1. (a) discrete 이다.

확률 변수의 개수는 무한 하지만 셀 수 있으며

$\sum_{i=1}^{\infty} p(x_i) = 1$ 이므로 discrete

(b) $w>0$ 일때 X가 취할 수 있는 값이 $[0, 1]$ 사이의 모든 값이므로

discrete이 아니다.

(c) X가 취할 수 있는 값이 $[0, 1]$ 사이의 모든 값이므로

discrete이 아니다.

(d) X가 취할 수 있는 값이 1과 -1 둘 뿐이므로

discrete 이다.

2. (a) $p_X(x)$ 들의 합은 1이 되어야 하므로

$c+4c+9c+16c=1 \quad \therefore c=\frac{1}{30}$

(b) $x=2, 4 \qquad p_X(2)=\frac{4}{30}, \; p_X(4)=\frac{16}{30}$

$p_X(2)+p_X(4)=\frac{2}{3} \qquad \therefore \frac{2}{3}$

(c) $x=3, 4 \qquad p_X(3)=\frac{9}{30}$

$p_X(3)+p_X(4)=\frac{9}{30}+\frac{16}{30}=\frac{5}{6} \qquad \therefore \frac{5}{6}$

(d) $E(X) = \frac{1}{30}\times 1 + \frac{4}{30}\times 2 + \frac{9}{30}\times 3 + \frac{16}{30}\times 4 = \frac{10}{3}$

$E(X^2) = \frac{1}{30}\times 1 + \frac{4}{30}\times 4 + \frac{9}{30}\times 9 + \frac{16}{30}\times 16 = \frac{59}{5}$

(e) $var(X) = E(X^2) - \{E(X)\}^2 = \frac{59}{5} - \frac{100}{9} = \frac{31}{45}$

(f) $var(X^2) > var(X) > var(\sqrt{X})$, 편차가 커진 것이므로 ...

3. (b)

$$E(X) = \sum_{x=0}^{n} x \cdot \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} = \sum_{x=0}^{n} x \cdot \frac{k!}{x!\,(k-x)!} \binom{N-k}{n-x} \div \binom{N}{n}$$

$$= \sum_{x=1}^{n} \frac{(k-1)!}{(x-1)!\,(k-x)!} k \cdot \binom{N-k}{n-x} \div \binom{N}{n} = k \sum_{x=1}^{n} \frac{(k-1)!}{(x-1)!\,(k-x)!} \cdot \binom{N-k}{n-x} \div \frac{N}{n}\binom{N-1}{n-1}$$

$$= \frac{nk}{N} \underbrace{\sum_{x=1}^{n} \binom{k-1}{x-1} \binom{(N-1)-(k-1)}{(n-1)-(x-1)} \div \binom{N-1}{n-1}}_{\text{주어진 식과 같은 구조}} = \frac{nk}{N} \times 1 = \frac{nk}{N}$$

x=0일때 항이 0이므로 1부터 시작해도 무관하다.

$\therefore \Sigma = 1$

(c) $var(X) = E(X^2) - (E(X))^2$

$$E(X^2) = \sum_{x=1}^{n} x^2 \frac{k!}{x!\,(k-x)!} \cdot \frac{\binom{N-k}{n-x}}{\binom{N}{n}} = k \sum_{x=1}^{n} x \frac{(k-1)!}{(x-1)!\,(k-x)!} \frac{\binom{N-k}{n-x}}{\binom{N}{n}}, \quad \binom{N}{n} = \frac{N}{n}\binom{N-1}{n-1}$$

$$k \sum_{x=1}^{n} x \frac{(k-1)!}{(x-1)!\,(k-x)!} \frac{n}{N} \frac{\binom{N-k}{n-x}}{\binom{N-1}{n-1}} = \frac{nk}{N} \sum_{x=1}^{n} \frac{x \binom{k-1}{x-1} \binom{(N-1)-(k-1)}{(n-1)-(x-1)}}{\binom{N-1}{n-1}}$$

$$= \frac{nk}{N} \sum_{x=0}^{n} (x+1) \frac{\binom{k-1}{x-1}\binom{(N-1)-(k-1)}{(n-1)-(x-1)}}{\binom{N-1}{n-1}} = \frac{nk}{N} \left\{ \underbrace{\sum_{x=0}^{n} \frac{x \binom{k-1}{x-1}\binom{(N-1)-(k-1)}{n-1-(x-1)}}{\binom{N-1}{n-1}}}_{\text{원래식에서 }(N-1,\ k-1,\ n-1)\text{인 기댓값}} + \underbrace{\sum_{x=0}^{n} \frac{\binom{k-1}{x-1}\binom{(N-1)-(k-1)}{(n-1)-(x-1)}}{\binom{N-1}{n-1}}}_{1} \right\}$$

$$= \frac{nk}{N} \left\{ \frac{(n-1)(k-1)}{N-1} + 1 \right\}$$

$$\therefore var(X) = \frac{nk}{N} \left\{ \frac{(n-1)(k-1)}{(N-1)} + 1 \right\} - \frac{n^2k^2}{N^2} = \frac{nk}{N} \left\{ \frac{(n-1)(k-1)}{(N-1)} + 1 - \frac{nk}{N} \right\} = \frac{N-n}{N-1} \cdot n \cdot \frac{k}{N}\left(1 - \frac{k}{N}\right)$$

4. (a) $\dfrac{1}{\binom{45}{6}} = \dfrac{1}{8145060}$

(b) $P_X(k) = P(X=k) = \binom{2n-1}{k}\left(\dfrac{1}{{}_{45}C_6}\right)^{k}\left(\dfrac{{}_{45}C_6-1}{{}_{45}C_6}\right)^{2n-1-k}$

(c) $W_k = \dfrac{2n \times 500}{k+1}$

$$E(X) = \sum_{k=0}^{2n-1} \frac{2n\times 500}{k+1}\binom{2n-1}{k}\left(\frac{1}{{}_{45}C_6}\right)^{k}\left(\frac{{}_{45}C_6-1}{{}_{45}C_6}\right)^{2n-1-k}$$

$$= 500\sum_{k=0}^{2n-1}\binom{2n}{k+1}\left(\frac{1}{{}_{45}C_6}\right)^{k}\left(\frac{{}_{45}C_6-1}{{}_{45}C_6}\right)^{2n-1-k}\cdot\frac{{}_{45}C_6}{{}_{45}C_6}$$

$$= 500\sum_{k=0}^{2n-1}\binom{2n}{k+1}\left(\frac{1}{{}_{45}C_6}\right)^{k+1}\left(\frac{{}_{45}C_6-1}{{}_{45}C_6}\right)^{2n-1-k}\cdot {}_{45}C_6$$

$$= 500\left\{\sum_{k=-1}^{2n-1}\binom{2n}{k+1}\left(\frac{1}{{}_{45}C_6}\right)^{k+1}\left(\frac{{}_{45}C_6-1}{{}_{45}C_6}\right)^{2n-1-k} - \binom{2n}{0}\left(\frac{1}{{}_{45}C_6}\right)^{0}\left(\frac{{}_{45}C_6-1}{{}_{45}C_6}\right)^{2n}\right\}\cdot {}_{45}C_6$$

$$= 500\cdot {}_{45}C_6\left(1-\left(\frac{{}_{45}C_6-1}{{}_{45}C_6}\right)^{2n}\right)$$

(d) 상금의 기댓값이 $500\cdot\binom{45}{6}\left\{1-\left(\dfrac{{}_{45}C_6-1}{{}_{45}C_6}\right)^{2n}\right\}$ 이므로

선형적으로 증가하지 않는다