Polarisation of the Z boson in the process $pp \to ZH$.

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Introduction

1. Polarisation
   - I wrote one paper on polarisation in 2012: " *TauDecay, a library to simulate polarized tau decays via FeynRules and MadGraph5*" with K. Hagiwara, T. Li and K. Mawatari. However, I did calculations without understanding physics...
   - I wanted to understand further details of it for a long time.
   - Now I have been enjoying to learn it this half a year.
   - Application? (or Motivation?) $\Rightarrow pp \to ZH$.
2. $pp \to ZH$
   - provides direct access to HZZ and $HZ\gamma$ couplings.
   - receives more attention after Butterworth, Davison, Rubin and Salam (2008).
3. Polarisation + $pp \to ZH$
   - Z boson is a spin 1 particle, thus can be in a polarised state.
   - Goal? $\Rightarrow$ a detailed study of HZZ and $HZ\gamma$ couplings by using the polarisation information of the Z boson

Outline

- Introduction
- Spin, polarisation and polarisation density matrices
- Helicity amplitudes and constraints from symmetries
- Decay angular distributions of a polarised Z boson

Spin, polarisation and polarisation density matrices

Spin angular momentum (Spin) induces additional degrees of freedom for a state of a particle. For a given spin s, a state vector is a $(2s+1)$-dimensional complex vector:

$$|1\rangle, |2\rangle, \cdots, |2s+1\rangle : \text{eigenfunctions of } \hat{s}_z \text{ for instance.}$$

$$\sum_{i=1}^{2s+1} |i\rangle\langle i| = 1,$$

$$|\alpha\rangle = \sum_{i=1}^{2s+1} |i\rangle\langle i|\alpha\rangle = a_1|1\rangle + a_2|2\rangle + \cdots + a_{2s+1}|2s+1\rangle = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{2s+1} \end{pmatrix},$$

where a_i $(i = 1, 2, \cdots, 2s+1)$ are complex numbers, which transform under spatial rotations (SU(2)).

d.o.f. of $|\alpha\rangle$ is $(2s+1) \times 2 - \underbrace{1}_{\langle\alpha|\alpha\rangle=1} - \underbrace{1}_{\text{overall phase}} = 4s$

$\rightarrow$ vector $|\alpha\rangle$ has a particular direction characterised by $4s$ real parameters.

= polarisation.

Spin, polarisation and polarisation density matrices

Let us consider a state of a spin 1/2 particle:

$$|\alpha\rangle = a_1|1\rangle + a_2|2\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \langle\alpha|\alpha\rangle = 1,$$

where eigenfunctions of $\hat{s}_z = \sigma_z/2$ are chosen as base vectors:

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Expectation values of the spin generators are

$$\langle\alpha|\hat{s}_z|\alpha\rangle = \frac{1}{2}\begin{pmatrix} a_1^* & a_2^* \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \frac{1}{2}(|a_1|^2 - |a_2|^2),$$

$$\langle\alpha|(\hat{s}_x, \hat{s}_y, \hat{s}_z)|\alpha\rangle = \Big(Re[a_1^* a_2], Im[a_1^* a_2], (|a_1|^2 - |a_2|^2)/2\Big).$$

The expectation values can be also derived by using a matrix:

$$\langle\alpha|\hat{s}_z|\alpha\rangle = \sum_{i=1,2}\langle\alpha|i\rangle\langle i|\hat{s}_z|\alpha\rangle = \sum_{i=1,2}\langle i|\hat{s}_z|\alpha\rangle\langle\alpha|i\rangle = tr(\hat{s}_z\rho_\alpha),$$

$$\rho_\alpha \equiv |\alpha\rangle\langle\alpha| = \begin{pmatrix} |a_1|^2 & a_1 a_2^* \\ a_1^* a_2 & |a_2|^2 \end{pmatrix}, \quad tr(\rho_\alpha) = 1, \; \rho_\alpha^\dagger = \rho_\alpha.$$

$\rightarrow$ polarisation density matrix

Spin, polarisation and pol. density matrices

A spin 1/2 particle in a mixed state which consists of

$$\begin{gathered} |\alpha\rangle \text{ with probability } p_\alpha, \\ |\beta\rangle \text{ with probability } p_\beta, \\ \vdots, \\ p_\alpha + p_\beta + \cdots = 1. \end{gathered}$$

Expectation value of $\hat{s}_z$ for the particle in this mixed state is

$$\begin{aligned} \langle \hat{s}_z \rangle &= \langle \alpha | \hat{s}_z | \alpha \rangle \times p_\alpha + \langle \beta | \hat{s}_z | \beta \rangle \times p_\beta + \cdots \\ &= tr\left(\hat{s}_z \rho_\alpha\right) \times p_\alpha + tr\left(\hat{s}_z \rho_\beta\right) \times p_\beta + \cdots \\ &= tr\left[\hat{s}_z(\rho_\alpha p_\alpha + \rho_\beta p_\beta + \cdots)\right] \\ &\equiv tr(\hat{s}_z \rho). \end{aligned}$$

The density matrix ρ satisfies (recall that $tr(\rho_\alpha) = 1,\ \rho_\alpha^\dagger = \rho_\alpha$)

$$tr(\rho) = 1,\ \rho^\dagger = \rho.$$

- Polarisation of a spin 1/2 particle in a mixed state is uniquely described by a single 2×2 density matrix ρ, whose d.o.f is 3.
- In general, for a given spin s, d.o.f of a density matrix ρ is $(2s+1)^2 - 1$.

Spin, polarisation and polarisation density matrices

A state vector of Z boson as a result of scattering $q\bar{q} \to ZH$ can be written in terms of scattering amplitudes in helicity basis as

$$|Z_\alpha\rangle = \frac{1}{n_\alpha} \sum_{\lambda=\pm,0} \mathcal{M}_\alpha^\lambda |\lambda\rangle, \quad \langle Z_\alpha | Z_\alpha \rangle = 1.$$

$$|\lambda\rangle : \text{helicity eigenvectors of the } Z \text{ boson; } \sum_{\lambda=\pm,0} |\lambda\rangle\langle\lambda| = 1.$$

$$n_\alpha^2 = |\mathcal{M}_\alpha^+|^2 + |\mathcal{M}_\alpha^-|^2 + |\mathcal{M}_\alpha^0|^2 : \text{normalisation factor.}$$

$$\alpha : \text{specifying a helicity state of } q\bar{q}.$$

To confirm this, notice

$$\langle \lambda = + | Z_\alpha \rangle = \frac{1}{n_\alpha} \mathcal{M}_\alpha^+, \quad |\langle \lambda = + | Z_\alpha \rangle|^2 = \frac{1}{n_\alpha^2} |\mathcal{M}_\alpha^+|^2, \quad \sum_{\lambda=\pm,0} |\langle \lambda | Z_\alpha \rangle|^2 = 1.$$

