Role of the Z polarization in the $H \to b\bar{b}$ measurement.

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D. Gonçalves and JN (arXiv:1805.06385), D. Gonçalves and JN (in preparation).

Introduction: VH, $H \rightarrow b\bar{b}$ channels.

- ► The SM Higgs decay $H \to b\bar{b}$ has the largest branching ratio, $\sim 58\%$.
- ▶ The largest sensitivity to $H \to b\bar{b}$ can be gained from the boosted VH production (V = W, Z), where V decays into leptons. (Butterworth et al 2008)
- ▶ ATLAS and CMS have reported evidences for the VH, $H \rightarrow b\bar{b}$ with 3.5σ and 3.3σ significance, respectively. (ATLAS 2017, CMS 2018)
- ► There are 3 channels, based on the number of charged leptons: $ZH \rightarrow \nu\nu b\bar{b}$, $WH \rightarrow \ell\nu b\bar{b}$, $ZH \rightarrow \ell\ell b\bar{b}$.

Dataset	1	p_0	Signif	icance
Dataset	Exp.	Obs.	Exp.	Obs.
0-lepton	4.2%	30%	1.7	0.5
1-lepton	3.5%	1.1%	1.8	2.3
2-lepton	3.1%	0.019%	1.9	3.6
Combined	0.12%	0.019%	3.0	3.5

Channels	Significance	Significance
	expected	observed
0-lepton	1.5	0.0
1-lepton	1.5	3.2
2-lepton	1.8	3.1
Combined	2.8	3.3

(ATLAS 2017, CMS 2018)

Introduction: $Z(\ell^+\ell^-)H(b\bar{b})$ channel.

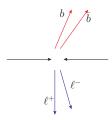
▶ When a search for the ZH, $ZH \rightarrow \ell\ell b\bar{b}$ channel is performed at the LHC, the dominant background after a signal extraction procedure is $Zb\bar{b}$, $Zb\bar{b} \rightarrow \ell\ell b\bar{b}$.

Process	0-lepton	1-lepton	2-lepton low- $p_{\rm T}({\rm V})$	2-lepton high- $p_{\rm T}({ m V})$
Vbb	216.8	102.5	617.5	113.9
Vb	31.8	20.0	141.1	17.2
V+udscg	10.2	9.8	58.4	4.1
t t	34.7	98.0	157.7	3.2
Single top quark	11.8	44.6	2.3	0.0
VV(udscg)	0.5	1.5	6.6	0.5
VZ(bb)	9.9	6.9	22.9	3.8
Total background	315.7	283.3	1006.5	142.7
VH	38.3	33.5	33.7	22.1
Data	334	320	1030	179
S/B	0.12	0.12	0.033	0.15

(CMS 2018)

Introduction: $Z(\ell^+\ell^-)b\bar{b}$ background in $Z(\ell^+\ell^-)H(b\bar{b})$ channel.

▶ Information on the Z polarization has not been exploited with the aim of discriminating the ZH signal from the $Zb\bar{b}$ background.



Differential cross section for $Z(\ell^+\ell^-) + X$ can be expanded in general as

$$\begin{split} \frac{d\sigma}{d\Omega d\cos\theta d\phi} &= F_1(1+\cos^2\theta) + F_2(1-3\cos^2\theta) + F_3\sin2\theta\cos\phi + F_4\sin^2\theta\cos2\phi \\ &\quad + F_5\cos\theta + F_6\sin\theta\cos\phi + F_7\sin\theta\sin\phi + F_8\sin2\theta\sin\phi + F_9\sin^2\theta\sin2\phi. \end{split}$$

▶ Eight functions F_i/F_1 (i=2 to 9) parametrize the Z polarization and determine $Z \to \ell^+\ell^-$ decay angular ($\cos\theta,\phi$) distribution.

In this work, we . . .

- ▶ show that the ZH signal and the $Zb\bar{b}$ dominant background exhibit different states of Z polarization. (i.e. different values for F_i/F_1 (i=2 to 9)).
- present a procedure to maximally exploit this information and estimate the possible sensitivity gains to the current analyses.

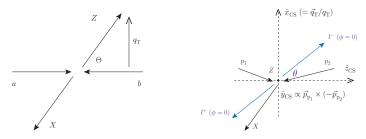
Outline

- ▶ Introduction.
- Kinematics and frame.
- ightharpoonup Z polarization in the ZH signal and the $Zbar{b}$ background.
- ▶ Lepton p_T.
- Analysis.
- Summary.

