A40410-Analog and Digital Communications

UNIT I Continuous Wave Modulation

Introduction: The communication Process, Communication Channels, Baseband and Passband Signals, Analog vs Digital Communications, Need for the modulation.

Amplitude Modulation (AM): AM and its modifications – DSB, SSB, VSB. Frequency Translation, Frequency Division Multiplexing (FDM).

Angle Modulation: Frequency Modulation (FM), Phase Modulation, PLL, Nonlinear Effects in FM, Super heterodyne Receivers.

Communication Process

Communication is process of transferring information from one place, person, or group to another. The fundamental components of the communication process include:

1. **Sender:** The entity that initiates the communication by encoding a message.
2. **Message:** The actual information being conveyed.
3. **Encoding:** The process of converting the message into a form suitable for transmission.
4. **Channel:** The medium used to transmit the message.
5. **Receiver:** The entity that receives and decodes the message.
6. **Decoding:** The process of interpreting the encoded message.
7. **Feedback:** The receiver's response that ensures the sender understands the message.

Communication Channels

Communication channels are the mediums through which a message is transmitted. Common types include:

1. **Wired Channels:** Use physical media such as cables (e.g., coaxial, twisted pair, fiber optics).
   - *Advantages:* High reliability, high speed, and security.
   - *Disadvantages:* Limited mobility and installation cost.
2. **Wireless Channels:** Use electromagnetic waves (e.g., radio waves, microwaves, infrared).
   - *Advantages:* Greater mobility and flexibility.
   - *Disadvantages:* Susceptible to interference and eavesdropping.

Examples of Communication Channels:

- Telecommunication (telephones, mobile networks)
- Radio and television broadcasting
- Internet and satellite communication

Baseband and Passband Signals

1. **Baseband Signals:**
   - These are signals that occupy the frequency spectrum close to zero.
   - Examples: Original signals such as voice, audio, and digital data.
   - Applications: Local area networks (LANs), Ethernet, and audio communication.
1. **Passband Signals:**
   - These signals are modulated to occupy a higher frequency range.
   - Used for long-distance communication to overcome attenuation and interference.
   - Applications

Analog vs. Digital Communications

Feature	**Analog Communication**	**Digital Communication**
Signal Type	Continuous	Discrete
Noise Susceptibility	High	Low
Bandwidth Usage	High	Efficient
Error Detection	Difficult	Easy

Examples	AM, FM, TV broadcasting	Internet, Digital TV, VoIP

Advantages of Digital Communication:

1. High noise immunity.
2. Easier encryption and security.
3. Flexibility and compatibility with modern devices.

Disadvantages of Digital Communication:

1. Requires higher bandwidth.
2. More complex circuitry.

Need for Modulation

Modulation is the process of varying a carrier signal to encode information.

Why Modulation is Necessary:

1. **Long-Distance Communication:** Baseband signals are low-frequency and cannot travel long distances.
2. **Multiplexing:** Enables multiple signals to share the same channel.
3. **Improved Signal Quality:** Reduces the impact of noise and interference.
4. **Antenna Size:** Higher frequency signals require smaller antennas.

Types of Modulation:

1. **Analog Modulation:** AM (Amplitude Modulation), FM (Frequency Modulation), PM (Phase Modulation).
2. **Digital Modulation:** ASK (Amplitude Shift Keying), FSK (Frequency Shift Keying), PSK (Phase Shift Keying).

Modulation Example:

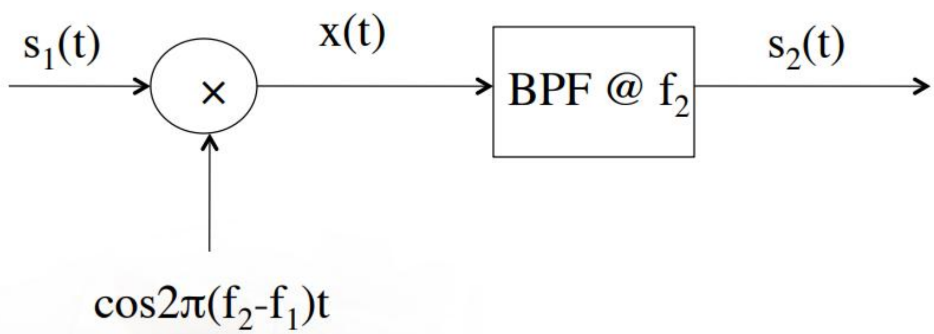
A carrier wave (high-frequency sinusoidal signal) is modulated with an audio signal:

- **Unmodulated Carrier Wave:**
- **Amplitude Modulated Signal:**

Frequency Translation

- Suppose we have a modulated wave $s_1(t)$ whose spectrum is centered around frequency f_1 and we wish to move it upward in frequency, so that its spectrum is centered around f_2.
- This can be accomplished by multiplying $s_1(t)$ by $\cos 2\pi(f_2-f_1)t$ and passing it through a BPF.

Frequency Translation



$$X(f) = 0.5S_1(f-f_2+f_1)+0.5S_1(f+f_2-f_1)$$

Downward Frequency Translation (Downconversion)

- We can also decrease the frequency of a modulated signal by multiplying by $\cos 2\pi(f_2-f_1)t$ and then filtering out the higher frequency (sum) component, and using the lower frequency (difference) component.

ADC

Unit - I

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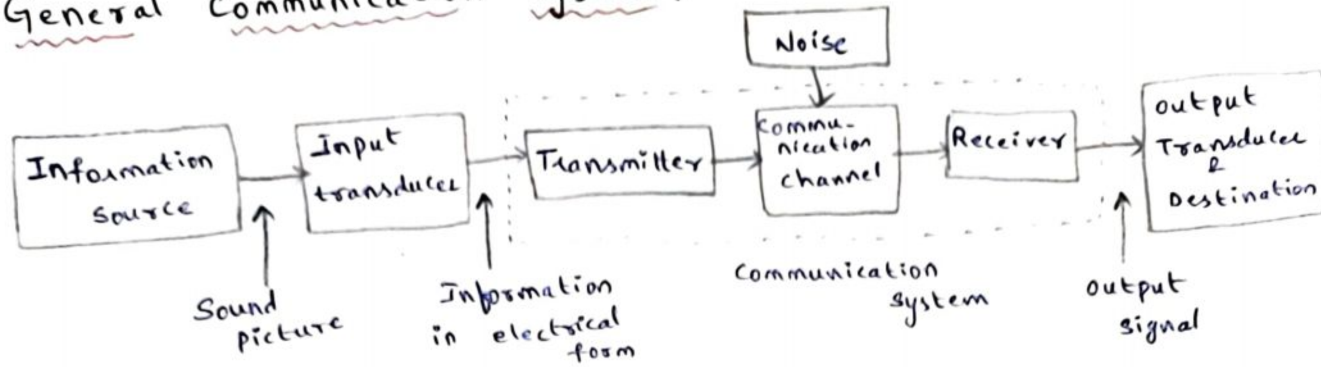
General Communication System :-



Information source :- this produces required message which has to be transmitted.

Input transducer :- A transducer is a device which converts sound signal (energy) to electrical signals (energy)

ex:- Microphone

Transmitter :- The main purpose of transmitter is to modify the message signal into a suitable for transmission over the channel. This modification is achieved by 'Modulation', which involves varying some parameter of a 'carrier wave (e.g:- Amplitude, frequency, phase of a sinusoidal wave) in accordance with the message signal.

Communication channel :- In communication channel may be a transmission line as in telephony and telegrapy, an optical fiber as in optical communications, or free space in which the signal is radiated as an electromagnetic wave as in radio and television broad casting.

In propagating through the channel, the transmitted signal is distorted due to nonlinearties or imperfections in the frequency response of the channel. During the process of transmission and reception the signal gets distorted due to noise introduced in the system.

Receiver:- The main function of the receiver is to reproduce the message signal in electrical form, from the distorted received signal. This reproduction of the original signal is accomplished by a process known as demodulation or detection, which is the reverse of the modulation process used in the transmitter

Destination:- This acts like a transducer which converts electrical signal into the sound signals.

Need for Modulation:

The purpose of communication system is to transmit information bearing signals as baseband signals through a communication channel separating the transmitter from the receiver.

The term "Base band" is used to designate the band of frequencies representing the original signal as delivered by a source of information.

The efficient utilization of communication channel requires a shift of the range of baseband frequencies into other frequency ranges suitable for transmission, this is possible by using 'Modulation' which is defined as "the process by which some characteristic of a carrier is varied in accordance with a modulating wave", and the resultant wave is converted to a modulated wave.

Modulation serves several purposes in communication systems.

1) Practicality of Antenna :-

when free space is used as communication media, messages are transmitted and received with the help of antennas.

The message signal is radiated by an antenna at the transmitter. the antenna radiates effectively when its size is of the order of the wavelength (meter) of the signal being transmitted.

In broadcast systems, the maximum audio frequency transmitted froms a radio station is of the order of 5KHZ, If the signal is transmitted without modulation the size of antenna will be

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^3} = 60KM \qquad c = \text{speed of light}$$

which is impracticable to constuct and instal such as antenna.

The size of antenna can be reduced by Analog modulation technique, which provides frequency shifting or frequency translation to much higher frequency spectrum.

Then the size of antenna is reduced and becomes practical to install.

for example :- An audio frequency is translated to a radio frequency carrier of frequency 10MHZ, the antenna height required will be

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^6} = 30\,\text{Meter}.$$

2) To remove Interference :-

In radio - broadcasting, there are several radio stations and all will transmit the audio signal with 5KHZ frequency. In case there is no modulation, the programmes of different stations will get mixed up.

To keep various signals separate, it is necessary to modulate the signals, and each station is allocated a band of frequency broad cast channels are placed adjacent to each other, each channel occupying 10 KHz band width. Hence, different stations are allotted bandwidths from 790 to 800 KHz, 800 to 810 KHz. In radio receiver, a tunned circuit at the input selects the desired station and rejects all other stations.

3) Reduction of Noise :

Noise is major limitation of any communication. with the help of several modulation schemes, the effect of noise can be minimized.

4) Narrow banding :-

Assume baseband signal in a broadcast system is radiated directly with the frequency range extending from 50 Hz to 10 KHz The ratio of highest to lowest wave length is 200

$$\lambda_H = \frac{3\times10^8}{50} = 6\times10^6 \qquad \frac{\lambda_H}{\lambda_L} = \frac{6\times10^6}{3\times10^4} = \frac{200}{1}$$

$$\lambda_L = \frac{3\times10^8}{10K} = 3\times10^4$$

∴ An Antenna suitable for use at one end of the range (10 KHz) would be entirely too short or too long for the other end (50 Hz).

→ Let audio frequency range is translated to 1 MHz ie, it has a range of (10^6+50) to (10^6+10^4) Hz. Then the ratio of highest to lowest wavelength is 1.01 which is approximately unity

$$\lambda_H = \frac{3 \times 10^8}{10^6 + 50} = 299.985 \text{ Hz}$$

$$\frac{\lambda_H}{\lambda_L} = \frac{1.01}{1}$$

$$\lambda_L = \frac{3 \times 10^8}{10^6 + 10^4} = 297.0297 \text{ Hz}$$

Some Antenna will be suitable for entire band extending from $(10^6 + 50)$ to $(10^6 + 10^4)$ Hz. Thus frequency translation converts a wideband signal to a narrow band. This is called "Narrow banding".

5) Multiplexing :- Simultaneous transmission of multiple messages over a channel is known as multiplexing. Here channel is pair of wires or free space. If the signals are transmitted without any modulation and if these signals are transmitted over a single channel then the signals will interfere with one another. So, multiplexing helps in transmitting a number of messages simultaneously over a single channel and therefore the number of channels needed will be less. This reduces the cost of installation and maintenance of more channels.

Amplitude Modulation :- (AM)

Defination: Amplitude modulation (AM) is defined as a process in which the amplitude of the carrier wave $c(t)$ is varied linearly with the baseband signal $m(t)$.