Density matrix ρ_α of the produced Z boson in helicity basis is

$$\rho_\alpha = |Z_\alpha\rangle\langle Z_\alpha| = \frac{1}{n_\alpha^2} \begin{pmatrix} |\mathcal{M}_\alpha^+|^2 & \mathcal{M}_\alpha^+ \mathcal{M}_\alpha^{-*} & \mathcal{M}_\alpha^+ \mathcal{M}_\alpha^{0*} \\ \mathcal{M}_\alpha^- \mathcal{M}_\alpha^{+*} & |\mathcal{M}_\alpha^-|^2 & \mathcal{M}_\alpha^- \mathcal{M}_\alpha^{0*} \\ \mathcal{M}_\alpha^0 \mathcal{M}_\alpha^{+*} & \mathcal{M}_\alpha^0 \mathcal{M}_\alpha^{-*} & |\mathcal{M}_\alpha^0|^2 \end{pmatrix}.$$

Spin, polarisation and polarisation density matrices

If another helicity state of $q\bar{q}$ is allowed, the Z boson is in a mixed state with respect to polarisation. Density matrix of the Z boson in such a state is

$$
\begin{aligned}
\rho &= |Z_\alpha\rangle\langle Z_\alpha| \times p_\alpha + |Z_\beta\rangle\langle Z_\beta| \times p_\beta \qquad (p_\alpha + p_\beta = 1) \\
&= \frac{1}{n}\left[\begin{pmatrix} |\mathcal{M}_\alpha^+|^2 & \mathcal{M}_\alpha^+\mathcal{M}_\alpha^{-*} & \mathcal{M}_\alpha^+\mathcal{M}_\alpha^{0*} \\ \mathcal{M}_\alpha^-\mathcal{M}_\alpha^{+*} & |\mathcal{M}_\alpha^-|^2 & \mathcal{M}_\alpha^-\mathcal{M}_\alpha^{0*} \\ \mathcal{M}_\alpha^0\mathcal{M}_\alpha^{+*} & \mathcal{M}_\alpha^0\mathcal{M}_\alpha^{-*} & |\mathcal{M}_\alpha^0|^2 \end{pmatrix} + \right. \\
&\qquad \left. \begin{pmatrix} |\mathcal{M}_\beta^+|^2 & \mathcal{M}_\beta^+\mathcal{M}_\beta^{-*} & \mathcal{M}_\beta^+\mathcal{M}_\beta^{0*} \\ \mathcal{M}_\beta^-\mathcal{M}_\beta^{+*} & |\mathcal{M}_\beta^-|^2 & \mathcal{M}_\beta^-\mathcal{M}_\beta^{0*} \\ \mathcal{M}_\beta^0\mathcal{M}_\beta^{+*} & \mathcal{M}_\beta^0\mathcal{M}_\beta^{-*} & |\mathcal{M}_\beta^0|^2 \end{pmatrix} \right],
\end{aligned}
$$

where we set $n = 1$ so that

$$
tr(\rho) = |\mathcal{M}_\alpha^+|^2 + |\mathcal{M}_\alpha^-|^2 + |\mathcal{M}_\alpha^0|^2 + |\mathcal{M}_\beta^+|^2 + |\mathcal{M}_\beta^-|^2 + |\mathcal{M}_\beta^0|^2.
$$

- $tr(\rho)$ gives just the $q\bar{q} \to ZH$ cross section. D.o.f of our DM ρ is $8 + 1 = 9$.
- DM ρ contains more information than the $q\bar{q} \to ZH$ cross section (8 additional information).
- "We use the full information of polarisation." = "We relate all the elements of ρ with measurable observables."

Summary

- Spin induces additional degrees of freedom for a state of a particle as polarisation (8 for a general state of a spin 1 particle).
- A state of polarisation of Z boson is uniquely described by a 3×3 density matrix ρ; $tr(\rho) = 1$ and $\rho^\dagger = \rho$.
- DM ρ can be constructed of scattering amplitudes for the process $q\bar{q} \to ZH$.
- DM ρ contains more information than the $q\bar{q} \to ZH$ cross section.
- "We use the full information of polarisation." $=$ "We relate all the elements of ρ with measurable observables."

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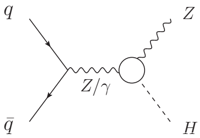
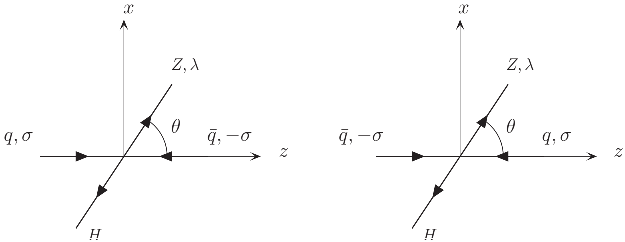
Helicity amplitudes and constraints from symmetries

Feynman diagram:



We assume the standard interaction for qqV and non-standard interactions for ZZH and $Z\gamma H$.

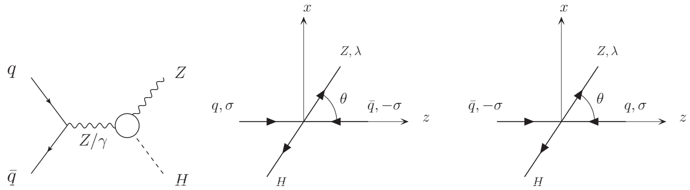
$q\bar{q}$ c.m. frame (left), where q moves along the positive direction of the z-axis.
$\bar{q}q$ c.m. frame (right), where $\bar{q}$ moves along the positive direction of the z-axis:



λ: helicity of Z boson.
σ: helicity of q.
$-\sigma$: helicity of $\bar{q}$.

Helicity amplitudes and constraints from symmetries

Feynman diagram, $q\bar{q}$ c.m. frame (left) and $\bar{q}q$ c.m. frame (right):



Helicity amplitudes in $q\bar{q}$ c.m. frame are

$$\mathcal{M}^{\lambda=\pm}_{\sigma=\pm}(q\bar{q}) = \sigma\frac{1+\sigma\lambda\cos\theta}{\sqrt{2}}\hat{M}^{\lambda=\pm}_{\sigma},$$

$$\mathcal{M}^{\lambda=0}_{\sigma=\pm}(q\bar{q}) = \sin\theta\hat{M}^{\lambda=0}_{\sigma}.$$

Helicity amplitudes in $\bar{q}q$ c.m. frame are

$$\mathcal{M}^{\lambda=\pm}_{\sigma=\pm}(\bar{q}q) = -\sigma\frac{1-\sigma\lambda\cos\theta}{\sqrt{2}}\hat{M}^{\lambda=\pm}_{\sigma},$$

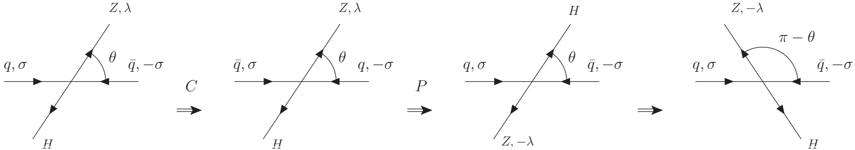
$$\mathcal{M}^{\lambda=0}_{\sigma=\pm}(\bar{q}q) = \sin\theta\hat{M}^{\lambda=0}_{\sigma}.$$

- $\hat{M}^{\lambda}_{\sigma}$ depend on an explicit form of ZVH interactions.
- Polar angle θ dependence is completely factorised.

