PHD Filtering for Multi-Source DOA Tracking With Extended Co-Prime Array: An Improved MUSIC Pseudo-Likelihood

Jun Zhao, Renzhou Gui, and Xudong Dong

***Abstract*— To solve the problem of multi-source direction of arrival (DOA) tracking in co-prime array, a multi-source DOA tracking algorithm based on probability hypothesis density (PHD) filtering is proposed, which can adapt to the scenario where DOA and number of sources change with time. In this letter, we use the minimum description length (MDL) method to estimate the number of sources and construct a new noise subspace by performing eigenvalue decomposition (EVD) on the reconstructed signal subspace. An improved multiple signal classification (MUSIC) pseudo-spectrum is utilized to calculate the likelihood function of the proposed method. The likelihood function is further exponentially weighted to increase the weight of particles. Simulation results show that compared with the existing methods, this algorithm has better tracking performance.**

***Index Terms*— Direction-of-arrival (DOA) tracking, probability hypothesis density (PHD), likelihood function, co-prime array.**

I. Introduction

Multi-source direction of arrival (DOA) estimation is an essential topic in array processing, which has been applied to radar, sonar, wireless communication, and source location [1]–[5]. Compared with a traditional uniform linear array (ULA), the sparse array [6]–[8] is helpful to obtain a larger array aperture, degree of freedom (DOF) and higher resolution. Among them, the extended co-prime array (ECA) [7] can use the same number of actual array elements to generate more virtual array elements, thus improving the estimation performance. However, the methods mentioned above need to know the number of static sources beforehand. In fact, the sources may be dynamic, and the number is unknown. Therefore, it is necessary to develop a real-time DOA tracking algorithm.

Recently, the compressed sensing (CS) method can be used to solve the DOA estimation problem [9]–[11], in which the orthogonal matching pursuit (OMP) algorithm [11] has better estimation performance under a low signal-to-noise ratio (SNR). However, these methods are based on grid class, Off-grid DOAs may lead to mismatches. In some applications, it is usually considered that the source is static. Fortunately, the DOA tracking problem has also received significant attention [12]–[14]. The particle filter (PF) method, as a Sequential Monte Carlo method based on recursive Bayesian estimation theory, which can effectively estimate nonlinear problems [15]. In [16], the PF algorithm is used to solve the DOA tracking problem of the co-prime array, in which the propagator method (PM) is used to the likelihood function of PF. However, there are some issues in the methods mentioned above: (1) PF method can only solve the real-time tracking problem of a single source. (2) The number of sources changes with time, which is no longer within the scope of PF. (3) How to model the process with a time-varying number of moving sources is a key issue.

Mahler introduced the random finite set (RFS) [17] in recent years, which can model the above sources well. Among them, the recursive method of probability hypothesis density (PHD) [18] propagates the posterior intensity of target RFS in time. In [19], PHD and cardinal probability hypothesis density (CPHD) algorithms are used for multi-target tracking of the network microphone array, which is helpful to solve the problem of multi-sources time-varying DOA tracking. The PF algorithm based on random RFS [20], can also solve the time-varying DOA tracking problem. However, most of these methods are based on ULAs and need to be improved in terms of DOA estimation performance.

Based on the above analysis, a DOA tracking algorithm with co-prime array based on a PHD filter is proposed, and our contributions are as follows: (1) Calculate the number of targets each time by using the minimum description length (MDL, [21]) algorithm. (2) We propose the measurement separation method and construct $P(k)$ noise subspaces. (3) Since the likelihood function tends to be flat in the update step when SNR is low or the snapshot is insufficient, an improved MUSIC spectrum likelihood function is proposed, which is exponentially weighted to make it easier for particles to move to the high likelihood region.

Notations: we use upper-case (lower-case) bold characters to indicate matrices (vectors). $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent the conjugate, transpose and conjugate transpose. $diag(\cdot)$ is the diagonal matrix and $vec(\cdot)$ is the vectorization operation. $\odot$ and $\otimes$ are the Khatri-Rao product and Kronecker product, respectively. $\mathbb{E}$, $\mathbf{I}_N$ and $|\cdot|$ stand for the expectation operator, $N \times N$ identity matrix and module of complex value.

