VE485, Optimization in Machine Learning (Summer 2020) Homework Five

5. Solving Non-constrained Problem

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Problem 1

The pure Newton method. Newton's method with fixed step size $t = 1$ can diverge if the initial point is not close to $x^\star$. In this problem we consider two examples.

1. $f(x) = \log(e^x + e^{-x})$ has a unique minimizer $x^\star = 0$. Run Newton's method with fixed step size $t = 1$, starting at $x^{(0)} = 1$ and at $x^{(0)} = 1.1$.
2. $f(x) = -\log x + x$ has a unique minimizer $x^\star = 1$. Run Newton's method with fixed step size $t = 1$, starting at $x^{(0)} = 3$.

Plot f and f', and show the first few iterates.

Answer.

1. The plot of f and f' can be seen in Figure 1. When $x^{(0)} = 1.0$, the first 10 iterations are shown in Table 1.

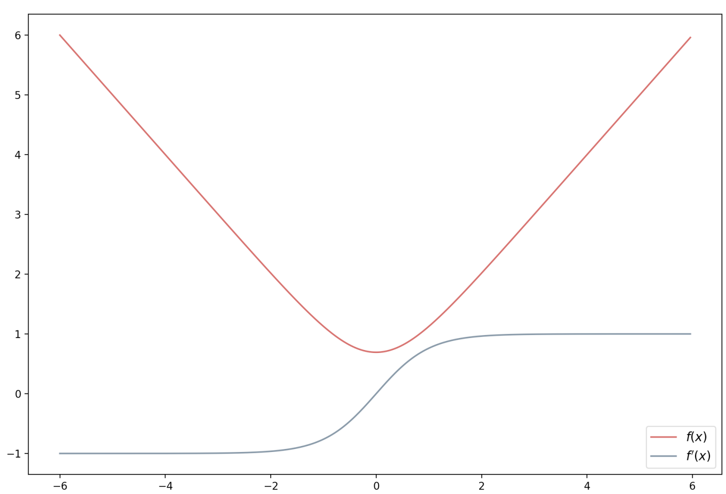

Figure 1: Plot of f and f', where $f(x) = \log(e^x + e^{-x})$

When $x^{(0)} = 1.1$, the first 10 iterations are shown in Table 2.

2. The plot of f and f' can be seen in Figure 2. At the beginning, $x^{(0)} = 3$ and $f(x^{(0)}) - f(x^*) = 0.901388$. For the first iteration, $x^{(1)} = -3$, which is out of the domain of $f(x) = -\log x + x$.

iteration k	$x^{(k)}$	$f(x^{(k)}) - f(x^*)$
0	1.000000	0.433781
1	0.412399	0.082730
2	0.200305	0.019928
3	0.099481	0.004940
4	0.049659	0.001232
5	0.024819	0.000308
6	0.012408	0.000077
7	0.006204	0.000019
8	0.003102	0.000005
9	0.001551	0.000001

Table 1: First 10 iterations when $x^{(0)} = 1.0$ and $f(x) = \log(e^x + e^{-x})$

iteration k	$x^{(k)}$	$f(x^{(k)}) - f(x^*)$
0	1.100000	0.511936
1	0.432176	0.090618
2	0.209298	0.021745
3	0.103883	0.005386
4	0.051848	0.001344
5	0.025913	0.000336
6	0.012955	0.000084
7	0.006477	0.000021
8	0.003239	0.000005
9	0.001619	0.000001

Table 2: First 10 iterations when $x^{(0)} = 1.1$ and $f(x) = \log(e^x + e^{-x})$

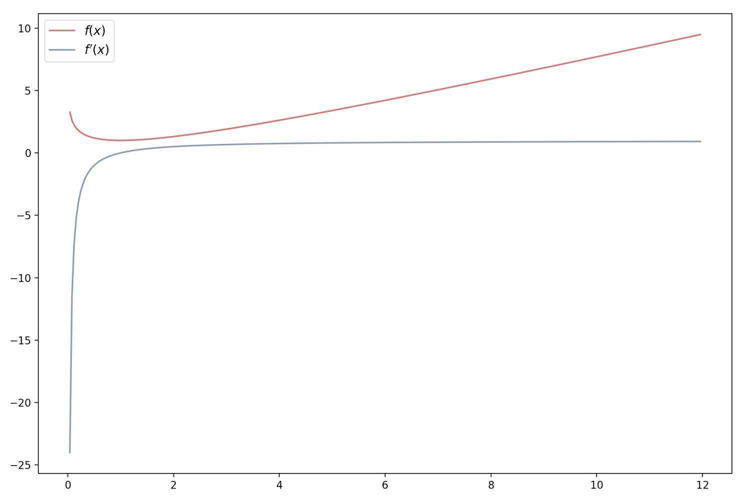

Figure 2: Plot of f and f', where $f(x) = -\log x + x$