1. (a) $\sum P(X=x_i, Y=y_i) = 1$

$\therefore \ c + 2c + 3c + 2c + 4c + 6c = 18c = 1 \qquad \therefore c = \frac{1}{18}$

(b) $P_X(x) = \begin{cases} x=1 & \frac{1}{3} \\ x=2 & \frac{2}{3} \end{cases} \qquad P_Y(y) = \begin{cases} y=1 & \frac{1}{6} \\ y=2 & \frac{1}{3} \\ y=3 & \frac{1}{2} \end{cases}$

(c) Independent 하다.

$P_X(1) \times P_Y(1) = P_{XY}(1,1)$ .... 이 전부 성립하므로.

(d) $P_X(x)$와 $P_Y(y)$가 독립이므로 $P_{X|Y}(x|y) = P_X(x)$, $P_{Y|X}(y|x) = P(y)$

$\therefore \ P_{X|Y}(x|y) = \begin{cases} \frac{1}{3} & (x=1) \\ \frac{2}{3} & (x=2) \end{cases} \qquad P_{Y|X}(y|x) = \begin{cases} \frac{1}{6} & (y=1) \\ \frac{1}{3} & (y=2) \\ \frac{1}{2} & (y=3) \end{cases}$

(e) $E(X^2Y^2) = \sum_x \sum_y x^2 y^2 P_{XY}(x,y) = \sum_x \sum_y x^2 y^2 P_X(x) P_Y(y)$

$= \sum_x x^2 P_X(x) \sum_y y^2 P_Y(y) = \left(\frac{1}{3} + \frac{8}{3}\right)\left(\frac{1}{6} + \frac{4}{3} + \frac{9}{2}\right) = 18$

2. $$P_{N_i}(n) = \begin{cases} n=1 & P_i \\ n=2 & P_i(1-P_i) \\ n=3 & P_i(1-P_i)^2 \\ \vdots & \\ n=k & P_i(1-P_i)^{k-1} \end{cases}$$

$$\sum_{n=1}^{\infty} P_N(n) = \sum_{n=1}^{\infty} P_i(1-P_i)^{n-1} = \frac{P_i}{1-P_i}\left(\sum_{n=0}^{\infty}(1-P_i)^n - 1\right)$$

$$= \frac{P_i}{1-P_i}\left(\frac{1}{P_i} - 1\right) = 1$$

$\therefore$ pmf

$$E(N) = \sum_{n=1}^{\infty} n P_N(n) = \sum_{n=1}^{\infty} n P_i(1-P_i)^{n-1} = P_i \underbrace{\sum_{n=1}^{\infty} n(1-P_i)^{n-1}}_{} = P_i \cdot \frac{1}{\{1-(1-P_i)\}^2}$$

$$= \frac{1}{P_i}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\frac{d}{dx}\sum_{n=0}^{\infty} x^n = \frac{d}{dx}(1-x)^{-1}$$

$$\boxed{\sum_{n=0}^{\infty} n x^{n-1}} = (-1)(1-x)^{-2}\cdot(-1)$$

$$= (1-x)^{-2}$$

주사위 눈이 나올 확률이 각각 $\frac{1}{6}$ 이므로 $P_I(i) = \begin{cases} E(N_1), & \frac{1}{6} \quad (i=1) \\ E(N_2) & \frac{1}{6} \quad (i=2) \\ \vdots & \\ E(N_6) & \frac{1}{6} \quad (i=6) \end{cases}$

$$E_I\left[E(N_i \mid I)\right] = E(N)$$

$$= \frac{1}{6P_1} + \frac{1}{6P_2} + \frac{1}{6P_3} + \frac{1}{6P_4} + \frac{1}{6P_5} + \frac{1}{6P_6}$$