Kinematics and frame: Z production and decay

Differential cross sections for ZH and $Zb\bar{b}$, $Z \to \ell^+\ell^-$ can be expanded in general as

$$\begin{split} \frac{d\sigma}{dq_{\mathrm{T}}^2 d\cos\Theta d\cos\theta d\phi} &= F_1(1+\cos^2\theta) + F_2(1-3\cos^2\theta) + F_3\sin2\theta\cos\phi + F_4\sin^2\theta\cos2\phi \\ &\quad + F_5\cos\theta + F_6\sin\theta\cos\phi + F_7\sin\theta\sin\phi + F_8\sin2\theta\sin\phi + F_9\sin^2\theta\sin2\phi. \end{split}$$



- ▶ Coefficients F_i (i = 1 to 9) are functions of only q_T and Θ .
- $lackbox{ }q_{
 m T}$: transverse momentum of the Z in the lab. frame. $(q_{
 m T}\equiv |ec{q}_{
 m T}|)$
- ullet : polar angle of the Z from the collision axis in the c.m. frame of the Z+X system.

We define the lepton angles θ , ϕ in the Collins-Soper (CS) frame (Collins, Soper 1977),

$$\ell^-: \frac{Q}{2} \big(1, \; \sin\theta\cos\phi, \; \sin\theta\sin\phi, \; \cos\theta \big), \quad \ \ \ell^+: \; \frac{Q}{2} \big(1, \; -\sin\theta\cos\phi, \; -\sin\theta\sin\phi, \; -\cos\theta \big)$$

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Z polarization in ZH and $Zb\bar{b}$: coefficients in the ZH signal.

We find that only F_1 and F_4 are nonzero in $q\bar{q} \to ZH$ at the LO (employing notation of Hagiwara et al 1984):

$$\frac{d\sigma}{dq_{\mathrm{T}}^2 d\cos\Theta d\cos\theta d\phi} = F_1(1+\cos^2\theta) + \sqrt{(1-3\cos^2\theta)} + \sqrt{\sin 2\theta \cos \phi} + F_4\sin^2\theta \cos 2\phi + \sqrt{\cos \theta} + \sqrt{\sin \theta \cos \phi} + \sqrt{\sin \theta \sin \phi} + \sqrt{\sin 2\theta \sin \phi} + \sqrt{\sin 2\theta \sin 2\phi}.$$

The scattering amplitudes in the CS frame are

$$\mathcal{M}_{\sigma}^{\lambda=\pm}(q\bar{q}) = -4m_Z^3 G_F(v_q + \sigma a_q) \frac{\sqrt{\hat{s}}}{\hat{s} - m_Z^2 + im_Z \Gamma_Z} \sigma \left(1 + \sigma \lambda \sqrt{1 + q_{\mathrm{T}}^2/m_Z^2}\right),$$
 $\mathcal{M}_{\sigma}^{\lambda=0}(q\bar{q}) = 0.$

where λ (= \pm , 0) are the eigenvalues of J_z for the Z boson.

$$F_{2,3,5,6,7,8,9}$$
 vanish because ...

$$ightharpoonup F_{7.8,9}$$
 are proportional to complex phases in the amplitudes.

F_{2,3,6,7,8} are proportional to
$$\mathcal{M}_{\sigma}^{\lambda=0}(q\bar{q})$$
.
F₅ vanishes in symmetric pp collisions:

$$F_5$$
 vanishes in symmetric pp collisions

$$F_{i} = \sum_{q=u,d,c,s}^{} \int dY \ q(x_{1})\bar{q}(x_{2})\hat{F}_{i}^{q\bar{q}} + \bar{q}(x_{1})q(x_{2})\hat{F}_{i}^{\bar{q}q}, \qquad \hat{F}_{5}^{q\bar{q}} = -\hat{F}_{5}^{\bar{q}q} \propto \sqrt{1 + q_{\mathrm{T}}^{2}/m_{Z}^{2}},$$

The θ , ϕ distribution depends only on $F_4/F_1 = -\frac{q_{\rm T}^2}{2m_{-}^2+q_{-}^2}$.

- It increases along the negative direction with q_T
- It is flat over $\cos \Theta$

Z polarization in ZH and $Zb\bar{b}$: difference between ZH and $Zb\bar{b}$.

We find that only F_1 , F_2 and F_4 are non-vanishing in $gg \to ZH$ and $Zb\bar{b}$ after integration over $\cos\Theta$, when the signal selections are imposed. Consequently, the angular distribution is determined from

$$\frac{d\sigma}{dq_{\mathrm{T}}^2 d\cos\theta d\phi} = \widehat{F}_1 \big[1 + \cos^2\theta + \frac{\mathbf{A_2}}{2} (1 - 3\cos^2\theta) + \frac{\mathbf{A_4}}{4}\sin^2\theta\cos2\phi \big],$$

where $A_2 = \hat{F}_2/\hat{F}_1$ and $A_4 = \hat{F}_4/\hat{F}_1$. We generate the LO events with MadGraph5.aMC@NLO (Alwall et al 2014) and apply the signal selection cuts,

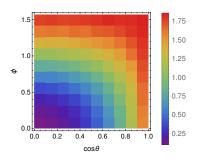
$$\begin{split} &75 < m_{\ell\ell} < 105 \text{ GeV}, \quad 115 < m_{bb} < 135 \text{ GeV}, \\ &\rho_{\text{Tb}} > 25 \text{ GeV}, \quad |y_b| < 2.5, \quad 0.3 < \Delta R_{bb} < 1.2, \quad q_{\text{T}} > 200 \text{ GeV}. \end{split}$$

	ZH_{DY}	$ZH_{ m GF}$	Zbb
A_2	0.00	0.03	0.47
A_4	-0.83	-0.97	0.45

- ▶ Table: Normalized angular coefficients A_2 , A_4 for the signal and the background.
- ▶ The signal shows very distinct values A_2 , A_4 from the background.
- ▶ These differences clearly appear in a $(\cos \theta, \phi)$ distribution.