Let $m(t)$ be the base band signal which carries the message

Consider a sinusoidal carrier wave $c(t)$ defined by

$$c(t) = A_c \cos \omega_c t \qquad \omega = 2\pi f_c$$

$$= A_c \cos 2\pi f_c t$$

A_c - carrier amplitude

f_c - carrier frequency

AM wave is described as

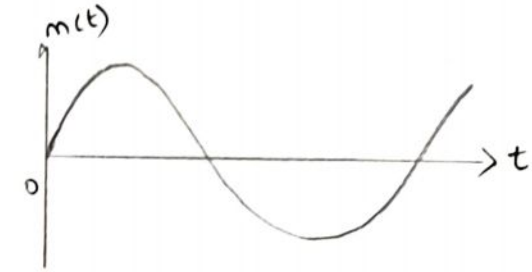
$$s(t) = A_c\,[1 + K_a\,m(t)]\cos\omega_c t$$

$$s(t) = A_c\,[1 + K_a\,m(t)]\cos 2\pi f_c t$$

$K_a \rightarrow$ is constant called "Amplitude sensitivity" of modulator

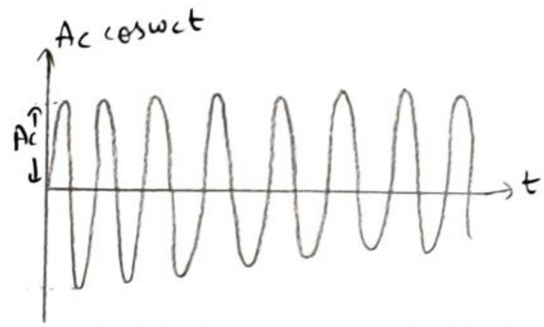
1) Amplitude of $K_a m(t)$ is less than unity i.e,

$|K_a m(t)| < 1$, for all 't'. (under modulation)



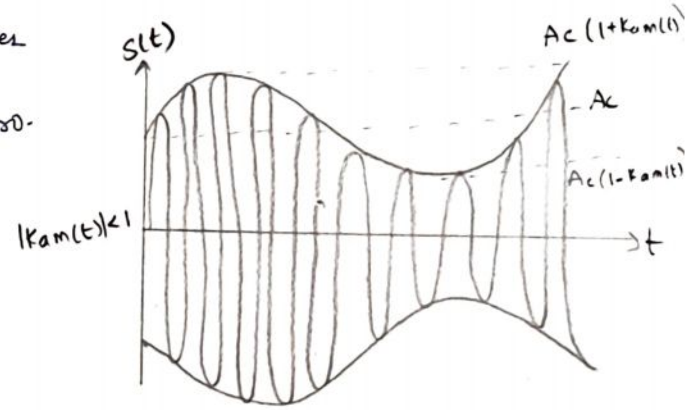
function $1 + K_a m(t)$ is positive

$\therefore s(t) = A_c\,[1 + K_a\,m(t)]$

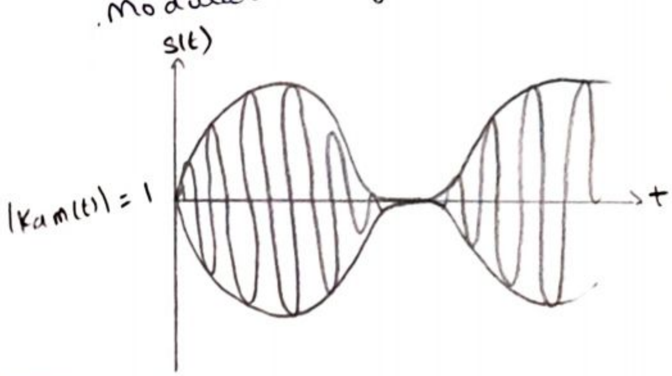


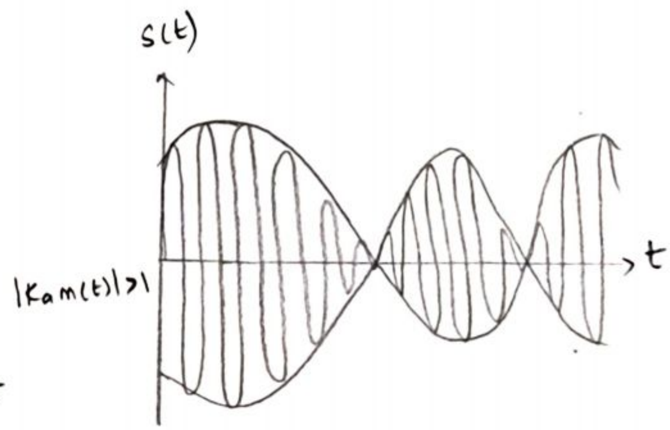
2) $|K_a m(t)| > 1$ for any 't'

the carrier wave becomes 'over modulated', resulting in carrier phase reversals whenever the factor $1 + K_a m(t)$ crosses zero. The modulated wave exhibits 'Envelope distortion'.



3) $|K_a m(t)| = 1$, then the carrier wave becomes perfect modulated signal.





The absolute Maximum value of $K_a m(t)$ multiplied by 100 is referred to as the 'percentage modulation'.

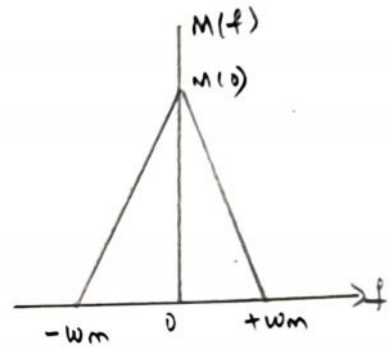
3)

$$s(t) = A_c\,[1 + K_a m(t)]\cos\omega_c t$$

$$S(f) = \frac{A_c}{2}\left(\delta(f-f_c) + \delta(f+f_c)\right) + \frac{K_a A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

The carrier frequency 'f_c' is much greater than the highest frequency component ω_m of message signal $m(t)$.

$\omega_c >> \omega_m$ ω_m – Message bandwidth.

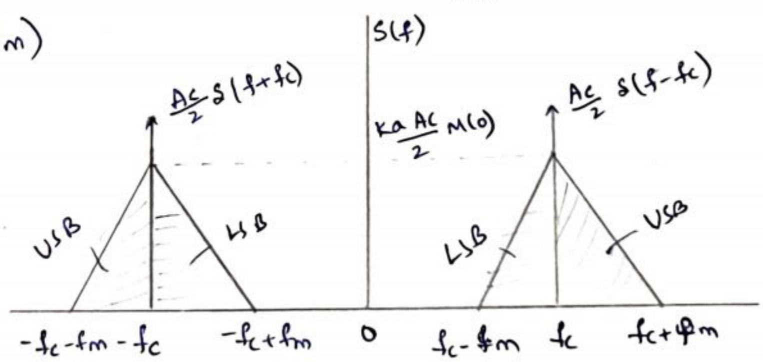


4) The difference between highest frequency component $f_c + f_m$ and lowest frequency component $f_c - f_m$ is "Transmission bandwidth B_T".

$$B_T = f_c + f_m - (f_c - f_m)$$

$$\boxed{B_T = 2f_m}$$

B_T which is twice of message bandwidth



Spectrum of AM wave.

Single-Tone Modulation :-

Consider a modulating signal $m(t)$ that consists of a single tone or frequency component i.e., $m(t) = A_m \cos\omega_m t = A_m \cos 2\pi f_m t$

AM wave is $s(t) = A_c\,[1 + K_a m(t)]\cos 2\pi f_c t$

$$= A_c\,(1 + K_a A_m \cos 2\pi f_m t)\cos 2\pi f_c t$$

$$= A_c\,[1 + \mu \cos 2\pi f_m t]\cos 2\pi f_c t$$

where $\mu = k_a A_m$ called "Modulation factor" (or) Modulation index (or) Depth of modulation (or) degree of modulation which is dimension less.

Modulation index is defined as the measure of extent of amplitude variation about an un modulated maximum carrier.

To avoid envelope distortion due to over modulation, the modulation factor 'μ' must be kept below unity ($\mu < 1$).

Let A_{max} and A_{min} represent the maximum and minimum values of the envelope of the modulated wave, then

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)} = \frac{A_c(1+M)}{A_c(1-M)}$$

$$\therefore \mu \text{ (or) } M = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \text{ (or) } \frac{Max = A_M}{Min : A_c} = \frac{A_M}{A_c}$$

$$\cos a \cos b = \frac{\cos(a+b) + \cos(a-b)}{2}$$

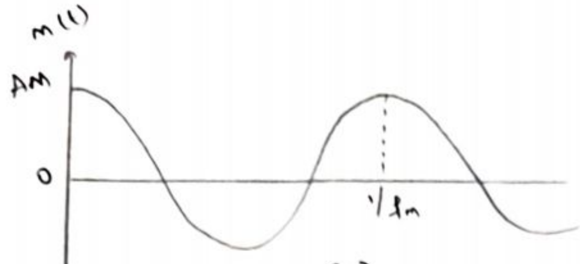
$$\therefore s(t) = A_c(1 + \mu \cos 2\pi f_m t) \cos 2\pi f_c t$$

$$s(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\left(\cos(2\pi(f_c + f_m)t\right] + \frac{\mu A_c}{2} \cos\left(2\pi(f_c - f_m)t\right]$$

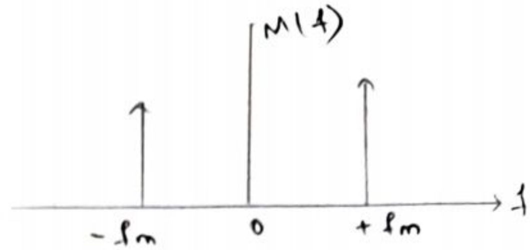
$$S(f) = \frac{A_c}{2}\left(\delta(f - f_c) + \delta(f + f_c)\right] + \frac{\mu A_c}{4}\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right]$$

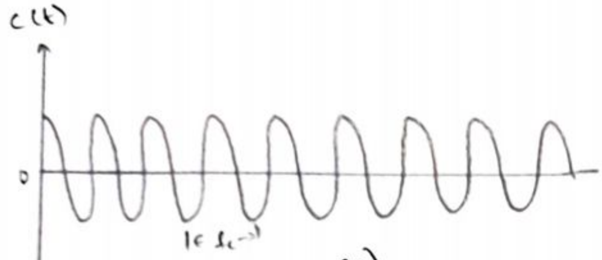
$$+ \frac{\mu A_c}{4}\left(\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right]$$

Time domain and frequency domain description :-



(a)





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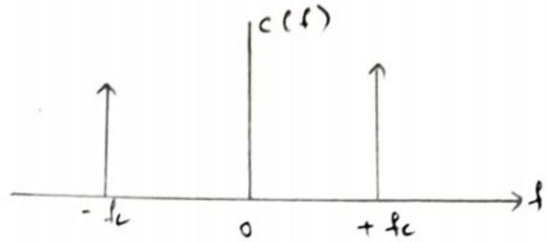
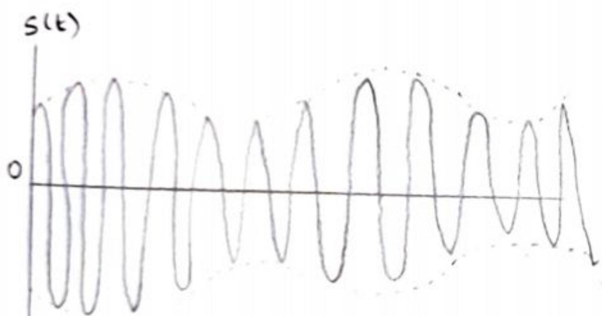
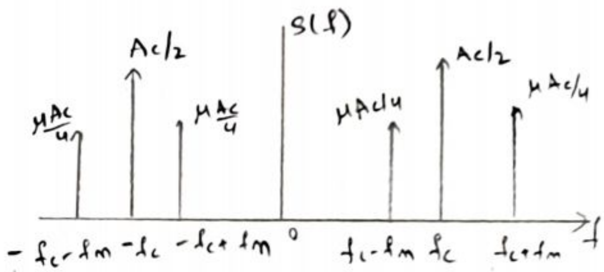
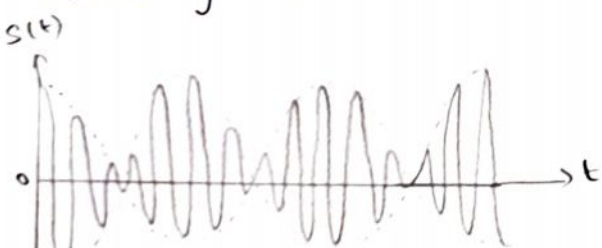




