2021.

1. (a) F (b) T (c) F (d) T (e) F (f) F (g) F (h) T (i) ~~T~~ F − uniform ~ (j) T

2. (a) $\left(\frac{1}{9}+a\right)\left(\frac{1}{6}\right)=\frac{1}{18}$, $\left(\frac{1}{9}+b\right)\left(\frac{1}{6}\right)=\frac{1}{18}$, $\left(\frac{1}{9}+c\right)\left(\frac{1}{6}\right)=\frac{1}{18}$

$\therefore a=b=c=\frac{2}{9}$

(b) $P_X(x)=\begin{cases}\frac{2}{3} & (x=1)\\ \frac{1}{6} & (x=2)\\ \frac{1}{6} & (x=3)\end{cases}$ $\quad P_Y(y)=\begin{cases}\frac{1}{3} & (y=1)\\ \frac{1}{3} & (y=2)\\ \frac{1}{3} & (y=3)\end{cases}$

(c) $P(A)=\frac{1}{3}$ $\quad P(B)=\frac{1}{3}$ $\quad P(A\cap B)=\frac{1}{18}+\frac{1}{18}=\frac{1}{9}$

$P(A)P(B)=\frac{1}{9}$ $\quad\therefore$ Independent

(d) $E(X|A)=\frac{5}{2}$ $\quad E(Y|B)=1$

3. (a) $\Omega=\{(1,H),(1,TH),(2,H),(2,TH)\cdots(6,H),(6,TH)\}$
$(1,HTH)\ (1,HTT)\ \cdots\ (6,HTH)\ (6,HTT)$
$\vdots$

(b) $\sum_{i=1}^{6}\left[1-\left\{\left(1-\frac{i}{8}\right)^3+\left(\frac{i}{8}\right)\left(1-\frac{i}{8}\right)^2\times 3\right\}\right]=\sum_{i=1}^{6}\left\{1-\left(1-\frac{i}{8}\right)^2\left(1+\frac{i}{4}\right)\right\}$

$=\sum_{i=1}^{6}\left(\frac{3}{64}i^2-\frac{i^3}{256}\right)=$

(c) ? A_i는 주사위, B는 2회의 Head $\quad P(B)\neq P(B|A_i)$ $\therefore$ x Inde~

(d) $\dfrac{\sum_{i=1}^{6}\left(\frac{i}{8}\right)^3\times\left(\frac{i}{8}\right)^3}{6}$ $\quad P(A_i|C)=\dfrac{P(A_i)P(C|A_i)}{\sum_{j=1}^{6}P(A_j)P(C|A_j)}=\dfrac{\frac{1}{6}P(C|A_i)}{\sum_{j=1}^{6}\frac{1}{6}P(C|A_j)}$

C = 3 Head

5.

(a) $\sum_{y=1}^{3}\sum_{x=1}^{3}\left(\frac{a+xy}{a^2}\right)=1$. $a^2-9a-36=0$ $a=12$ or -3

$P_X(x)\geq 0$ $a=12$

(b) $P_X(1)=\frac{42}{144}=\frac{7}{24}$

$P_X(2)=\frac{48}{144}=\frac{1}{3}$

$P_X(3)=\frac{54}{144}=\frac{3}{8}$

(c) $P_{X|Y=1}(1)=\frac{13}{42}$

$P_{X|Y=1}(2)=\frac{14}{42}=\frac{2}{6}=\frac{1}{3}$

$P_{X|Y=1}(3)=\frac{15}{42}=\frac{5}{14}$

(d) $P(X|Y=1)=\frac{P(X)P(Y)}{P(Y=1)}$ = if independent $\frac{P(X)P(Y)}{P(Y)}=P(X)$. 1/2/3x

∴ X independent

(e)

y/x	1	2	3
3	$\frac{15}{144}$	$\frac{18}{144}$	$\frac{21}{144}$
2	$\frac{14}{144}$	$\frac{16}{144}$	$\frac{18}{144}$
1	$\frac{13}{144}$	$\frac{14}{144}$	$\frac{15}{144}$