Z polarization in ZH and $Zb\bar{b}$: $(\cos\theta,\phi)$ distribution.

$$\frac{d\sigma}{dq_{rr}^2 d\cos\theta d\phi} = \widehat{F}_1 \big[1 + \cos^2\theta + \frac{\mathbf{A_2}}{2} (1 - 3\cos^2\theta) + \frac{\mathbf{A_4}}{4}\sin^2\theta\cos2\phi \big],$$



	ZH_{DY}	$ZH_{ m GF}$	Zbb
A_2	0.00	0.03	0.47
A_4	-0.83	-0.97	0.45

- Figure: Ratio of the normalized $(\cos \theta, \phi)$ distribution for ZH_{DY} to that for $Zb\bar{b}$.
- ▶ The angles can be defined in the restricted ranges $0 \le \theta \le \pi/2$, $0 \le \phi \le \pi/2$.
- lt is observed that ZH signal events are more distributed at $\phi \sim \pi/2$ and $Zb\bar{b}$ more at $\phi \sim 0$. This is the consequence of the large difference in A_4 .
- We perform a two dimensional binned log-likelihood analysis based on the $(\cos \theta, \phi)$ distribution.

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Lepton $p_{\rm T}$: difference between ZH and $Zb\bar{b}$

 $\textit{p}_{\mathrm{T}\ell}$ have simple expressions in terms of $\theta,\,\phi$ defined in the CS frame and \textit{q}_{T} :

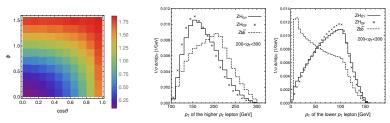
$$p_{{\rm T}\ell 1(2)} = \frac{1}{2} \sqrt{q_{\rm T}^2 + Q^2 \sin^2 \theta + q_{\rm T}^2 \sin^2 \theta \cos^2 \phi \pm 2 q_{\rm T} \sqrt{Q^2 + q_{\rm T}^2} \sin \theta |\cos \phi|}.$$

The ZH signal predicts more events at $\phi \sim \pi/2$:

$$p_{{\rm T}\ell 1} = p_{{\rm T}\ell 2} = \frac{1}{2} \sqrt{q_{\rm T}^2 + Q^2 \sin^2 \theta}, \qquad \quad p_{{\rm T}\ell 1} - p_{{\rm T}\ell 2} = 0.$$

The $Zb\bar{b}$ background predicts more events at $\phi \sim$ 0:

$$p_{\mathrm{T}\ell 1(2)} = \frac{1}{2} \Big| q_{\mathrm{T}} \pm \sqrt{Q^2 + q_{\mathrm{T}}^2} \sin \theta \Big|, \qquad p_{\mathrm{T}\ell 1} - p_{\mathrm{T}\ell 2} = \min \Big\{ q_{\mathrm{T}}, \sqrt{Q^2 + q_{\mathrm{T}}^2} \sin \theta \Big\}.$$



- ▶ The higher (lower) p_T lepton in $Zb\bar{b}$ is predicted to be harder (softer) than that in ZH; a consequence of the difference in the Z polarization.
- A lepton p_T cut can partially capture the difference in the Z polarization and improve the sensitivity to the signal.

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Analysis: simulation setup.

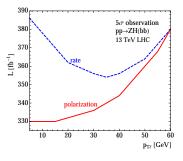
- ▶ We simulate the ZH signal and backgrounds at the hadron level with Sherpa+OpenLoops. (Gleisberg et al 2009, Cascioli et al 2012, Denner et al 2017)
- We use the BDRS analysis as a well understood benchmark in order to achieve a high signal sensitivity; $p_{TJ} > 200$ GeV, $|\eta_J| < 2.5$, $|m_H^{\rm BDRS} m_H| < 10$ GeV. (Butterwirth et al 2008)
- We require two charged leptons (e or μ) with $|\eta_\ell| < 2.5$, which reconstruct a boosted Z boson: 75 $< m_{\ell\ell} < 105$ GeV, and $q_{\rm T} \equiv p_{{\rm T}\ell\ell} > 200$ GeV.

	$ZH_{ m DY}$				
Rates (fb) after BDRS, $p_{\mathrm{T}\ell} >$ 30 GeV	0.16	0.03	0.35	0.02	0.02

- ▶ Table: Signal and background rates after the BDRS analysis, with the lepton $p_{\rm T}$ threshold $p_{\rm T\ell} >$ 30 GeV. 70% b-tagging efficiency and 1% misstag rate are taken into account.
- After the BDRS analysis, we perform a two dimensional binned log-likelihood analysis based on the $(\cos\theta,\phi)$ distribution, invoking the CL_s method (Read 2002).