II. Background

A. The Model of Extended Co-Prime Array

Fig. 1 shows an extended co-prime array (ECA), and the position of array sensors can be written as

$$\mathbb{L} = \{Nnd, 0 \leqslant n \leqslant 2M-1\} \cup \{Mnd, 0 \leqslant n \leqslant N-1\}, \quad (1)$$

where N and M are co-prime integers, $d = \lambda/2$ is unit array spacing and λ represents the wavelength of signal. Let $\mathrm{O} = \{l_1, l_2, \ldots, l_{N+2M-1}\}$, $l \in \mathbb{L}$ represents the sensors location and $l_i < l_j, l_1 = 0$ if $i < j$.

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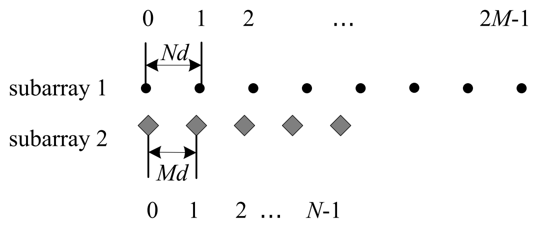

Fig. 1. The extended co-prime array (ECA).

Assuming that $P(t)$ narrow band sources $s_p(t), p = 1, \ldots, P(t)$ with DOA θ_p impinging on the ECA at time *t*, the received signal can be expressed as

$$\mathbf{Z}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{A} = \left[\mathbf{a}(\theta_1), \ldots, \mathbf{a}\left(\theta_{P(t)}\right)\right]$ is steering matrix with direction vector $\mathbf{a}(\theta_p) = \left[1, \ldots, e^{-j\pi l_{N+2M-1}\sin\theta_p}\right]^T$. $\mathbf{s}(t) = \left[s_1(t), s_2(t), \ldots, s_{P(t)}(t)\right] \in \mathbb{C}^{P(t)\times 1}$ is the signal vector, $\mathbf{n}(t)$ represents additive white Gaussian noise with mean value $\mathbf{0}$ and variance σ_n^2.

The covariance matrix of the received signal [7] can be expressed as

$$\mathbf{R}_X = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma_n^2\mathbf{I}_{N+2M-1}, \tag{3}$$

where $\mathbf{R}_s = \mathbb{E}\left[\mathbf{s}(t)\mathbf{s}^H(t)\right] = diag\left(\sigma_1^2, \ldots, \sigma_p^2, \ldots, \sigma_{P(t)}^2\right)$ denoting the source covariance matrix with σ_p^2 denoting the *p*-th signal power. In practice, covariance matrix is estimated using the L available samples

$$\mathbf{R}_X = \sum_{l=1}^{L} \frac{\mathbf{Z}(t_l)\mathbf{Z}^H(t_l)}{L}. \tag{4}$$

According to [7], by vectorizing (3), we have

$$\mathbf{z} = vec(\mathbf{R}_X) = (\mathbf{A}^* \odot \mathbf{A})\mathbf{p} + \sigma_n^2 vec(\mathbf{I}_{N+2M-1}), \tag{5}$$

where $\mathbf{A}^* \odot \mathbf{A} = \left[\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \ldots, \mathbf{a}^*\left(\theta_{P(t)}\right) \otimes \mathbf{a}\left(\theta_{P(t)}\right)\right]$ is the extended direction matrix, $\mathbf{p} = \left[\sigma_1^2, \sigma_2^2, \ldots, \sigma_{P(t)}^2\right]^T$ is the signal power. From [22], $\mathbf{z}$ contains many redundant items. After the redundant items are removed, new received data $\tilde{\mathbf{z}}$ can be obtained

$$\tilde{\mathbf{z}} = \tilde{\mathbf{A}}\mathbf{p} + \sigma_n^2\widetilde{\mathbf{I}}, \tag{6}$$

where $\tilde{\mathbf{A}}$ is the direction matrix by removing the redundant items of $(\mathbf{A}^* \odot \mathbf{A})$ and $\widetilde{\mathbf{I}}$ is corresponding matrix.