$\cancel{\frac{13}{144}+\frac{28}{144}+\frac{45}{144}=\frac{86}{144}}=$

$E(X|Y=1)=\frac{13}{42}\times1+\frac{14}{42}\times2+\frac{15}{42}\times3=\frac{13+28+45}{42}=\frac{86}{42}$

$=\frac{43}{21}$

6. $f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}, \quad \lambda > 0$

(a) $\int_0^a \lambda e^{-\lambda x} dx = -e^{-\lambda x} \Big]_0^a = 1 - e^{-\lambda a}$

(b) PDF는 CDF를 미분하면 됨

$y < 0 \quad P(Y \leq y) = 0$

$y \geq 0 \quad P(\max(X_1, X_2, \cdots X_n) \leq y)$

$= P(X_1 \leq y, X_2 \leq y, X_3 \leq y \cdots X_n \leq y)$

$= P(X_1 \leq y) \cdot P(X_2 \leq y) \cdots P(X_n \leq y)$

$= (1 - e^{-\lambda y})^n \Leftarrow$ CDF

$f_Y(y) = \frac{dF_Y(y)}{dy} = n(1 - e^{-\lambda y})^{n-1} \cdot \lambda e^{-\lambda y}$

7. (a) $\int_a^\infty \lambda e^{-\lambda x} dx = -e^{-\lambda x} \Big]_a^\infty = e^{-\lambda a}$

(b) $P(X_1 \geq y) \overset{or}{} P(X_2 \geq y) \overset{or}{} \cdots \overset{or}{} P(X_n \geq y)$

$\downarrow$

$1 - P(X_1 < y) P(X_2 < y) \cdots P(X_n < y)$

$= 1 - e^{-n\lambda y}, \quad f_Y(y) = n\lambda e^{-n\lambda y} \ (y \geq 0)$

8. (a) $E(X_1) = \cancel{0.9 \times 1.2 Y_0 + 0.1 \times 0.8 Y_0 = 1.16 Y_0}$ 0.16

$E(X_2) = \cancel{0.1 \times 1.2 Y_1 + 0.9 \times 0.8 Y_1 = 0.84 Y_1}$ -0.16

(b) $\cancel{E(Y_n) = Y_0 (1+X_1)^{\frac{n}{2}+\frac{1}{2}} (1+X_2)^{\frac{n}{2}} = Y_0 (1.16)^{\frac{n}{2}+\frac{1}{2}} (0.84)^{\frac{n}{2}}}$

if (n이 홀수)

$Y_n = Y_0 (1+X_1)^{\frac{n}{2}+\frac{1}{2}} (1+X_2)^{\frac{n}{2}-\frac{1}{2}}$

if (n이 짝수) $Y_n = Y_0 (1+X_1)^{\frac{n}{2}} (1+X_2)^{\frac{n}{2}}$

(c) $E(Y_{21}) = Y_0 (1.16)^{11} (0.84)^{10}$

(d) $E(Y_{30}) = Y_0 (1.16)^{15} (0.84)^{15} = Y_0 (0.9744)^{15} \fallingdotseq Y_0 (0.67)$

$\therefore$ 원금의 33%를 잃는다

4 (a) $E(Y) = \sum_{k=0}^{n} (4k-3n)\binom{n}{k} p^k (1-p)^{n-k}$

$= \sum_{k=0}^{n} \{4k \binom{n}{k} p^k (1-p)^{n-k}\} - 3n \underbrace{\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}}_{=1}$

$= \sum_{k=1}^{n} \{4k \binom{n}{k} p^k (1-p)^{n-k}\} - 4k(1-p)^n - 3n$

$= \sum_{k=1}^{n} \{4k \cdot \frac{n}{k} \binom{n-1}{k-1} p \cdot p^{k-1} (1-p)^{n-k}\} - 4k(1-p)^n - 3n$

