Problems from Foundations of Machine Learning by Mohri et al.

- 1. Simplified AdaBoost. Suppose we simplify AdaBoost by setting the parambeter  $\alpha_t$  to a fixed value  $\alpha_t = \alpha > 0$ , independent of round t.
  - (a) Let  $\gamma$  be such that  $\left(\frac{1}{2} \epsilon_t\right) \geq \gamma > 0$ . Find the optimal  $\alpha$  as a function of  $\gamma$  wrt empirical error.
  - (b) For the value of  $\alpha$  that you found, does the algorithm assign the same probability mass to correctly classified and misclassified examples at each round? If not, which set has higher probability?
  - (c) Using the previous value of  $\alpha$ , give a bound on the empirical error of the algorithm as a function only of  $\gamma$  and the number of rounds of boosting (T).
- 2. Update guarantee. Assume that the main weak learner assumption of AdaBoost holds. Let  $h_t$  be the base learner selected at round t. Show that the base learner  $h_{t+1}$  selected at round t+1 must be different from  $h_t$ .
- 3. Fix  $\epsilon \in (0, .5)$ . Let the training data be m points.  $\frac{m}{4}$  negative points are each at (1, 1) and (-1, -1);  $\frac{m(1-\epsilon)}{4}$  points are each at (1, -1); and  $\frac{m(1+\epsilon)}{4}$  points are at (-1, +1). What does AdaBoost do with boosting stumps as the base learner? What solution does the algorithm return after T rounds?