Helicity amplitudes and constraints from symmetries

Conditions imposed by symmetries lead to certain relations between amplitudes.

CP transformation:



CP invariance leads to

$$\mathcal{M}_{\sigma}^{\lambda}(q\bar{q})(\theta) = \mathcal{M}_{\sigma}^{-\lambda}(q\bar{q})(\pi - \theta).$$

In terms of $\hat{M}_{\sigma}^{\lambda}$, this relation is simply

$$\hat{M}_{\sigma}^{\lambda} = \hat{M}_{\sigma}^{-\lambda}.$$

Violation of this relation immediately signals *CP* violation in *ZVH* interactions.

Helicity amplitudes and constraints from symmetries

S-matrix: $S = 1 + iT$

$$S^\dagger S = 1 \text{ (unitarity)},$$
$$-i(T - T^\dagger) = T^\dagger T,$$
$$-i\big(\langle f|T|i\rangle - \langle f|T^\dagger|i\rangle\big) = \sum_n \langle f|T^\dagger|n\rangle\langle n|T|i\rangle,$$
$$-i(T_{fi} - T^*_{if}) = \sum_n T^*_{nf} T_{ni},$$
$$-i(\mathcal{M}_{fi} - \mathcal{M}^*_{if}) = (2\pi)^4 \sum_n \delta^4(P_i - P_n)\mathcal{M}^*_{nf}\mathcal{M}_{ni}.$$

CPT invariance leads to

$$\mathcal{M}_{fi} = \mathcal{M}_{\hat{i}\hat{f}},$$

where $\hat{i}(\hat{f})$ denotes the *CPT* conjugate state of $i(f)$. The above unitarity condition becomes

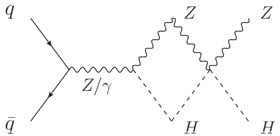
$$-i(\mathcal{M}_{fi} - \mathcal{M}^*_{\hat{f}\hat{i}}) = (2\pi)^4 \sum_n \delta^4(P_i - P_n)\mathcal{M}^*_{nf}\mathcal{M}_{ni}.$$

Unitarity *and* *CPT* invariance tell us that $\mathcal{M}_{fi} \neq \mathcal{M}^*_{\hat{f}\hat{i}}$ indicates the existence of rescattering effects (Hagiwara et al 1987).

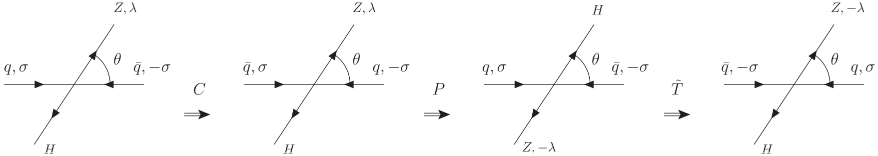
Helicity amplitudes and constraints from symmetries

$$-i(\mathcal{M}_{fi} - \mathcal{M}^*_{\hat{f}\hat{i}}) = (2\pi)^4 \sum_n \delta^4(P_i - P_n)\mathcal{M}^*_{nf}\mathcal{M}_{ni}.$$

An example of rescattering processes is the following, due to strong final state ZH interaction:



CPT transformation without $i \leftrightarrow j$:



The relation $\mathcal{M}_{fi} = \mathcal{M}^*_{\hat{f}\hat{i}}$ applied to our process is

$$\mathcal{M}^{\lambda}_{\sigma}(q\bar{q}) = -\left\{\mathcal{M}^{-\lambda}_{\sigma}(\bar{q}q)\right\}^*, \quad \text{or } \hat{M}^{\lambda}_{\sigma} = \left(\hat{M}^{-\lambda}_{\sigma}\right)^*.$$

Violation of this relation immediately indicates rescattering effects. We call it $CP\widetilde{T}$ violation.

Summary

- We obtained a set of scattering helicity amplitudes for $q\bar{q} \to ZH$ in the two c.m. frames: in one frame q moves along the positive direction of the z-axis and in another frame $\bar{q}$ moves along the positive direction of the z-axis.
- We derived relations between the amplitudes imposed by symmetries:

  CP invariance leads to $\hat{M}_\sigma^\lambda = \hat{M}_\sigma^{-\lambda}$,

  $CP\widetilde{T}$ invariance leads to $\hat{M}_\sigma^\lambda = \left(\hat{M}_\sigma^{-\lambda}\right)^*$.

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Decay angular distributions of polarised Z

Question: How to measure a state of polarisation at experiments?

Recall that

- "Z boson is in a polarised state." = "the state has a particular direction specified by a density matrix ρ (i.e. 9 real parameters)."
- Decay angular distributions of a Z boson is spherically not symmetric.

Let us consider the decay $Z \to f\bar{f}$

$$\begin{aligned} Z &: \; \left(m_Z, 0, 0, 0\right) \\ f &: \; \frac{m_Z}{2}\left(1,\; \sin\widehat{\theta}\cos\widehat{\phi},\; \sin\widehat{\theta}\sin\widehat{\phi},\; \cos\widehat{\theta}\right) \\ \bar{f} &: \; \frac{m_Z}{2}\left(1,\; -\sin\widehat{\theta}\cos\widehat{\phi},\; -\sin\widehat{\theta}\sin\widehat{\phi},\; -\cos\widehat{\theta}\right). \end{aligned}$$

Differential cross sections have 9 independent angular distributions:

$$\begin{aligned} \frac{d\sigma}{d\Omega\; d\cos\widehat{\theta}\; d\widehat{\phi}} =& F_1\left(1+\cos^2\widehat{\theta}\right) + F_2\left(1-3\cos^2\widehat{\theta}\right) + F_3\cos\widehat{\theta} \\ &+ F_4\sin\widehat{\theta}\cos\widehat{\phi} + F_5\sin 2\widehat{\theta}\cos\widehat{\phi} + F_6\sin^2\widehat{\theta}\cos 2\widehat{\phi} \\ &+ F_7\sin\widehat{\theta}\sin\widehat{\phi} + F_8\sin 2\widehat{\theta}\sin\widehat{\phi} + F_9\sin^2\widehat{\theta}\sin 2\widehat{\phi}, \end{aligned}$$