B. Toeplitz Reconstruction Method

According to (6), the signal no longer satifies the irrelevant condition. In this section, the Toeplitz reconstruction method [22] is used for decoherence. As mentioned in [7], the received signal on the virtual ULA located in $[-Y+1, Y-1]$ $(Y = M(N+1))$ of $\tilde{\mathbf{z}}$ can be written as

$$\tilde{\mathbf{z}}_1 = \tilde{\mathbf{A}}_1\mathbf{p} + \sigma_n^2\widetilde{\mathbf{I}}_1, \tag{7}$$

where $\tilde{\mathbf{A}}_1$ is the direction matrix of virtual ULA, and $\widetilde{\mathbf{I}}_1$ is a $(2Y-1)\times 1$ vector.

By using Toeplitz reconstruction method [22] on (7), we can obtain

$$\mathbf{R}_T = \begin{bmatrix} \tilde{\mathbf{z}}_1(0) & \tilde{\mathbf{z}}_1(-1) & \cdots & \tilde{\mathbf{z}}_1(1-Y) \\ \tilde{\mathbf{z}}_1(1) & \tilde{\mathbf{z}}_1(0) & \cdots & \tilde{\mathbf{z}}_1(2-Y) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{z}}_1(Y-1) & \tilde{\mathbf{z}}_1(Y-2) & \cdots & \tilde{\mathbf{z}}_1(0) \end{bmatrix}. \tag{8}$$

Then the rank restored data covariance matrix $\mathbf{R}_T \in \mathbb{C}^{Y\times Y}$ is obtained. And subspace based method can be applied for DOA estimation.

C. Source Motion Model and PHD Theory

1) Source Motion Model: Suppose $\mathbf{x}_t = \left[\theta_t, \dot{\theta}_t\right]^T$ denotes the source state at time *t*, where θ_t and $\dot{\theta}_t$ stand for the DOA and velocity, and the constant velocity (CV) model

$$\begin{aligned} \mathbf{x}_t &= \mathbf{F}_t\mathbf{x}_{t-1} + \mathbf{G}_t\mathbf{v}_t \\ &= \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix}\mathbf{x}_{t-1} + \begin{bmatrix} \Delta T^2/2 \\ \Delta T \end{bmatrix}\mathbf{v}_t, \end{aligned} \tag{9}$$

where ΔT represents the sampling time interval, where $\mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_t)$ is the noise term following the zero-mean Gaussian process.

2) PHD Theory Based on RFS: Since traditional DOA estimation methods can not solve the tracking problem of time-varying sources, we try to use a new method based on RFS to solve the time-varying DOA. According to [17], the RFS $\mathbf{X}$ can be expressed as

$$\mathbf{X}_t = \left\{\mathbf{x}_{t,1}, \mathbf{x}_{t,2}, \ldots, \mathbf{x}_{t,P(t)}\right\}. \tag{10}$$

Similarly, the outputs of ECA can only form a set, which can be represented as

$$\mathbf{Z}_t = \{\mathbf{Z}(t)\}. \tag{11}$$

From [18], the expressions of PHD filters recursion in the prediction and update steps are defined as

$$\begin{aligned} &D_{t|t-1}(\mathbf{x}) \\ &= \int p_{S,t}(\xi) f_{t|t-1}(\mathbf{x}|\xi) D_{t-1}(\xi) d\xi + \gamma_t(\mathbf{x}), \end{aligned} \tag{12}$$

$$\begin{aligned} &D_t(\mathbf{x}) \\ &= q_{D,t}(\mathbf{x})D_{t|t-1}(\mathbf{x}) + \frac{p_{D,t}(\mathbf{x}) g_t(\mathbf{Z}_t|\mathbf{X}_t) D_{t|t-1}(\mathbf{x})}{\int p_{D,t}(\xi) g_t(\mathbf{Z}_t|\xi) D_{t|t-1}(\xi) d\xi}, \end{aligned} \tag{13}$$