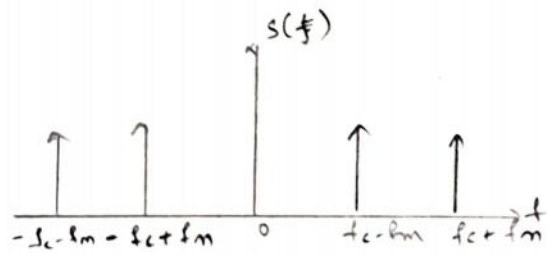
(c)

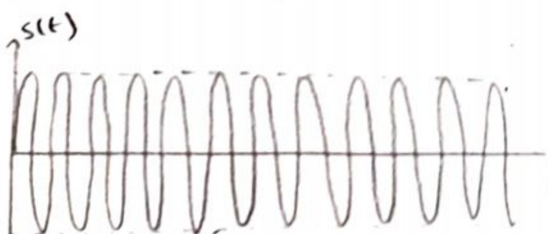


(a) modulating signal b) carrier signal c) AM wave

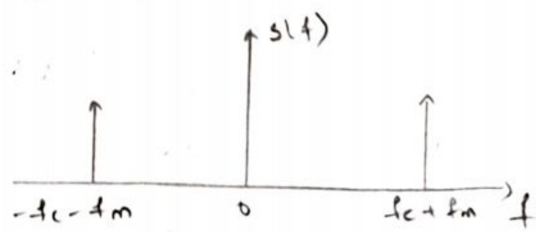


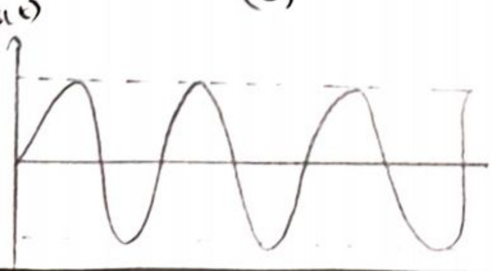
(d)





(e)





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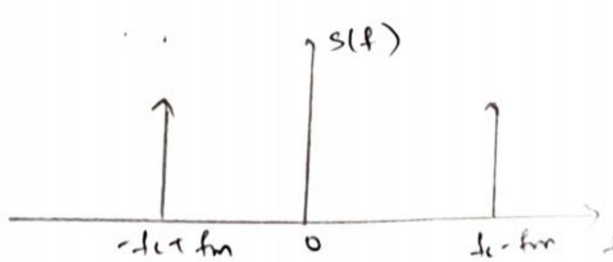
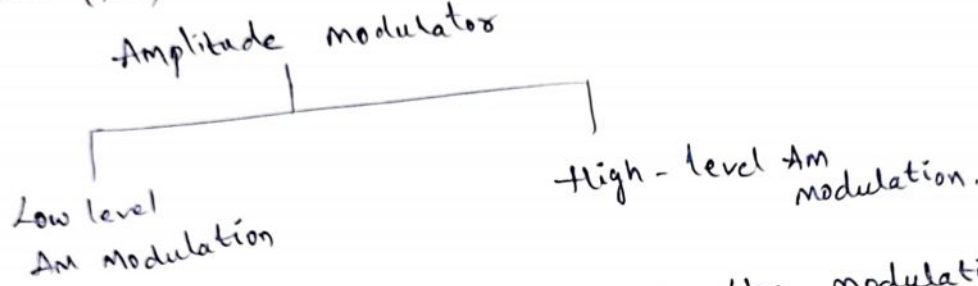


d) DSBSC wave e) SSB wave with upper side frequency transmitted

f) SSB wave with lower-side frequency transmitted

Generation of AM waves :-

The device which is used to generate an amplitude Modulated (AM) wave is Known as Amplitude modulator.



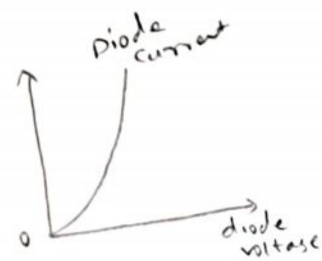
In low level AM modulation system, the modulation is due to a low power level i.e., Carrier and modulating signal are associated with very small prower.

Ex:- Square-Law Modulator & switching modulator

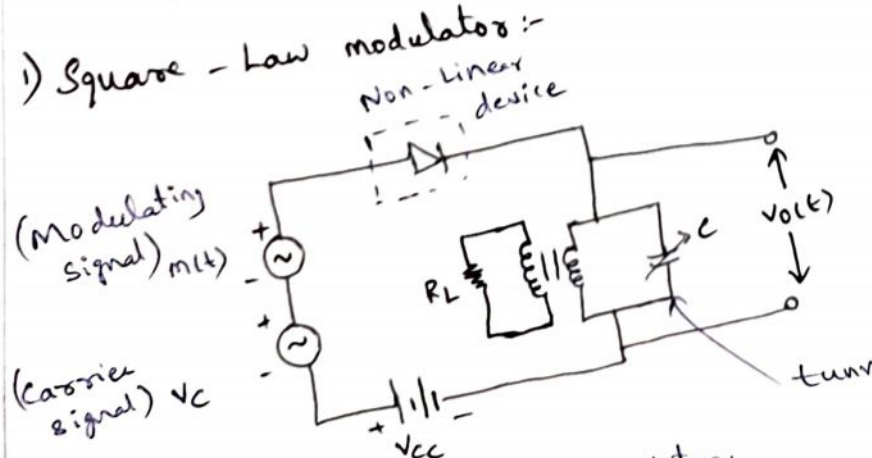
In high-level AM Modulation system, the modulation is done at high power level. i.e., carrier and modulating signal are associated with very high power.

Ex:- Collector modulation Method.

1) Square - Law modulator :-



V-I characteristic of diode



Square-Law modulator.

Square - Law diode modulation circuit uses non-linear current voltage characteristics of diode. this method is suited at low voltage levels because of the fact that current-voltage characteristic of a diode is highly non linear particularly in the low voltage region as shown in figure.

Square - law modulator requires 3 features:

(i) summing carrier and modulating waves

(ii) A non-linear element

(iii) A Band pass filter for extracting the desired modulation products, which is tunned to center frequency, fc

→ Semiconductor diodes and transistors are the most common non-linear devices used for implementing square-Law modulations, the filtering requirement is usually satisfied by using a single or double-tuned filter.

A D.C battery Vcc is connected across diode to get a fixed Operating point on the V-I characteristics of diode. i.e., when the signal applied to the diode is relatively weak, we find that the transfer characteristic of the diode-load resistor combination can be represented closely by a "square-law".

$$v_0(t) = a_1 v_1(t) + a_2 v_1^2(t) \quad ; \; a_1 \,\&\, a_2 \text{ are constants}$$

$v_1(t)$ is input valtage $\quad v_1(t) = v_c(t) + m(t)$

$$= A_c \cos w_c t + m(t)$$

$$= A_c \cos w_c t + A_m \cos w_m t.$$

$$v_0(t) = a_1 [A_c \cos w_c t + A_m \cos w_m t] + a_2 [A_c \cos w_c t + A_m \cos w_m t]^2$$

$$= a_1 A_c \cos w_c t + a_1 A_m \cos w_m t + a_2 \left(A_c^2 \cos^2 w_c t + A_m^2 \cos^2 w_m t + 2 A_c A_m \cos w_c t \cos w_m t \right]$$

$= a_1 A_c \cos w_c t + a_1 A_m \cos w_m t + a_2 A_c^2 \cos^2 w_c t + a_2 A_m^2 \cos^2 w_m t$

$+ 2 a_2 A_c A_m \cos w_c t \cos w_m t.$

$$V_0(t) = a_1 A_c \cos w_c t \left(1 + 2\frac{a_2}{a_1} A_m \cos w_m t\right) + a_1 A_m \cos w_m t + a_2 A_c^2 \cos^2 w_c t + a_2 A_m^2 \cos^2 w_m t.$$

$$= a_1 A_c \left[1 + k_a A_m \cos w_m t\right) \cos w_c t + a_1 A_m \cos w_m t + a_2 A_c^2 \cos^2 w_c t + a_2 A_m^2 \cos^2 w_m t.$$

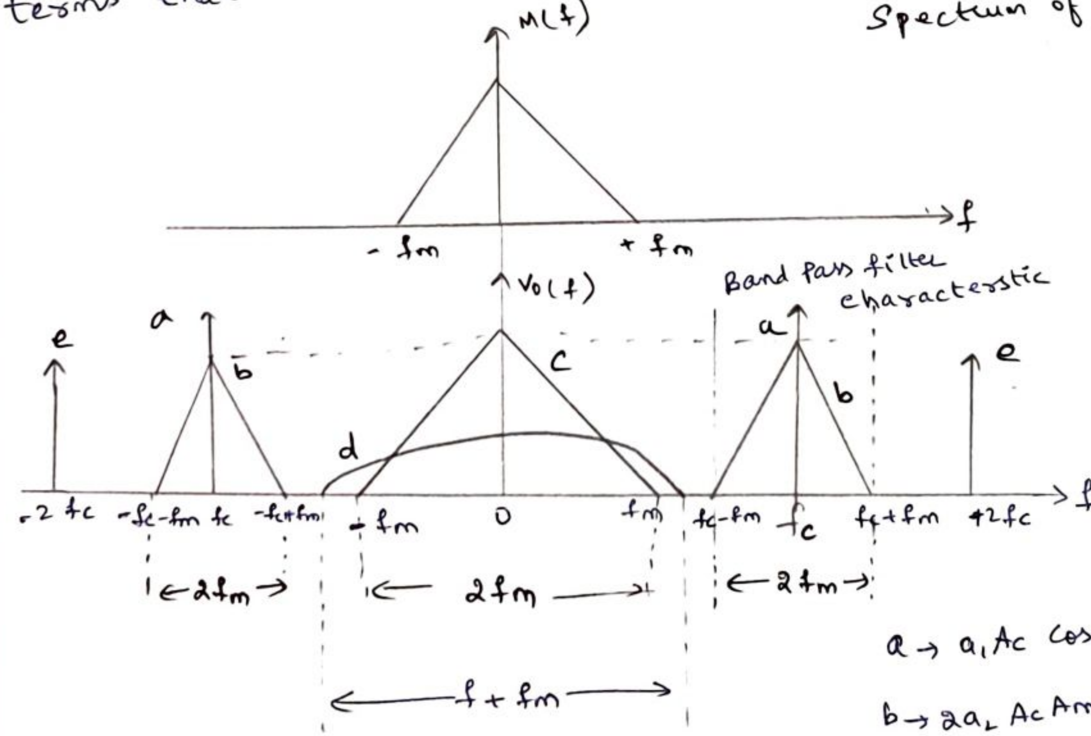
$\boxed{k_a = \frac{2a_2}{a_1}}$ = Amplitude sensitivity

$$\therefore v_0(t) = \underbrace{a_1 A_c \left(1 + M \cos w_c t\right] \cos w_c t}_{1} + a_1 A_m \cos w_m t + a_2 A_c^2 \cos^2 w_c t + a_2 A_m^2 \cos^2 w_m t.$$

$$v_0(t) = a_1 A_c \cos w_c t + a_1 \frac{A_c M}{2} \left(\cos(w_m + w_c)t + \cos(w_m - w_c)t\right) + \text{------}$$

first term $a_1 A_c (1 + m \cos w_m t) \cos w_c t$ is the desired AM wave with Modulation index 'M'. Remaining three terms are unwanted terms that are removed by filtering

Spectum of base band signal



Spectum of vo(t).