Analysis: results.

Figure: Luminosities required for a 5σ observation of the $pp \to Z(\ell\ell)H(bb)$ channel as functions of the lepton $p_{\rm T}$ lower cut. The solid curve takes the Z polarization into account by performing a two dimensional binned log-likelihood analysis on the $(\cos\theta,\phi)$ distribution, while the dashed curve does not.



- ▶ The rate result continuously improves from 385 $\rm fb^{-1}$ to 355 $\rm fb^{-1}$ by changing the $p_{\rm T}\ell$ threshold from 5 to 35 GeV, as expected.
- ► The pol. result is always better than the rate-only result.
- The required luminosity monotonically increases with the $p_{T\ell}$ threshold, because the pol. information is disturbed by the $p_{T\ell}$ cut. This selection can capture the difference in pol. only partially, therefore it is never better than the explicit use of the $(\cos\theta,\phi)$ distribution.
- It is suggested to define $p_{T\ell}$ threshold as small as possible and to exploit the difference in the (cos θ , ϕ) distribution between the signal and the background.

Summary.

- We have studied the potential of the Z polarization to improve the sensitivity to the signal $pp \to Z(\ell\ell)H(b\bar{b})$.
- At first, we have shown that the signal and the $Zb\bar{b}$ dominant background exhibit different states of Z polarization, which appear as the large difference in the lepton angles $(\cos\theta,\phi)$ distribution.
- ▶ This difference can be partially captured by a suitable value for the $p_{T\ell}$ lower threshold, and fully taken into account by explicitly analyzing the $(\cos\theta,\phi)$ distribution.
- ▶ We have performed simulations at hadron level and estimated the impact of these two approaches for the 5σ $Z(\ell\ell)H(b\bar{b})$ observation, and found relevant improvements.
- Because our proposal relies only on lepton reconstruction, which has small experimental uncertainty, it can be readily included in the current ATLAS and CMS studies. The current ATLAS (CMS) study uses the $p_{T\ell}$ threshold 7 GeV (20 GeV), in which case the benefit of exploiting the Z polarization is estimated to be $\sim 15\%$ ($\sim 10\%$) in the required luminosity for 5σ observation.

Thank you so much for your attention.



- ► The current ATLAS and CMS results use multivariate analyses after basic event selections, in order to maximize the sensitivity to the signal.
- ▶ The variables used for the multivariate analyses include only $m(\ell^+\ell^-)$ as the information of the charged leptons:

Variable	0-lepton	1-lepton	2-lepton
p_{T}^{V}	$\equiv E_{\mathrm{T}}^{\mathrm{miss}}$	×	×
$E_{\mathrm{T}}^{\mathrm{miss}}$	×	×	×
$p_{\mathrm{T}}^{b_{1}^{1}}$ $p_{\mathrm{T}}^{b_{2}}$	×	×	×
$p_{\mathrm{T}}^{b_2}$	×	×	×
m_{bb}	×	×	×
$\Delta R(\vec{b}_1, \vec{b}_2)$	×	×	×
$ \Delta \eta(ec{b}_1,ec{b}_2) $	×		
$\Delta\phi(\vec{V}, b\vec{b})$	×	×	×
$ \Delta \eta(\vec{V}, b\vec{b}) $			×
$m_{ m eff}$	×		
$\min[\Delta\phi(\vec{\ell}, \vec{b})]$		×	
$m_{ m T}^W$		×	
$m_{\ell\ell}$			×
m_{top}		×	
$ \Delta \hat{Y}(\vec{V}, \vec{bb}) $		×	
	Only	in 3-jet ev	vents
$p_{\mathrm{T}}^{\mathrm{jet_{3}}}$	×	×	×
m_{bbj}	×	×	×

(ATLAS 2017)

- ► The current ATLAS and CMS results use multivariate analyses after basic event selections, in order to maximize the sensitivity to the signal.
- ▶ The variables used for the multivariate analyses include only $m(\ell^+\ell^-)$ as the information of the charged leptons:

