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MOD( $v, d$ ):  $\mathbb{Z}, \mathbb{Z} \rightarrow \mathbb{Z}$ 
   $res = (v + d) \bmod d$ 
  return ( $res$ )

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SIMPS( $f, h$ ):  $\mathbb{R}^1, \mathbb{R}^1 \rightarrow \mathbb{R}^1$ 
   $r \in \mathbb{R}_{100,80}^2$ 
   $n = \text{SHAPE}(f)_0$ 
   $undef = \mathfrak{G}^{\text{DROP}([1], \text{SHAPE}(f))} 0.0$ 
   $r_i \mid i \in [0, n - 1] = \begin{cases} \frac{11.0 \cdot f_0 + 14.0 \cdot f_1 - f_2}{24.0} & i = 0 \\ \frac{f_{i-1} + 4.0 \cdot f_i + f_{i+1}}{3.0} & 1 \leq i \leq n - 2 \\ undef & \text{otherwise} \end{cases}$ 
   $rs = r_2$ 
   $r_2 = r_1$ 
   $r_1 = r_0$ 
   $r_0 = undef$ 
   $r_i^{[i]} = r_{i-2}^{[i-2]} + rs^{[i-1]}$ 
   $rs^{[i]} = r_{i-1}^{[i-1]}$ 
  filter( $r^{[i]} \mid i = n - 1$ )
  return ( $r \cdot h$ )

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$$\begin{aligned}
& \mathbf{N}(u, dx, dy): \mathbb{R}^2, \mathbb{R}^1, \mathbb{R}^1 \rightarrow \mathbb{R}^2 \\
& \mathit{undef} = \mathfrak{G}^{\text{DROP}([1], \text{SHAPE}(u))} 0.0 \\
& z_{i,j} \mid [i, j : 0 \leq i \leq s_0 \wedge 0 \leq j \leq s_1] = \begin{cases} \frac{u_{i,j+1} - 2.0 \cdot u_{i,j} + u_{i,s_1-1}}{dy \cdot dy} & [i, j : 0 \leq i \leq s_0 \wedge 0 \leq j \leq 1] \\ \frac{u_{i,j+1} - 2.0 \cdot u_{i,j} + u_{i,j-1}}{dy \cdot dy} & [i, j : 0 \leq i \leq s_0 - 1 \wedge 0 \leq j \leq s_0] \\ \frac{u_{i,0} - 2.0 \cdot u_{i,j} + u_{i,j-1}}{dy \cdot dy} & [i, j : 0 \leq i \leq s_0 \wedge s_1 - 1 \leq j \leq s_1] \\ 0.0 & \text{otherwise} \end{cases} \\
& du_i \mid i \in [0, \text{TAKE}([1], s)] = \begin{cases} \frac{u_{i+1} - u_{i-1}}{2.0 \cdot dx} & 1 \leq i \leq s_0 - 2 \\ \mathit{undef} & \text{otherwise} \end{cases} \\
& du_0 = \frac{u_1 - u_0}{dx} \\
& du_{s_0-1} = \frac{u_{s_0-1} - u_{s_0} - 2}{dx} \\
& \mathbf{return} (3.0 \cdot \text{SIMPS}(z, dx) - 6.0 \cdot u \cdot du)
\end{aligned}$$

$$\begin{aligned}
& \mathbf{L}(u, dx): \mathbb{R}^1, \mathbb{R}^1 \rightarrow \mathbb{R}^1 \\
& \mathit{undef} = \mathfrak{G}^{\text{DROP}([1], \text{SHAPE}(u))} 0.0 \\
& s = \text{SHAPE}(u)_0 \\
& z_i \mid i \in [0, \text{TAKE}([1], \text{SHAPE}(u))] = \begin{cases} -\frac{u_{i+2} - 2.0 \cdot u_{i+1} + 2.0 \cdot u_{i-1} - u_{i-2}}{2.0 \cdot dx \cdot dx \cdot dx} & 2 \leq i \leq s - 3 \\ \mathit{undef} & \text{otherwise} \end{cases} \\
& z_0 = -\frac{u_0 - 2.0 \cdot u_1 + u_2}{dx \cdot dx \cdot dx} \\
& z_1 = -\frac{-u_0 + 3.0 \cdot u_1 - 3.0 \cdot u_2 + u_3}{dx \cdot dx \cdot dx} \\
& z_{s-2} = -\frac{-u_{s-4} + 3.0 \cdot u_{s-3} - 3.0 \cdot u_{s-2} + u_{s-1}}{dx \cdot dx \cdot dx} \\
& z_{s-1} = \frac{u_{s-3} - 2.0 \cdot u_{s-2} + u_{s-1}}{dx \cdot dx \cdot dx} \\
& \mathbf{return} (x)
\end{aligned}$$