$= 4np \underbrace{\sum_{k=1}^{n} \{\binom{n-1}{k-1} p^{k-1} (1-p)^{n-k}\}}_{=1} - 4k(1-p)^n - 3n$

$\therefore 4np - 4\underset{k=0}{k}(1-p)^n - 3n = 4np - 3n$

(b) $E(Y) = 4np - 3n = n(4p-3)$

if $p > \frac{3}{4}$, $E(Y) > 0$

$\therefore \frac{3}{4}$

(c) $n=10 \rightarrow P_Y(4k-30)\binom{10}{k}\left(\frac{3}{4}\right)^k\left(\frac{1}{4}\right)^{10-k}$ $k = 8, 9, 10$

$n=20 \rightarrow P_Y = (4k-60)\binom{20}{k}\left(\frac{3}{4}\right)^k\left(\frac{1}{4}\right)^{20-k}$ $k = 15, 16 \cdots 20$

$n=30 \rightarrow P_Y = (4k-90)\binom{30}{k}\left(\frac{3}{4}\right)^k\left(\frac{1}{4}\right)^{30-k}$ $k = 23, 24 \cdots 30$

$\therefore \frac{1}{5} \times \sum_{k=8}^{10} \{(4k-30)\binom{10}{k}\left(\frac{3}{4}\right)^k\left(\frac{1}{4}\right)^{10-k}\} + \frac{2}{5} \times \sum_{k=15}^{20} \{(4k-60)\binom{20}{k}\left(\frac{3}{4}\right)^k\left(\frac{1}{4}\right)^{20-k}\}$

$+ \frac{2}{5} \sum_{k=23}^{30} \{(4k-90)\binom{30}{k}\left(\frac{3}{4}\right)^k\left(\frac{1}{4}\right)^{30-k}\}$

작거나 같음 . . .

2022.

1. (a) T (b) T (c) T (d) T (e) T

2. (a) $\Omega = \{5 \le M \le 8,\ 6 \le F \le 8 \mid (M, M<F \le M+1),\ (M, M+1<F),\ (F<M \le F+1, F),\ (F+1<M, F)\}$

(b) 확률공리 : $\begin{cases} P(\Omega)=1 \\ 0 \le P(A) \le 1 \\ P(A \cup B) = P(A)+P(B) \quad (A,B\text{는 배반.}) \end{cases}$

$P(S) = \dfrac{\text{area}(S)}{6} \quad S \subseteq \Omega$

(c) if $(M, M<F \le M+1)$, $5 \le M \le 7$ $P(F) = \dfrac{M+1-M}{8-6}$, $\dfrac{2}{3} \times \dfrac{1}{8-6} = \dfrac{1}{3}$

$|x-y| \le 1$

$7 \le M \le 8$ $P(F) = \dfrac{8-M}{8-6}$ $\dfrac{1}{3} \times \dfrac{8-M}{2} = \dfrac{8-M}{6}$

if $(F<M \le F+1, F)$ $6 \le F \le 7$ $P(M) = \dfrac{F+1-F}{8-5}$ $\dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$

$\dfrac{6-2-\frac{1}{2}}{6} = \dfrac{7}{12}$

$7 \le F \le 8$ $P(M) = \dfrac{8-F}{8-5}$ $\dfrac{1}{2} \times \dfrac{8-F}{3} = \dfrac{8-F}{6}$

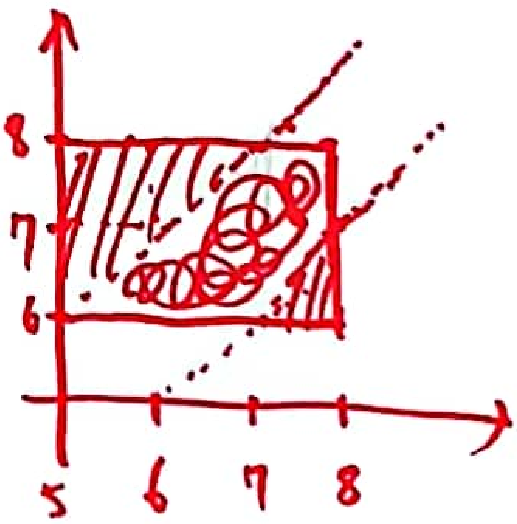
if $(M, M+1<F)$ $5 \le M < 7$ $P(F) = \dfrac{8-(M+1)}{2}$ $\dfrac{2}{3} \times \dfrac{8-(M+1)}{2}$