Answer: "Measurement of all F_i ($i = 1, 2, \cdots, 9$)" = "Measurement of a state of polarisation"

Decay angular distributions of polarised Z

The full process is $q(\sigma) + \bar{q}(-\sigma) \to Z(\lambda) + H; \quad Z(\lambda) \to f(\tau) + \bar{f}(-\tau)$.
The full amplitudes are

$$\mathcal{T}_\sigma^\tau(q\bar{q}) = P_Z \sum_{\lambda=\pm,0} \mathcal{M}_\sigma^\lambda(q\bar{q})\ D_\lambda^\tau, \quad \mathcal{T}_\sigma^\tau(\bar{q}q) = P_Z \sum_{\lambda=\pm,0} \mathcal{M}_\sigma^\lambda(\bar{q}q)\ D_\lambda^\tau,$$

where

$$P_Z = (Q^2 - m_Z^2 + i m_Z \Gamma_Z)^{-1}, \quad D_\lambda^\tau = g_{Zf\bar{f}}^\tau m_Z\ d_\lambda^\tau \text{ : decay amplitude.}$$

Squared amplitude gives the probability:

$$\begin{aligned}
\sum_{\sigma=\pm} |\mathcal{T}_\sigma^\tau(q\bar{q})|^2 &= \left|P_Z m_Z g_{Zf\bar{f}}^\tau\right|^2 \sum_\sigma \sum_{\lambda',\lambda} (d_{\lambda'}^\tau)^* \{\mathcal{M}_\sigma^{\lambda'}(q\bar{q})\}^* \mathcal{M}_\sigma^\lambda(q\bar{q})\ d_\lambda^\tau \\
&= \left|P_Z m_Z g_{Zf\bar{f}}^\tau\right|^2 \sum_\sigma \sum_{\lambda',\lambda} (d_{\lambda'}^\tau)^*\ \rho_\sigma^{\lambda'\lambda}(q\bar{q})\ d_\lambda^\tau \\
&= \left|P_Z m_Z g_{Zf\bar{f}}^\tau\right|^2\ d^{\tau\dagger}\ \rho(q\bar{q})\ d^\tau,
\end{aligned}$$

where

$$\rho^{\lambda'\lambda}(q\bar{q}) \equiv \sum_\sigma \rho_\sigma^{\lambda'\lambda}(q\bar{q}) \equiv \sum_\sigma \{\mathcal{M}_\sigma^{\lambda'}(q\bar{q})\}^* \mathcal{M}_\sigma^\lambda(q\bar{q}) \text{ ; DM elements.}$$

Do the same for another amplitude:

$$\sum_{\sigma=\pm} |\mathcal{T}_\sigma^\tau(\bar{q}q)|^2 = \left|P_Z m_Z g_{Zf\bar{f}}^\tau\right|^2 d^{\tau\dagger} \rho(\bar{q}q)\ d^\tau, \quad \rho^{\lambda'\lambda}(\bar{q}q) \equiv \sum_\sigma \{\mathcal{M}_\sigma^{\lambda'}(\bar{q}q)\}^* \mathcal{M}_\sigma^\lambda(\bar{q}q).$$

Decay angular distributions of polarised Z

The complete differential cross section is

$$\frac{d\sigma}{d\hat{s}\ dy\ d\cos\theta\ d\cos\widehat{\theta}\ d\widehat{\phi}} \propto q(x_1)\bar{q}(x_2)\ d^{\tau\dagger}\rho(q\bar{q})\ d^{\tau} + \bar{q}(x_1)q(x_2)\ d^{\tau\dagger}\rho(\bar{q}q)\ d^{\tau}.$$

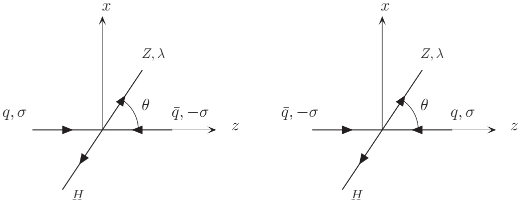
$q(x_i)$: PDF of a quark with energy fraction x_i.

$\hat{s} = sx_1x_2$: $\mathrm{q\bar{q}}$ and $\mathrm{\bar{q}q}$ c.m. energy squared; $(m_Z + m_H)^2 < \hat{s} < s$.

$y = \frac{1}{2}\ln\frac{x_1}{x_2}$: rapidity of the $\mathrm{q\bar{q}}$ and $\mathrm{\bar{q}q}$ c.m. frames; $-\ln\sqrt{\frac{s}{\hat{s}}} < y < \ln\sqrt{\frac{s}{\hat{s}}}$.

θ : polar angle.

$\widehat{\theta}, \widehat{\phi}$: Z decay angles.



We perform $\cos\theta$ integration and y integration.

Decay angular distributions of polarised Z

First, $\cos\theta$ integration:

$$\frac{d\sigma}{d\hat{s}\ dy\ d\cos\widehat{\theta}\ d\widehat{\phi}} \propto \int_{-\epsilon}^{\epsilon} d\cos\theta\ q(x_1)\bar{q}(x_2)\ d^{\tau\dagger}\rho(q\bar{q})\ d^{\tau} + \bar{q}(x_1)q(x_2)\ d^{\tau\dagger}\rho(\bar{q}q)\ d^{\tau}$$
$$= q(x_1)\bar{q}(x_2)\ d^{\tau\dagger}\langle\rho(q\bar{q})\rangle\ d^{\tau} + \bar{q}(x_1)q(x_2)\ d^{\tau\dagger}\langle\rho(\bar{q}q)\rangle\ d^{\tau},$$

where

$$\langle\rho(q\bar{q})\rangle \equiv \int_{-\epsilon}^{\epsilon} d\cos\theta\rho(q\bar{q}) = \sum_{\sigma}\begin{pmatrix} c_1\left|\hat{M}_\sigma^+\right|^2 & \frac{c_2}{2}\left(\hat{M}_\sigma^+\right)^*\hat{M}_\sigma^- & c_3\sigma\left(\hat{M}_\sigma^+\right)^*\hat{M}_\sigma^0 \\ \frac{c_2}{2}\hat{M}_\sigma^+\left(\hat{M}_\sigma^-\right)^* & c_1\left|\hat{M}_\sigma^-\right|^2 & c_3\sigma\left(\hat{M}_\sigma^-\right)^*\hat{M}_\sigma^0 \\ c_3\sigma\hat{M}_\sigma^+\left(\hat{M}_\sigma^0\right)^* & c_3\sigma\hat{M}_\sigma^-\left(\hat{M}_\sigma^0\right)^* & c_2\left|\hat{M}_\sigma^0\right|^2 \end{pmatrix},$$

$$\langle\rho(\bar{q}q)\rangle \equiv \int_{-\epsilon}^{\epsilon} d\cos\theta\rho(\bar{q}q) = \sum_{\sigma}\begin{pmatrix} c_1\left|\hat{M}_\sigma^+\right|^2 & \frac{c_2}{2}\left(\hat{M}_\sigma^+\right)^*\hat{M}_\sigma^- & -c_3\sigma\left(\hat{M}_\sigma^+\right)^*\hat{M}_\sigma^0 \\ \frac{c_2}{2}\hat{M}_\sigma^+\left(\hat{M}_\sigma^-\right)^* & c_1\left|\hat{M}_\sigma^-\right|^2 & -c_3\sigma\left(\hat{M}_\sigma^-\right)^*\hat{M}_\sigma^0 \\ -c_3\sigma\hat{M}_\sigma^+\left(\hat{M}_\sigma^0\right)^* & -c_3\sigma\hat{M}_\sigma^-\left(\hat{M}_\sigma^0\right)^* & c_2\left|\hat{M}_\sigma^0\right|^2 \end{pmatrix}$$

c_i $(i = 1, 2, 3)$ are constant values which depend on ϵ.

