



G.PULLAIAH COLLEGE OF ENGINEERING & TECHNOLOGY: KURNOOL
(Autonomous)

II B. Tech Ist SEM (R23) I MID Examination- September 2024

Department of Electronic and Communication Engineering

Sub: Signals, System and Stochastic Processes (A40401)

Time: 90 min

Date: 24-09-2024

Max. Marks: 30


Answer all the Questions (3X10=30M)

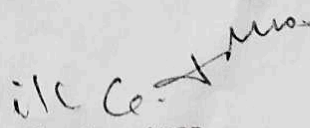
SET NO: 4

S. No	Questions	Marks	Unit	CO	Cognitive Level
1.	What are the basic operations on signals? Illustrate with an example.	10	I	CO.1	Remember
OR					
2.	Give the Exponential Fourier series representation of a periodic signal $f(t)$ with time period $T = \frac{2\pi}{\omega_0}$.	10	I	CO.1	Apply

3.a.	Write short notes on Fourier spectrum?	7	II	CO.2	Apply
3.b.	What are the effects of sampling rate?	3	II	CO.2	Understand
OR					
4.a.	Derive the expression inverse Fourier transform.	6	II	CO.2	Evaluate
4.b.	State the properties of Fourier transforms	4	II	CO.2	Remember

5.	Define a system. How are systems classified? Define each one of them.	10	I	CO.1	Remember
OR					
6.a.	Find the Fourier transform of $e^{-at}u(t)$	5	II	CO.2	Understand
6.b.	State and prove the sampling theorem.	5	II	CO.2	Remember


Signature of faculty


Signature of HOD



G.PULLAIAH COLLEGE OF ENGINEERING & TECHNOLOGY: KURNOOL
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II B. Tech I SEM (R23) II MID Examination- NOVEMBER 2024
Department of Electronics and Communication Engineering

Sub: Signal Systems and Stochastic Processor (A40401)
Time: 90 min

Date: 28-11-2024
Max. Marks: 30

Answer all the Questions (3X10=30M)

SET NO: 4

- 1 With neat sketches explain the characteristics of ideal LPF, HPF, BPF and BRF.

(OR)

- 2 Derive the relation between bandwidth and rise time.

- 3 Explain about joint distribution and density functions of random processes.

(OR)

- 4 A random process is given as $X(t) = A \sin(\omega_0 t + \Theta)$ is a wide sense stationary random process, where ω_0 is constant and Θ is a uniformly distributed random variable on the interval $(0, 2\pi)$. Find a) $E[X(t)]$ b) $R_{XX}(\tau)$ c) average power d) mean square value of $X(t)$.

- 5 State and prove Wiener – Khintchine relations.

(OR)

- 6 Derive the expression for Cross Power Spectral Density.

Marks	Unit	CO	Cognitive Level
10M	III	CO3	Understanding
10M	III	CO4	Evaluating
10M	IV	CO5	Understanding
10M	IV	CO5	Applying
10M	V	CO6	Analyzing
10M	V	CO6	Evaluating

Signature of faculty

Signature of HOD

CODE: A40401

R23

H.T.No: 23 A 7 A 0 4 0 0 9

G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY
(AUTONOMOUS)

B. Tech II Year I Semester Regular Examinations December 2024
Signals, Systems, and Stochastic Processes
Electronics and Communication Engineering

SET-1**Time: 3 Hours****Max. Marks: 70****Instructions:**

1. Answer all 10 questions from Part-A. Each question carries two marks
2. Answer one full question from each unit in Part-B. Each full question carries 10marks

PART-A

1	a	List the classification of systems.	2M	CO1	BTL1
	b	Define the Convolution of two signals	2M	CO1	BTL1
	c	What is CTFT?	2M	CO2	BTL1
	d	Define Region of Convergence in Laplace Transform	2M	CO2	BTL1
	e	Sketch ideal and practical LPF characteristics	2M	CO3	BTL5
	f	What is distortion of a signal?	2M	CO3	BTL1
	g	State Ergodic theorem.	2M	CO4	BTL1
	h	Explain the first-order stationary process briefly	2M	CO4	BTL2
	i	List any three Properties of Power Spectral Density.	2M	CO5	BTL1
	j	Define the relation between power spectral density and autocorrelation function.	2M	CO5	BTL1

PART-B**UNIT-I**

2	a	Write about the classification of the signals with examples.	5M	CO1	BTL2
	b	Explain the analogy between vectors and signals.	5M	CO1	BTL2

OR

3		Check the system $y(n) = a u(n)$, is (i) Static or dynamic. (ii) Linear or non-linear. (iii) Causal or non-causal. (iv) Time invariant or time variant. (v) Stable or not stable.	10M	CO1	BTL2
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UNIT-II

4	a	State and prove any two properties of Fourier Transform	5M	CO2	BTL5
	b	Apply time convolution theorem of Fourier Transform to compute the frequency domain of a continuous time domain signal $f(t) = e^{-3t}u(t) * e^{-2t}u(t)$	5M	CO2	BTL3

OR

5		Determine the Laplace transform and the associate region convergence for each of the following functions: (i) $x(t) = 1$; $0 \leq t \leq 1$ (ii) $x(t) = t$ for $0 \leq t \leq 1$ and (iii) $x(t) = 2-t$ for $1 \leq t \leq 2$.	10M	CO2	BTL4
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UNIT-III

6	a	Distinguish between LTI and LTV systems with an example.	5M	CO3	BTL4
	b	If the transfer function of an LTI system is $\frac{1}{j\omega+2}$, then determine the system's output for an input $(0.8)^t u(t)$.	5M	CO3	BTL4

OR

7	a	Explain about filter characteristics of linear systems	5M	CO3	BTL2
	b	Derive the relationship between rise time and bandwidth.	5M	CO3	BTL5

UNIT IV

8	a	Define the terms, time average and ergodicity? Discuss about mean-ergodic and correlation ergodic processes	5M	CO4	BTL2
	b	Define and explain any five properties of autocorrelation function.	5M	CO4	BTL2

OR

9	a	Define and explain any five properties of cross correlation function.	5M	CO4	BTL2
	b	Discuss the evaluation process of (i) Mean value of system response and (ii) Mean-squared value of system response.	5M	CO4	BTL2

UNIT V

10	a	Drive the relation between the autocorrelation function and PSD	5M	CO5	BTL5
	b	State and prove the properties of Power spectral density of random process	5M	CO5	BTL5

OR

11	a	Drive the relation between the Cross-correlation function and CPSD	5M	CO5	BTL5
	b	Derive an equation for Autocorrelation function of the output response of an LTI system	5M	CO5	BTL5

SIGNALS AND SYSTEMS
2 MARKS QUESTIONS WITH ANSWERS & 10 MARKS QUESTIONS

UNIT-I
CLASSIFICATION OF SIGNALS AND SYSTEMS

1. Define signal and system.

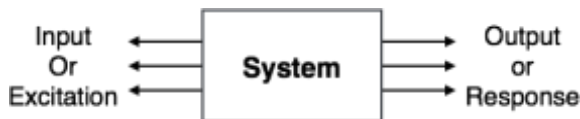
Signal : A signal describes a time varying physical phenomenon which is intended to convey information. (or) Signal is a function of time or any other variable of interest. (or) Signal is a function of one or more independent variables, which contain some information.

Example: voice signal, video signal, signals on telephone wires, EEG, ECG etc. Signals may be of continuous time or discrete time signals.

System : System is a device or combination of devices, which can operate on signals and produces corresponding response. Input to a system is called as excitation and output from it is called as response. (or) System is a combination of sub units which will interact with each other to achieve a common interest.

For one or more inputs, the system can have one or more outputs.