$a \rightarrow a_1 A_c \cos 2\pi f_c t$

$b \rightarrow 2a_2 A_c A_m \cos 2\pi f_c t + \cos 2\pi f_m t$

$c \rightarrow a_1 A_m \cos 2\pi f_m t$

$d \rightarrow a_2 A_m^2 \cos^2 w_m t$

$e \rightarrow a_2 A_c^2 \cos^2 w_c t$

o/p waveform will be : which is tuned to f_c.

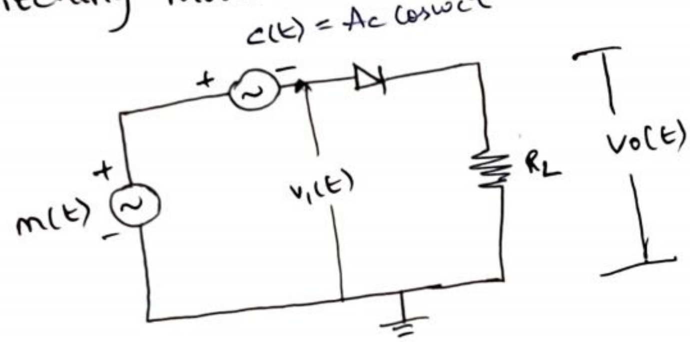
$$V_o(t) = a_1 A_c \cos\omega_c t + \frac{a_1 A_c M}{2}\cos(\omega_c+\omega_m)t + \frac{a_1 A_c M}{2}\cos(\omega_c-\omega_m)t$$

(cos a b formula.)

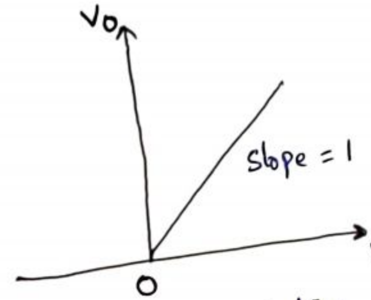
$$= a_1 A_c \cos\omega_c t + a_1 A_c m \cos\omega_c t \cdot \cos\omega_m t$$

$$\boxed{V_o(t) = a_1 A_c (1+m\cos\omega_m t)\cos\omega_c t}$$

(ii) Switching Modulator :-



Circuit diagram



input output relation.

Here, it is assumed that the carrier wave c(t) applied to the diode is large in Amplitude, so, that it seeing right across the characteristic curve of the diode.

The diode acts as ideal switch

$c(t) > 0$ forward biased

$c(t) < 0$ reserved biased

The transfer characteristic of the diode - load resistor consideration can be approximated by a piece wise - linear characteristic as shown in fig

for an input voltage $V_1(t)$; $V_1(t) = A_c\cos\omega_c t + m(t)$

where $|m(t)| << A$:- the resulting load voltage $V_o(t)$ is

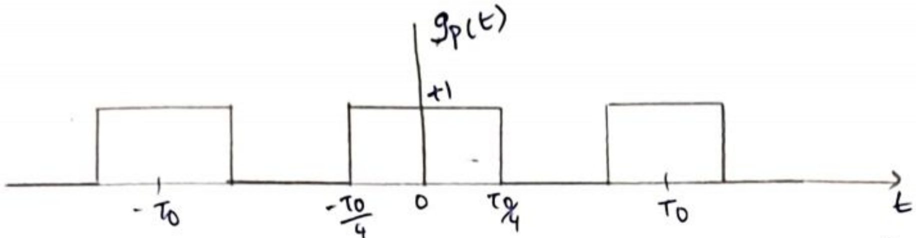
$$V_o(t) = \begin{cases} V_1(t) ; & c(t) > 0 \\ 0 ; & c(t) < 0 \end{cases}$$

i.e., the load voltage $v_o(t)$ varies periodically between the values $v_1(t)$ & zero at a rate equal to the carrier frequency f_c

So, Mathematically $v_o(t)$ can be expressed as

$$v_o(t) = (A_c \cos w_c t + m(t))\, g_p(t) \longrightarrow ①$$

where $g_p(t)$ is a periodic pulse train of duty cycle equal to one-half, and period $T_0 = 1/f_c$ shown in fig



periodic pulse train

Representing this $g_p(t)$ by its fourier series, we have

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left(2\pi f_c t\,(2n-1)\right)$$

Substituting this equation in ① we have find that the load voltage $v_o(t)$ consists of the sum of two components:

$$v_o(t) = \left[A_c \cos 2\pi f_c t + m(t)\right]\left(\frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}\cos\left(2\pi f_c t\,(2n-1)\right)\right)$$

$$v_o(t) = \frac{A_c}{2}\cos 2\pi f_c t + \frac{m(t)}{2} + \frac{2A_c}{\pi}\cos 2\pi f_c t \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}\cos\left(2\pi f_c t\,(2n-1)\right)$$

$$+ \frac{2m(t)}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left(2\pi f_c t\,(2n-1)\right)$$

for $n = 1$

$$v_o(t) = \frac{A_c}{2}\cos 2\pi f_c t + \frac{2m(t)}{\pi}\cos 2\pi f_c t + \frac{2A_c}{\pi}\cos^2 2\pi f_c t.$$

$$v_o(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right]\cos 2\pi f_c t + \frac{2A_c}{\pi}\cos^2 2\pi f_c t +$$

$(n=2)$

$$- \frac{2A_c}{3\pi}\cos w_c t \cdot \cos 3 w_c t - \frac{2}{3}\frac{m(t)}{\pi}\cos 3 w_c t + \ldots$$

(i) The component

$$\frac{A_c}{2}\left[1+\frac{4}{\pi A_c}m(t)\right]\cos 2\pi f_c t$$ which is the desired

AM wave with amplitude sensitivity $K_a = \frac{4}{\pi A_c}$.

(ii) An unwanted component, the spectum of which contains delta functions at $0, \pm 2f_c, \pm 4f_c$ ± and so.., and which occupied frequency intervals of bandwidth $2f_m$ centered at $0, \pm 3f_c \pm 5f_c$ where f_m is message bandwidth

These, again the unwanted terms are removed from the load voltage $v_o(t)$ by means of a BPF ~~which~~ with mid band frequency 'f_c' and bandwidth $2f_m$, provided $f_c > 2f_m$

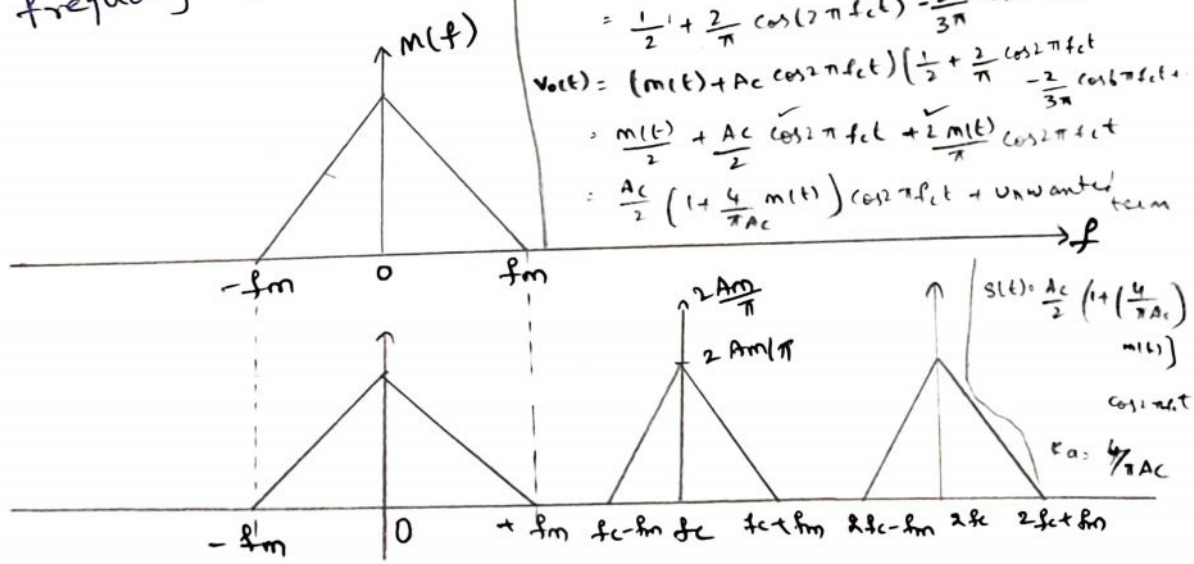
$$x(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos 2\pi f_c t(2n-1)$$

$$= \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos 6\pi f_c t + \cdots$$

$$v_o(t) = (m(t)+A_c\cos 2\pi f_c t)\left(\frac{1}{2}+\frac{2}{\pi}\cos 2\pi f_c t - \frac{2}{3\pi}\cos 6\pi f_c t + \cdots\right)$$

$$= \frac{m(t)}{2} + \frac{A_c}{2}\cos 2\pi f_c t + \frac{2\,m(t)}{\pi}\cos 2\pi f_c t$$

$$= \frac{A_c}{2}\left(1+\frac{4}{\pi A_c}m(t)\right)\cos 2\pi f_c t + \text{unwanted term}$$



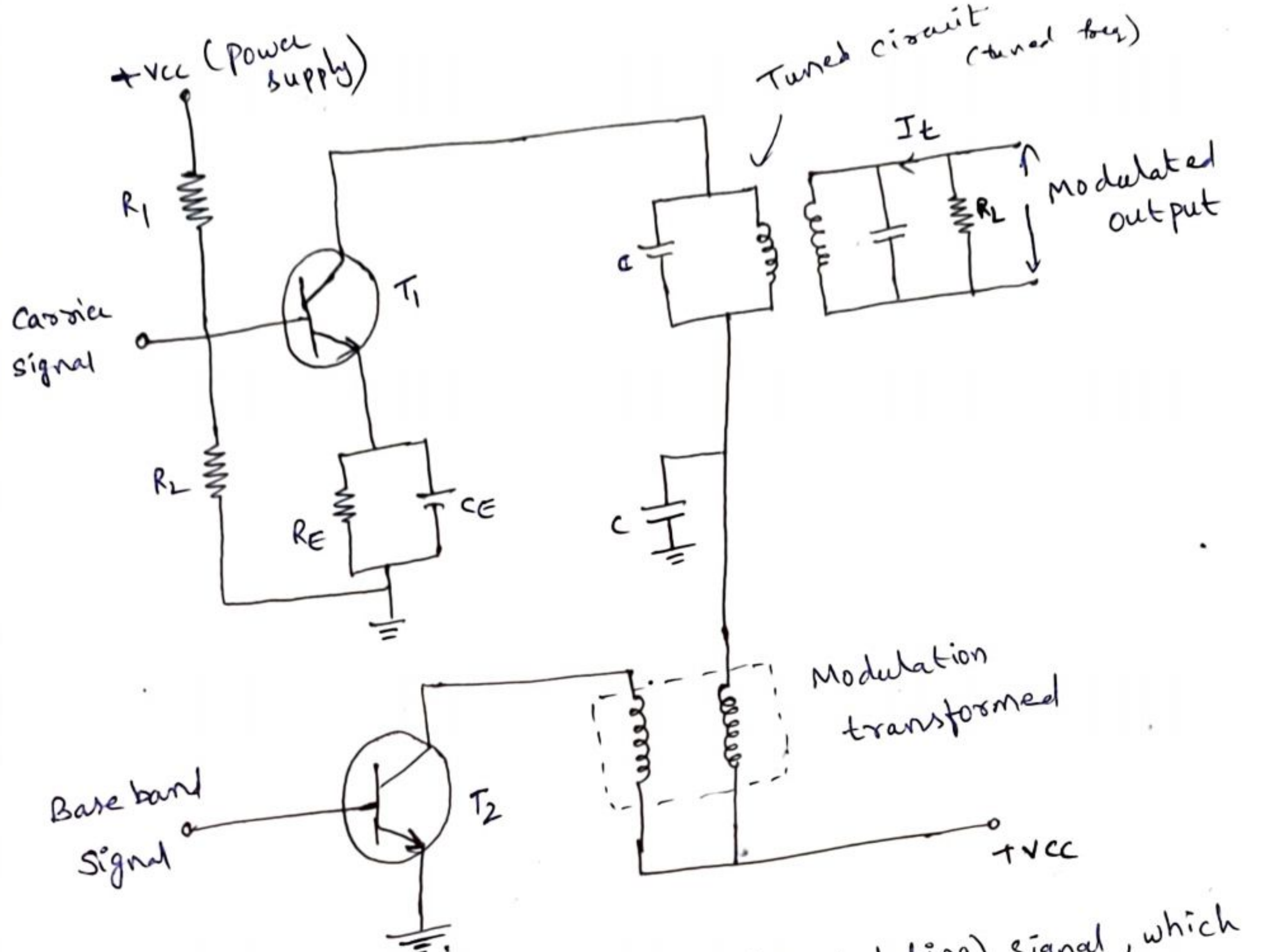
(iii) Collector Modulation Method:-

Collector modulation method is a very popular method for AM generation which is shown in below fig