Recall that the θ dependence is factorised as $\mathcal{M}_{\sigma=\pm}^{\lambda=\pm}(q\bar{q}) = \sigma\frac{1+\sigma\lambda\cos\theta}{\sqrt{2}}\hat{M}_\sigma^{\lambda=\pm}$ for e.g..

- $\langle\rho(q\bar{q})\rangle + \langle\rho(\bar{q}q)\rangle \to$ "the σ terms" vanish.
- $\langle\rho(q\bar{q})\rangle - \langle\rho(\bar{q}q)\rangle \to$ only "the σ terms" survive.

Decay angular distributions of polarised Z

Second, y integration:

$$\frac{d\sigma}{d\hat{s}\ d\cos\widehat{\theta}\ d\widehat{\phi}} \propto \int_{-y_{\rm cut}}^{y_{\rm cut}} dy\ q(x_1)\bar{q}(x_2)\ d^{\tau\dagger}\langle\rho(q\bar{q})\rangle\ d^{\tau} + \bar{q}(x_1)q(x_2)\ d^{\tau\dagger}\langle\rho(\bar{q}q)\rangle\ d^{\tau}$$

$$= \underbrace{\int_{0}^{y_{\rm cut}}}_{x_1>x_2} dy\ \underbrace{q(x_1)\bar{q}(x_2)}_{A}\ d^{\tau\dagger}\langle\rho(q\bar{q})\rangle\ d^{\tau} + \underbrace{\bar{q}(x_1)q(x_2)}_{B}\ d^{\tau\dagger}\langle\rho(\bar{q}q)\rangle\ d^{\tau}$$

$$+ \underbrace{\int_{-y_{\rm cut}}^{0}}_{x_2>x_1} dy\ \underbrace{q(x_1)\bar{q}(x_2)}_{B}\ d^{\tau\dagger}\langle\rho(q\bar{q})\rangle\ d^{\tau} + \underbrace{\bar{q}(x_1)q(x_2)}_{A}\ d^{\tau\dagger}\langle\rho(\bar{q}q)\rangle\ d^{\tau}$$

$$= \int_{0}^{y_{\rm cut}} dy\ \left\{q(x_1)\bar{q}(x_2)+\bar{q}(x_1)q(x_2)\right\}\ d^{\tau\dagger}\left\{\langle\rho(q\bar{q})\rangle + \langle\rho(\bar{q}q)\rangle\right\}\ d^{\tau}$$

$$= \int_{0}^{y_{\rm cut}} dy\ 2\left\{q(x_1)\bar{q}(x_2)+\bar{q}(x_1)q(x_2)\right\}$$

$$\times\ d^{\tau\dagger}\sum_{\sigma}\begin{pmatrix} c_1\left|\hat{M}_\sigma^+\right|^2 & \frac{c_2}{2}\left(\hat{M}_\sigma^+\right)^*\hat{M}_\sigma^- & 0 \\ \frac{c_2}{2}\hat{M}_\sigma^+\left(\hat{M}_\sigma^-\right)^* & c_1\left|\hat{M}_\sigma^-\right|^2 & 0 \\ 0 & 0 & c_2\left|\hat{M}_\sigma^0\right|^2 \end{pmatrix}.d^{\tau}$$

- $y = \frac{1}{2}\ln\frac{x_1}{x_2}$.
- 4 of the 9 elements of the DM vanish after the $\cos\theta$ and y integration, due to the sign difference in $\langle\rho(q\bar{q})\rangle$ and $\langle\rho(\bar{q}q)\rangle$.

Decay angular distributions of polarised Z

Second, y integration in a different way:

$$
\begin{aligned}
\left.\frac{d\sigma}{d\hat{s}d\cos\widehat{\theta}d\widehat{\phi}}\right|_{\mathcal{B}} &\propto \left(\int_0^{y_{\rm cut}} - \int_{-y_{\rm cut}}^{0}\right) dy\, q(x_1)\bar{q}(x_2) d^{\tau\dagger}\langle\rho(q\bar{q})\rangle d^{\tau} + \bar{q}(x_1)q(x_2)\; d^{\tau\dagger}\langle\rho(\bar{q}q)\rangle\; d^{\tau} \\
&= \underbrace{\int_0^{y_{\rm cut}}}_{x_1>x_2} dy\; \underbrace{q(x_1)\bar{q}(x_2)}_{A}\; d^{\tau\dagger}\langle\rho(q\bar{q})\rangle\; d^{\tau} + \underbrace{\bar{q}(x_1)q(x_2)}_{B}\; d^{\tau\dagger}\langle\rho(\bar{q}q)\rangle\; d^{\tau} \\
&\quad - \underbrace{\int_{-y_{\rm cut}}^{0}}_{x_2>x_1} dy\; \underbrace{q(x_1)\bar{q}(x_2)}_{B}\; d^{\tau\dagger}\langle\rho(q\bar{q})\rangle\; d^{\tau} + \underbrace{\bar{q}(x_1)q(x_2)}_{A}\; d^{\tau\dagger}\langle\rho(\bar{q}q)\rangle\; d^{\tau} \\
&= \int_0^{y_{\rm cut}} dy\; \left\{q(x_1)\bar{q}(x_2) - \bar{q}(x_1)q(x_2)\right\}\; d^{\tau\dagger}\left\{\langle\rho(q\bar{q})\rangle - \langle\rho(\bar{q}q)\rangle\right\}\; d^{\tau} \\
&= \int_0^{y_{\rm cut}} dy\; 2\left\{q(x_1)\bar{q}(x_2) - \bar{q}(x_1)q(x_2)\right\} \\
&\qquad \times d^{\tau\dagger}\sum_{\sigma}\begin{pmatrix} 0 & 0 & c_3\sigma\left(\hat{M}_\sigma^{+}\right)^*\hat{M}_\sigma^0 \\ 0 & 0 & c_3\sigma\left(\hat{M}_\sigma^{-}\right)^*\hat{M}_\sigma^0 \\ c_3\sigma\hat{M}_\sigma^{+}\left(\hat{M}_\sigma^0\right)^* & c_3\sigma\hat{M}_\sigma^{-}\left(\hat{M}_\sigma^0\right)^* & 0 \end{pmatrix} d^{\tau}.
\end{aligned}
$$

