Probing the scalar sector in $e^+e^- \to ZHH$.

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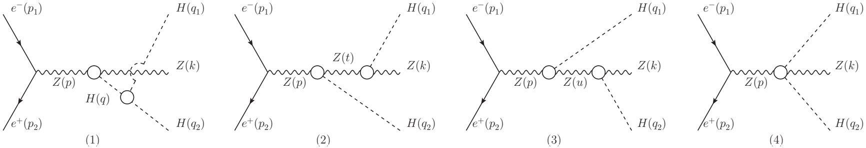
Kepler Center Workshop, 27.9.2017 - 29.9.2017,
Freudenstadt-Lauterbad

1. The Higgs boson
   - Its existence is predicted by the standard model (SM) of elementary particle physics.
   - It has been discovered in 2012 by the ATLAS and CMS collaborations at the large hadron collider (LHC).
   - The measurements of its couplings to the SM particles are essential tests of the SM.
2. Electron (e^-)-Positron (e^+) colliders
   - The precision measurements of the Higgs couplings are expected due to their cleaner environments than the LHC.
   - They are the next generation high energy colliders: the international linear collider (ILC) and the compact linear collider (CLIC) are planned.
3. The Higgs boson + e^+e^- colliders $\rightarrow$ the process $e^+e^- \rightarrow ZHH$

3. The Higgs boson + e^+e^- colliders → the process $e^+e^- \to ZHH$

- It is a reaction producing the Z boson (Z) and the 2 Higgs bosons (HH).
- It provides direct access to the Higgs self-couplings: $ZZHH$, $Z\gamma HH$ and HHH (see the figure below).
- It is the most promising reaction to measure these couplings in the first stage of an e^+e^- collider ($\sqrt{s} = 500$ GeV).
- The goal of this work is (1) to understand the angular distributions of the Z and the 2 Higgs bosons, and (2) to show how these distributions can be used to study the Higgs (self-)couplings.



→ The Feynman diagrams in momentum space, which represent the leading contributions to the scattering amplitude. The small circles denote the Higgs (self-)couplings, which we wish to measure.

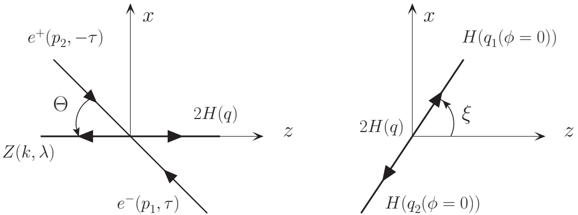
Outline

- Introduction
- Angular distributions of the Z, H and H
- Requests from symmetries

Angular distributions of the Z, H and H

$$e^-(p_1) + e^+(p_2) \to Z(k) + H(q_1) + H(q_2).$$

$$\xrightarrow[\text{decompose}]{} e^-(p_1) + e^+(p_2) \to Z(k) + 2H(q) \text{ and } 2H(q) \to H(q_1) + H(q_2).$$



- Θ : The polar angle of the Z boson from the e^- direction.
- ξ : The polar angle of the H boson from the z-axis.
- ϕ : The azimuthal angle of the the H boson from the x-axis.

- Q : The mass of $2H$. It determines the magnitude of the Higgs bosons' momenta.
- E : The e^+e^- center-of-mass (c.m.) energy. $E \geq m_Z + 2m_H \simeq 342$ GeV. A fixed value ($E = 500$ GeV in the first stage of the ILC).

These 5 are the only independent kinematic variables (^^):
$\{\Theta, \xi, \phi, Q, E\} \longleftrightarrow$ all the momenta of the Z, H and H bosons.

Angular distributions of the Z, H and H

Θ, ξ, ϕ, Q and E are the only independent kinematic variables. We regard E as a fixed value.

The complete differential cross section is given by

$$\frac{d\sigma}{d\cos\Theta dQ^2 d\cos\xi d\phi} = \frac{1}{1024\pi^4}\frac{l}{E^3}\sqrt{1-\frac{4m_H^2}{Q^2}}\sum_{\tau=\pm}\sum_{\lambda=\pm,0}|\mathcal{M}_\tau^\lambda|^2,$$

where the scattering amplitudes $\mathcal{M}_\tau^\lambda$ are the functions of Θ, ξ, ϕ, Q and E: $\mathcal{M}_\tau^\lambda = \mathcal{M}_\tau^\lambda(\Theta, \xi, \phi, Q, E)$.