$\text{PREPENT}(a, b, c, d, e): \mathbb{R}^1, \mathbb{R}^1, \mathbb{R}^1, \mathbb{R}^1, \mathbb{R}^1 \rightarrow \mathbb{R}^1, \mathbb{R}^1, \mathbb{R}^1, \mathbb{R}^1$

$n = \text{SHAPE}(a)_0$

$undef = \mathfrak{G}^{\text{DROP}([1], \text{SHAPE}(a))} 0.0$

$p = q = bet = den = \mathfrak{G}^n undef$

$bet_0 = \frac{1.0}{c_0}$

$p_0 = -d_0 \cdot bet_0$

$q_0 = -e_0 \cdot bet_0$

$bet_1 = -\frac{1.0}{c_1 + b_1 \cdot p_0}$

$p_1 = (d_1 + b_1 \cdot q_0) \cdot bet_1$

$q_1 = e_1 \cdot bet_1$

$den_1 = b_1$

$p_0^{[0]} = p_0, p_1^{[1]} = p_1, q_0^{[0]} = q_0, q_1^{[1]} = q_1$

$bet_i^{[i]} = b_i + a_i \cdot p_{i-2}^{[i-2]}$

$den_i^{[i]} = -\frac{1.0}{c_i + a_i \cdot q_{i-2}^{[i-2]} + bet_i^{[i]} \cdot p_{i-1}^{[i-1]}}$

$p_i^{[i]} = (d_i + bet_i^{[i]} \cdot q_{i-1}^{[i-1]})_{i-1} \cdot den_i^{[i]}$

$q_i^{[i]} = e_i \cdot den_i^{[i]}$

filter($p^{[i]}, q^{[i]}, bet^{[i]}, den^{[i]} \mid i = n - 1$)

return (p, q, bet, den)

$\text{PENT}(p, q, bet, den, a, u): \mathbb{R}_{100}^1, \mathbb{R}_{100}^1, \mathbb{R}_{100}^1, \mathbb{R}_{100}^1, \mathbb{R}_{100}^1, \mathbb{R}_{100,80}^2 \rightarrow \mathbb{R}^2$

$n = \text{SHAPE}(a)_0$

$n_0 = u_0 \cdot bet_0$

$u_1 = (den_1 \cdot u_0 - u_1) \cdot bet_1$

$u_1^{[0]} = u_1$

$u_2^{[1]} = u_2$

$u_3^{[2]} = u_3$

$u_i^{[i]} = (a_i \cdot u_{i-2}^{[i-3]} + bet_i \cdot u_{i-1}^{[i-2]} - u_i^{[i-1]}) \cdot den_i$

filter($u^{[i]} \mid i = n - 1$)

$u^{[0]} = u$

$u_{n-2} = u_{n-2} + p_{n-2} \cdot u_{n-1}$

$u_{n-3-i}^{[i]} = u_{n-3-i}^{[i-1]} + p_i \cdot u_{n-4-i}^{[i-1]} + q_i \cdot u_{n-5-i}^{[i-1]}$

filter($u^{[i]} \mid i = n - 3$)

return (u)

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SOLITON( $x, y$ ):  $\mathbb{R}^1, \mathbb{R}^1 \rightarrow \mathbb{R}^1$ 
 $k = \text{frac}(\sqrt{6.0}, 4.0)$ 
 $num = -4.0 \cdot x \cdot x + 15.0 \cdot k \cdot k \cdot y \cdot y + \frac{1.0}{k \cdot k}$ 
 $denom = 4.0 \cdot x \cdot x + 16.0 \cdot k \cdot k \cdot y \cdot y + \frac{1.0}{k \cdot k}$ 
return ( $\frac{16.0 \cdot num}{denom \cdot denom}$ )

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