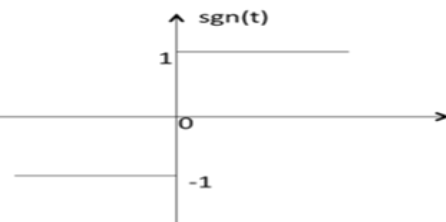
Example: Communication System



2. Deduce the relation between signum function and unit step function.

Signum function is denoted as $\text{sgn}(t)$.

$$\text{It is defined as } \text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



$$\text{sgn}(t) = 2u(t) - 1$$

3. State the classification of continuous time signals.

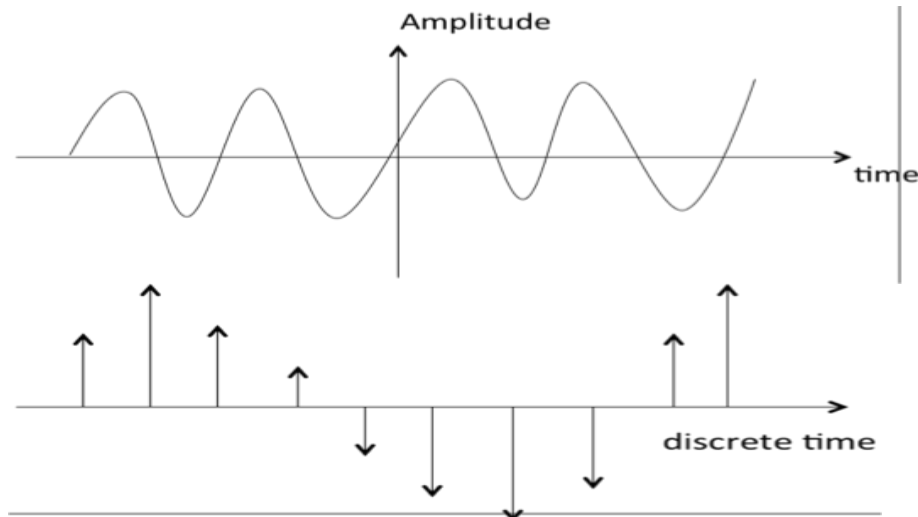
The CT signals are classified as follows

- (i) Periodic and non-periodic signals

- (ii) Analog and digital signals
- (iii) Even and odd signals
- (iv) Energy and power signals
- (v) Deterministic and random signals.

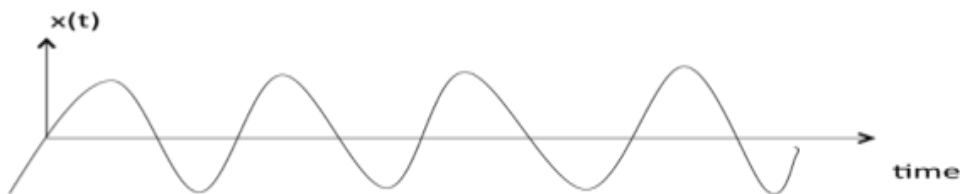
4. Distinguish between continuous time (CT) and discrete time (DT) signals.

A signal is said to be continuous when it is defined for all instants of time.

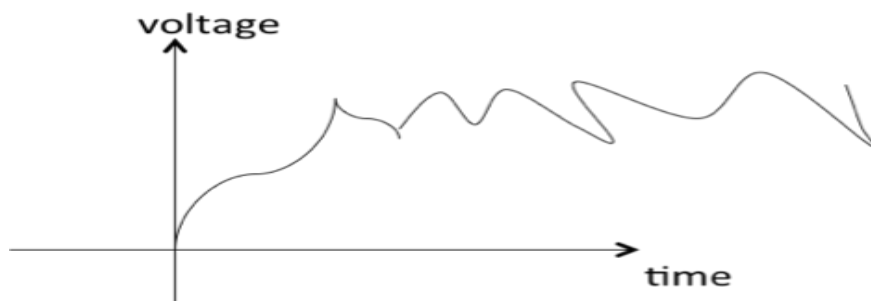


5. Distinguish between deterministic and random signals.

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.



A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modeled in probabilistic terms.



6. Distinguish between energy and power signals.

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} x^2(t) dt$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x^2(t) dt$$

NOTE: A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

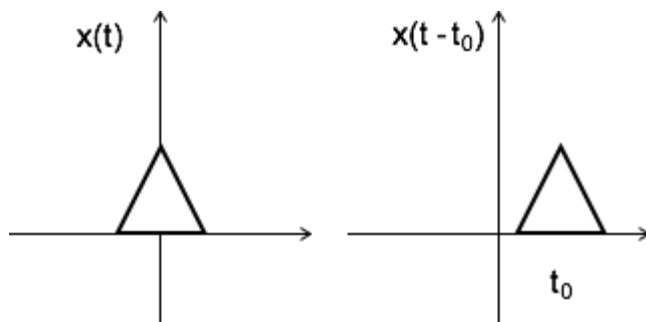
Power of energy signal = 0

Energy of power signal = ∞

7. Explain about time shifting and time scaling operations on signals.

$x(t \pm t_0)$ is time shifted version of the signal $x(t)$. $x(t + t_0) \rightarrow$ negative shift

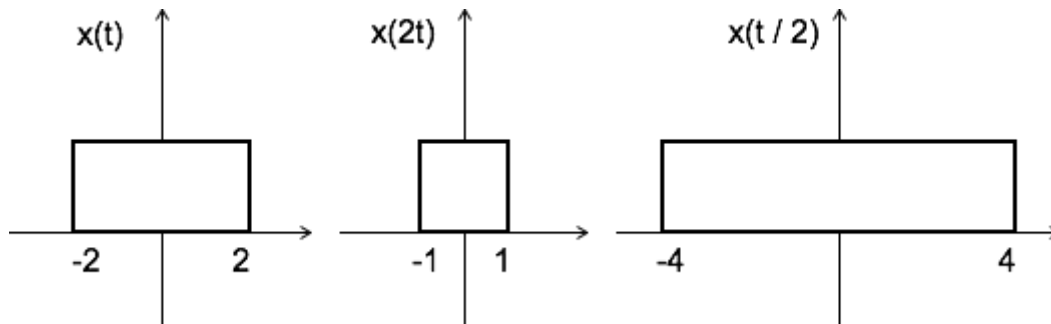
$x(t - t_0) \rightarrow$ positive shift



$x(At)$ is time scaled version of the signal $x(t)$. where A is always positive

$|A| > 1 \rightarrow$ Compression of the signal

$|A| < 1 \rightarrow$ Expansion of the signal



8. List the classification of systems.

Systems are classified into the following categories:

- Liner and Non-liner Systems
- Time Variant and Time Invariant Systems
- Liner Time variant and Liner Time invariant systems
- Static and Dynamic Systems
- Causal and Non-causal Systems
- Invertible and Non-Invertible Systems
- Stable and Unstable Systems

9. Distinguish between linear and non-linear systems.

A system is said to be linear when it satisfies superposition and homogenate principles.

Consider two systems with inputs as $x_1(t)$, $x_2(t)$, and outputs as $y_1(t)$, $y_2(t)$ respectively.

Then, according to the superposition and homogenate principles,

$$T[a_1 x_1(t) + a_2 x_2(t)] = a_1 T[x_1(t)] + a_2 T[x_2(t)]$$

$$\therefore T[a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t)$$

Example: $y(t) = x^2(t)$

Solution

$$y_1(t) = T[x_1(t)] = x_1^2(t)$$

$$y_2(t) = T[x_2(t)] = x_2^2(t)$$

$$T[a_1 x_1(t) + a_2 x_2(t)] = [a_1 x_1(t) + a_2 x_2(t)]^2$$

Which is not equal to $a_1 y_1(t) + a_2 y_2(t)$. Hence the system is said to be nonlinear.

10. Distinguish between time invariant and time varying systems.

A system is said to be time variant if its input and output characteristics vary with time.

Otherwise, the system is considered as time invariant.

The condition for time invariant system is:

$$y(n, t) = y(n-t)$$

The condition for time variant system is:

$y(n, t) \neq y(n-t)$ Where $y(n, t) = T[x(n-t)] = \text{input change}$

$y(n-t) = \text{output change}$

$y(n) = x(-n)$

$y(n, t) = T[x(n-t)] = x(-n-t)$

$y(n-t) = x(-(n-t)) = x(-n + t)$

$\therefore y(n, t) \neq y(n-t)$. Hence, the system is time variant.

11. Distinguish between memory and memory less system.

Static system is memory-less whereas dynamic system is a memory system.

Example 1: $y(t) = 2x(t)$

For present value $t=0$, the system output is $y(0) = 2x(0)$. Here, the output is only dependent upon present input. Hence the system is memory less or static.

Example 2: $y(t) = 2x(t) + 3x(t-3)$

For present value $t=0$, the system output is $y(0) = 2x(0) + 3x(-3)$.

Here $x(-3)$ is past value for the present input for which the system requires memory to get this output. Hence, the system is a dynamic system.

12. Distinguish between causal and non-causal systems.

A system is said to be causal if its output depends upon present and past inputs, and does not depend upon future input.

For non-causal system, the output depends upon future inputs also.

Example 1: $y(n) = 2x(n) + 3x(n-3)$

For present value $t=1$, the system output is $y(1) = 2x(1) + 3x(-2)$.

Here, the system output only depends upon present and past inputs. Hence, the system is causal.

