SVD Decomposition.

$$A = U \Sigma V^T$$

(U) $m \times m$ orthogonal

$m \times n$ diagonal

$n \times n$ orthogonal

$$A = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix}$$

如果用 registration 的角度 $\Rightarrow$ (rotate) (stretch) (rotate)

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orthogonal

$\Rightarrow A A^T = A^T A = I$

$A = U \Sigma V^T$ (singular vector, singular value)

$\Sigma \Rightarrow \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \end{bmatrix}$

$\Rightarrow A^T A = (V \Sigma^T U^T) U \Sigma V^T$

$U^T U = I$

$= V(\Sigma^T \Sigma) V^T$

eigen value $\begin{bmatrix} \lambda & & \\ & \lambda & \\ & & \end{bmatrix}$ λ for $A^T A$, σ^2 for A

$\Rightarrow A A^T = U \Sigma V^T (V \Sigma^T U^T)$

$V^T V = I$

$= U(\Sigma \Sigma^T) U$

eigen vector

example:

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} \sqrt{10} & \\ & 0 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

important.

left singular vector — singular value. — right singular vector

example 2:

person 1 2 3

gene 1 2 3 4

$$\begin{bmatrix} & & \\ & & \\ & & \\ & & \end{bmatrix} = [\;|\;][\;\cdot\;][\;—\;]$$

most important / most information

biggest variance

PCA $\Longleftarrow$ $u_1 \sigma_1 v_1^T$

unit — 變異數 — unit (單位向量)