The 4th week number theory solution

1. $4,849,845 = 3\cdot 5\cdot 7\cdot 11\cdot 13\cdot 17\cdot 19$

2. Integers of the form p^2 where p is prime have exactly three positive divisors.

   Integers of the form pq or p^3 where p and q are distinct primes have exactly four positive divisors.

3. $\sqrt{2}+\sqrt{3}$ is irrational

   pf) Suppose that $x=\sqrt{2}+\sqrt{3}$. Then $x^2 = 2+2\sqrt{2}\sqrt{3}+3 = 5+2\sqrt{6}$.

   Hence, $x^2-5=2\sqrt{6}$

   It follows that $x^4-10x^2+25=24$.

   $\therefore\ x^4-10x^2+1=0$

   Therefore $\sqrt{2}+\sqrt{3}$ is irrational since $\sqrt{2}+\sqrt{3}$ is an integer.

   $(3<\sqrt{2}+\sqrt{3}<4)$

4. $2x+6y=18$

   $x_0=9,\ y_0=0$ is a particular solution of the equation

   $\therefore\ x=9+3k \qquad y=0-k=-k. \quad (k\in\mathbb{Z})$

   $x=9+3k,\ y=-k \quad (k\in\mathbb{Z})$

5. $1485x + 1745y = 15$

By the Euclidean algorithm, we know $(1485, 1745) = 5$

$$
(\because \quad
\begin{aligned}
1745 &= 1 \cdot 1485 + 260 \\
1485 &= 5 \cdot 260 + 185 \\
260 &= 1 \cdot 185 + 75 \\
185 &= 2 \cdot 75 + 35 \\
75 &= 2 \cdot 35 + 5 \\
35 &= 7 \cdot 5 + 0
\end{aligned}
\quad)
$$

Since $5 \mid 15$, the equation has a solution.

$$
\begin{aligned}
5 &= 75 - 2 \cdot 35 \\
&= 75 - 2 \cdot (185 - 2 \cdot 75) \\
&= 5 \cdot 75 - 2 \cdot 185 \\
&= 5 \cdot (260 - 185) - 2 \cdot 185 \\
&= 5 \cdot 260 - 7 \cdot 185 \\
&= 40 \cdot 260 - 7 \cdot 1485 \\
&= 5 \cdot 260 - 7 \cdot (1485 - 5 \cdot 260) \\
&= 40 \cdot (1745 - 1485) - 7 \cdot 1485 \\
&= 40 \cdot 1745 - 47 \cdot 1485
\end{aligned}
$$

Hence $x_0 = -141$, $y_0 = 120$ is a particular solution.

$\therefore \quad x = -141 + 349k, \quad y = 120 - 297k \quad (k \in \mathbb{Z})$.