Probability and Statistics, S2023	Posted: April 11, 2023
Problem Set 5	
Lecturer: Hyang-Won Lee	**Not to be turned in**

Continuous random variables, CDFs

- You may refer to the table of integrals (for instance, https://www.integral-table.com) and complete calculations, but do not use integral calculators (적분 계산을 완료하기 위해 적분 테이블을 참고해도 괜찮지만, 적분 계산기는 사용하지 마시오)

1. Consider the following function:

$$f_X(x) = \frac{c}{1+x^2}, \forall x \in \mathbb{R}.$$

   (a) Determine the value of c so that f_X can be a probability density function (PDF) of continuous random variable X. (f_X가 연속확률변수 X의 확률밀도함수가 되도록 c 값을 정하시오)

   (b) Calculate the expectation of X.

2. Consider the following function ($\sigma > 0$):

$$f_X(x) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, & x \geq 0 \\ 0, & x < 0. \end{cases}$$

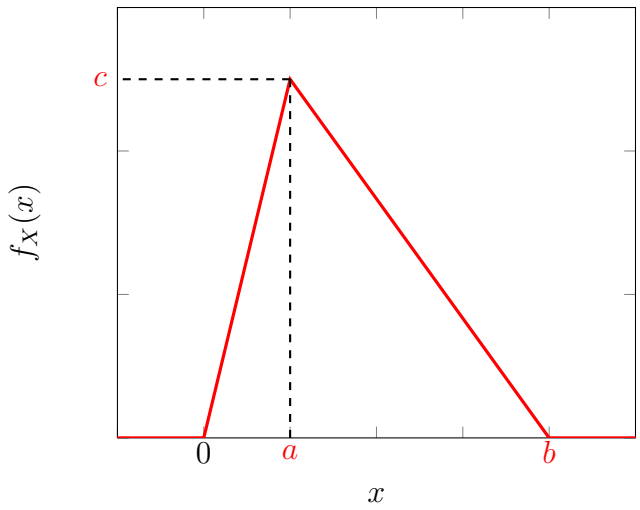
   (a) Prove that $f_X(x)$ is a PDF of continuous random variable X.

(b) Calculate the expectation of X.

(c) Calculate the variance of X.

(d) Find the CDF of X.

3. Consider the following PDF of a continuous random variable X where a, b, c are positive constants. Assume that $a + b = 3$.



(a) Calculate the expectation of X.

(b) Show (through calculation) that the variance of X is minimized when $a = b = 1.5$. What is your intuition that this is true?