Probability and Statistics, S2023	Posted: April 11, 2023
Problem Set 4	
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Joint PMF, conditioning, independence

1. Two random variables X and Y have the following joint PMF:

$$p_{X,Y}(x,y) = \begin{cases} cxy, & \text{if } (x,y) = (1,1),(1,2),(1,3),(2,1),(2,2),(2,3) \\ 0, & \text{otherwise} \end{cases}$$

   (a) Calculate the value of constant c.

   (b) Find the marginal PMFs $p_X(x)$ and $p_Y(y)$.

   (c) Determine whether X and Y are independent.

   (d) Find the conditional PMFs $p_{X|Y}(x|y)$ and $p_{Y|X}(y|x)$.

(e) Calculate the expectation of X^2Y^2.

2. There are six different coins numbered 1 through 6. Coin i has a probability of p_i of landing heads. You will flip one of these coins according to the following procedure: (6개의 동전이 있다. 동전 i는 앞면이 나올 확률이 p_i이다. 다음과 같이 동전 하나를 던진다고 가정하자)

   - You roll a fair six-sided die. Let i be the number rolled on the die. (6개 면을 갖는 공평한 주사위를 던진다. i를 나온 숫자라 하자)
   - Pick the coin i, and flip it until it lands heads. (동전 i를 선택해서 앞면이 나올 때까지 계속 던진다)

   We are interested in the number of tosses until the coin lands heads. Let N denote this number. (여기서 우리는 동전을 얼마나 많이 던지게 되는지를 계산하고자 한다. 이 수를 N이라 정의함)

   - Given that the number rolled on the die is i, what is the conditional expectation of N?

   - Calculate the expectation of N?