1.(a) $\int_{-\infty}^{\infty}\frac{c}{1+x^2}dx = 1$, $c\tan^{-1}x\Big]_{-\infty}^{\infty} = c\left(\lim_{x\to\infty}\tan^{-1}x - \lim_{x\to-\infty}\tan^{-1}x\right)$

$= c\cdot\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right) = \pi\cdot c \quad \therefore c = \frac{1}{\pi}$

(b) $\int_{-\infty}^{\infty}\frac{x}{\pi(1+x^2)}dx = \frac{1}{\pi}\left\{x\tan^{-1}x\Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\tan^{-1}x\right\}$

$= \frac{1}{\pi}\left\{x\tan^{-1}x - x\tan^{-1}x + \frac{\ln(x^2+1)}{2}\right\} = \frac{\ln(x^2+1)}{2\pi}\Big]_{-\infty}^{\infty}$

$\therefore$ 0? 존재X?

2.(a) if $f_X(x)$ is PDF → $\int_{-\infty}^{\infty} f_X(x)dx = 1$, $\int_0^{\infty}\frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}dx = 1$

$\int_0^{\infty}\frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}} = -e^{-\frac{x^2}{2\sigma^2}}\Big]_0^{\infty} = -e^{-\infty}-(-e^{0}) = 0-(-1) = 1 \quad \therefore$ PDF

(b) $\int_0^{\infty}\frac{x^2}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}} = -e^{-\frac{x^2}{2\sigma^2}}\cdot x\Big]_0^{\infty} - \int_0^{\infty}\left(-e^{-\frac{x^2}{2\sigma^2}}\right)dx = \frac{-x}{e^{\frac{x^2}{2\sigma^2}}}\Big]_0^{\infty} + \int_0^{\infty}e^{-\frac{x^2}{2\sigma^2}}dx$

$\int_0^{\infty}e^{-x^2} = \frac{\sqrt{\pi}}{2} \quad \therefore \int_0^{\infty}e^{-\frac{x^2}{2\sigma^2}} = \frac{\sqrt{2\pi}}{2}\sigma$

$\lim_{x\to\infty}\left(-\frac{x}{e^{\frac{x^2}{2\sigma^2}}}\right) = \lim_{x\to\infty}\frac{-1}{\frac{x}{\sigma^2}e^{\frac{x^2}{2\sigma^2}}} = 0$

$\therefore \int_0^{\infty}f(x)dx = \frac{\sqrt{\pi}}{2}$ $\quad \int_0^{\infty}f\left(\frac{x}{\sqrt{2}\sigma}\right)dx = \frac{\sqrt{2\pi}}{2}\sigma$

$\therefore 0-0+\frac{\sqrt{2\pi}}{2}\sigma = \frac{\sqrt{2\pi}}{2}\sigma$

(c) $\int_0^{\infty}\frac{x^3}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}} = -e^{-\frac{x^2}{2\sigma^2}}x^2\Big]_0^{\infty} - \int_0^{\infty}-2xe^{-\frac{x^2}{2\sigma^2}} = 0-2\sigma^2e^{-\frac{x^2}{2\sigma^2}}\Big]_0^{\infty} = 2\sigma^2 = E(X^2)$

$\lim_{x\to\infty}\frac{-x^2}{e^{\frac{x^2}{2\sigma^2}}} = \lim_{x\to\infty}\frac{-2x}{\frac{1}{\sigma^2}e^{\frac{x^2}{2\sigma^2}}} = \frac{-2}{\frac{1}{\sigma^2}e^{\frac{x^2}{2\sigma^2}}} = 0$

$\therefore Var(X) = E(X^2)-E(X)^2$

$= 2\sigma^2-\frac{2\pi}{4}\sigma^2 = \left(\frac{8-2\pi}{4}\right)\sigma^2$

(d) CDF: $F(x) = \int_0^{\infty} \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} = \left(-e^{-\frac{x^2}{2\sigma^2}}\right)\Big]_0^{x} = -e^{-\frac{x^2}{2\sigma^2}} + 1$

3. (a) $E(x) = \int_0^a x \cdot \frac{c}{a} x\,dx + \int_a^b x\left(\frac{c}{a-b}x - \frac{bc}{a-b}\right)dx$

$\int_0^a \frac{c}{a} x\,dx + \int_a^b \left(\frac{c}{a-b}x - \frac{b}{a-b}\right)dx = \frac{1}{2}bc = 1 \quad \therefore bc = 2, \quad a+b=3, \quad a = 3-b$

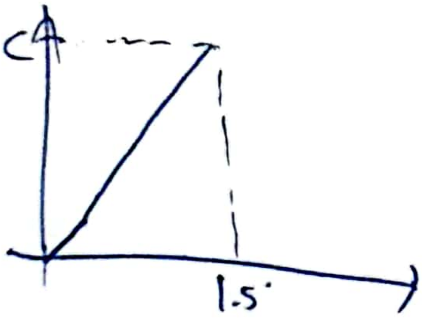
$\frac{c}{a}\frac{1}{3}a^3 + \frac{c(b^3-a^3)}{3(a-b)} - \frac{bc(b^2-a^2)}{2(a-b)} = \frac{a^2c}{3} + \frac{bc(a+b)}{2} - \frac{c\overbrace{(a^2+b^2+ab)}^{(a+b)^2-ab}}{3} = \frac{(3-b)^2c}{3} + 3 - \frac{c(9-(3-b)b)}{3}$

$= 3 + \frac{c\{9-6b+b^2-9+3b-b^2\}}{3} = 3 + \frac{-3bc}{3} = 3-2 = 1$

(b) 분산이 줄어들려면 값들이 대푯값을 중심으로 몰려야 한다.

b가 줄어들수록 c는 커지는데, ~~[illegible]~~ b가 줄어들고 c가 커질수록 분산은 줄어든다.

b를 최대한으로 줄인게 b=1.5 이므로 a=b=1.5 일때 분산이 최소다.



1.5 근처로 몰림 ($p_x(1.5)$가 가장 크므로..)