Transistor T_1 makes a radio frequency (RF) class-c amplifier

At the base of T_1 carried signal is applied.

Vcc makes collector supply used for biasing purpose.



→ (Through T_2 we are sending baseband (modulating) signal, which acts as class B amplifier, the modulating signal is appeared through modulation transformer.

→ The capacitor is used to creat low impedance path, to create high frequency for carrier signal and to remove distortions from carrier signal.

The carrier signal and modulating signal are mixed at tuned circuit and modulated signal can be received at R_L.

(Carrier signal will be modulated according to message signal) then only the required modulating signal can be received.)

Collector Modulation Method is a very popular method for Am generation which is shown in above fig.

Transistor T_1 Makes a radio frequency (RF) class-C amplifier. at the base of 'T_1' carrier signal is applied.

Vcc makes collector supply used for biasing purpose.

Transistor 'T_2' makes a class-B amplifier which is used to amplify the audio or Modulating signal.

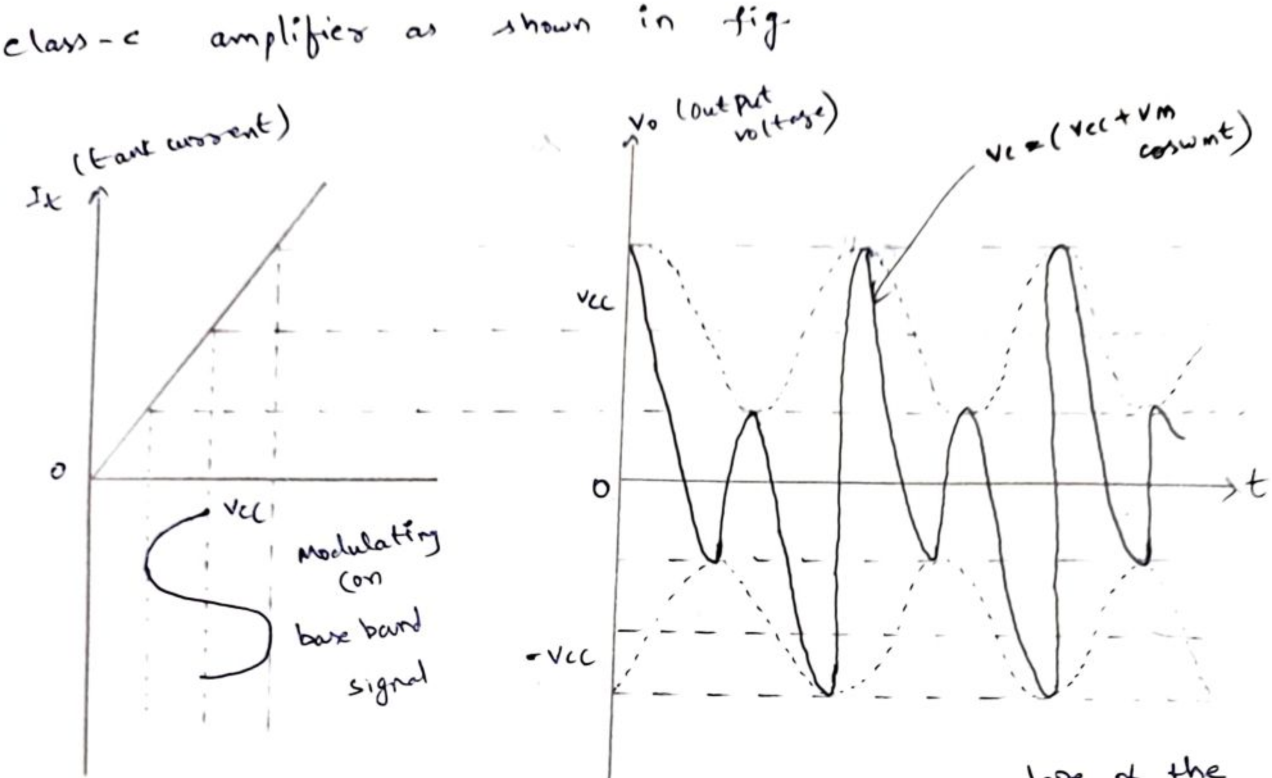
The base band or Modulating signal appears across the Modulation transformed after amplification. This amplified based band signal appears in series with the collector supply Vcc.

Function of capacitor 'C' is to offer low impedence path for the high frequency carrier signal and hence the carrier signal is prevented from frowing through the modulation transformer.

In class-C amplifier the output voltage will be an exact replica of the input voltage wave forms and the magnitude of the output voltage will be approximately equal to the carrier supply voltage 'Vcc'

output voltage $V_{cc} = I_t \cdot R_L$

Base band signal $V_m = A_m \cos w_m t$ is added to carrier supply voltage 'Vcc'. This results in a quite slow variation in carrier supply voltage 'Vcc'. This type of slow-variation in carrier supply voltage changes the magnitude of the carrier signal, voltage at the output of the modulated class-c amplifier as shown in fig.



In the above figure it is observed that the envelope of the output voltage is identical with the base band or modulating signal voltage and hence an Am signal is generated.

Analysis:-

Slowly changing carrier supply voltage 'V_c' is expressed as

$$V_c = V_{cc} + V_m$$

$$V_c = V_{cc} + A_m \cos\omega_m t$$

Modulation index $m_a = \frac{V_m}{V_{cc}}$ (or) $\frac{A_m}{V_{cc}}$ $\Rightarrow A_m = m_a V_{cc}$

$$V_c = V_{cc} + m_a V_{cc} \cdot \cos\omega_m t \; ; \; V_c = V_{cc}\,[1 + m_a \cos\omega_m t]$$

Carrier voltage $V_c = V_c \cos\omega_c t$

Modulated output voltage will be $V_o = V_c \cos\omega_c t$; $\boxed{V_o = V_{cc}(1 + m_a \cos\omega_m t)\cos\omega_c t}$

Which is required expression for Am wave.

Collector modulation has the advantages over base modulation of better linearity, higher collector efficiency and higher power output per transistor.

Demodulation of AM waves :-

The process of extracting a base band (Modulating) signal from the modulated signal is known as demodulation.

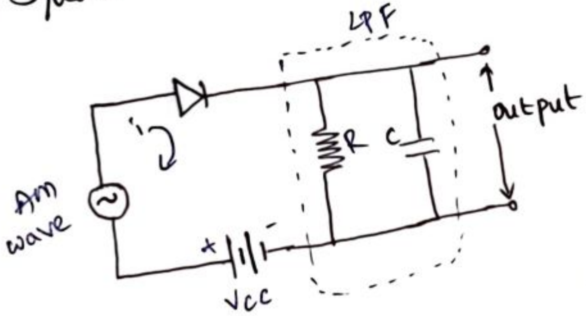
AM signals with large carrier are detected by using the 'Envelope detector'.

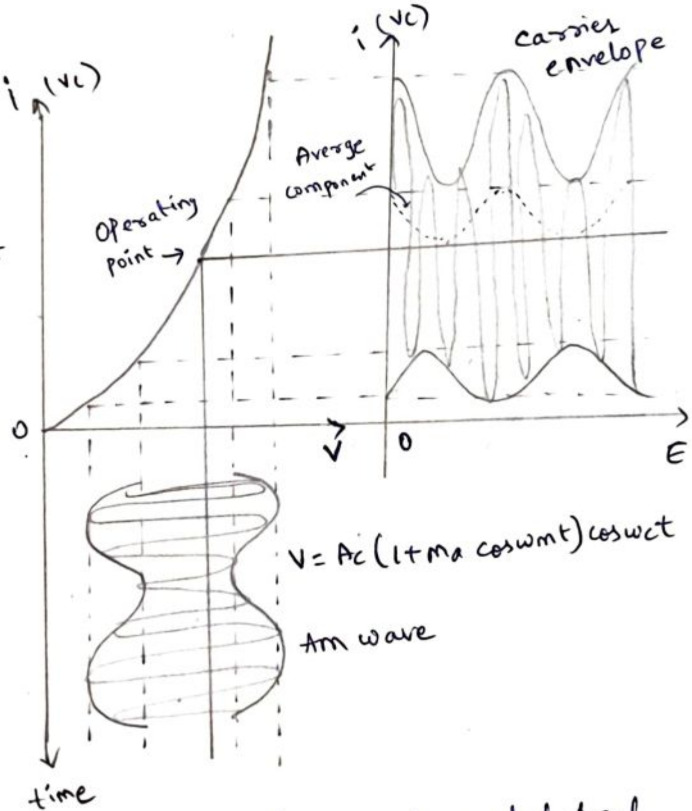
At low level modulated signal can be detected by using "square-law detector".

Thus detectors are of two types:

1) Square - Law detector   2) Envelope detector

1) Square - Law detector :





Square law detectors are used for detecting low-level modulated signals so that operating region of device - characteristic is restricted to non-Linear region.

The circuit is very similar to square law modulator except filter circuit difference. In detector LPF is used instead of BPF

The DC source V_{cc} is ~~en~~ used to adjust the operating point. The non-linear characteristic, modulated input voltage and resulting diode current waveforms are shown in the fig.

The operation is limited to the non-linear region due to which the lower half portion of the current waveform is compressed. This causes 'Envelope distortion'. The average value of diode current does not remain constant, rather it varies with ~~a~~ time as shown in fig.

The distorted diode current (voltage) is given by non-linear (square-law) relations:

$$i = a_1 v_1(t) + a_2 v_1^2(t)$$

$$v_o(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

where $v_1(t) \rightarrow$ input voltage

$v_o(t) \rightarrow$ output voltage

$a_1, a_2 \rightarrow$ constants

Input modulated voltage $v_1(t) =$

$$v_1(t) = A_c(1 + K_a m(t)) \cos 2\pi f_c t \qquad \left(\begin{matrix} 2\cos^2\theta = \\ \cos 2\theta + 1 \end{matrix}\right)$$

$$v_o(t) = a_1 (A_c (1+K_a m(t)) \cos 2\pi f_c t + a_2 [A_c (1+K_a m(t)) \cos w_c t]^2$$

$$v_o(t) = a_1 A_c (1+K_a m(t)) \cos 2\pi f_c t + a_2 A_c^2 (1+K_a m(t))^2 \cos^2 w_c t$$

$$v_o(t) = a_1 A_c (1+K_a m(t)) \cos 2\pi f_c t + a_2 A_c^2 (1 + K_a^2 m^2(t) + 2 K_a m(t)) \cos^2 w_c t$$

$$v_o(t) = a_1 A_c (1+K_a m(t)) \cos 2\pi f_c t + a_2 A_c^2 (1 + 2K_a m(t) + K_a^2 m^2(t))(1 + \cos 4\pi f_c t)$$

$$v_o(t) = a_1 A_c (1+K_a m(t)) \cos 2\pi f_c t + \frac{a_2 A_c^2}{2} (1 + 2K_a m(t) + K_a^2 m^2(t))$$

The desired signal $a_2 A_c^2 K_a m(t)$ is due to $a_2 v_1^2(t)$ term - hence this is the description of "Square-Law detector". This component is extracted by means of a low-pass filter (lpf).

