VE 492 Homework8

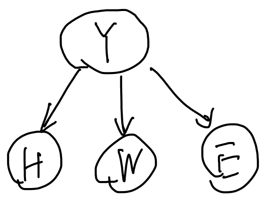
Due: 23:59, July 28

Q1. Naive Bayes

Your friend claims that he can write an effective Naive Bayes spam detector with only three features: the hour of the day that the email was received ($H \in \{1,2,...,24\}$), whether it contains the word 'viagra' ($W \in \{yes, no\}$), and whether the email address of the sender is Known in his address book, Seen before in his inbox, or Unseen before ($E \in \{K,S,U\}$).

(a) Flesh out the following information about this Bayes net:

Graph structure:



$Y \in \{spam, ham\}$

Parameters: $P(Y)$, $P(H|Y)$, $P(W|Y)$, $P(E|Y)$

Size of the set of parameters:

$2 + (24 + 2 + 3) \times 2 = 60$

Suppose now that you labeled three of the emails in your mailbox to test this idea:

spam or ham?	H	W	E
spam	3	yes	S
ham	14	no	K
ham	15	no	K

(b) $P(Y=spam) = 1/3$, $P(Y=ham) = 2/3$
$P(H=3 | Y=spam) = 1$, $P(H=14 | Y=ham) = 1/2$
$P(W=yes | Y=spam) = 1$, $P(E=S | Y=spam) = 1$
$P(E=K | Y=ham) = 1$, others are 0

(b) Use the three instances to estimate the maximum likelihood parameters.

(c) Using the maximum likelihood parameters, find the predicted class of a new datapoint with $H = 3, W = no, E = U$. can not predict the class.

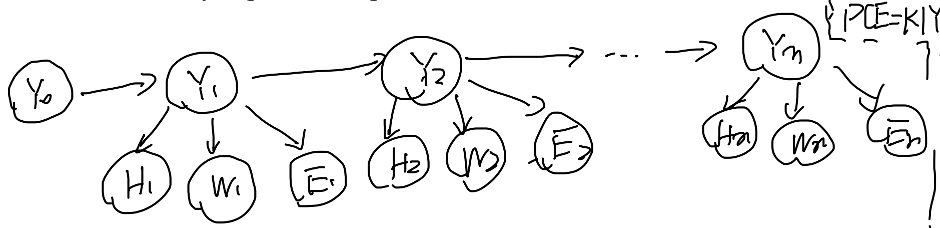
(d) Now use the three to estimate the parameters using Laplace smoothing and $k = 2$. Do not forget to smooth both the class prior parameters and the feature values parameters.

$P(Y=spam) = 3/7$, $P(Y=ham) = 3/49$; $P(H=3 | Y=spam) = 3/49$; $P(H | Y=spam) = 2/49$ for all other H.
$P(H=14 | Y=ham) = 3/50$, $P(H=15 | Y=ham) = 3/50$,
$P(H | Y=ham) = 1/25$ for all other H, $P(W=yes | Y=spam) = 3/5$
$P(W=yes | Y=ham) = 1/3$
$P(E=S | Y=spam) = 3/7$,
$P(E=K | Y=spam) = 2/7$,
$P(E=S | Y=ham) = 1/4$
$P(E=K | Y=ham) = 1/2$

(e) Using the parameters obtained with Laplace smoothing, find the predicted class of a new datapoint with $H = 3, W = no, E = U$. Ham

(f) You observe that you tend to receive spam emails in batches. In particular, if you receive one spam message, the next message is more likely to be a spam message as well. Explain a new graphical model which most naturally captures this phenomena.

Graph structure:



Parameters:

$P(Y_0)$, $P(Y_t | Y_{t-1})$, $P(H_t | Y_t)$, $P(W_t | Y_t)$, $P(E_t | Y_t)$

Size of the set of parameters:

$2 + 2 \times 2 + 60 = 66$

Q2. Perceptron

A (a) Suppose you have a binary perceptron in 2D with weight vector $\mathbf{w} = r\,[w_1, w_2]^T$. You are given w_1 and w_2 and are given that r > 0, but otherwise not told what r is. *Assume that ties are broken as positive*. Can you determine the perceptron's classifification of a new example x with known feature vector f(x)?

A. Always

B. Sometimes

C. Never

(b) Now you are learning a multi-class perceptron between 4 classes. The weight vectors are currently $[1,0]^T$,$[0,1]^T$,$[-1,0]^T$,$[0,-1]^T$ for the classes A, B, C, and D. The next training example *x* has a **label of A** and feature vector *f*(*x*).

For the following questions, *do not make any assumptions about tie-breaking.* (Do not write down a solution that creates a tie.)

If the answer does not exist, write down **Not possible**

$$f(x) = \begin{bmatrix} \\ \\ \end{bmatrix}$$ ○ Not possible

(i) Write down a feature vector in which no weight vectors will be updated. $f(x) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(ii) Write down a feature vector in which **only** $\mathbf{w}_A$ will be updated by the perceptron. Not possible.

(iii) Write down a feature vector in which **only** $\mathbf{w}_A$ and $\mathbf{w}_B$ will be updated by the perceptron. $f(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(iv) Write down a feature vector in which **only** $\mathbf{w}_A$ and $\mathbf{w}_C$ will be updated by the perceptron.

$f(x) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

The weight vectors are the same as before, but now there is a bias feature with value of 1 for all *x* and the weight of this bias feature is 0, -2, 1, - 1 for classes A, B, C, and D respectively. As before, the next training example *x* has a **label of A** and a feature vector *f*(*x*). The always "1" bias feature is the first entry in *f*(*x*).

If the answer does not exist, write down **Not possible**

$$f(x) = \begin{bmatrix} 1 \\ \\ \\ \end{bmatrix}$$ ○ Not possible

(v) Write down a feature vector in which **only** $\mathbf{w}B$ and $\mathbf{w}C$ will be updated by the perceptron. Not possible

(vi) Write down a feature vector in which **only** $\mathbf{w}A$ and $\mathbf{w}C$ will be updated by the perceptron.

$f(x) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$