Example 2: $y(n) = 2x(n) + 3x(n-3) + 6x(n+3)$

For present value $t=1$, the system output is $y(1) = 2x(1) + 3x(-2) + 6x(4)$. Here, the system output depends upon future input. Hence the system is non-causal system.

13. Distinguish between stable and unstable systems.

The system is said to be stable only when the output is bounded for bounded input. For a bounded input, if the output is unbounded in the system then it is said to be unstable.

Note: For a bounded signal, amplitude is finite.

Example 1: $y(t) = x^2(t)$

Let the input is $u(t)$ (unit step bounded input) then the output

$y(t) = u^2(t) = u(t)$ = bounded output.

Hence, the system is stable.

14. Derive the relation between unit impulse and unit step functions.

$$\int_{-\infty}^{\infty} \delta(t) dt = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$

15. Define unit step, ramp and delta functions.

Unit step function is defined as

$U(t) = 1$ for $t \geq 0$

0 otherwise

Unit ramp function is defined as $r(t) = t$ for $t \geq 0$

0 for $t < 0$

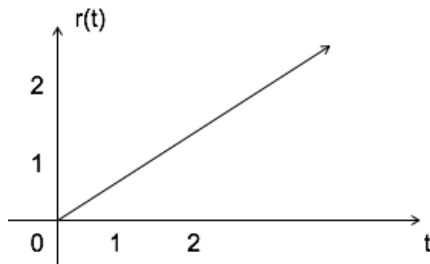
Unit delta function is defined as

$\delta(t) = 1$ for $t = 0$

0 otherwise

16. Give the relation between ramp and step signals.

Ramp signal is denoted by $r(t)$, and it is defined as $r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$



$$\int u(t) = \int 1 = t = r(t)$$

$$u(t) = \frac{dr(t)}{dt}$$

17. Derive the expression for parabolic signal using step signal.

$$\iint u(t)dt = \int r(t)dt = \int tdt = \frac{t^2}{2} = \text{parabolic signal}$$

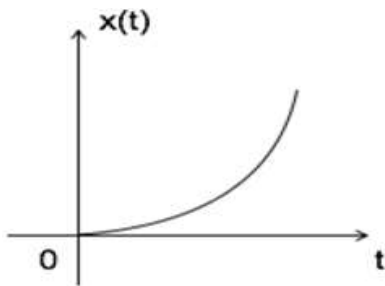
$$\Rightarrow u(t) = \frac{d^2 x(t)}{dt^2}$$

$$\Rightarrow r(t) = \frac{dx(t)}{dt}$$

where $x(t)$ is the parabolic signal.

18. Sketch the parabolic signal.

Parabolic signal can be defined as $x(t) = \begin{cases} t^2/2 & t \geq 0 \\ 0 & t < 0 \end{cases}$



19. Draw the graphical form of decaying rising and double exponential signals.

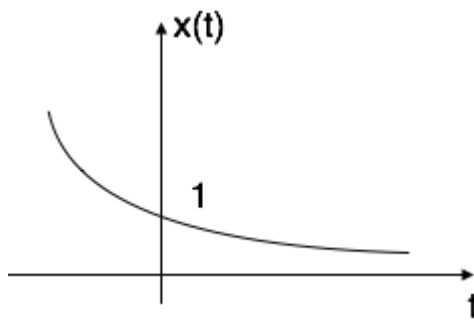


Figure: Decaying Exponential

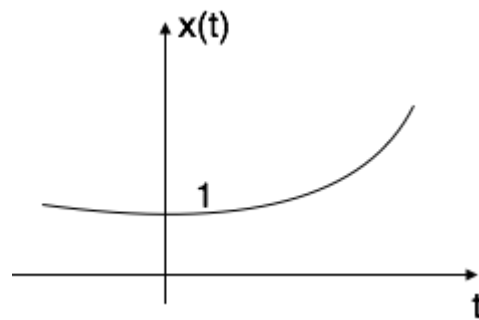


Figure: Raising Exponential

20. Draw the triangular pulse.

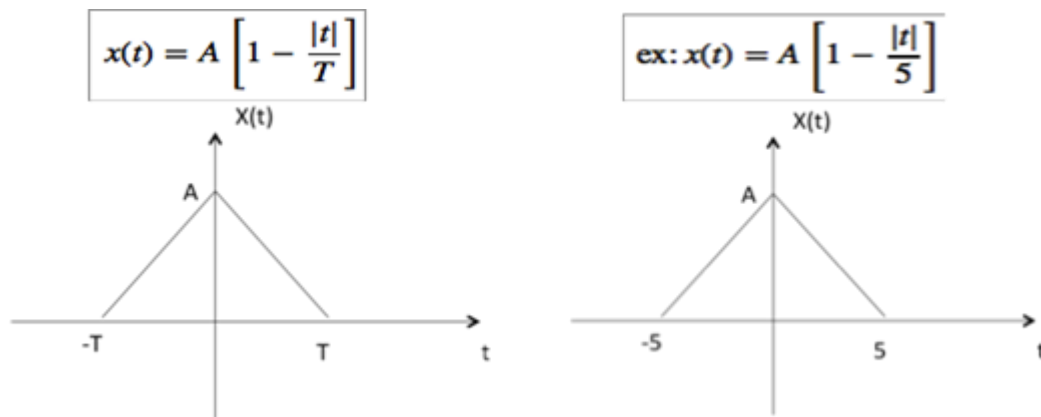
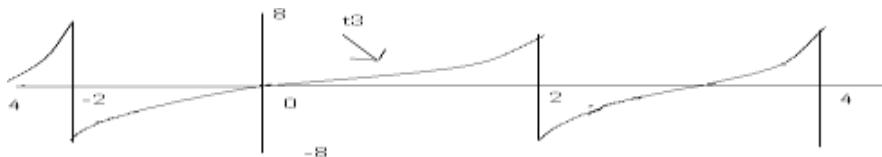


Figure: Triangular pulse

10 MARKS QUESTIONS

- Check whether the following signals are periodic or not.
 (i) $\cos^2(2\pi t)$ (ii) $\sin^3(2t)$ (iii) $e^{-2t} \cos(2\pi t)$ (iv) $(-1)^n$
- Prove that
 (i) The energy of power signal is infinite over an infinite interval.
 (ii) The power of energy signal is zero over an infinite interval.
 (iii) Determine the power and rms value of the signal $x(t) = A \sin(\omega_0 t + \phi)$.
- Categorize each of the following signals as power or energy signals. Also determine either power or energy of the signals
 (i) $x(t) = \begin{cases} 2 - t & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$ (ii) $x[n] = \begin{cases} n & 0 \leq n \leq 5 \\ 0 & \text{otherwise} \end{cases}$
 (i) $x(t) = \begin{cases} 2 - t & 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$ (ii) $x[n] = \begin{cases} 10 - n & 5 \leq n \leq 10 \\ 0 & \text{otherwise} \end{cases}$
- Find even and odd part of the following signals.
 (i) $X(t) = \cos t + \sin t + \sin t \cos t$ (ii) $x[n] = 1 \quad n \geq 0$
- Find and sketch even and odd components of the following
 (i) $u(t)$ (ii) $tu(t)$ (iii) $\sin(\omega_0 t)$
- Determine whether or not each of the following signals is periodic or not. If the signal is periodic, then determine the fundamental period.
 (i) $x(n) = 1 + e^{j(4\pi n/7)} - e^{j(2\pi n/5)}$ (ii) $x(t) = e^{(-1+j)t}$ (iii) $e^{j7\pi n}$
- Sketch the following signal: $f(t) = 3u(t) + tu(t) - u(t-1) + u(t+1) - 5u(t-2)$.
- Find the power of periodic signal $x(t)$ shown below.