where

- $p_{S,t}(\xi)$ is the survival probability that a source ξ still exist at time *t*;
- $\gamma_t(\mathbf{x})$ is the intensity of the birth RFS at time *t*;
- $p_{D,t}(\mathbf{x})$ is the detection probability of state $\mathbf{x}$ at time *t*, and $q_{D,t} = 1 - p_{D,t}$;
- $f_{t|t-1}(\cdot|\cdot)$ represents the multi-target transition density;
- $g_t(\cdot|\cdot)$ represents the multi-target likelihood function.

III. Innovation

In this section, we will propose the measurement separation method and calculate the likelihood function for the proposed PHD method.

A. Measurement Separation (MSS) Method

By performing eigenvalue decomposition (EVD) on (8), we can get

$$\mathbf{R}_T = \mathbf{\Lambda}_s \mathbf{\Sigma}_s \mathbf{\Lambda}_s^H + \mathbf{\Lambda}_n \mathbf{\Sigma}_n \mathbf{\Lambda}_n^H , \tag{14}$$

where $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_n$ represent the matrix composed of eigenvectors corresponding to $P(t)$ maximum eigenvalues and the remaining $Y - P(t)$ eigenvalues, respectively. In addition, $\mathbf{\Sigma}_s = diag\left\{\tau_1, \ldots, \tau_p, \ldots, \tau_{P(t)}\right\}$ is the diagonal matrix with τ_p denoting the p-th eigenvalues, and $\mathbf{\Sigma}_n = diag\left\{\tau_{P(t)+1}, \ldots, \tau_Y\right\}$ represents a diagonal matrix consisting of the remaining $Y - P(t)$ eigenvalues.

Remark 1: It is worth noting that we are considering the problem of DOA time-varying tracking, and the source will change with time. We use the MDL [21] method to estimate the number of signal sources $P(t)$ at time t, then the noise subspace $\mathbf{\Lambda}_s$ and signal subspace $\mathbf{\Lambda}_n$ can be obtained.

We calculate the covariance matrix again by the MSS method, and the new covariance matrix is reconstructed as

$$\mathbf{\Gamma}_p = \mathbf{\Lambda}_{s,p} \mathbf{\Sigma}_{s,p} \mathbf{\Lambda}_{s,p}^H, \tag{15}$$

where $\mathbf{\Lambda}_{s,p}$ is the p-th eigenvector (column) of $\mathbf{\Lambda}_s$ and $\mathbf{\Gamma}_p$ is the new covariance matrix, $p = 1, 2, \ldots, P(t)$. By performing EVD again of $\mathbf{\Gamma}_p$, we can obtain the new noise subspace $\mathbf{U}_{n,p}$.

B. The Improved MUSIC Pseudo-Likelihood Function

As mentioned above, $\mathbf{\Gamma}_p, p = 1, 2, \ldots, P\left(t\right)$ is the reconstructed covariance matrix. Perform EVD on $\mathbf{\Gamma}_p$, and only the p-th column corresponding to the eigenvector is considered as the signal subspace $\mathbf{U}_{s,p}$ each time, and the rest eigenvectors are regarded as the noise subspace $\mathbf{U}_{n,p}$, which satisfies $\mathbf{U}_{n,p}^H \tilde{\mathbf{a}}\left(\theta_p\right) = \mathbf{0}$. $\tilde{\mathbf{a}}\left(\theta_p\right) = \left[1, \ldots, e^{-j\pi(Y-1)\sin\theta_p}\right]^T$ denotes the direction vector of the virtual ULA.

