

## Problem Set 1

**Sample space, probability law, probabilistic models**

1. Prove that  $\mathbb{P}((A \cap B)^c) = 1 + \mathbb{P}(A \cup B) - \mathbb{P}(A) - \mathbb{P}(B)$ .
2. Consider an experiment of rolling a six-sided die, and then tossing a coin as many times as the number rolled on the die.
  - (a) What is the sample space?
  - (b) Assuming that each outcome is equally likely, what is the probability of a single outcome?
  - (c) Assuming that each outcome is equally likely, determine whether the die is fair or not.
3. Consider an experiment of sampling a point from the unit square  $[0, 1] \times [0, 1]$ . Denote by  $(X, Y)$  the outcome of the experiment. Assume that each outcome is equally likely.
  - (a) What is the sample space?
  - (b) Calculate the probability that  $|X - Y| \leq 0.5$ .
  - (c) Calculate the probability that  $X = Y$  or  $X + Y = 1$ .
4. Consider an experiment of tossing a fair coin five times and then rolling a fair 6-sided die. Assume that each outcome is equally likely.
  - (a) What is the sample space  $\Omega$ ?
  - (b) What is the probability that at least two heads come up and the roll is even?
  - (c) Compute the probability that at most one head comes up or the roll is odd.
5. A six-sided die is loaded in a way that each even face is three times as likely as each odd face. All even faces are equally likely, and all odd faces are equally likely.
  - (a) Specify a probabilistic model  $(\Omega, \mathbb{P})$  for a single roll of this die.
  - (b) Find the probability that the outcome is less than 4.

6. A four-sided die is rolled repeatedly, until the first time (if ever) that an even number is obtained. What is the sample space for this experiment?