

Problem Set 2

Conditional probability, Bayes' rule, independence, counting

1. Consider four events A , B , C and D associated with the same experiment. Let $\mathbb{P}(A) = 1/4$, $\mathbb{P}(B) = 1/8$, $\mathbb{P}(C) = 5/8$ and $\mathbb{P}(D) = 3/8$. Assume that A and B are disjoint, and C and D are independent.

(a) Is it possible for the four events to be a partition of the sample space?

(b) Calculate $\mathbb{P}(A \cap B)$, $\mathbb{P}(A \cup B)$, $\mathbb{P}(A \cap B^c)$ and $\mathbb{P}(A \cup B^c)$.

(c) Are A and B independent?

(d) Calculate $\mathbb{P}(C \cap D)$, $\mathbb{P}(C \cap D^c)$, and $\mathbb{P}(C^c \cap D^c)$.

(e) Are C^c and D^c independent? Justify your answer.

2. Kunwoo is a freshman at Konkuk University, and is taking the Probability and Statistics (PS) class. He plays League of Legends (LoL, a computer game) every day. If he loses more than 50% of the games he played, he gets upset and tosses a fair coin to make a decision on whether he will attend the PS class. If a head comes up, he attends the class. Otherwise, he practices LoL instead of attending the class. If he loses no more than 50% of the games he played, he feels proud of himself and attends the class. Suppose that Kunwoo's winning percentage is 80% all the time (i.e., he wins a game with probability 0.8 independently of other games), and he plays 4 games.

(a) Calculate the probability that he loses more than 50% of the games.

(b) Given that he loses more than 50% of the games, what is the conditional probability that he attends the class?

(c) Given that he loses no more than 50% of the games, what is the conditional probability that he attends the class?

(d) Calculate the probability that he attends the class.

(e) Given that he misses the class, what is the probability that he has lost more than 50% the games?

- (f) Given that he attends the class, what is the probability that he has lost more than 50% the games?

3. In the kitchen in your apartment, you put all your 25 forks in the left drawer and all 20 knives in the right drawer. Your roommate, who does not agree with your organizational approach, comes in, takes five forks from the left drawer and tosses them into the right drawer. She then takes at random an item (knife or fork) from the right drawer and tosses it in the left drawer. After this exchange, you come in and randomly pick up an item from a randomly chosen drawer.

- (a) Let L be the event that you have opened the left drawer. What is the probability of event L ?

- (b) Let K be the event you picked up a knife. Given that you have opened the left drawer, what is the conditional probability of K ? (Hint. We want to calculate the probability $\mathbb{P}(K|L)$. Define K_R (or F_R) to be the event that your roommate picked up a knife (or fork) from the right drawer and tossed it into the left. By the total probability theorem, $\mathbb{P}(K|L) = \mathbb{P}(K_R|L)\mathbb{P}(K|L, K_R) + \mathbb{P}(F_R|L)\mathbb{P}(K|L, F_R)$).

- (c) Given that you have opened the right drawer, what is the conditional probability of K ?

- (d) Given you have picked up a knife, what is the probability that you have opened the left drawer? (Hint. The probability we want to calculate is $\mathbb{P}(L|K)$. Apply Bayes' rule, together with the results in parts (a), (b) and (c).)

4. Consider the experiment of rolling a fair six-sided die. Let R_i be the event that the roll is i . Let G_j be the event that the roll is greater than j . Let E be the event that the roll is even-numbered.

- (a) What is the conditional probability that 3 is rolled, given that the roll is greater than 1

- (b) What is the conditional probability that 6 is rolled, given that the roll is greater than 3?

- (c) What is the conditional probability that the roll is greater than 3, given that the roll is even?

(d) Given that the roll is greater than 3, what is the conditional probability that the roll is even?

5. There is a deck of 52 cards (13 hearts, 13 spades, 13 diamonds, 13 clovers). You and your friend both pick two cards at random. Do not consider the order in which cards are picked.

(a) How many outcomes are there in the sample space?

(b) Calculate the probability that you get two hearts?

(c) Given that you have got two hearts, what is the conditional probability that you friend has got two clovers?

(d) What is the probability that you get two hearts and your friend get two clovers?

(e) What is the probability that you and your friend get the same shape? (that is, all four cards are heart, spade, diamond or clover)