In traditional MUSIC algorithm [3], if the noise subspace is obtained, the DOA estimation is realized by the minimum optimization search, namely

$$\theta_{\text{MUSIC},p} = \arg\min_{\theta} \tilde{\mathbf{a}}^H\left(\theta_p\right) \mathbf{U}_{n,p} \mathbf{U}_{n,p}^H \tilde{\mathbf{a}}\left(\theta_p\right). \tag{16}$$

Then, the spectral peak search becomes

$$P_{\text{MUSIC},p} = \frac{1}{\tilde{\mathbf{a}}^H\left(\theta_p\right) \mathbf{U}_{n,p} \mathbf{U}_{n,p}^H \tilde{\mathbf{a}}\left(\theta_p\right)}. \tag{17}$$

Remark 2: MUSIC spatial spectrum is used to determine the direction of arrival, which shows a peak near the actual signal. In the process of particle filtering, if the likelihood function of a particle is significant, its weight value will be more considerable. Similarly, in PHD filtering, assuming that a particle is close to the true DOA, it will show that the peak value of MUSIC spatial spectrum is close to the true DOA, which means that a larger weight value can be obtained in the process of particle filtering. It is feasible for us to consider using MUSIC spatial spectrum function instead of the likelihood function of particles. In addition, when the SNR is low and there are few snapshots, the likelihood function becomes flat. The particle likelihood function value becomes smaller, which is unfavourable for subsequent resampling. Therefore, we chose exponential weighting (17) to improve the peak performance of the likelihood function. The enhanced likelihood function can be expressed as

$$g_p\left(\mathbf{\Gamma}_p|\mathbf{x}\right) = \left|\frac{1}{\tilde{\mathbf{a}}^H(\mathbf{Cx}) \mathbf{U}_{n,p} \mathbf{U}_{n,p}^H \tilde{\mathbf{a}}\left(\mathbf{Cx}\right)}\right|^r, \tag{18}$$

where $p = 1, \ldots, P(t)$ and $\mathbf{C} = [\mathbf{1}, \mathbf{0}]$, $\mathbf{Cx}$ denotes the DOA information of the source and $r \in \mathbf{R}^+$ represents the exponential.

C. The Proposed PHD Method

Assuming that there are N_{t-1} particles at time $t-1$, then the survival particles and weight can be expressed as

$$\mathbf{x}_{p,t|t-1}^i = \mathbf{F}_t \mathbf{x}_{t-1}^i + \mathbf{G}_t \mathbf{v}_t, \quad i = 1, 2, \ldots, N_{t-1}, \tag{19}$$

$$\omega_{p,t|t-1}^i = \omega_{t-1}^i \cdot p_{S,t}\left(\mathbf{x}_{t-1}^i\right), \tag{20}$$

where $p_{S,t}\left(\cdot\right)$ represents the survival probability.

Similarly, assuming that the number of sampled particles of the newborn target at time t is J_t, then the particles of the newborn target are sampled according to the Gaussian distribution, which can be expressed as

$$\left(\mathbf{x}_{t,b}^i, \omega_{t,b}^i\right) \sim \mathcal{N}\left(\mathbf{m}, \mathbf{P}\right), \tag{21}$$

where $i = 1, 2, \ldots, J_t$, $\mathbf{m}$ is the mean and $\mathbf{P}$ is the corresponding covariance. Then the particles in the prediction step can be united and expressed as

$$\begin{aligned}&\left\{\mathbf{x}_{t|t-1}^i, \omega_{t|t-1}^i\right\}_{i=1}^{N_{t-1}+J_t}\\ &\quad = \left\{\mathbf{x}_{p,t|t-1}^i, \omega_{p,t|t-1}^i\right\}_{i=1}^{N_{t-1}} + \left\{\mathbf{x}_{t,b}^i, \omega_{t,b}^i\right\}_{i=1}^{J_t}.\end{aligned} \tag{22}$$