- The vanished elements of the DM in the previous approach are revived (^^)

Decay angular distributions of polarised Z

$$\frac{d\sigma}{d\hat{s}d\cos\widehat{\theta}d\widehat{\phi}}\bigg|_{i(=\mathcal{A},\mathcal{B})} = F_{i1}\left(1+\cos^2\widehat{\theta}\right) + F_{i2}\left(1-3\cos^2\widehat{\theta}\right) + F_{i3}\cos\widehat{\theta} + F_{i4}\sin\widehat{\theta}\cos\widehat{\phi}$$

$$+ F_{i5}\sin 2\widehat{\theta}\cos\widehat{\phi} + F_{i6}\sin^2\widehat{\theta}\cos 2\widehat{\phi} + F_{i7}\sin\widehat{\theta}\sin\widehat{\phi} + F_{i8}\sin 2\widehat{\theta}\sin\widehat{\phi} + F_{i9}\sin^2\widehat{\theta}\sin 2\widehat{\phi},$$

$$F_{\mathcal{A}(\mathcal{B})a} \propto \int_0^{\ln\sqrt{\frac{s}{\hat{s}}}} dy\; 2\Big[q(x_1)\bar{q}(x_2) \pm \bar{q}(x_1)q(x_2)\Big] \sum_{\sigma} f_{\mathcal{A}(\mathcal{B})a},$$

$$f_{\mathcal{A}1} = \frac{1}{2}\left(c_1|\hat{M}_\sigma^+|^2 + c_1|\hat{M}_\sigma^-|^2 + c_2|\hat{M}_\sigma^0|^2\right), \qquad f_{\mathcal{B}1} = 0,$$

$$f_{\mathcal{A}2} = \frac{1}{2}c_2|\hat{M}_\sigma^0|^2, \qquad f_{\mathcal{B}2} = 0,$$

$$f_{\mathcal{A}3} = c_1\left(|\hat{M}_\sigma^+|^2 - |\hat{M}_\sigma^-|^2\right)\tau, \qquad f_{\mathcal{B}3} = 0,$$

$$f_{\mathcal{A}4} = 0, \qquad f_{\mathcal{B}4} = \sqrt{2}\sigma c_3 Re\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 + (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right]\tau,$$

$$f_{\mathcal{A}5} = 0, \qquad f_{\mathcal{B}5} = \frac{1}{\sqrt{2}}\sigma c_3 Re\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right],$$

$$f_{\mathcal{A}6} = \frac{1}{2}c_2 Re\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^-\right], \qquad f_{\mathcal{B}6} = 0,$$

$$f_{\mathcal{A}7} = 0, \qquad f_{\mathcal{B}7} = \sqrt{2}\sigma c_3 Im\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 + (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right]\tau,$$

$$f_{\mathcal{A}8} = 0, \qquad f_{\mathcal{B}8} = \frac{1}{\sqrt{2}}\sigma c_3 Im\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right],$$

$$f_{\mathcal{A}9} = \frac{1}{2}c_2 Im\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^-\right], \qquad f_{\mathcal{B}9} = 0.$$

- The 9 non-zero observables correspond to the 9 independent combinations of the DM elements, i.e. these 9 observables are all independent.
- This means that we are making use of the full information of polarisation (^^).

Decay angular distributions of polarised Z

Perform a translation $\widehat{\theta} \to \pi - \widehat{\theta}$ *and* $\widehat{\phi} \to \widehat{\phi} + \pi$:

$$
\begin{aligned}
\left.\frac{d\sigma}{d\hat{s}\ d\cos\widehat{\theta}\ d\widehat{\phi}}\right|_{i(=\mathcal{A},\mathcal{B})} \propto\ & F_{i1}\left(1+\cos^2\widehat{\theta}\right) + F_{i2}\left(1-3\cos^2\widehat{\theta}\right) - F_{i3}\cos\widehat{\theta} \\
& - F_{i4}\sin\widehat{\theta}\cos\widehat{\phi} + F_{i5}\sin 2\widehat{\theta}\cos\widehat{\phi} + F_{i6}\sin^2\widehat{\theta}\cos 2\widehat{\phi} \\
& - F_{i7}\sin\widehat{\theta}\sin\widehat{\phi} + F_{i8}\sin 2\widehat{\theta}\sin\widehat{\phi} + F_{i9}\sin^2\widehat{\theta}\sin 2\widehat{\phi},
\end{aligned}
$$

where we observe the change of the sign in front of the F_{i3}, F_{i4} and F_{i7} terms.

This means

- The F_{i3}, F_{i4} and F_{i7} terms are statistically zero, if we do not distinguish the fermion f from the antifermion $\bar{f}$.
- The events with $Z \to jj$ cannot be used to measure the coefficients $F_{\mathcal{A}3}$, $F_{\mathcal{B}4}$ and $F_{\mathcal{B}7}$, in view of difficulty of flavor identification of the jets. (- -)

Decay angular distributions of polarised Z

First, $\cos\theta$ integration in a different way:

$$\frac{d\sigma}{d\hat{s}dyd\cos\widehat{\theta}d\widehat{\phi}} \propto \left(\int_0^{\epsilon} - \int_{-\epsilon}^{0}\right) d\cos\theta\ q(x_1)\bar{q}(x_2)\ d^{\tau\dagger}\rho(q\bar{q})\ d^{\tau} + \bar{q}(x_1)q(x_2)\ d^{\tau\dagger}\rho(\bar{q}q)\ d^{\tau}$$

$$= q(x_1)\bar{q}(x_2)\ d^{\tau\dagger}\overline{\langle\rho(q\bar{q})\rangle}\ d^{\tau} + \bar{q}(x_1)q(x_2)\ d^{\tau\dagger}\overline{\langle\rho(\bar{q}q)\rangle}\ d^{\tau},$$

where

$$\overline{\langle\rho(q\bar{q})\rangle} \equiv \left(\int_0^{\epsilon} - \int_{-\epsilon}^{0}\right) d\cos\theta\ \rho(q\bar{q})$$

$$= \sum_{\sigma} \begin{pmatrix} c_5\sigma\left|\hat{M}_{\sigma}^{+}\right|^2 & 0 & c_4\left(\hat{M}_{\sigma}^{+}\right)^*\hat{M}_{\sigma}^{0} \\ 0 & -c_5\sigma\left|\hat{M}_{\sigma}^{-}\right|^2 & -c_4\left(\hat{M}_{\sigma}^{-}\right)^*\hat{M}_{\sigma}^{0} \\ c_4\hat{M}_{\sigma}^{+}\left(\hat{M}_{\sigma}^{0}\right)^* & -c_4\hat{M}_{\sigma}^{-}\left(\hat{M}_{\sigma}^{0}\right)^* & 0 \end{pmatrix},$$