- Find also the power of (i) $-x(t)$ (ii) $2x(t)$ (iii) $Cx(t)$
9. Evaluate the following integrals
 (i) $\int_0^5 \delta(t) \sin 2\pi t dt$ (ii) $\int_{-\infty}^{\infty} e^{-at^2} \delta(t-1) dt$
10. Check the system $y(n) = a^n u(n)$, is
 (i) Static or dynamic. (ii) Linear or non-linear. (iii) Causal or non-causal.
 (iv) Time invariant or time variant. (v) Stable or not stable.
11. Simplify the following expressions
 (i) $\left(\frac{\sin t}{t}\right) \delta(t)$ (ii) $\left(\frac{j\omega+2}{\omega^2+9}\right) \delta(\omega)$ (iii) $\{e^{-t} \cos(3t - 60^\circ)\} \delta(t)$
 (iv) $\left(\frac{\sin^* \frac{t+2\pi}{2}(t-2)}{t^2+4}\right) \delta(1-t)$ (v) $\left(\frac{1}{j\omega+2}\right) \delta(\omega+3)$ (vi) $\left(\frac{\sin k\omega}{\omega}\right) \delta(\omega)$
12. Check whether the given system
 $\frac{d^3}{dt^3} y(t) + 2 \frac{d^2}{dt^2} y(t) + 4 \frac{d}{dt} y(t) + 3 y^2(t) = x(t+1)$ is
 (i) Static or dynamic (ii) Linear or non-linear (iii) Causal or non-causal (iv) Time-invariant or Time-variant

UNIT-II

FOURIER SERIES AND FOURIER TRANSFORM OF CONTINUOUS TIME SIGNALS

1. Express one vector in terms of the other vector.

Consider two vectors V_1 and V_2 as shown in the following diagram. Let the component of V_1 along with V_2 is given by $C_{12}V_2$. The component of a vector V_1 along with the vector V_2 can be obtained by taking a perpendicular from the end of V_1 to the vector V_2 as shown in diagram:

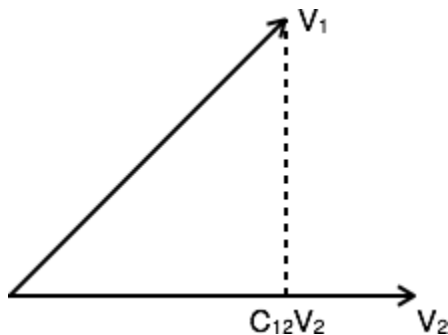


Figure: Projection of v_1 on to v_2

From the diagram, components of V_1 along $V_2 = C_{12} V_2$

where $C_{12} = \frac{V_1 \cdot V_2}{V_2^2}$

2. Let us consider two signals $f_1(t)$ and $f_2(t)$. Approximate $f_1(t)$ in terms of $f_2(t)$.

$$f_1(t) = C_{12} f_2(t) \quad \text{where} \quad C_{12} = \frac{\int_{t_1}^{t_2} f_1(t) f_2(t) dt}{\int_{t_1}^{t_2} f_2^2(t) dt}$$

3. Derive the expression for mean square error.

$$\begin{aligned} \varepsilon &= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_e(t)]^2 dt \\ &= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_e(t) - \sum_{r=1}^n C_r x_r(t)]^2 dt \\ &= \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} [f_e^2(t)] dt + \sum_{r=1}^n C_r^2 \int_{t_1}^{t_2} x_r^2(t) dt - 2 \sum_{r=1}^n C_r \int_{t_1}^{t_2} x_r(t) f(t) dt \right] \end{aligned}$$

You know that $C_r^2 \int_{t_1}^{t_2} x_r^2(t) dt = C_r \int_{t_1}^{t_2} x_r(t) f(t) dt = C_r^2 K_r$

$$\begin{aligned} \varepsilon &= \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} [f^2(t)] dt + \sum_{r=1}^n C_r^2 K_r - 2 \sum_{r=1}^n C_r^2 K_r \right] \\ &= \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} [f^2(t)] dt - \sum_{r=1}^n C_r^2 K_r \right] \end{aligned}$$

$$\therefore \varepsilon = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} [f^2(t)] dt + (C_1^2 K_1 + C_2^2 K_2 + \dots + C_n^2 K_n) \right]$$

4. Give the Trigonometric Fourier series representation of a periodic signal $x(t)$ with time period $T = \frac{2\pi}{\omega_0}$.

$$\text{Where } a_0 = \frac{\int_{t_0}^{t_0+T} x(t) \cdot 1 dt}{\int_{t_0}^{t_0+T} 1^2 dt} = \frac{1}{T} \cdot \int_{t_0}^{t_0+T} x(t) dt$$

$$a_n = \frac{\int_{t_0}^{t_0+T} x(t) \cdot \cos n\omega_0 t dt}{\int_{t_0}^{t_0+T} \cos^2 n\omega_0 t dt}$$

$$b_n = \frac{\int_{t_0}^{t_0+T} x(t) \cdot \sin n\omega_0 t dt}{\int_{t_0}^{t_0+T} \sin^2 n\omega_0 t dt}$$

$$\text{Here } \int_{t_0}^{t_0+T} \cos^2 n\omega_0 t dt = \int_{t_0}^{t_0+T} \sin^2 n\omega_0 t dt = \frac{T}{2}$$

$$\therefore a_n = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \cos n\omega_0 t dt$$

$$b_n = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \sin n\omega_0 t dt$$

The

above

equation represents trigonometric Fourier series representation of $x(t)$.

5. Give the Exponential Fourier series representation of a periodic signal $f(t)$ with time period $T = \frac{2\pi}{\omega_0}$.

$$f(t) = F_0 + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t} + \dots + F_n e^{jn\omega_0 t} + \dots \\ F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + \dots + F_{-n} e^{-jn\omega_0 t} + \dots$$

$$\therefore f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \quad (t_0 < t < t_0 + T) \dots \dots \dots (1)$$

where

$$F_n = \frac{\int_{t_0}^{t_0+T} f(t) (e^{jn\omega_0 t})^* dt}{\int_{t_0}^{t_0+T} e^{jn\omega_0 t} (e^{jn\omega_0 t})^* dt} \\ = \frac{\int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt}{\int_{t_0}^{t_0+T} e^{-jn\omega_0 t} e^{jn\omega_0 t} dt} \\ = \frac{\int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt}{\int_{t_0}^{t_0+T} 1 dt} = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt$$

$$\therefore F_n = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt$$

6. Derive the relation between Trigonometric and Exponential Fourier series representations.

Consider a periodic signal $x(t)$, the TFS & EFS representations are given below respectively

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \dots \dots \dots (1)$$

$$x(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

$$= F_0 + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t} + \dots + F_n e^{jn\omega_0 t} + \dots$$

$$F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + \dots + F_{-n} e^{-jn\omega_0 t} + \dots$$

$$= F_0 + F_1 (\cos \omega_0 t + j \sin \omega_0 t) + F_2 (\cos 2\omega_0 t + j \sin 2\omega_0 t) + \dots + F_n (\cos n\omega_0 t + j \sin n\omega_0 t) + \dots$$

$$+ F_{-1} (\cos \omega_0 t - j \sin \omega_0 t) + F_{-2} (\cos 2\omega_0 t - j \sin 2\omega_0 t) + \dots + F_{-n} (\cos n\omega_0 t - j \sin n\omega_0 t) + \dots$$

$$= F_0 + (F_1 + F_{-1}) \cos \omega_0 t + (F_2 + F_{-2}) \cos 2\omega_0 t + \dots + j(F_1 - F_{-1}) \sin \omega_0 t + j(F_2 - F_{-2}) \sin 2\omega_0 t + \dots$$

$$\therefore x(t) = F_0 + \sum_{n=1}^{\infty} ((F_n + F_{-n}) \cos n\omega_0 t + j(F_n - F_{-n}) \sin n\omega_0 t) \dots \dots (2)$$

Comparing equations 1 and 2.

$$\begin{array}{l} F_n = \frac{1}{2}(a_n - jb_n) \\ F_{-n} = \frac{1}{2}(a_n + jb_n) \end{array} \quad \begin{array}{l} a_0 = F_0 \\ a_n = F_n + F_{-n} \\ b_n = j(F_n - F_{-n}) \end{array}$$

7. What is Fourier spectrum?

The Fourier spectrum of a periodic signal $x(t)$ is a plot of its Fourier coefficients versus frequency ω . It is in two parts: (a) the amplitude spectrum and (b) The phase spectrum. The plot of the amplitude of Fourier coefficients versus frequency is known as the amplitude spectra, and the plot of the phase of Fourier coefficients versus frequency is known as the phase spectra. The two plots together are known as Fourier frequency of spectra of $x(t)$

8. What are the merits of Fourier transform?

The merits of Fourier transform are

The Fourier transform is the most useful tool for analyzing signals.

The original time function can be uniquely recovered from it i.e inverse Fourier transform is unique

Convolution integrals can be evaluated using Fourier transform.

9. What are the limitations of Fourier transform?

The limitations of the Fourier transform are as follows

There are many functions for which the Laplace transform exists, but the Fourier transform does not exist.

It is less powerful than Laplace transform

10. Write the formulae for Fourier transform and inverse Fourier transform.

The formula for Fourier transform also called the equation for direct transform or analysis equation is:

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

The formula for inverse Fourier transform also called the equation for inverse transform or synthesis equation is:

$$x(t) = \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$$

11. What is the relation between Fourier transform and Laplace transform?

Laplace transform is a complex Fourier transform. The Fourier transform of a function can be obtained from its Laplace transform by replacing $j\omega$, i.e. Fourier transform is the Laplace transform evaluated along the imaginary axis of the s -plane.