Variable	Description	Channels
M(jj)	dijet invariant mass	All
$p_{\mathrm{T}}(\mathbf{j}\mathbf{j})$	dijet transverse momentum	All
$p_T(j_1), p_T(j_2)$	transverse momentum of each jet	0- and 2-lepton
$\Delta R(jj)$	distance in η – ϕ between jets	2-lepton
$\Delta \eta(jj)$	difference in η between jets	0- and 2-lepton
$\Delta \phi(jj)$	azimuthal angle between jets	0-lepton
$p_{\mathrm{T}}(\mathrm{V})$	vector boson transverse momentum	All
$\Delta \phi(V, jj)$	azimuthal angle between vector boson and dijet directions	All
$p_{\mathrm{T}}(jj)/p_{\mathrm{T}}(\mathrm{V})$	p _T ratio between dijet and vector boson	2-lepton
$M(\ell\ell)$	reconstructed Z boson mass	2-lepton
CMVA _{max}	value of CMVA discriminant for the jet	0- and 2-lepton
	with highest CMVA value	
$CMVA_{min}$	value of CMVA discriminant for the jet	All
	with second highest CMVA value	
CMVA _{add}	value of CMVA for the additional jet	0-lepton
	with highest CMVA value	
$p_{\mathrm{T}}^{\mathrm{miss}}$	missing transverse momentum	1- and 2-lepton
$\Delta \phi(\vec{p}_{T}^{miss},j)$	azimuthal angle between $\vec{p}_{T}^{\text{miss}}$ and closest jet ($p_{T} > 30 \text{GeV}$)	0-lepton
$\Delta \phi(\vec{p}_{T}^{miss}, \ell)$	azimuthal angle between \vec{p}_{T}^{miss} and lepton	1-lepton
m_{T}	mass of lepton $\vec{p}_T + \vec{p}_T^{miss}$	1-lepton
m_{top}	reconstructed top quark mass	1-lepton
$N_{\rm aj}$	number of additional jets	1- and 2-lepton
$p_{T}(add)$	transverse momentum of leading additional jet	0-lepton
SA5	number of soft-track jets with $p_T > 5 \text{ GeV}$	Áll

Signal regions		pton	1-lepton		2-lepton			
Signal regions	$p_{\rm T}^V > 150$ C	GeV, 2-b-tag	$p_{\rm T}^V > 150$ (GeV, 2-b-tag	75 GeV < 1	$p_{\rm T}^{V} < 150 \text{ GeV}, 2\text{-}b\text{-tag}$	$p_{\rm T}^{V} > 150 \text{ GeV}, 2\text{-}b\text{-tag}$	
Sample	2-jet	3-jet	2-jet	3-jet	2-jet	≥3-jet	2-jet	≥3-jet
Z + ll	9.0 ± 5.1	15.5 ± 8.1	< 1	-	9.2 ± 5.4	35 ± 19	1.9 ± 1.1	16.4 ± 9.3
Z + cl	21.4 ± 7.7	42 ± 14	2.2 ± 0.1	4.2 ± 0.1	25.3 ± 9.5	105 ± 39	5.3 ± 1.9	46 ± 17
Z + HF	2198 ± 84	3270 ± 170	86.5 ± 6.1	186 ± 13	3449 ± 79	8270 ± 150	651 ± 20	3052 ± 66
W + ll	9.8 ± 5.6	17.9 ± 9.9	22 ± 10	47 ± 22	< 1	< 1	< 1	< 1
W + cl	19.9 ± 8.8	41 ± 18	70 ± 27	138 ± 53	< 1	< 1	< 1	< 1
W + HF	460 ± 51	1120 ± 120	1280 ± 160	3140 ± 420	3.0 ± 0.4	5.9 ± 0.7	< 1	2.2 ± 0.2
Single top quark	145 ± 22	536 ± 98	830 ± 120	3700 ± 670	53 ± 16	134 ± 46	5.9 ± 1.9	30 ± 10
$t\bar{t}$	463 ± 42	3390 ± 200	2650 ± 170	20640 ± 680	1453 ± 46	4904 ± 91	49.6 ± 2.9	430 ± 22
Diboson	116 ± 26	119 ± 36	79 ± 23	135 ± 47	73 ± 19	149 ± 32	24.4 ± 6.2	87 ± 19
Multi-jet e sub-ch.	_	-	102 ± 66	27 ± 68	-	-	_	_
Multi-jet μ sub-ch.	-	-	133 ± 99	90 ± 130	-	-	_	_
Total bkg.	3443 ± 57	8560 ± 91	5255 ± 80	28110 ± 170	5065 ± 66	13600 ± 110	738 ± 19	3664 ± 56
Signal (fit)	58 ± 17	60 ± 19	63 ± 19	65 ± 21	25.6 ± 7.8	46 ± 15	13.6 ± 4.1	35 ± 11
Data	3520	8634	5307	28168	5113	13640	724	3708

(ATLAS 2017)