$7 \le M \le 8$ $P(F) = 0$

if $(F+1<M, F)$ $6 \le F < 7$ $P(M) = \dfrac{8-(F+1)}{3}$ $\dfrac{1}{2} \times \dfrac{8-(F+1)}{3}$

$7 \le F \le 8$ 0

(d) $\dfrac{3}{4}$ $\left(\dfrac{\frac{1}{4}}{\frac{1}{3}}\right)$



(e) Discrete.

$p_Z(z) = \begin{cases} \frac{7}{12}, & x=1 \\ \frac{5}{12}, & x=0 \end{cases}$ $E(Z) = \dfrac{7}{12}$

$\text{var}(Z) = \dfrac{7}{12} \times \dfrac{5}{12} = \dfrac{35}{144}$

$p(1-p)$ 베르누이 분포

3. (a) $\Omega = \{(1,T),(1,H),(2,TT),(2,TH),(2,HT),2,HH)\cdots\}$

(b) $\binom{3}{2}\left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} = \frac{3}{8}$

(c) $\frac{1}{4} \times \frac{3}{8} = \frac{3}{32}$

(d) if $k=2$ $\left(\frac{1}{2}\right)^2$

if $k=3$ $\frac{3}{8}$

if $k=4$ $\binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

$\frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{8} + \frac{1}{4} \cdot \frac{3}{8} = \frac{1}{4}$

(e) 0

(f) 1

(g) $$p_X(x) = \begin{cases} \frac{15}{64} & x=0 \\ \frac{13}{32} & x=100 \\ \frac{8}{32} & x=200 \\ \frac{3}{32} & x=300 \\ \frac{1}{64} & x=400 \end{cases}$$

(h) $\frac{13}{32} \times 100 + \frac{8}{32} \times 200 + \frac{3}{32} \times 300 + \frac{1}{64} \times 400$

(i) $\frac{13}{32} \times (100^2 - 100) + \frac{8}{32}(200^2 - 200) + \frac{3}{32}(300^2 - 300) + \frac{1}{64}(400^2 - 400)$

4. (a) $P_Y(y) = \begin{cases} \frac{1}{3} & y=1 \\ \frac{1}{9} & y=2 \\ \frac{5}{9} & y=3 \end{cases}$ $P_X(x) = \begin{cases} \frac{2}{3} & x=1 \\ \frac{1}{9} & x=2 \\ \frac{2}{9} & x=3 \end{cases}$

(b) $P_{X|Y}(x|1) = \begin{cases} \frac{2}{3} & x=1 \quad // \\ 0 & x=2 \quad // \\ \frac{1}{3} & x=3 \quad // \end{cases}$

(c) $\overset{P_X(x) \cap P_Y(y)}{\cancel{P_{X|Y}(x|1)}} = P_X(x) \times P_Y(1)$ (if independent)

$\frac{1}{6} \neq \frac{1}{9} \times$ $\therefore$ X independent

(d) $P_X(2) \cap P_Y(1) \neq \frac{1}{9} \times \frac{1}{3} = \frac{1}{27}$

$\therefore$ ~~X independent~~

$(X \geq 2)(Y < 2)$

$= \frac{1}{9} = \frac{1}{3} \times \frac{1}{3}$

$\therefore$ independent

(e) $E(X|A) = \frac{2}{3} + \frac{2}{3} = \frac{8}{3}$

$E(Y|B) = \frac{1}{3} \times 1 = \frac{1}{3}$

5\. (a) $E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum E[X_i] = \frac{1}{n}\times\frac{1}{2}n = \frac{1}{2}$ $P(Y) = \begin{cases} \frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{cases}$