I have found that the amplitude-squared has the 9 independent angular distributions:

$$\begin{aligned}\sum_{\lambda=\pm,0}|\mathcal{M}_\tau^\lambda|^2 =& F_1\left(1+\cos^2\Theta\right) + F_2\left(1-3\cos^2\Theta\right) + F_3\cos\Theta \\ &+ F_4\sin\Theta\cos\phi + F_5\sin 2\Theta\cos\phi + F_6\sin^2\Theta\cos 2\phi \\ &+ F_7\sin\Theta\sin\phi + F_8\sin 2\Theta\sin\phi + F_9\sin^2\Theta\sin 2\phi,\end{aligned}$$

where the 9 coefficients are $F_i = F_i(\xi, Q, E)$, therefore the Θ and ϕ dependences are totally factorized!

Angular distributions of the Z, H and H

Our complete formula:

$$\frac{d\sigma(\tau)}{d\cos\Theta dQ^2 d\cos\xi d\phi} = \frac{1}{1024\pi^4}\frac{I}{E^3}\sqrt{1-\frac{4m_H^2}{Q^2}}\Big[F_1(1+\cos^2\Theta) + F_2(1-3\cos^2\Theta)$$
$$+ F_3\cos\Theta + F_4\sin\Theta\cos\phi + F_5\sin 2\Theta\cos\phi + F_6\sin^2\Theta\cos 2\phi$$
$$+ F_7\sin\Theta\sin\phi + F_8\sin 2\Theta\sin\phi + F_9\sin^2\Theta\sin 2\phi\Big],$$

where $F_i = F_i(\xi, Q, E)$.

(1) The $\cos\Theta$ distribution provides us access to only $F_{1,2,3}$ (i.e. the other terms vanish after the integration over ϕ):

$$\frac{d\sigma(\tau)}{d\cos\Theta dQ^2 d\cos\xi} = \frac{1}{512\pi^3}\frac{I}{E^3}\sqrt{1-\frac{4m_H^2}{Q^2}}\Big[F_1(1+\cos^2\Theta) + F_2(1-3\cos^2\Theta) + F_3\cos\Theta\Big].$$

(2) The total cross section, or the differential cross section with respect to Q^2 and $\cos\xi$, provides us access to only F_1:

$$\frac{d\sigma(\tau)}{dQ^2 d\cos\xi} = \frac{1}{192\pi^3}\frac{I}{E^3}\sqrt{1-\frac{4m_H^2}{Q^2}}F_1.$$

- In order to benefit from all of the 9 coefficients F_i, we need to measure the $\cos\Theta$ *and* ϕ distributions (^^).

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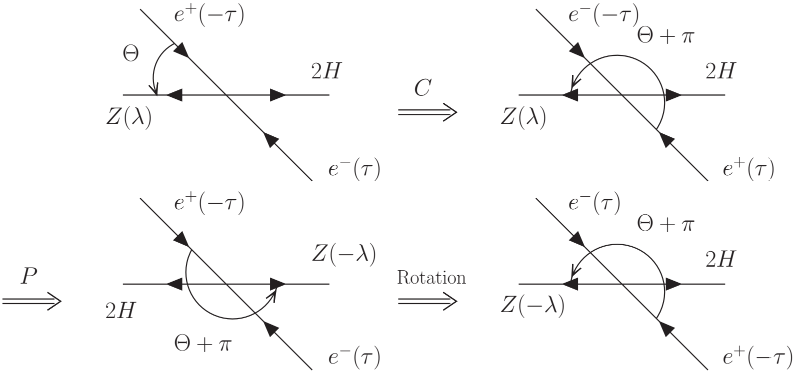
Requests from symmetries

Symmetries lead to certain constraints on the angular distributions.

Charge-conjugation (C) and parity (P) transformation:



1. C : $Q_{\rm EM} \to -Q_{\rm EM}$
2. P : $\vec{p} \to -\vec{p}$, Helicity $\to -$ Helicity (Helicity $= \vec{p}\cdot\vec{s}/|\vec{p}|$)
3. A rotation around the y-axis by π; it does not affect final results.

$$CP : \cos\Theta \to -\cos\Theta, \sin\Theta \to -\sin\Theta, \lambda \to -\lambda.$$

This is not all, because the 2 Higgs bosons also transform...