Selection	0-lepton	1-l	epton	2-lepton
		e sub-channel	μ sub-channel	
Trigger	$E_{\mathrm{T}}^{\mathrm{miss}}$	Single lepton	$E_{\mathrm{T}}^{\mathrm{miss}}$	Single lepton
Leptons	0 loose leptons	1 tight electron	1 medium muon	2 loose leptons with $p_T > 7 \text{ GeV}$
	with $p_T > 7 \text{ GeV}$	$p_T > 27 \text{ GeV}$	$p_T > 25 \text{ GeV}$	\geq 1 lepton with $p_T > 27 \text{ GeV}$
$E_{\mathrm{T}}^{\mathrm{miss}}$	> 150 GeV	> 30 GeV	-	_
$m_{\ell\ell}$	-		-	$81 \text{ GeV} < m_{\ell\ell} < 101 \text{ GeV}$
Jets	Exactly	2 or 3 jets		Exactly 2 or \geq 3 jets
Jet p_T		>	> 20 GeV	
b-jets		Exactly	2 b-tagged jets	
Leading b-tagged jet p_T		>	→ 45 GeV	
H_{T}	> 120 (2 jets), >150 GeV (3 jets)		-	-
$\min[\Delta\phi(\vec{E}_{T}^{miss}, jets)]$	> 20° (2 jets), > 30° (3 jets)		-	_
$\Delta \phi(\vec{E}_{T}^{miss}, b\vec{b})$	> 120°		-	-
$\Delta \phi(\vec{b}_1, \vec{b}_2)$	< 140°	-		-
$\Delta \phi(\vec{E}_{T}^{miss}, \vec{E}_{T,trk}^{miss})$	< 90°		-	_
p_{T}^{V} regions	> 15	50 GeV		(75, 150] GeV, > 150 GeV
Signal regions	✓	$m_{bb} \ge 75 \text{ GeV}$	or $m_{\rm top} \le 225 \text{ GeV}$	Same-flavour leptons
			•	Opposite-sign charge ($\mu\mu$ sub-channel)
Control regions	-	$m_{bb} < 75 \text{ GeV as}$	$m_{top} > 225 \text{ GeV}$	Different-flavour leptons

(ATLAS 2017)

Variable	0-lepton	1-lepton	2-lepton
$p_{\mathrm{T}}(\mathrm{V})$	>170	>100	[50, 150], >150
$M(\ell\ell)$	_	_	[75, 105]
p_{T}^{ℓ}	_	(> 25, > 30)	>20
$p_{\mathrm{T}}(\mathbf{j}_1)$	>60	>25	>20
$p_{\mathrm{T}}(\mathbf{j}_2)$	>35	>25	>20
$p_{\mathrm{T}}(\mathbf{j}\mathbf{j})$	>120	>100	_
M(jj)	[60, 160]	[90, 150]	[90, 150]
$\Delta \phi(V, jj)$	>2.0	>2.5	>2.5
CMVA _{max}	$>$ CMVA $_{\rm T}$	$>$ CMVA $_{\rm T}$	$>$ CMVA $_{\rm L}$
CMVA _{min}	>CMVA _L	>CMVA _L	$>$ CMVA $_{\rm L}$
$N_{\rm aj}$	<2	<2	_
$N_{\mathrm{a}\ell}$	=0	=0	_
pmiss	>170	_	_
$\Delta \phi(\vec{p}_{\rm T}^{\rm miss}, j)$	>0.5	_	_
$\Delta \phi(\vec{p}_{\mathrm{T}}^{\mathrm{miss}}, \vec{p}_{\mathrm{T}}^{\mathrm{miss}}(\mathrm{trk}))$	< 0.5	_	_
$\Delta \phi(\vec{p}_{\mathrm{T}}^{\mathrm{miss}}, \ell)$	_	< 2.0	_
Lepton isolation	_	< 0.06	(< 0.25, < 0.15)

(CMS 2018)

> -0.8

The amplitudes $\mathcal{M}_{\sigma}^{\lambda}(qar{q})$ can be written in general as

$$\mathcal{M}_{\sigma}^{\lambda}(q\bar{q}) = \frac{2m_Z^2}{v} \frac{1}{\hat{s} - m_Z^2 + im_Z \Gamma_Z} \Gamma_{\sigma}(q\bar{q}) \cdot \epsilon_{\lambda}.$$

In the c.m. frame of the Z + H system, where the z axis is chosen along the collision axis and the y axis is chosen perpendicular to the scattering plane, we find

$$\Gamma_{\sigma}(qar{q})^{\mu}=rac{2m_{Z}}{v}(v_{q}+\sigma a_{q})\sqrt{\hat{s}}\;(0,-1,-i\sigma,0).$$

The currents in the CS frame can be easily obtained by the two boost steps:

$$\Gamma_{\sigma}(qar{q})^{\mu}=rac{2m_{Z}}{v}(v_{q}+\sigma a_{q})\sqrt{\hat{s}}\;\Big(q_{\mathrm{T}}/m_{Z},-\sqrt{1+q_{\mathrm{T}}^{2}/m_{Z}^{2}},-i\sigma,0\Big).$$

This procedure is justified because the initial quark and anti-quark are assumed to be massless and the helicity of a massless particle is frame-independent. We choose the following as the Z polarization vectors:

$$\begin{split} \epsilon^{\mu}_{\lambda=\pm} &= (0,-\lambda,-i,0)/\sqrt{2}, \\ \epsilon^{\mu}_{\lambda=0} &= (0,0,0,1). \end{split}$$