In the update step, according to section III-A and III-B, we use measurement separation technology to calculate each particle weights of $P(t)$ new covariance matrix, then the updated weight at time t can be expressed as

$$\begin{aligned}\omega_{t,p}^i =& \left(1 - p_{D,t}\left(\mathbf{x}_{t|t-1}^i\right)\right) \cdot \omega_{t|t-1}^i\\ &+ \left(\frac{p_{D,t}\left(\mathbf{x}_{t|t-1}^i\right) g_{t,p}\left(\mathbf{\Gamma}_p|\mathbf{x}_{t|t-1}^i\right) \omega_{t|t-1}^i}{\sum_i p_{D,t}\left(\mathbf{x}_{t|t-1}^i\right) g_{t,p}\left(\mathbf{\Gamma}_p|\mathbf{x}_{t|t-1}^i\right) \omega_{t|t-1}^i}\right) \omega_{t|t-1}^i,\end{aligned} \tag{23}$$

where $g_{t,p}\left(\mathbf{\Gamma}_p|\mathbf{x}_{t|t-1}^i\right), i = 1, \ldots, N_{t-1} + J_t$ represents the likelihood function of particle $\mathbf{x}_{t|t-1}^i$.

According to the size of the weight, the re-sampling step [18] is utilized to select effective particles, which can be defined as

$$\left[\left\{\mathbf{x}_t^i, \omega_t^i\right\}_{i=1}^{N_t}\right] = \text{re-sampling}\left\{\mathbf{x}_{t|t-1}^i, \omega_{t|t-1}^i\right\}_{i=1}^{N_{t-1}+J_t}.$$

It should be noted that the number and state of the targets can be clustered by performing K-means operation on the updated particles [19]. Assuming that particle set $\left\{\left\{\mathbf{x}_p^i, \omega_p^i\right\}_{i=1}^{N_p}\right\}_{p=1}^{\hat{P}(t)}$ is obtained by K-means method, then the DOA estimation value can be calculated by

$$\mathbf{x}_{p,t} = \sum_{i=1}^{N_p} \mathbf{x}_p^i \omega_p^i, \quad p = 1, \ldots, \hat{P}\left(t\right), \tag{24}$$

where $\hat{P}\left(t\right)$ is the estimated number of sources by performing K-means operation at time t.

IV. Simulation Results

A. Computational Complexity

In this section, the multiplication times are used as the standard of complexity. The computational complexity of the proposed method is about $O\{T[2Y^3 + 2Y^2 + Y + N_t(P_t(Y^2(Y-1) + Y^2 + 2Y + 2) + 6 + \hat{P}_t)]\}$, including the EVD, prediction, update and the state extraction, where $Y = M(N+1)$, $T = 50s$ denotes the total observation time, N_t is the total number of particle at time t. $P_t = P(t)$ is the number of signals estimated by MDL method and $\hat{P}_t = \hat{P}(t)$ is the estimated number of signals by k-means method at time t. By constract, the computational complexity of RFS-PF [20] method based on ULA is $O\{T[P^3 + N_t(P^2 P_t + P^2 + 2P + P^2 + 6)]\}$, where $P = 2M + N - 1$ is the number of array elements. The OMP [11] method computational complexity is $O\{T[2\Delta(2Y-1) + \sum_{i=1}^{P_t} i^3 + (2Y-1)(i^2 + 3i)]\}$, where Δ denote the grid number of DOA, in next subsection, the grid DOA ranges from -90° to 90° with increase of 0.1°, then $\Delta = 1801$. The computational complexity of the SS-PAST [14] method is $O\{T[3(Y-1)^2(2Y-1) + (Y-1)[2Y - 1 + 2(2Y-1)^2 + (2Y-1)^3] + (2Y-1)^2]\}$. Obviously, for RFS-PF, OMP and SS-PAST, the computational complexity grows exponentially with respect to P_t and Y. Therefore, when P_t and Y takes very large values, the RFS-PF, OMP and SS-PAST methods have heaver computational load than the proposed method.

B. RMSE Versus Different Scenarios

To highlight the good performance of our proposed method, we compare the proposed method with the existing DOA tracking algorithms, such as SS-PAST [14], PF-PM [16], OMP [11], and RFS-PF [20]. Two simulation scenarios (single-source tracking scenario and multi-source tracking scenario) are provided, and CRLB [23] was added. In the following simulations, $M = 3, N = 5$, and the number of ULA elements is 10. The root mean square error (RMSE) is utilized to judge the estimation performance and is defined as:

$$\mathrm{RMSE} = \frac{1}{\hat{P}(t)} \sum_{p=1}^{\hat{P}(t)} \frac{1}{MC} \sum_{j=1}^{MC} \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\mathbf{x}_{p,tj} - \bar{\mathbf{x}}_{p,t})^2}, \quad (25)$$

where $\mathbf{x}_{p,tj}$ is the estimated DOA of p-th sources in j-th monte carlo (*MC*) trails at time t, T is the total observation time.