? (b) $var(X) = \frac{1}{n}\sum_{i=1}^{n}(X_i)^2 - \left(\frac{1}{n}\sum_{i=1}^{n}(X_i)\right)^2 = \cancel{\frac{1}{n}\left\{\sum_{i=1}^{n}(X_i) - (X_i)\right.}$

$\cancel{\frac{1}{n}\sum_{i=1}^{n}\left((X_i)^2 - (X_i)\right)^2} = \frac{1}{n}\left\{\sum_{i=1}^{n}(X_i)^2 - \frac{1}{n}\left(\sum_{i=1}^{n}(X_i)\right)^2\right\}$

$n \to n^2$? $var(X) = \frac{1}{n^2}\sum_{i=1}^{n}(X_i)^2 -$

$u[a,b]$ 균일분포 $var = \frac{(b-a)^2}{12}$

$Var\left(\frac{1}{n}\sum X_i\right) = \frac{1}{n^2}\sum var(X_i) = \frac{1}{n^2}\cdot\frac{n}{12} = \frac{1}{12n}$, n 증가 $\to 0$

n 증가 $\to$ 평균으로 몰린다

(c) $E(Y) = \frac{1}{2}$

d) $var(Y) = \frac{1}{4n}$

베르누이 $\to$ $var = p(1-p)$

$\frac{1}{n^2}$ [illegible] $[Y] = \frac{1}{n^2}\cdot\frac{1}{4}n = \frac{1}{4n}$

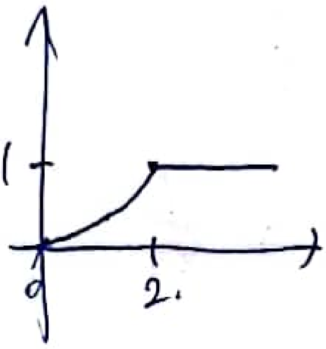
6. (a) $\int_0^2 cx\,dx = \frac{1}{2}cx^2\Big]_0^2 = 2c = 1 \qquad c = \frac{1}{2}$

(b) $\int_0^2 \frac{1}{2}x^2\,dx = \frac{1}{6}x^3\Big]_0^2 = \frac{4}{3} = E(X)$

(c) $var(X) = E(X^2) - [E(X)]^2$

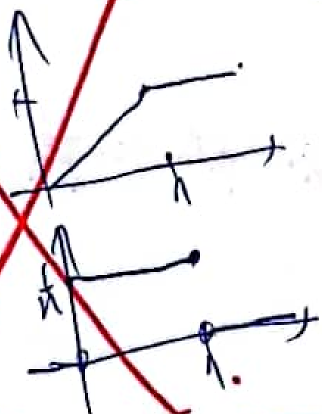
$= \int_0^2 \frac{1}{2}x^3\,dx - \frac{16}{9} = \frac{1}{8}x^4\Big]_0^2 - \frac{16}{9} = 2 - \frac{16}{9} = \frac{2}{9}$

(d) $\int_0^x \frac{1}{2}x\,dx = \frac{1}{4}x^2$



2 (a) $\cancel{F(x) = \frac{1}{n}x}$

$\cancel{f(x) = \frac{1}{n}}$



Order statistics

$F_{X_{(n)}}(x) = P(X_{(n)} \le x)$

$x_1, x_2 \cdots x_i \le \cdots x_n$

$\sum_{i=1}^{n}\binom{n}{i}[F(x)]^i(1-F(x))^{n-i}$

(b) $F(x) = 1 - e^{-\lambda x}$

$\therefore n(e^{-\lambda x})^{n-1}\lambda e^{-\lambda x} = n\lambda e^{-n\lambda x}$

$= 1 - (1-F(x))^n$

$\therefore$ PDF $= n(1-F(x))^{n-1}\cdot f(x)$

(C)

$\sum_{j=k}^{n}\binom{n}{k}F(x)^i(1-F(x))^{n-i}$