The angles θ (0 $\leq \theta \leq \pi/2)$ and ϕ (0 $\leq \phi \leq \pi/2)$ can be obtained from

$$\begin{split} |\cos\theta| &= \frac{2|q^0p_\ell^3 - q^3p_\ell^0|}{Q\sqrt{Q^2 + |\vec{q}_\mathrm{T}|^2}}\,,\\ |\cos\phi| &= \frac{2}{\sin\theta} \frac{\left|Q^2\vec{p}_{\mathrm{T}\ell} \cdot \vec{q}_{\mathrm{T}} - |\vec{q}_{\mathrm{T}}|^2p_\ell \cdot q\right|}{Q^2|\vec{q}_{\mathrm{T}}|\sqrt{Q^2 + |\vec{q}_{\mathrm{T}}|^2}} \end{split}$$

where $q^\mu=(q^0,\vec{q}_{\rm T},q^3)$ and $p_\ell^\mu=(p_\ell^0,\vec{p}_{{
m T}\ell},p_\ell^3)$ are four-momenta of the Z boson and one of the leptons, respectively, in the laboratory frame. We stress that p_ℓ^μ can be the momentum of either ℓ^- or ℓ^+ (i.e. either gives the same θ and ϕ values). This is

simply because interchanging ℓ^- and ℓ^+ corresponds to $\theta \to \pi - \theta$ and $\phi \to \phi + \pi$

(i.e. $\cos \theta \rightarrow -\cos \theta$ and $\cos \phi \rightarrow -\cos \phi$).

The coefficients F_i (i = 2, 9) can be numerically calculated from

$$\frac{F_2}{F_1} = \frac{1}{3} + \frac{16}{9} \frac{\sigma(1 - 3\cos^2\theta > 1/4) - \sigma(1 - 3\cos^2\theta < 1/4)}{\sigma(1 - 3\cos^2\theta > 1/4) + \sigma(1 - 3\cos^2\theta < 1/4)},\tag{2a}$$

$$\frac{F_3}{F_1} = \pi \frac{\sigma(\sin 2\theta \cos \phi > 0) - \sigma(\sin 2\theta \cos \phi < 0)}{\sigma(\sin 2\theta \cos \phi > 0) + \sigma(\sin 2\theta \cos \phi < 0)},$$
(2b)

$$\frac{F_4}{F_1} = \pi \frac{\sigma(\sin^2\theta\cos 2\phi > 0) - \sigma(\sin^2\theta\cos 2\phi < 0)}{\sigma(\sin^2\theta\cos 2\phi > 0) + \sigma(\sin^2\theta\cos 2\phi < 0)},$$
(2c)

$$\frac{F_5}{F_1} = \frac{8}{3} \frac{\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)}{\sigma(\cos \theta > 0) + \sigma(\cos \theta < 0)},\tag{2d}$$

$$\frac{F_6}{F_1} = \frac{8}{3} \frac{\sigma(\sin\theta\cos\phi > 0) - \sigma(\sin\theta\cos\phi < 0)}{\sigma(\sin\theta\cos\phi > 0) + \sigma(\sin\theta\cos\phi < 0)},\tag{2e}$$

$$\frac{F_7}{F_1} = \frac{8}{3} \frac{\sigma(\sin\theta\sin\phi > 0) - \sigma(\sin\theta\sin\phi < 0)}{\sigma(\sin\theta\sin\phi > 0) + \sigma(\sin\theta\sin\phi < 0)},$$
(2f)

$$\frac{F_8}{F_1} = \pi \frac{\sigma(\sin 2\theta \sin \phi > 0) - \sigma(\sin 2\theta \sin \phi < 0)}{\sigma(\sin 2\theta \sin \phi > 0) + \sigma(\sin 2\theta \sin \phi < 0)},$$
(2g)

$$\frac{F_9}{F_1} = \pi \frac{\sigma(\sin^2\theta \sin 2\phi > 0) - \sigma(\sin^2\theta \sin 2\phi < 0)}{\sigma(\sin^2\theta \sin 2\phi > 0) + \sigma(\sin^2\theta \sin 2\phi < 0)},\tag{2h}$$

which measure the differences in the numbers of events.

In terms of the scattering amplitudes $\mathcal{M}^{\lambda}_{\lambda_1\lambda_2}$, where $\lambda_{1,2}$ denote the helicity of the initial gluons, the functions f_i can be written as

$$egin{align} f_1 &= \overline{\sum_{\lambda_1,\lambda_2}} rac{1}{2} ig(|\mathcal{M}^+_{\lambda_1\lambda_2}|^2 + |\mathcal{M}^-_{\lambda_1\lambda_2}|^2 + |\mathcal{M}^0_{\lambda_1\lambda_2}|^2 ig) \ f_2 &= \overline{\sum_{\lambda_1,\lambda_2}} rac{1}{2} |\mathcal{M}^0_{\lambda_1\lambda_2}|^2 \ \end{aligned}$$

$$egin{aligned} f_2 &= \sum_{\lambda_1,\lambda_2} rac{1}{2} |\mathcal{M}_{\lambda_1\lambda_2}^0|^2 \ f_3 &= \sum_{\lambda_1,\lambda_2} rac{1}{\sqrt{2}} Reig[\mathcal{M}_{\lambda_1\lambda_2}^0(\mathcal{M}_{\lambda_1\lambda_2}^+)^* - \mathcal{M}_{\lambda_1\lambda_2}^-(\mathcal{M}_{\lambda_1\lambda_2}^0)^*ig] \ f_4 &= \sum_{\lambda_1,\lambda_2} Reig[\mathcal{M}_{\lambda_1\lambda_2}^-(\mathcal{M}_{\lambda_1\lambda_2}^+)^*ig] \ f_5 &= \sum_{\lambda_1,\lambda_2} ig(|\mathcal{M}_{\lambda_1\lambda_2}^+|^2 - |\mathcal{M}_{\lambda_1\lambda_2}^-|^2ig) \end{aligned}$$