1) Case 1: A Single Source Tracking Scenario: The survival time of the single source is $1-50$s, and initial source state is $\mathbf{x}(0) = [-50; 0.5]$. Other parameters: $p_{D,t}(\cdot) = 0.98$, $p_{S,t}(\cdot) = 0.99$, $r = 10$. We assume that there is only one newborn source at time t, which obeys Gaussian distribution $(\mathbf{x}_{t,b}, \omega_{t,b}) \sim \mathcal{N}(\mathbf{m}, \mathbf{P}), i = 1, 2, \ldots, J_t$, where $\mathbf{m} = [-50, 0]$ and $J_t = 1000$.

Fig. 2(a) show the RMSE under different SNR, where $L = 200, MC = 100$. It can be seen from Fig. 2(a) that with the increase of SNR, the RMSE of the four algorithms decreases. The performance of the proposed method is better than the other three algorithms, and it can also show good tracking performance when SNR $= -10$ dB. Fig. 2(b) depicts the RMSE under the different number of snapshots. Again,

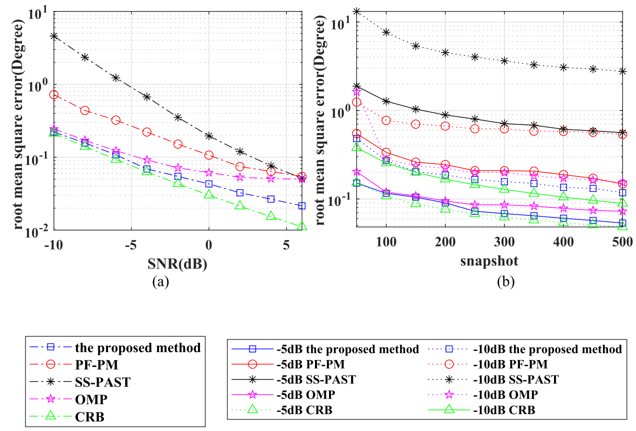

Fig. 2. (a) The RMSE under different SNR, $L = 200$. (b) The RMSE under different SNR versus snapshots.

TABLE I

Target Motion States

source	Survival time/s	Initial source	Velocity(rad/s)
1	1-40	-60	2.0
2	10-50	40	-3.0
3	20-30	50	-1.0
4	25-50	80	-1.5

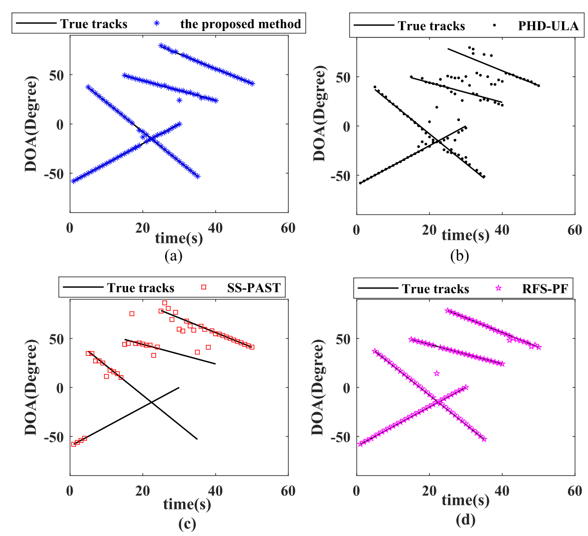

Fig. 3. The trajectory tracking versus time, $MC = 1$.

the performance of these algorithms when SNR is -5 dB is better than that when SNR is -10 dB. Comparatively speaking, the proposed method can always show better DOA estimation performance.