12. Determine the Fourier series coefficients for the signal $\cos(\pi t)$.

The Fourier series coefficients for the signal $\cos(\pi t)$ is determined by simply expand the cosine function as a linear combination of complex exponentials and identify by inspection of Fourier series coefficients. The $\cos(\pi t)$ can be expressed as

$$\cos(\pi t) = \frac{e^{j\pi t} + e^{-j\pi t}}{2} \quad \text{----- (1)}$$

In general $x(t)$ can be represented as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{----- (2)}$$

Compare equation number (1) and (2) possible values of k are $k=1$, $k=-1$ and $k \neq 1 \text{ or } -1$

$$a_1 = \frac{1}{2}, \quad a_{-1} = \frac{1}{2} \text{ and } a_k = 0, k \neq 1 \text{ or } -1$$

10 MARKS QUESTIONS

- Find the Fourier transform of
(i) $e^{-at}u(t)$ (ii) $te^{-at}u(t)$
- Find the Fourier transform of double exponential pulse $e^{-a|t|}$.
- Find the Fourier transform of (i) $e^{j\omega_0 t}$ (ii) $e^{-j\omega_0 t}$
- Obtain the Fourier transform of the following functions:
(i) Impulse function $\delta(t)$ (ii) DC signal (iii) Unit step function
- Find the Fourier transform of $\sin\omega_0 t$ and sketch its spectrum.
- Find the Fourier transform of the signal
$$f(t) = \frac{1}{2} [\delta(t+1) + \delta(t + \frac{1}{2}) + \delta(t - \frac{1}{2}) + \delta(t-1)]$$
- State and prove the following properties of continuous-time Fourier transform
(i) Linearity (ii) Time-shifting (iii) Frequency shifting (iv) Differentiation and integration in time domain
- Find the Fourier transform of $\cos\omega_0 t$ and sketch its spectrum.
- State the properties of Fourier transform?

Here are the properties of Fourier Transform:

Linearity Property

$$\text{If } x(t) \leftrightarrow F.TX(\omega) \text{ If } x(t) \leftrightarrow F.TX(\omega)$$

$$\&y(t) \leftrightarrow F.TY(\omega) \&y(t) \leftrightarrow F.TY(\omega)$$

Then linearity property states that

$$ax(t)+by(t) \leftrightarrow F.TaX(\omega)+bY(\omega) \quad ax(t)+by(t) \leftrightarrow F.TaX(\omega)+bY(\omega)$$

Time Shifting Property

$$If x(t) \leftrightarrow F.TX(\omega) \text{ then } If x(t-t_0) \leftrightarrow F.TX(\omega)$$

Then Time shifting property states that

$$x(t-t_0) \leftrightarrow F.Te^{-j\omega t_0}X(\omega) \text{ and } x(t+t_0) \leftrightarrow F.Te^{j\omega t_0}X(\omega)$$

Frequency Shifting Property

$$If x(t) \leftrightarrow F.TX(\omega) \text{ then } If x(t)e^{j\omega_0 t} \leftrightarrow F.TX(\omega-\omega_0)$$

Then frequency shifting property states that

$$e^{j\omega_0 t}x(t) \leftrightarrow F.TX(\omega-\omega_0) \text{ and } e^{-j\omega_0 t}x(t) \leftrightarrow F.TX(\omega+\omega_0)$$

Time Reversal Property

$$If x(t) \leftrightarrow F.TX(\omega) \text{ then } If x(-t) \leftrightarrow F.TX(-\omega)$$

Then Time reversal property states that

$$x(-t) \leftrightarrow F.TX(-\omega)$$

Time Scaling Property

$$If x(t) \leftrightarrow F.TX(\omega) \text{ then } If x(at) \leftrightarrow \frac{1}{|a|}F.TX\left(\frac{\omega}{a}\right)$$

Then Time scaling property states that

$$x(at) \leftrightarrow \frac{1}{|a|}F.TX\left(\frac{\omega}{a}\right)$$

Differentiation and Integration Properties

$$If x(t) \leftrightarrow F.TX(\omega) \text{ then } If \frac{dx(t)}{dt} \leftrightarrow F.T(j\omega)X(\omega)$$

Then Differentiation property states that

$$\frac{dx(t)}{dt} \leftrightarrow F.T(j\omega)X(\omega)$$

$$\int x(t) dt \leftrightarrow F.\frac{1}{j\omega}X(\omega) \text{ and } \int \frac{dx(t)}{dt} dt \leftrightarrow F.X(\omega)$$

and integration property states that

$$\int x(t) dt \leftrightarrow F.\frac{1}{j\omega}X(\omega)$$

$$\int \frac{dx(t)}{dt} dt \leftrightarrow F.X(\omega)$$

Multiplication and Convolution Properties

$$If x(t) \leftrightarrow F.TX(\omega) \text{ and } y(t) \leftrightarrow F.TY(\omega) \text{ then } x(t)y(t) \leftrightarrow F.TX(\omega) * TY(\omega)$$

$$x(t) * y(t) \leftrightarrow F.TX(\omega) . TY(\omega)$$

Then multiplication property states that

$$x(t)y(t) \leftrightarrow F.TX(\omega) * TY(\omega)$$

and convolution property states that

$$x(t) * y(t) \leftrightarrow F.TX(\omega) . TY(\omega)$$

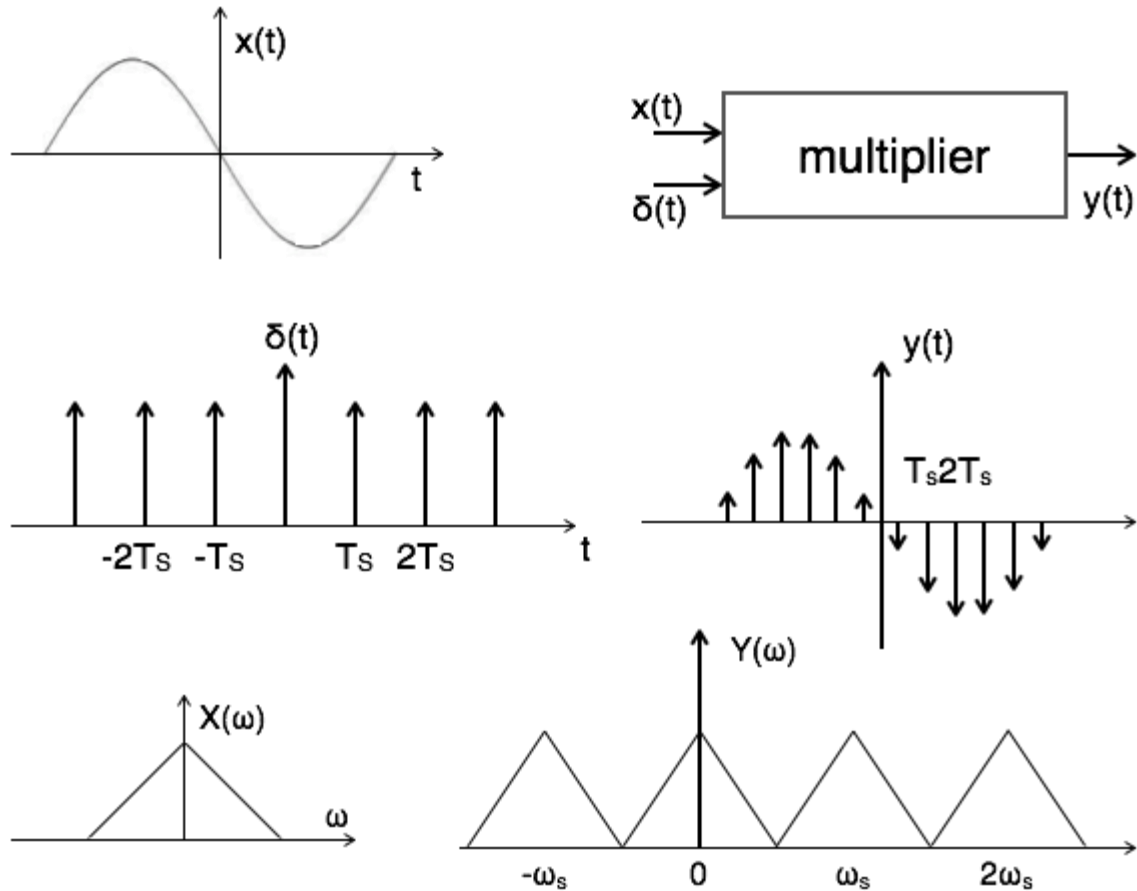
10. State and prove the sampling theorem.