 $\mathit{f}_{5} = \overline{\sum_{\lambda_{1},\lambda_{2}}} \big(|\mathcal{M}_{\lambda_{1}\lambda_{2}}^{+}|^{2} - |\mathcal{M}_{\lambda_{1}\lambda_{2}}^{-}|^{2} \big)$ $\mathit{f}_{6} = \overline{\sum} \sqrt{2} Re \big[\mathcal{M}^{0}_{\lambda_{1} \lambda_{2}} (\mathcal{M}^{+}_{\lambda_{1} \lambda_{2}})^{*} + \mathcal{M}^{-}_{\lambda_{1} \lambda_{2}} (\mathcal{M}^{0}_{\lambda_{1} \lambda_{2}})^{*} \big]$

 $\mathit{f}_{7} = \sum_{\lambda_{1},\lambda_{2}}^{} \sqrt{2} \mathit{Im} \big[\mathcal{M}^{0}_{\lambda_{1}\lambda_{2}} (\mathcal{M}^{+}_{\lambda_{1}\lambda_{2}})^{*} + \mathcal{M}^{-}_{\lambda_{1}\lambda_{2}} (\mathcal{M}^{0}_{\lambda_{1}\lambda_{2}})^{*} \big]$

 $f_8 = \overline{\sum_{\lambda = \lambda}} \frac{1}{\sqrt{2}} Im \left[\mathcal{M}^0_{\lambda_1 \lambda_2} (\mathcal{M}^+_{\lambda_1 \lambda_2})^* - \mathcal{M}^-_{\lambda_1 \lambda_2} (\mathcal{M}^0_{\lambda_1 \lambda_2})^* \right]$

 $f_9 = \sum Im[\mathcal{M}_{\lambda_1\lambda_2}^-(\mathcal{M}_{\lambda_1\lambda_2}^+)^*].$

CP conservation and Bose symmetry gives the relation,

$$\mathcal{M}_{\lambda_1 \lambda_2}^{\lambda} = (-1)^{1-\lambda} \mathcal{M}_{-\lambda_1 - \lambda_2}^{-\lambda}.$$

This relation leads to

$$f_5 = f_6 = f_8 = f_9 = 0.$$

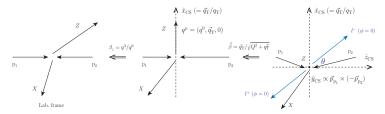
We have numerically found that F_3 and F_7 are totally antisymmetric around $\cos\Theta=0$, namely

$$F_i(\cos\Theta) = -F_i(-\cos\Theta)$$
 for $i = 3, 7$.

This indicates that F_3 and F_7 do not contribute to the angular distribution after integration over $\cos \Theta$.

Lepton $p_{\rm T}$: general formula.

Defining the lepton angles θ , ϕ in the CS frame has a great advantage: $p_{\mathrm{T}\ell}$ have simple expressions in terms of these angles and q_{T} .



In the CS frame:

$$\ell^-: \frac{Q}{2} \big(1, \; \sin\theta\cos\phi, \; \sin\theta\sin\phi, \; \cos\theta \big), \quad \ \ \ell^+: \frac{Q}{2} \big(1, \; -\sin\theta\cos\phi, \; -\sin\theta\sin\phi, \; -\cos\theta \big).$$

We boost them along the x-axis with a boost factor $\beta=q_{\rm T}/\sqrt{Q^2+q_{\rm T}^2}$; the vectorial transverse momenta are

$$\vec{p}_{\mathrm{T}\ell^-(\ell^+)} = \frac{1}{2} \Big(q_{\mathrm{T}} \pm \sqrt{Q^2 + q_{\mathrm{T}}^2} \sin\theta \cos\phi, \ \pm Q \sin\theta \sin\phi \Big).$$

which are now independent of a choice of the x-axis in the lab. frame.

Their absolute values are given by

$$\rho_{\mathrm{T}\ell^-(\ell^+)} \equiv \left| \vec{p}_{\mathrm{T}\ell^-(\ell^+)} \right| = \frac{1}{2} \sqrt{q_{\mathrm{T}}^2 + Q^2 \sin^2 \theta + q_{\mathrm{T}}^2 \sin^2 \theta \cos^2 \phi \pm 2 q_{\mathrm{T}} \sqrt{Q^2 + q_{\mathrm{T}}^2} \sin \theta \cos \phi},$$