$$\overline{\langle\rho(\bar{q}q)\rangle} \equiv \left(\int_0^{\epsilon} - \int_{-\epsilon}^{0}\right) d\cos\theta\ \rho(\bar{q}q)$$

$$= \sum_{\sigma} \begin{pmatrix} -c_5\sigma\left|\hat{M}_{\sigma}^{+}\right|^2 & 0 & c_4\left(\hat{M}_{\sigma}^{+}\right)^*\hat{M}_{\sigma}^{0} \\ 0 & c_5\sigma\left|\hat{M}_{\sigma}^{-}\right|^2 & -c_4\left(\hat{M}_{\sigma}^{-}\right)^*\hat{M}_{\sigma}^{0} \\ c_4\hat{M}_{\sigma}^{+}\left(\hat{M}_{\sigma}^{0}\right)^* & -c_4\hat{M}_{\sigma}^{-}\left(\hat{M}_{\sigma}^{0}\right)^* & 0 \end{pmatrix},$$

c_i $(i = 4, 5)$ are constant values which depend on ϵ.

- $\overline{\langle\rho(q\bar{q})\rangle} + \overline{\langle\rho(\bar{q}q)\rangle} \to$ "the σ terms" vanish.
- $\overline{\langle\rho(q\bar{q})\rangle} - \overline{\langle\rho(\bar{q}q)\rangle} \to$ only "the σ terms" survive.

Decay angular distributions of polarised Z

Second, y integration in the 2 different ways as before:

$$\left.\frac{d\sigma}{d\hat{s}d\cos\widehat{\theta}d\widehat{\phi}}\right|_{\mathcal{C}} \propto \int_{-\ln\sqrt{\frac{s}{\hat{s}}}}^{\ln\sqrt{\frac{s}{\hat{s}}}} dy\; q(x_1)\bar{q}(x_2)\; d^{\tau\dagger}\overline{\langle\rho(q\bar{q})\rangle}\; d^{\tau} + \bar{q}(x_1)q(x_2)\; d^{\tau\dagger}\overline{\langle\rho(\bar{q}q)\rangle}\; d^{\tau}$$

$$= \int_0^{\ln\sqrt{\frac{s}{\hat{s}}}} dy\; 2\{q(x_1)\bar{q}(x_2)+\bar{q}(x_1)q(x_2)\}$$

$$\times\, d^{\tau\dagger}\sum_{\sigma}\begin{pmatrix} 0 & 0 & c_4\left(\hat{M}_{\sigma}^{+}\right)^{*}\hat{M}_{\sigma}^{0} \\ 0 & 0 & -c_4\left(\hat{M}_{\sigma}^{-}\right)^{*}\hat{M}_{\sigma}^{0} \\ c_4\hat{M}_{\sigma}^{+}\left(\hat{M}_{\sigma}^{0}\right)^{*} & -c_4\hat{M}_{\sigma}^{-}\left(\hat{M}_{\sigma}^{0}\right)^{*} & 0 \end{pmatrix} d^{\tau},$$

$$\left.\frac{d\sigma}{d\hat{s}d\cos\widehat{\theta}d\widehat{\phi}}\right|_{\mathcal{D}} \propto \left(\int_0^{\ln\sqrt{\frac{s}{\hat{s}}}} - \int_{-\ln\sqrt{\frac{s}{\hat{s}}}}^{0}\right) dy\, q(x_1)\bar{q}(x_2)\; d^{\tau\dagger}\overline{\langle\rho(q\bar{q})\rangle} d^{\tau} + \bar{q}(x_1)q(x_2) d^{\tau\dagger}\overline{\langle\rho(\bar{q}q)\rangle}$$

$$= \int_0^{\ln\sqrt{\frac{s}{\hat{s}}}} dy\; 2\{q(x_1)\bar{q}(x_2)-\bar{q}(x_1)q(x_2)\}$$

$$\times\, d^{\tau\dagger}\sum_{\sigma}\begin{pmatrix} c_5\sigma\left|\hat{M}_{\sigma}^{+}\right|^2 & 0 & 0 \\ 0 & -c_5\sigma\left|\hat{M}_{\sigma}^{-}\right|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} d^{\tau}.$$

Decay angular distributions of polarised Z

$$\left.\frac{d\sigma}{d\hat{s}d\cos\hat{\theta}d\hat{\phi}}\right|_{i(=\mathcal{C},\mathcal{D})} = F_{i1}\left(1+\cos^2\hat{\theta}\right) + F_{i2}\left(1-3\cos^2\hat{\theta}\right) + F_{i3}\cos\hat{\theta} + F_{i4}\sin\hat{\theta}\cos\hat{\phi} + F_{i5}\sin 2\hat{\theta}\cos\hat{\phi}$$
$$+ F_{i6}\sin^2\hat{\theta}\cos 2\hat{\phi} + F_{i7}\sin\hat{\theta}\sin\hat{\phi} + F_{i8}\sin 2\hat{\theta}\sin\hat{\phi} + F_{i9}\sin^2\hat{\theta}\sin 2\hat{\phi},$$

$$F_{\mathcal{C}(\mathcal{D})a} \propto \int_0^{\ln\sqrt{\frac{s}{\hat{s}}}} dy\ 2\left[q(x_1)\bar{q}(x_2) \pm \bar{q}(x_1)q(x_2)\right]\sum_{\sigma} f_{\mathcal{C}(\mathcal{D})a},$$

$$f_{\mathcal{C}1} = 0, \qquad f_{\mathcal{D}1} = \frac{1}{2}\sigma c_5\left(|\hat{M}_\sigma^+|^2 - |\hat{M}_\sigma^-|^2\right),$$
$$f_{\mathcal{C}2} = 0, \qquad f_{\mathcal{D}2} = 0,$$
$$f_{\mathcal{C}3} = 0, \qquad f_{\mathcal{D}3} = \sigma c_5\left(|\hat{M}_\sigma^+|^2 + |\hat{M}_\sigma^-|^2\right)\tau,$$
$$f_{\mathcal{C}4} = \sqrt{2}c_4 Re\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right]\tau, \qquad f_{\mathcal{D}4} = 0,$$
$$f_{\mathcal{C}5} = \frac{1}{\sqrt{2}}c_4 Re\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 + (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right], \qquad f_{\mathcal{D}5} = 0,$$
$$f_{\mathcal{C}6} = 0, \qquad f_{\mathcal{D}6} = 0,$$
$$f_{\mathcal{C}7} = \sqrt{2}c_4 Im\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right]\tau, \qquad f_{\mathcal{D}7} = 0,$$
$$f_{\mathcal{C}8} = \frac{1}{\sqrt{2}}c_4 Im\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 + (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right], \qquad f_{\mathcal{D}8} = 0,$$
$$f_{\mathcal{C}9} = 0, \qquad f_{\mathcal{D}9} = 0.$$

- $F_{\mathcal{D}1}$, $F_{\mathcal{C}5}$ and $F_{\mathcal{C}8}$ measure the same combinations of the DM elements with $F_{\mathcal{A}3}$, $F_{\mathcal{B}4}$ and $F_{\mathcal{B}7}$, respectively.
- Observation of these coefficients does not require the charge (flavor) identification of the final fermion (^^).
- By means of these additional observables, we can make use of the full information of polarisation even in the process $Z \to jj$ (^^).