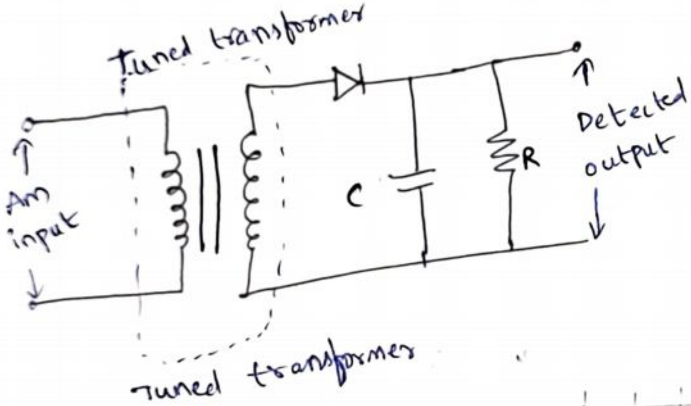
To make $K_a^2 m^2(t)$ small we have to follow $|K_a m(t)| < 1$

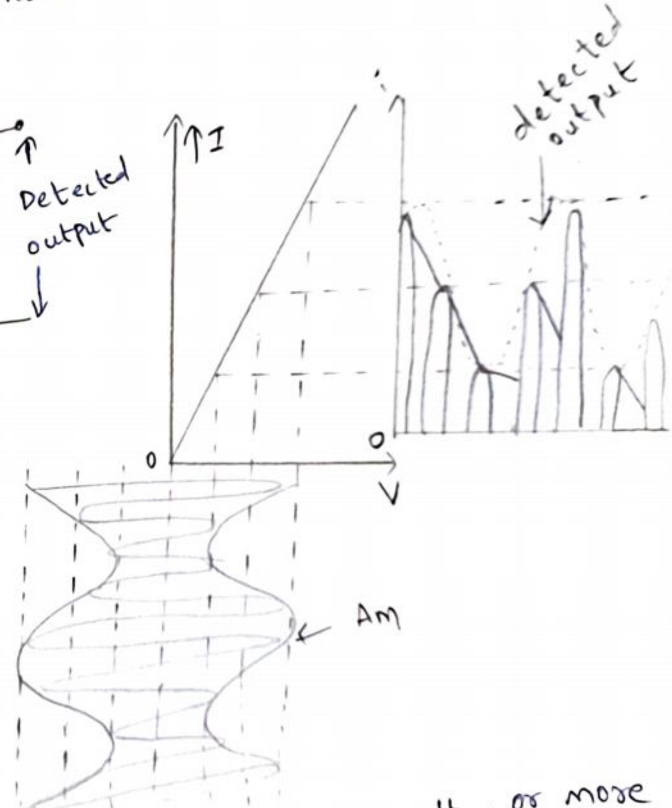
$\therefore$ distortionless recovery of base band signal $m(t)$ is possible only when the applied AM wave is weak and if $M < 1$. i.e., perfect modulation is very small.

(ii) Envelope detector :- (or) (linear diode detector).

A diode operating in a linear region of its characteristic can extract the envelop of an AM wave. Such a detector is called "Envelope detector". This detector is extremely popular in commercial receiver circuits because it is very simple and less expensive, and at the same time provides satisfactory performance for the reception of broad cast programmes.

The circuit diagram is shown in the below fig





Operation :-

Tuned transformer provides Perfect tuning at desired frequency

R-C turns time constant network.

when modulated carrier at input of detector is 1volt, or more the operation takes place in the linear region of diode characteristic which is shown in fig above.

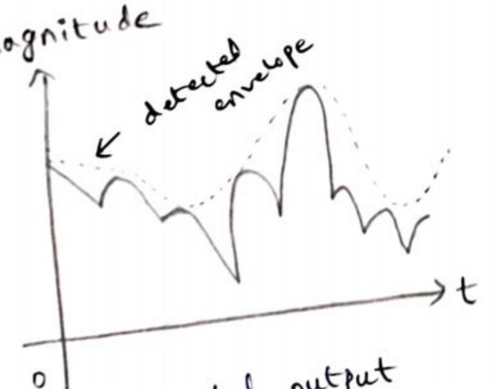
If the capacitor is absent output waveforms will be a half - rectified carrier wave as shown in fig. If the capacitor is ON for the positive half cycle of a rectified

wave, Capacitor is charged to peak value of carrier volt...
but, for a negative half cycle, the diode is reversed biased, and the carrier is disconnected from R-c circuit.

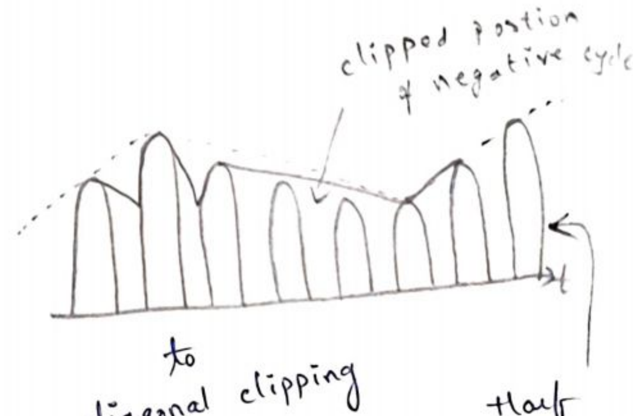
Now capacitor discharges with the constant $\tau = Rc$ and before it decreases to a small value next positive cycle appears. This positive cycle further charges the capacitor to the peak value of carrier voltage and the process continues.

∴ Voltage across capacitor is same as envelope of modulated carrier, but spikes are introduced due to charging and discharging of the capacitor which is shown in fig.

These spikes can be reduced by keeping R-c large, so that the capacitor discharges negligibly small. But large value of R-c creates a problems called "diagonal clipping".



Detected output



to diagonal clipping

Choice of Time constant :-

If R-C is very high, the discharge curve becomes approximately horizontal (to minimize spikes or fluctuations)

In that case, negative peaks of the detected envelope completely are missing.

This type of distortion is called "diagonal clipping".

(i) The spike, or fluctuation in a detected envelope should be minimum

(ii) Negative peaks of detected envelope should not be missed partially (i.e., diagonal clipping).

for this, "the rate of discharge of capacitor $\geq$ rate of decrease of modulation envelope."

Power calculations of Am wave :-

Consider the following equation of amplitude modulated wave.

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2} \cos(2\pi(f_c + f_m)t) + \frac{A_c \mu}{2} \cos(2\pi(f_c - f_m)t)$$

Power of Am wave ~~is~~ equal to the sum of powers of carrier, upper side band, and lower sideband frequency components.

$$P_t = P_c + P_{USB} + P_{LSB}$$

we know that standard formula for power of cos signal is

$$P = \frac{V_{rms}^2}{R} = \frac{(V_m / \sqrt{2})^2}{R}$$

where,

V_{rms} is the rms voltage of cos signal.

V_m is the peak value of cos signal.

First, let us find the power of the carrier, the upper and lower sideband one by one.

carrier power : $P_c = \frac{(A_c / \sqrt{2})^2}{R} = \frac{A_c^2}{2R}$

upper sideband power

$$P_{USB} = \frac{(A_c \mu / 2\sqrt{2})^2}{R} = \frac{A_c^2 \mu^2}{8R}$$

similarly, we will get the lower sideband power same as that of of the upper side band power.

$$P_{LSB} = \frac{A_c^2 \mu^2}{8R}$$

Now, Let us add these three powers in order to get the power of Am wave.

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{8R} + \frac{A_c^2 \mu^2}{8R}$$

Total power of Am signal is

$$P_T = P_c + P_s$$

$$P_T = P_c + P_s$$

$$= P_c + P_{s;\,LSB} + P_{s;\,USB}$$

$$= \frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{8} + \frac{\mu^2 A_c^2}{8}$$

$$= \frac{A_c^2}{2} + \frac{2\mu^2 A_c^2}{8}$$

$$= \frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{4} = \frac{A_c^2}{2}\left[1+\frac{\mu^2}{2}\right]$$

$$\boxed{P_T = P_c\left[1+\frac{\mu^2}{2}\right]}$$

Transmission efficiency of AM signal :-

~~Out of this~~ the total power 'P_T', is the useful message (base band) power, 'P_s' is the power carried by the side bands.

The large carrier power 'P_c' does not carry any message and this power is a waste from information point of view.

$\therefore$ P_s is the only useful message power 'P_s' present in AM wave is expressed by a term called "Transmission efficiency". Hence transmission efficiency of AM wave is defined as "percentage of total power contributed by the side bands

$$\mu = \frac{P_s}{P_T} \times 100 = \frac{\mu^2 A_c^2/4}{\frac{A_c^2}{2}\left[1+\frac{\mu^2}{2}\right]} = \frac{\mu^2}{2+\mu^2} \qquad \boxed{\therefore \mu = \frac{\mu^2}{2+\mu^2}}$$

If $M=1$; i.e., 100% Modulation is used; then $\mu = \frac{1}{2+1} = 1/3 = 33.33\%$.

i.e; the total power in the two side bands of AM wave is only ($1/3^{rd}$) (one-third) of total power in the Modulated wave; remaning of $(2/3)^{rd}$ is a waste.

Power Content in Multi tone AM:-

A multitone AM is that type of modulation in which the modulating signal consists of more than one frequency components

let $m(t) = V_{m_1} \cos\omega_1 t + V_{m_2} \cos\omega_2 t + V_{m_3} \cos\omega_3 t + \cdots$

General Expression for AM wave is

$$s(t) = A_c (1 + K_a m(t)) \cos\omega_c t$$

$$s(t) = A_c (1 + K_a V_{m_1} \cos\omega_1 t + K_a V_{m_2} \cos\omega_2 t + \cdots] \cos\omega_c t$$

$$= A_c [1 + \mu_1 \cos\omega_1 t + \mu_2 \cos\omega_2 t + \cdots] \cos\omega_c t$$

$$P_c = \frac{A_c^2}{2} \quad ; \quad P_s = \frac{A_c^2 \mu_1^2}{4} + \frac{A_c^2 \mu_2^2}{4} + \cdots$$

$$= \frac{A_c^2}{4} [\mu_1^2 + \mu_2^2 + \cdots]$$

$$P_T = P_c + P_s$$

$$= \frac{A_c^2}{2} + \frac{A_c^2}{4} [\mu_1^2 + \mu_2^2 + \cdots)$$

$$= \frac{A_c^2}{2} \left[1 + \frac{(\mu_1^2 + \mu_2^2 + \cdots)}{2} \right]$$

$$\boxed{\because \mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \cdots}}$$

$$P_T = P_c \left[1 + \frac{\mu_t^2}{2} \right]$$

μ_t = total Modulation index

Problems:

1) A 400 watts carrier is modulated to depth of 75%, find the total power in the amplitude modulated wave. Assume the modulating signal to be a sinusoidal one.

Sol:

$P_T = P_c\left(1+\frac{m^2}{2}\right)$ $P_c = 400$; $M = 75\% = 0.75$

$P_T = 400\left[1+\frac{(0.75)^2}{2}\right]$

$P_T = 512.5$ watts.