2) Case 2: Multi-Source Tracking Scenario: Consider a nonlinear multi-target scenario with four sources. The survival time and initial state parameters of these sources are shown in Table I. The newborn sources at each time step are defined as $\left(\mathbf{x}_{t,b}^i, \omega_{t,b}^i\right) \sim \mathcal{N}(\mathbf{m_c}, \mathbf{P}), c = 1, 2, 3, 4, i = 1, 2, \ldots, J_t$, where $\mathbf{m}_1 = [50, 0]$, $\mathbf{m}_2 = [40, 0]$, $\mathbf{m}_3 = [80, 0]$, $\mathbf{m}_4 = [-60, 0]$, $\mathbf{P} = diag\{[4, 2]\}$, and $J_t = 1000$. Other experimental parameters are the same as case 1.

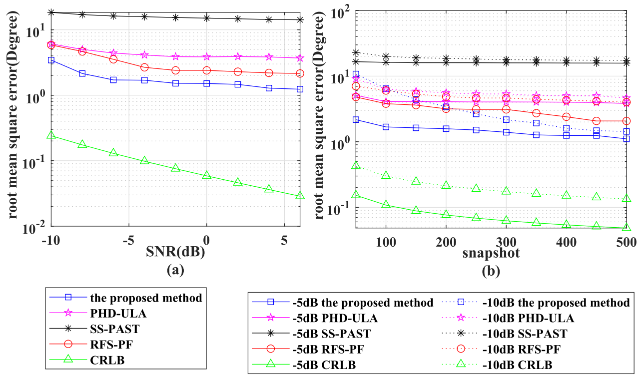

Fig. 4. (a) The RMSE under different SNR, $L = 200$, $MC = 100$. (b) The RMSE under different SNR versus snapshots, $L = 200$, $MC = 100$.

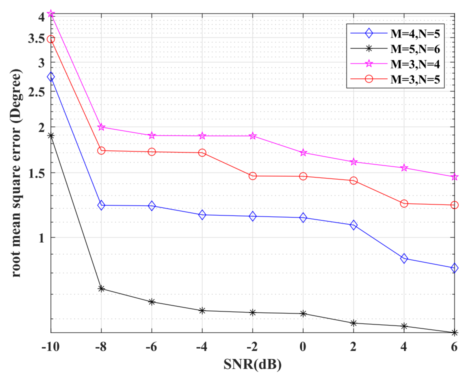

Fig. 5. The RMSE versus SNR with different number of array elements, $L = 200$, $MC = 100$.

Fig. 3(a)-(d) show the trajectory tracking and RMSE versus time, where SNR $= -10$ dB, $L = 200$. From Fig. 3, the proposed method and RFS-PF [20] can effectively track the time-varying direction of arrival. The tracking trajectories of PHD-ULA and SS-PAST methods are far from the true trajectories. However, the RFS-PF method appears to be inaccurate on the 20th.

To investigate the advantages of the proposed method, the RMSE results versus SNR and snapshots L is illustrated in Fig. 4(a)-(b). And Fig. 5 presents the RMSE performance versus SNR with different number of array elements. As can be seen from Fig. 4, the RMSE of the proposed method is lower than that of the PHD-ULA, RFS-PF, and SS-PAST algorithms, even in the negative SNR scenario. In Fig. 5, the RMSE performance improves with the improvement of the number of array elements. There is still a gap between the proposed method and CRLB, which indicates that the accuracy of the proposed method needs to be improved.

V. Conclusion

In order to solve the problem of multi-source DOA tracking with the extended co-prime array, a PHD filtering based on RFS is proposed. We propose a measurement separation (MSS) method, which has been applied to the likelihood function of particles. Furthermore, the improved MUSIC pseudo-likelihood is used to calculate the likelihood function of the algorithm, and it is exponentially weighted to increase the number of effective particles in the resampling process. Compared with the existing four algorithms, SS-PAST [14], PF-PM [16], OMP [11] and RFS-PF [20], the advantages of this algorithm are verified by simulation experiments.

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