Statement: A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal. i. e.

$$f_s \geq 2f_m.$$

Proof: Consider a continuous time signal $x(t)$. The spectrum of $x(t)$ is a band limited to f_m Hz i.e. the spectrum of $x(t)$ is zero for $|\omega| > \omega_m$.

Sampling of input signal $x(t)$ can be obtained by multiplying $x(t)$ with an impulse train $\delta(t)$ of period T_s . The output of multiplier is a discrete signal called sampled signal which is represented with $y(t)$ in the following diagrams:



Here, you can observe that the sampled signal takes the period of impulse. The process of sampling can be explained by the following mathematical expression:

$$\text{Sampled signal } y(t) = x(t) \cdot \delta(t) \dots (1)$$

The trigonometric Fourier series representation of $\delta(t)$ is given by

$$\delta(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_s t + b_n \sin n\omega_s t) \dots (2)$$

$$\text{Where } a_0 = \frac{1}{T_s} \int_{T/2 - T/2}^{T/2} \delta(t) dt = \frac{1}{T_s} \delta(0) = \frac{1}{T_s}$$

$$a_n = \frac{2}{T_s} \int_{T/2 - T/2}^{T/2} \delta(t) \cos n\omega_s t dt = \frac{2}{T_s} \delta(0) \cos n\omega_s 0 = \frac{2}{T_s}$$

$$b_n = \frac{2}{T_s} \int_{T/2 - T/2}^{T/2} \delta(t) \sin n\omega_s t dt = \frac{2}{T_s} \delta(0) \sin n\omega_s 0 = 0$$

Substitute above values in equation 2.

$$\therefore \delta(t) = \frac{1}{T_s} + \sum_{n=1}^{\infty} \left(\frac{2}{T_s} \cos n\omega_s t + 0 \right)$$

Substitute $\delta(t)$ in equation 1.

$$\rightarrow y(t) = x(t) \cdot \delta(t)$$

$$=x(t)[1T_s+\sum_{n=1}^{\infty}(2T_s\cos n\omega_s t)]$$

$$=1T_s[x(t)+2\sum_{n=1}^{\infty}(\cos n\omega_s t)x(t)]$$

$$y(t)=1T_s[x(t)+2\cos\omega_s t.x(t)+2\cos2\omega_s t.x(t)+2\cos3\omega_s t.x(t).....]$$

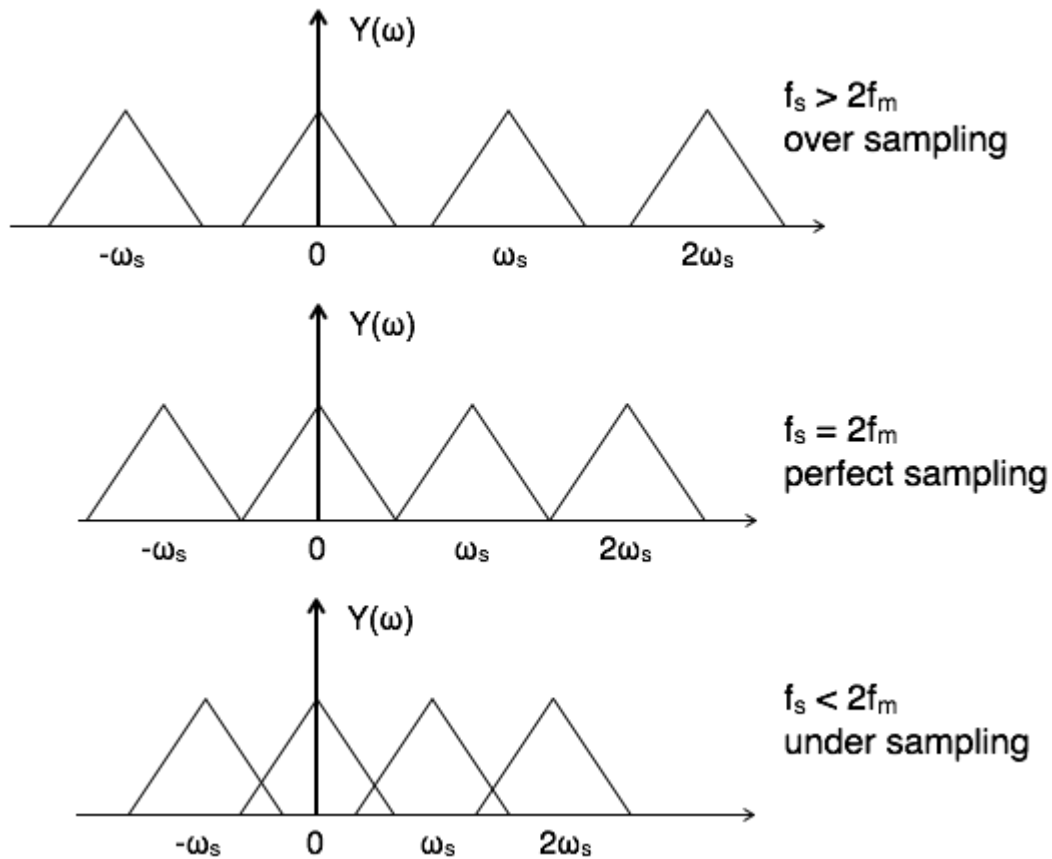
Take Fourier transform on both sides.

$$Y(\omega)=1T_s[X(\omega)+X(\omega-\omega_s)+X(\omega+\omega_s)+X(\omega-2\omega_s)+X(\omega+2\omega_s)+...]$$

$$\therefore Y(\omega)=1T_s\sum_{n=-\infty}^{\infty}X(\omega-n\omega_s)\text{ where }n=0,\pm1,\pm2,...$$

To reconstruct $x(t)$, you must recover input signal spectrum $X(\omega)$ from sampled signal spectrum $Y(\omega)$, which is possible when there is no overlapping between the cycles of $Y(\omega)$.

Possibility of sampled frequency spectrum with different conditions is given by the following diagrams:



Aliasing Effect

The overlapped region in case of under sampling represents aliasing effect, which can be removed by

- considering $f_s > 2f_m$
- By using anti aliasing filters.

11. Write short notes on Fourier spectrum?

The graph plotted between the Fourier coefficients of a periodic function $x(t)$ and the frequency (ω) is known as the **Fourier spectrum** of a periodic signal.

The Fourier spectrum of a periodic function has two parts –

- **Amplitude Spectrum** – The amplitude spectrum of the periodic signal is defined as the plot of amplitude of Fourier coefficients versus frequency.
- **Phase Spectrum** – The plot of the phase of Fourier coefficients versus frequency is called the *phase spectrum* of the signal.

The amplitude spectrum and phase spectrum together are known as **Fourier frequency spectra** of the periodic signal $x(t)$. This type of representation of a periodic signal is known as *frequency domain representation*.

The Fourier frequency spectra exists only at discrete frequencies, i.e., at ω_n , where, $n = 0, 1, 2, 3, \dots$. Therefore, the Fourier frequency spectra is also known as *discrete spectra or line spectra*. The envelope of the Fourier frequency spectra depends only upon shape of the pulse, but not upon the period of repetition.

The trigonometric Fourier series representation of a periodic function $x(t)$ contains both sine and cosine terms with positive and negative amplitude coefficients a_n and b_n but does not have phase angles.

Single-Sided Spectra

The cosine Fourier series representation of a periodic signal $x(t)$ has only positive amplitude coefficients A_n with phase angle θ_n . Hence, we can plot amplitude spectrum (A_n versus ω) and the phase spectrum (θ_n versus ω).

In the cosine representation, the Fourier coefficients exist only for positive frequencies. Therefore, this spectra is called the **single-sided spectra**.

Figure-1 represents the spectrum of a trigonometric (cosine) Fourier series extending from 0 to ∞ , producing a one sided spectrum because no negative frequencies exist in this representation.

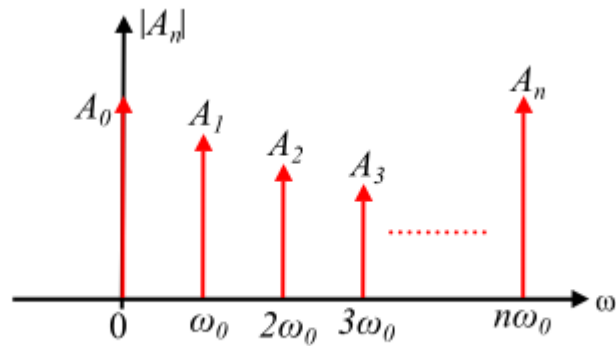


Figure-1

Two-Sided Spectra

The exponential Fourier series representation of a periodic function $x(t)$ has amplitude coefficients C_n which are complex and can be represented by magnitude and phase. Hence, we can plot the amplitude spectrum ($|C_n|$ versus ω) and the phase spectrum ($\angle C_n$ versus ω).