Requests from symmetries

The momentum of one of the 2 Higgs bosons transforms as

$$\vec{q}_1 = \Big(r\sin\xi\cos\phi,\ r\sin\xi\sin\phi,\ l/2 + (E-w)/Q \times r\cos\xi \Big),$$

$$\xrightarrow[CP]{} \Big(-r\sin\xi\cos\phi,\ -r\sin\xi\sin\phi,\ -l/2-(E-w)/Q \times r\cos\xi \Big),$$

$$\xrightarrow[\text{Rotation}]{} \Big(+r\sin\xi\cos\phi,\ -r\sin\xi\sin\phi,\ +l/2+(E-w)/Q \times r\cos\xi \Big).$$

The CP transformation can be, therefore, interpreted as

$$CP : \cos\Theta \to -\cos\Theta,\ \sin\Theta \to -\sin\Theta,\ \lambda \to -\lambda,\ \sin\phi \to -\sin\phi.$$

We apply this transformation to the amplitude-squared:

$$\sum_{\lambda=\pm,0} |\mathcal{M}_\tau^\lambda|^2 = F_1\big(1+\cos^2\Theta\big) + F_2\big(1-3\cos^2\Theta\big) + F_3\cos\Theta + \cdots$$

$$\xrightarrow[CP]{} F_1\big(1+\cos^2\Theta\big) + F_2\big(1-3\cos^2\Theta\big) - F_3\cos\Theta$$

$$-F_4\sin\Theta\cos\phi + F_5\sin 2\Theta\cos\phi + F_6\sin^2\Theta\cos 2\phi$$

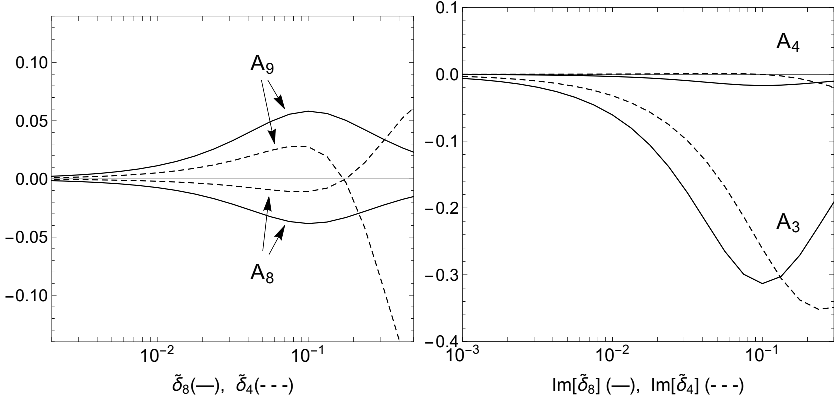
$$+F_7\sin\Theta\sin\phi - F_8\sin 2\Theta\sin\phi - F_9\sin^2\Theta\sin 2\phi$$

- CP invariance requires $F_{3,4,8,9} = 0$.
- Observation of non-zero values of $F_{3,4,8,9}$ indicates CP nonconservation in the scalar sector (^^).

Requests from symmetries

- The scalar sector of the standard model (SM) is *CP* conserving.
- Observation of CP nonconservation signals the existence of physics beyond the SM (^^).

Numerical result - an example: the coefficients $F_{3,4,8,9}$ are shown as deviations from the SM predictions caused by adding small *CP* violating effects in the Higgs (self-)couplings.



Summary

- The Higgs boson has been discovered in 2012 and the measurements of its couplings to the SM particles are essential tests of the SM.
- e^+e^- colliders are the next generation high energy colliders: ILC and CLIC. The precision measurements of the Higgs couplings are expected.
- The reaction $e^+e^- \to ZHH$ provides direct access to the Higgs self-couplings: $ZZHH$, $Z\gamma HH$ and HHH.
- By appropriately choosing the coordinate system and parameterizing the momenta, the differential cross section exhibits non-trivial angular distributions.
- These angular distributions can be used to extract important information on the Higgs (self-)couplings, such as CP nonconservation in the scalar sector.