we are familiar with. Think about this: $(10000 - 1) * (10000 - 1)$. Wow! It's an easy way to solve it because we are familiar with $10^n * 10^n$, and the addition or subtraction is easier than multiplication and division.

So, follow this idea, we come up with the Gaussian elimination. Of course, they have many different faces, but I think it gives us inspiration to solve the systems of linear equations.

Consider about the answer to 4 equations with 4 unknowns; the end answer will look like this:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

We would like to reduce our function to look like this, but it's not easy to do this. First, we should ensure that when we operate our liner equations, the answer of the original systems of equations is the same as the new systems of equations. So, now think about a linear equation.

$$ax + by = c$$

I know that all of you are familiar with this simple equation. Now, if we **add a constant** on the same side of this equation, will it change the solution of this equation?

$$ax + by + m = c + m$$

Of course, it will not change the solution because we can minus m on both sides to get the original linear equation. Then think about the two equations:

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

If we multiply the first equation by m and then add it to the second equation, will it change the solution of systems of linear equations? If you can't understand it, think about the inverse operation if we have that two equations? And if we do the inverse operation, we will get the same two equations as the original. The end idea is to change the location of two equations. It doesn't need any explanation, because we all know it.