As in the exponential representation, the spectra can be plotted for both positive and negative frequencies. Therefore, this spectra is known as **two-sided spectra**.

Figure-2 represents the spectrum of a complex exponential Fourier series extending from $(-\infty$ to $\infty)$, producing a **two-sided spectrum**.

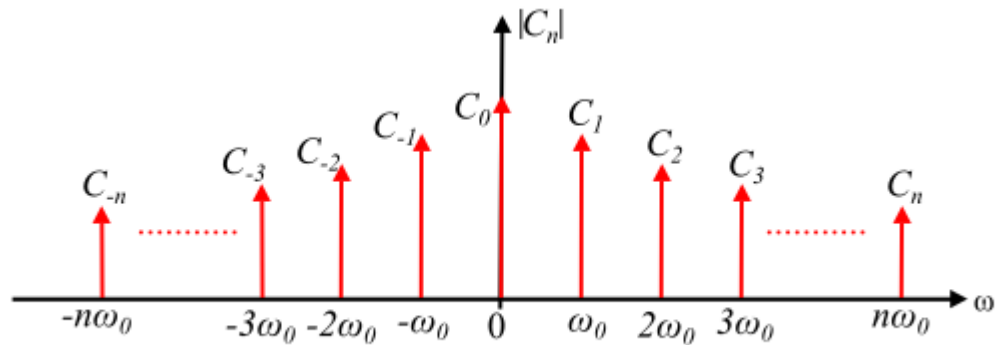


Figure-2

Also, if C_n is complex number, then

$$C_n = |C_n| e^{j\theta_n}$$

$$C_{-n} = |C_n| e^{-j\theta_n}$$

And

$$C_n = |C_{-n}|$$

Therefore, the amplitude spectrum of the exponential Fourier series is symmetrical about the vertical axis passing through the origin, i.e., the magnitude spectrum exhibits even

symmetry and the phase spectrum is antisymmetrical about the vertical axis passing through the origin, i.e., the phase spectrum exhibits odd symmetry.

Also, when the periodic signal $x(t)$ is real, then

$$C_{-n} = C_n^*$$

i.e. C_{-n} is the complex conjugate of the exponential coefficient C_n .

12. Derive the expression Fourier transform

The main drawback of Fourier series is, it is only applicable to periodic signals. There are some naturally produced signals such as nonperiodic or aperiodic, which we cannot represent using Fourier series. To overcome this shortcoming, Fourier developed a mathematical model to transform signals between time (or spatial) domain to frequency domain & vice versa, which is called 'Fourier transform'.

Fourier transform has many applications in physics and engineering such as analysis of LTI systems, RADAR, astronomy, signal processing etc.

Deriving Fourier transform from Fourier series

Consider a periodic signal $f(t)$ with period T . The complex Fourier series representation of $f(t)$ is given as

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \\ = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k T_0 t} \dots (1) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k T_0 t} \dots (1)$$

Let $1/T_0 = \Delta f$, then equation 1 becomes

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k \Delta f t} \dots (2) \quad f(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k \Delta f t} \dots (2)$$

but you know that

$$a_k = \frac{1}{T_0} \int_{t_0}^{t_0+T} f(t) e^{-jk\omega_0 t} dt \quad a_k = \frac{1}{T_0} \int_{t_0}^{t_0+T} f(t) e^{-jk\omega_0 t} dt$$

Substitute in equation 2.

$$(2) \Rightarrow f(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0} \int_{t_0}^{t_0+T} f(t) e^{-jk\omega_0 t} dt e^{j2\pi k \Delta f t} \Rightarrow f(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0} \int_{t_0}^{t_0+T} f(t) e^{-jk\omega_0 t} dt e^{j2\pi k \Delta f t}$$

Let $t_0 = T/2$, $t_0 + T = 3T/2$

$$= \sum_{k=-\infty}^{\infty} \left[\int_{T/2}^{3T/2} f(t) e^{-j2\pi k \Delta f t} dt \right] e^{j2\pi k \Delta f t} \Delta f = \sum_{k=-\infty}^{\infty} \left[\int_{-T/2}^{T/2} f(t) e^{-j2\pi k \Delta f t} dt \right] e^{j2\pi k \Delta f t} \Delta f$$

In the limit as $T \rightarrow \infty, \Delta f T \rightarrow \infty, \Delta f$ approaches differential $df, k \Delta f df, k \Delta f$ becomes a continuous variable f , and summation becomes integration

$$f(t) = \lim_{T \rightarrow \infty} \left\{ \sum_{k=-\infty}^{\infty} \left[\int_{-T/2}^{T/2} f(t) e^{-j2\pi k \Delta f t} dt \right] e^{j2\pi k \Delta f t} \Delta f \right\}$$

$$f(t) = \lim_{T \rightarrow \infty} \left\{ \sum_{k=-\infty}^{\infty} \left[\int_{-T/2}^{T/2} f(t) e^{-j2\pi k \Delta f t} dt \right] e^{j2\pi k \Delta f t} \Delta f \right\}$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(t) e^{-j2\pi f t} dt \right] e^{j2\pi f t} df$$

$$f(t) = \int_{-\infty}^{\infty} F[\omega] e^{j\omega t} d\omega \quad f(t) = \int_{-\infty}^{\infty} F[\omega] e^{j\omega t} d\omega$$

Where $F[\omega] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$ Where $F[\omega] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

Fourier transform of a signal

$$f(t) = F[\omega] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \quad f(t) = F[\omega] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Inverse Fourier Transform is

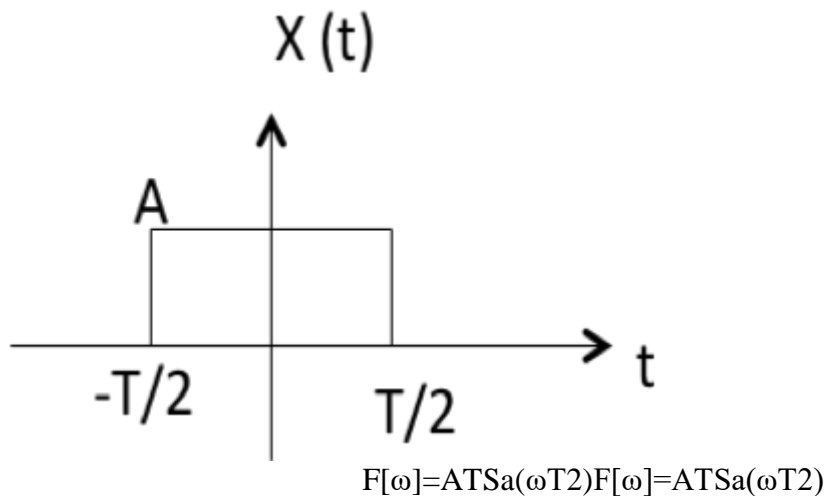
$$f(t) = \int_{-\infty}^{\infty} F[\omega] e^{j\omega t} d\omega \quad f(t) = \int_{-\infty}^{\infty} F[\omega] e^{j\omega t} d\omega$$



Fourier Transform of Basic Functions

Let us go through Fourier Transform of basic functions:

FT of GATE Function



Conditions for Existence of Fourier Transform

Any function $f(t)$ can be represented by using Fourier transform only when the function satisfies Dirichlet's conditions. i.e.