Decay angular distributions of polarised Z

Combinations of the density matrix elements	Symmetry properties CP	$CP\widetilde{T}$	Observables	f charge
$c_1\left\|\hat{M}_\sigma^+\right\|^2 + c_1\left\|\hat{M}_\sigma^-\right\|^2 + c_2\left\|\hat{M}_\sigma^0\right\|^2$	$+$	$+$	$F_{\mathcal{A}1}$	-
$\left\|\hat{M}_\sigma^0\right\|^2$	$+$	$+$	$F_{\mathcal{A}2}$	-
$\left\|\hat{M}_\sigma^+\right\|^2 + \left\|\hat{M}_\sigma^-\right\|^2$	$+$	$+$	$F_{\mathcal{D}3}$	$\circ$
$\left\|\hat{M}_\sigma^+\right\|^2 - \left\|\hat{M}_\sigma^-\right\|^2$	$-$	$-$	$F_{\mathcal{A}3}$	$\circ$
			$F_{\mathcal{D}1}$	-
$Re\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 + (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right]$	$+$	$+$	$F_{\mathcal{B}4}$	$\circ$
			$F_{\mathcal{C}5}$	-
$Re\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right]$	$-$	$-$	$F_{\mathcal{B}5}$	-
			$F_{\mathcal{C}4}$	$\circ$
$Re\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^-\right]$	$+$	$+$	$F_{\mathcal{A}6}$	-
$Im\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 + (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right]$	$-$	$+$	$F_{\mathcal{B}7}$	$\circ$
			$F_{\mathcal{C}8}$	-
$Im\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^*\hat{M}_\sigma^-\right]$	$+$	$-$	$F_{\mathcal{B}8}$	-
			$F_{\mathcal{C}7}$	$\circ$
$Im\left[(\hat{M}_\sigma^+)^*\hat{M}_\sigma^-\right]$	$-$	$+$	$F_{\mathcal{A}9}$	-

- Among the 36 ($= 4 \times 9$) coefficients, only the 15 coefficients can be non-zero.
- Observation of the 9 coefficients does not require the charge identification of f.
- These 9 coefficients are enough and sufficient to determine all the 9 independent combinations of the DM elements.

Decay angular distributions of polarised Z

Some of the 15 coefficients are strictly zero, if the amplitudes satisfy the restrictions

$$\hat{M}_\sigma^\lambda = \hat{M}_\sigma^{-\lambda} \text{ from CP invariance}$$
$$\hat{M}_\sigma^\lambda = (\hat{M}_\sigma^{-\lambda})^* \text{ from CP}\widetilde{\text{T}} \text{ invariance}$$

$F_{\mathcal{A}9} \propto Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^-\big]$:

$$CP \text{ invariance}: Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^-\big] = Im\big[|\hat{M}_\sigma^-|^2\big] = 0,$$
$$CP\widetilde{T} \text{ invariance}: Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^-\big] = Im\big[(\hat{M}_\sigma^-)^2\big] \neq 0,$$

$\longrightarrow$ Observation of a non-zero value of $F_{\mathcal{A}9}$ signals CP violation.

$F_{\mathcal{B}8}$ and $F_{\mathcal{C}7} \propto Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^* \hat{M}_\sigma^-\big]$:

$$CP \text{ invariance}: Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^* \hat{M}_\sigma^-\big] = Im\{2i Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0\big]\} \neq 0,$$
$$CP\widetilde{T} \text{ invariance}: Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^* \hat{M}_\sigma^-\big] = Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0 - \hat{M}_\sigma^0 (\hat{M}_\sigma^+)^*\big] = 0,$$

$\longrightarrow$ Observation of a non-zero value of $F_{\mathcal{B}8}$ or $F_{\mathcal{C}7}$ signals $CP\widetilde{T}$ violation.

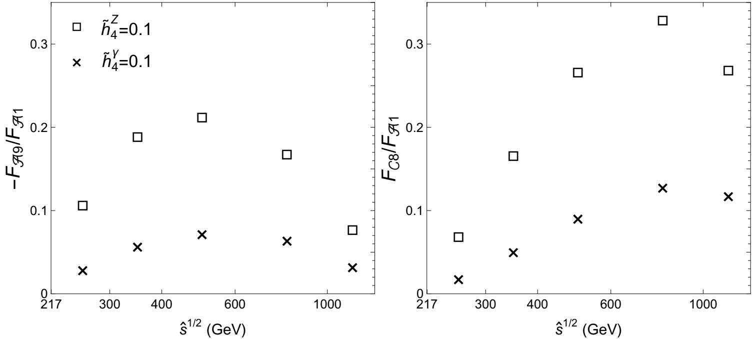
$F_{\mathcal{B}5}$ and $F_{\mathcal{C}4} \propto Re\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^* \hat{M}_\sigma^-\big]$:

$$CP \text{ invariance}: Re\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^* \hat{M}_\sigma^-\big] = Re\{2i Im\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0\big]\} = 0,$$
$$CP\widetilde{T} \text{ invariance}: Re\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0 - (\hat{M}_\sigma^0)^* \hat{M}_\sigma^-\big] = Re\big[(\hat{M}_\sigma^+)^* \hat{M}_\sigma^0 - \hat{M}_\sigma^0 (\hat{M}_\sigma^+)^*\big] = 0.$$

$\longrightarrow$ Observation of a non-zero value of $F_{\mathcal{B}5}$ and $F_{\mathcal{C}4}$ signals CP and $CP\widetilde{T}$ violation.

Decay angular distributions of polarised Z

$F_{\mathcal{A}9}$ and $F_{\mathcal{C}8}$; $(CP, CP\widetilde{T}) = (-,+)$



- identically zero in the SM due to CP invariance.
- a non-zero value immediately signals CP violation.

Summary

- The decay $Z \to f\bar{f}$ has 9 independent angular distributions, and measurement of these 9 distributions corresponds to measurement of a state of polarisation.
- We obtained the 15 observables as the coefficients of the Z decay angular distributions.
- Observation of the 9 observables among these 15 observables does not require the charge identification of the final fermion f.
- These 9 observables are necessary and sufficient to determine all of the 9 independent combinations of the DM elements.
- Symmetry properties of these 15 observables are clarified.