2) An AM broadcast radio transfer radiates 10K watts of power if modulation percentage is 60. Calculate how much of this is carrier power.

Sol:

$P_T = 10kW$; $M = 60\% = 0.6$

$P_T = P_c\left(1+\frac{M^2}{2}\right)$

$10K = P_c\left(1+\frac{(0.6)^2}{2}\right)$

$\therefore P_c = 8.47 kW$

3) An Am transmitter radiates 9K watts of power when the carrier is unmodulated and 10.125K watts when the carrier is sinusoidally modulated. Find the modulation index, percentage of modulation. Now, if another sine wave, corresponding to 40 percentage modulation is transmitted simultaneously, then calculate the total radiated power.

Sol:

$P_c = 9kW$ $P_T = 10.125kW$

$P_T = P_c\left(1+\frac{m^2}{2}\right) \Rightarrow 10.125K = 9K\left(1+\frac{m^2}{2}\right)$

$\frac{10.125K}{9K} = 1+\frac{m^2}{2}$

$1.125 = 1+\frac{m^2}{2}$

$1.125 - 1 = \frac{m^2}{2}$

$0.125 = \frac{m^2}{2}$, $m^2 = 0.25$

$m = \sqrt{0.25} = 0.5$

$\boxed{M_1 = 0.5}$

(ii) $M_2 = 40\% = 0.4$

$$m = \sqrt{m_1^2 + m_2^2} = \sqrt{0.5^2 + 0.4^2} = 0.64$$

$$P_T = 9K\left(1 + \frac{0.64^2}{2}\right) = 1.084\ KW$$

4) How many AM broad cast stations can be accommadated in 100 KHZ bandwidth if the highest frequency modulating a carrier is 5 KHZ?

Sol:-

$B.W = 100\ KHZ$; $f_m = 5\ KHZ$

signal B.W $B_T = 2f_m = 2 \times 5K = 10\ KHZ.$

$$\text{No. of stating accommadated} = \frac{\text{total B.W}}{\text{B.W per station}} = \frac{100K}{10K} = 10 \text{ stations}$$

5) The total power content of an AM signal is 1000W Determine the power being transmitted at the carrier frequency and at each of the sidebands when the percent modulation is 100%.

$P_T = 1KW$ $m = 100\% = 1.$

[Same problem just charge $M = 50\%$]

$$P_T = P_c\left(1 + \frac{m^2}{2}\right)$$

$$1K = P_c\left(1 + \tfrac{1}{2}\right) \quad ; \quad P_c = 666.67\ W$$

$$P_{USB} = \frac{A_c^2 M^2}{8} = \frac{m^2}{4} P_c = \frac{1}{4} \times 666.67 = 166.66\ W$$

$$P_{LSB} = \frac{m^2}{4} P_c = 166.66W$$

6) Find total modulated power, sideband power, and net modulation index for the AM signal $P_{AM} = 10\cos(2\pi 10^6 t) + 5\cos(2\pi 10^6 t)\cos 2\pi 10^3 t + 2\cos(2\pi 10^6 t)\cos 4\pi 10^3 t).$

Expression can be represented or expressed as:

$$P_{AM} = 10\left[1 + 0.5\cos 2\pi 10^3 t + 0.2\cos 4\pi 10^3 t\right]\cos 2\pi 10^6 t$$

$$P_{AM} = A_c\left[1 + m_1\cos\omega_{m_1} t + m_2\cos\omega_{m_2} t\right]\cos 2\pi f_c t$$

$A_c = 10$; $M_1 = 0.5$; $M_2 = 0.2$; $f_c = 10^6$

unmodulated carrier power $P_c = \frac{A_c^2}{2} = \frac{10^2}{2} = 50$ watts

Power of modulated signal $P_T = P_c\left(1 + \frac{m_1^2 + m_2^2}{2}\right)$

$$P_T = 50\left(1 + \frac{0.5^2 + 0.2^2}{2}\right)$$

$$= 57.25 \text{ watts}$$

sideband power $P_S = P_T - P_c = 57.25 - 50 = 7.25\,W$

Net modulation index $M = \sqrt{m_1^2 + m_2^2} = \sqrt{0.5^2 + 0.2^2} = 0.539$.

7) A Group of modulating signals are modulated with a dept of modulations are 0.5, 0.8 & 0.9 and the carrier is 50 sin 2000πt. Find the total power transmitted for a group of modulated signals

$M_1 = 0.5$; $M_2 = 0.8$; $M_3 = 0.9$; $M_t = \sqrt{m_1^2 + m_2^2 + m_3^2}$

$$= \sqrt{0.5^2 + 0.8^2 + 0.9^2}$$

$$m_t = 1.304$$

$v_c(t) = 50 \sin 2000\pi t = A_c \sin 2\pi f_c t \Rightarrow A_c = 50$; $f_c =$.

$$P_T = P_c\left(1 + \frac{m_t^2}{2}\right) = \frac{A_c^2}{2}\left(1 + \frac{m_t^2}{2}\right)$$

$$= \frac{50^2}{2}\left(1 + \frac{1.304^2}{2}\right)$$

$$= 2.312\ KW$$

8) The Antenna of an Am wave transmitted is 8A if only the carrier is sent, but it increases to 8.93A if the carrier is modulated by a single sinusoidal wave. Determine the percentage modulation. Also find the antenna current is the percent of modulation changes to 0.8

Sol. (i) $I_c = 8A$ $\quad I_T = 8.93A$

$$I_T = I_c\sqrt{1+\frac{m^2}{2}}$$

$$8.93 = 8\sqrt{1+\frac{m^2}{2}} \Rightarrow M = 70\%$$

(ii) $M = 0.8$; $I_c = 8A$

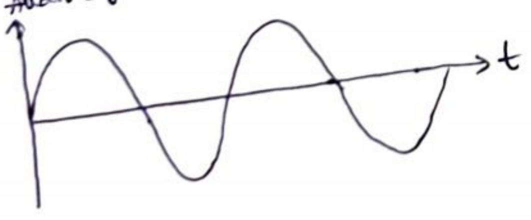
$$I_T = I_c\sqrt{1+\frac{m^2}{2}} \Rightarrow I_T = 8\sqrt{1+\frac{0.8^2}{2}} = 9.19\,A$$

9) An audio signal given as $15\sin 2\pi(1500t)$ amplitude modulates a carrier given as $60\sin 2\pi(100,000t)$ determine the following:

a) sketch the audio signal
b) sketch the carrier signal
c) construct the modulated wave
d) determine the modulation index & percentage modulation
e) what are the frequencies of audio signal and carrier?
f) what frequencies would present in a spectrum analysis of the modulation wave?

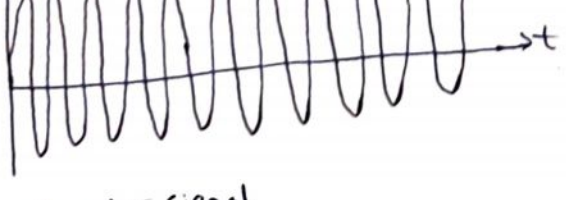
$$V_m(t) = V_m \sin 2\pi f_m t$$
$$= 15\sin 2\pi(1500t)$$
$$V_m = 15 \ ; \ f_m = 1500$$

Sol: a) Audio signal



$$V_c(t) = V_c \sin 2\pi f_c t$$
$$= 60\sin 2\pi(100,000t)$$
$$V_c = 60 \quad f_c = 100,000$$

b)



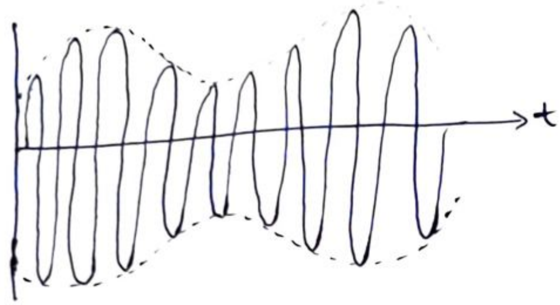
Carrier signal

d) $M = \frac{V_m}{V_c} = \frac{15}{60} = 0.25 = 25\%$

$A_{max} = A_c(1+M) = 60(1+0.25) = 75$

$A_{min} = A_c(1-M) = 60(1-0.25) = 45$

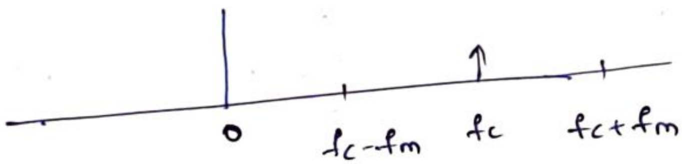
c)



e) $f_m = 1500\,Hz$

$f_c = 100{,}000\,Hz$

f)



$f_c = 100\,kHz$; $f_c + f_m = 100K + 1.5K \Rightarrow 101.5\,kHz$

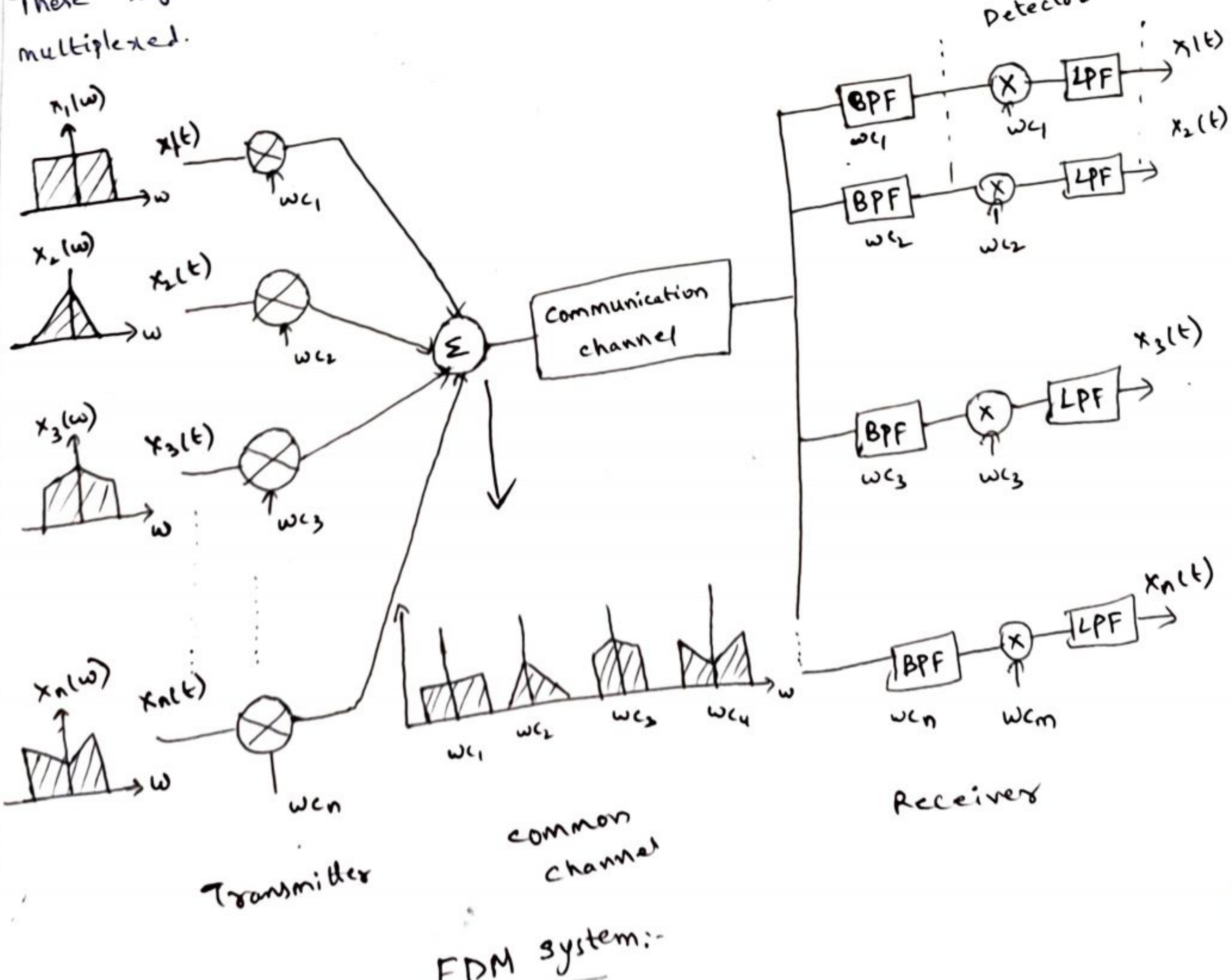
$f_c - f_m = 100K - 1.5K \Rightarrow 98.5\,kHz$

Frequency Division Multiplexing :-

Multiplexing is the Process of Simultaneous transmission of several messages over a common channel without interference. Let us transmit 'M' messages each one is band limited to ω_m,

eg:- telephone signals has a band width of 3KHz and radio signals has a bandwidth of 5KHz. Each Message is centered near zero frequency. The spectra of individual messages are shown in the figure below.