- The function $f(t)$ has finite number of maxima and minima.
- There must be finite number of discontinuities in the signal $f(t)$, in the given interval of time.
- It must be absolutely integrable in the given interval of time i.e.

$$\int_{-\infty}^{\infty} |f(t)| dt < \infty \quad \int_{-\infty}^{\infty} |f(t)| dt < \infty$$

13. Derive the expression inverse Fourier transform.

The Fourier series of a periodic function $g(t)$ is defined as,

$$g(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\therefore C_n = \frac{1}{T} \int_0^T g(t) e^{-jn\omega_0 t} dt \quad [\text{from eq. (5)}]$$

$$\therefore g(t) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{T} \int_0^T g(t) e^{-jn\omega_0 t} dt \right) e^{jn\omega_0 t}$$

$$\therefore n\omega_0 = \omega \text{ and } T = \frac{2\pi}{\omega_0}$$

$$\therefore g(t) = \sum_{n=-\infty}^{\infty} X(\omega) (2\pi/\omega_0) e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} X(\omega) 2\pi e^{jn\omega_0 t} \omega_0 \dots (7)$$

Hence, from equations (3) & (7), we have

$$x(t) = \lim_{T \rightarrow \infty} g(t) = \lim_{T \rightarrow \infty} \sum_{n=-\infty}^{\infty} X(\omega) e^{jn\omega_0 t} \omega_0 \dots (8)$$

As $T \rightarrow \infty$, we have,

$$\omega_0 = \frac{2\pi}{T} \xrightarrow{T \rightarrow \infty} 0$$

Thus, ω_0 can be represented by $d\omega$ and the summation becomes integration. Therefore, eq. (8) can be written as,

$$x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \dots (9)$$

Here, $x(t)$ is known as the **inverse Fourier transform** of $X(\omega)$.

The expressions in equation (6) for $X(\omega)$ and in eq. (9) for $x(t)$ are known as **Fourier transform pair** and can be represented as,

$$X(\omega) = F[x(t)]$$

And

$$x(t) = F^{-1}[X(\omega)]$$

The Fourier transform pair can also be represented as

$$x(t) \xleftrightarrow{FT} X(\omega)$$

- 1
 - a. Discuss the properties of LTI systems.
 - b. Explain the causality of a system.

- 2
 - a. Write notes on filter characteristic of linear systems.
 - b. Explain physical realizability of a system.

- 3
 - a. State the properties of auto correlation functions.
 - b. Write about mean ergodic process.

- 4
 - a. Distinguish between stationary and non stationary random process.
 - b. Explain how random processors are classified with neat sketches.

- 5
 - a. Find the autocorrelation of random process whose PSD is $16/(4+\omega^2)$.
 - b. Derive the expression for mean value of output response of a LTI system.

- 6
 - a. Prove that $S_{XY}(\omega) = 0$ and $S_{YX}(\omega) = 0$, If $X(t)$ and $Y(t)$ are orthogonal.
 - b. Derive the expression for mean square value of output response of a LTI system.

- 1
 - a. Find the response $y(t)$ of an LTI system with impulse response $h(t)=u(t-1)$ and input $x(t)= e^{-2t}u(t)$.
 - b. What are the conditions for a system to be LTI system?

- 2
 - a. The system function of a causal LTI system is $H(s) = \frac{s+1}{s^2+2s+2}$ determine the response $y(t)$ when input $x(t) = e^{-|t|}$
 - b. Explain physical realizability of a system.

- 3

Explain about Correlation function (auto & cross correlation).

- 4

A random process is given as $X(t) = A \cos(\omega_0 t + \Theta)$ is a wide sense stationary random process, where ω_0 is constant and Θ is a uniformly distributed random variable on the interval $(0, 2\pi)$. Find a) $E[X(t)]$ b) $R_{XX}(\tau)$

- 5

State and prove properties of the power spectral density of random process.

- 6

Derive the expression for response of LTI systems to random signals.

 - a) Autocorrelation b) Cross correlation.

- 1
 - a. The output of $y(t)$ of a stable LTI system characterised by the equation $\frac{dy(t)}{dt} + 2y(t) = x(t)$ Find its impulse response.

 - b. Derive the expression for step response of a LTI system.

- 2
 - a. What is ideal filter? explain.
 - b. Explain stability criteria of a linear system.

- 3
 - a. Explain the statistical properties of random processes.
 - b. If a random process, $X(t) = A\cos\omega t + B\sin\omega t$ is given, where A and B are uncorrelated, zero mean random variables having the variance σ^2 . Find autocorrelation.

- 4
 - a. A random process is given as $X(t) = At$, where A is an uniformly distributed random variable on (0, 2). Find whether $X(t)$ is WSS or not.
 - b. Explain about Auto covariance function.

- 5
 - a. If $R_{xx}(\tau) = a \cdot e^{-b|\tau|}$. Show that power spectrum density is $\frac{2ab}{b^2 + \omega^2}$.
 - b. If $S_{xx}(\omega)$ PSD of $X(t)$ prove that PSD of derivative of $X(t)$ is $\omega^2 S_{xx}(\omega)$.

- 6

Derive the expression for mean and PSD of output response of a LTI system.
- 1

With neat sketches explain the characteristics of ideal LPF, HPF, BPF and BRF.

- 2

Derive the relation between bandwidth and rise time.

- 3

Explain about joint distribution and density functions of random processes.

- 4

A random process is given as $X(t) = A \sin(\omega_0 t + \Theta)$ is a wide sense stationary random process, where ω_0 is constant and Θ is a uniformly distributed random variable on the interval (0, 2π). Find a) $E[X(t)]$ b) $R_{XX}(\tau)$ c) average power d) mean square value of $X(t)$.

- 5 State and prove Wiener – Khintchine relations.
- 6 Derive the expression for Cross Power Spectral Density.

Signals, Systems & Stochastic Process

SHORT ANSWER QUESTIONS [2MARKS]

UNIT –I

1. Define signal & system.
2. What are the major classifications of the signal?
3. Define discrete time signal.
4. Define continuous time signal.
6. Define discrete time unit step and unit impulse (Dirac Delta).
7. Define continuous time unit step and unit impulse (Dirac Delta).
8. Define unit ramp signal.
9. Define periodic signal and non-periodic signal.
10. Define even and odd signal.
11. Define energy and power signal.
12. Define unit pulse function.
13. Define continuous time complex exponential signal.
14. What is continuous time real exponential signal?
17. What are the types of Fourier series? Write down the exponential form of the Fourier series representation of a periodic signal?
18. Write down the trigonometric form of the Fourier series representation of a Periodic signal?
19. Write short notes on dirichlets conditions for Fourier series.
20. State Time Shifting property in relation to Fourier series.
21. State Parseval's theorem for continuous time periodic signals.
22. Explain in detail discrete time signal and continuous time signal.
23. Explain in detail complex exponential CT signal.
24. Find the energy of the signal $x[n] = (1/2)^n u[n]$
25. Find the odd and even components of the signal: $\cos t + \sin t + \cos t \sin t$.
26. Find odd and even components of $x[n] = \{1, 2, 2, 3, 4\}$.

27. Find the energy of the signal $e^{-2t} u(t)$.
28. Test whether the signal $y(t) = ax(t) + b$ is linear or nonlinear.
29. Find power and rms value of the signal: $x(t) = 20\cos 2\pi t$
30. Explain the following signals i. Periodic and aperiodic ii. Even and odd.
31. What is the relation between impulse, step, ramp and parabolic signals?
32. Distinguish causal and anti-causal signals.
33. Define Linear and Non-Linear System.
34. Define time-variant and time-invariant systems
35. How are systems classified?
36. Define stable and unstable systems
37. Define causal and non-causal systems.
38. What is the relation between convolution and correlation?
39. What is correlation and types of correlation?

UNIT-II

1. What are the Dirichlets conditions of Fourier Transform? State them.
2. Differentiate the Fourier series and Fourier transform.
3. What is Fourier transform?
4. Define Linearity Property of Fourier Transform
5. What are the Merits of Fourier Transform?
6. Define Fourier transform and Inverse Fourier transform of discrete time signal.
7. Define sampling and sampling period?
8. State Sampling theorem
9. What is Nyquist rate and Nyquist interval?
10. What is anti-aliasing filter?
12. What is the Region of Convergence (ROC)?
13. What is the relation between Laplace transform and Fourier transform?
14. State initial value theorem and final value theorem of Laplace transform.

15. What are the properties of ROC?

UNIT-III

1. What are the properties of LTI systems?
2. Define transfer function of a system?
3. Define impulse response of a system.
4. What is a filter? How are filters classified?
5. What is the Relation between unit step and impulse response?
6. State the polywiener criterion for physical realization.
7. Differentiate ESD and PSD?
8. State Parseval's energy theorem?
9. State Parseval's power theorem?

UNIT-IV

1. Define wide sense stationary process?
2. State autocorrelation properties.
3. What are mean ergodic processes?
4. Define covariance of two random variables.
5. Define cross correlation function of two variables.
6. Explain about strict-sense stationery processes.
7. Where the Poisson random processes are used? Explain.
8. Give any two examples for poison random process.
9. State any two differences between random variable and random process.

UNIT-V

1. What is power spectrum density?
2. Define cross power density spectrum.
3. State Wiener – Khintchine theorem.
4. Give any two spectral characteristics of the system response.