If these signals are transmitted simultaneously over a common channel without multiplexing, they will interfere with each other and no useful information will be produced at the receiver. These signals can be transmitted without interference if they are multiplexed.



FDM system:-

In FDM, each baseband signal is translated by Analog modulation (Amplitude or Angle) to different carrier frequencies. Each carrier is separated from the neighbouring one by at atleast $2\omega_m$.

At the receiver, the various carrier frequencies are selected using band pass filters tunned to appropriate carrier frequencies, and demodulated by separate detectors.

DSB Modulation :-

The carrier wave $c(t)$ is completely independent of the information - carrying signal (or) baseband signal $m(t)$ i.e., carrier wave transmission represents a waste of power.

$\therefore$ suppression of carrier component from the modulated wave results in double - side band suppressed carrier (modulation) (DSBSC)

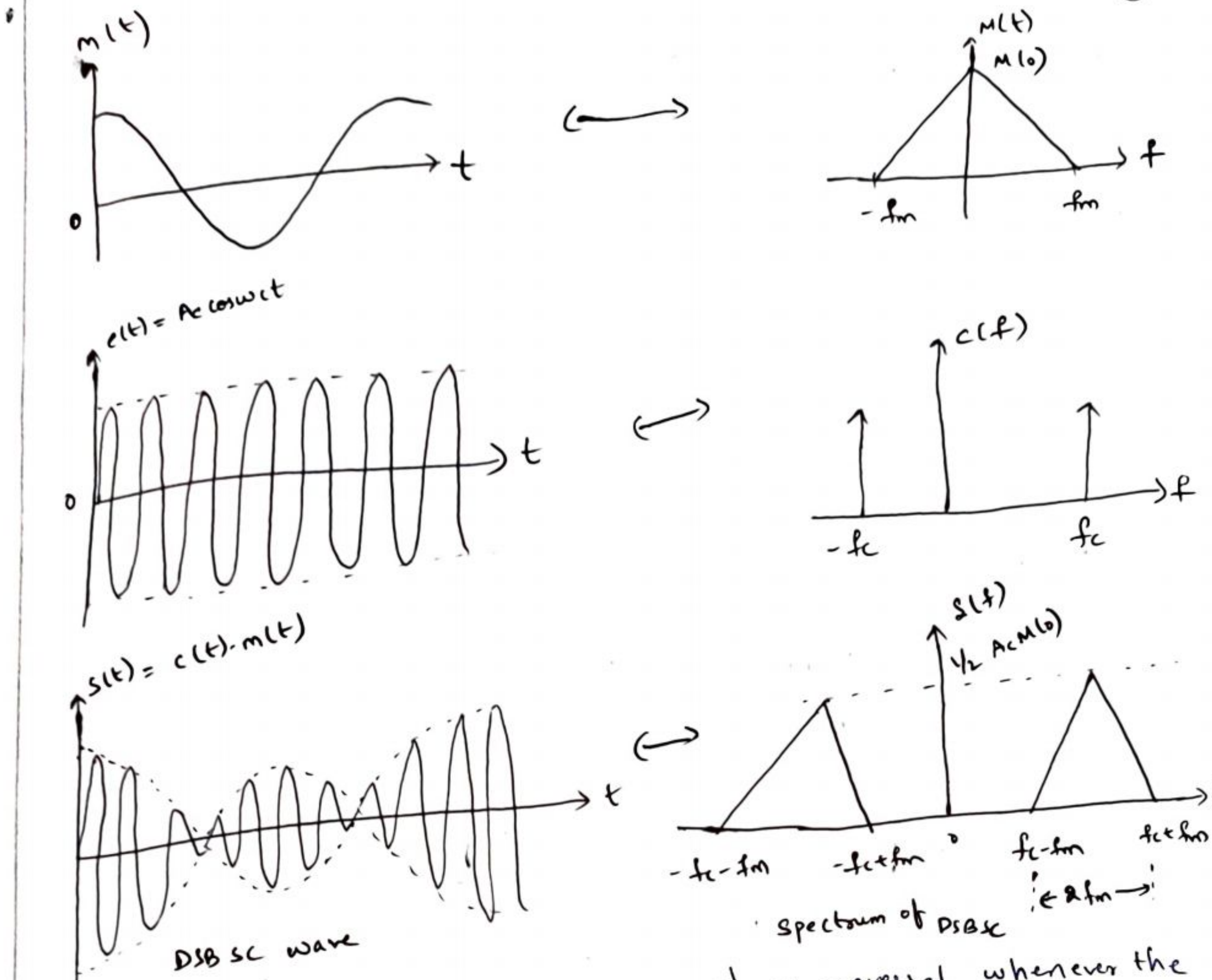
Suppressing the carrier, we obtain a modulated wave that is Propotional to the product of the carrier wave and the baseband signal.

$$s(t) = A_c[1 + K_a m(t)] \cos\omega_c t$$

$$= A_c(1 + M\cos\omega_m t]\cos\omega_c t$$

DSB-SC wave is $s(t) = A_c \cos\omega_m t \cos\omega_c t \quad [m<1]$

$$s(t) = \frac{A_c}{2}[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$$



This modulated wave undergoes a phase reversal whenever the baseband signal m(t) crosses zero ~~wave~~ axis as shown in fig above.

$$s(t) = A_c \cos\omega_c t \, m(t)$$

Fourier transform of s(t) is $\quad S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$

$$x(t) \leftrightarrow X(\omega)$$
$$e^{j\omega t} x(t) \leftrightarrow X(\omega-\omega_c)$$

The transmission bandwidth required by DSB-SC modulation is the same as that for Amplitude modulation i.e., $2f_m$

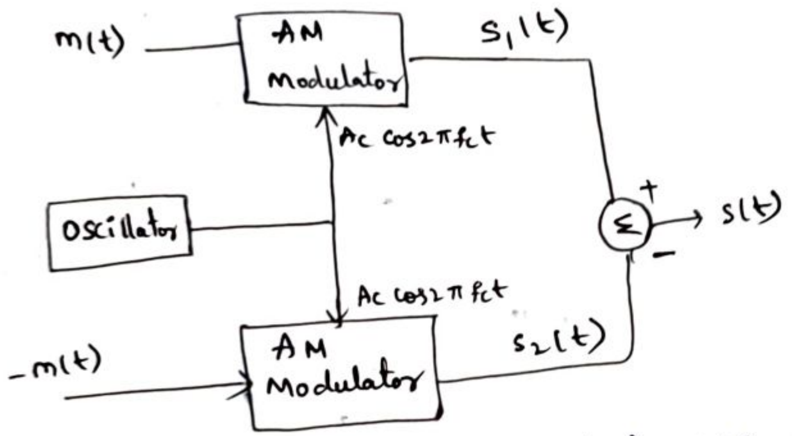
Generation of DSB-SC waves :-

A DSB-SC Modulated wave consists of a product of the baseband signal and the carrier wave. A device for achieving this requirement is called "product modulator"

Two forms of product modulator are:-

(i) The balanced Modulator (ii) Ring Modulator

1) Balanced Modulator:-



Two AM modulators are taken which are identical, except for the sign reversal of the modulating wave applied to the input of one of the modulators. This outputs are ;

$$s_1(t) = A_c\left[1 + k_a m(t)\right] \cos 2\pi f_c t$$

$$s_2(t) = A_c\left(1 - k_a m(t)\right) \cos 2\pi f_c t$$

Substracting $s_2(t)$ from $s_1(t)$, we obtain

$$s(t) = s_1(t) - s_2(t)$$

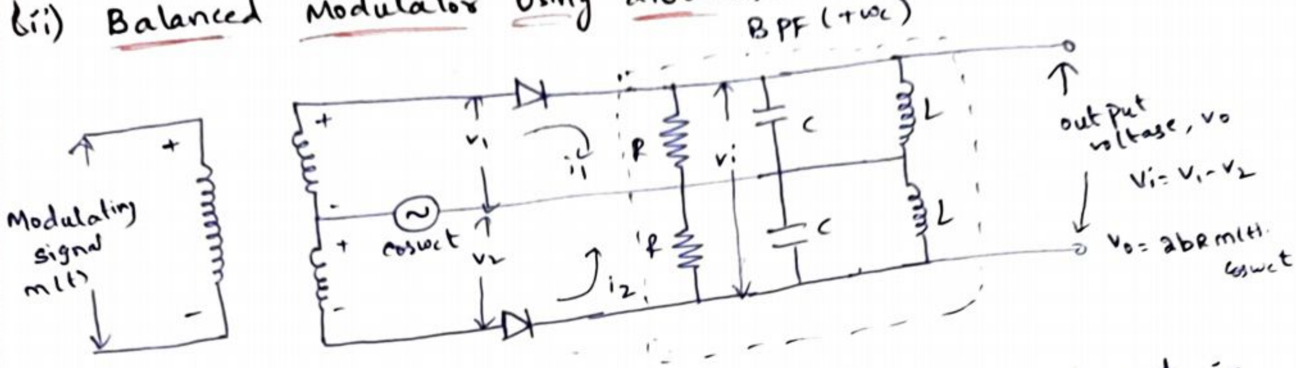
$$= A_c\left(1 + k_a m(t)\right) \cos 2\pi f_c t - A_c\left(1 - k_a m(t)\right) \cos 2\pi f_c t$$

$$= \left(A_c + A_c k_a m(t)\right) \cos 2\pi f_c t - A_c \cos 2\pi f_c t + k_a m(t) A_c \cos 2\pi f_c t$$

$$\boxed{s(t) = 2k_a A_c m(t) \cos 2\pi f_c t}$$

Hence $2k_a$ acts as scaling factor, the remaining $A_c\, m(t) \cos 2\pi f_c t$ is product of modulating wave & carrier is the output of balanced Modulator

(ii) Balanced Modulator using diodes :-



A balanced modulator using two diodes as non-linear elements is shown in fig. In this figure, voltages e_1 & e_2 are expressed as,

$$e_1 = m(t) + \cos\omega_c t \quad \& \quad e_2 = \cos\omega_c t - m(t)$$

Current flowing through diode 'D_1' is $i_1 = a e_1 + b e_1^2$

$$= a(m(t) + \cos\omega_c t) + b(\cos\omega_c t + m(t))^2$$

$v_1 = i_1 R$

$v_2 = -i_2 R$

and $i_2 = a e_2 + b e_2^2$

$$= a(\cos\omega_c t - m(t)) + b[\cos\omega_c t - m(t)]^2$$

The voltage 'v_i' at the input of the band pass filter is given by

$$v_i = v_1 - v_2 = i_1 R - i_2 R = R[i_1 - i_2]$$

$$v_i = R[4b\,m(t) \cdot \cos\omega_c t]$$

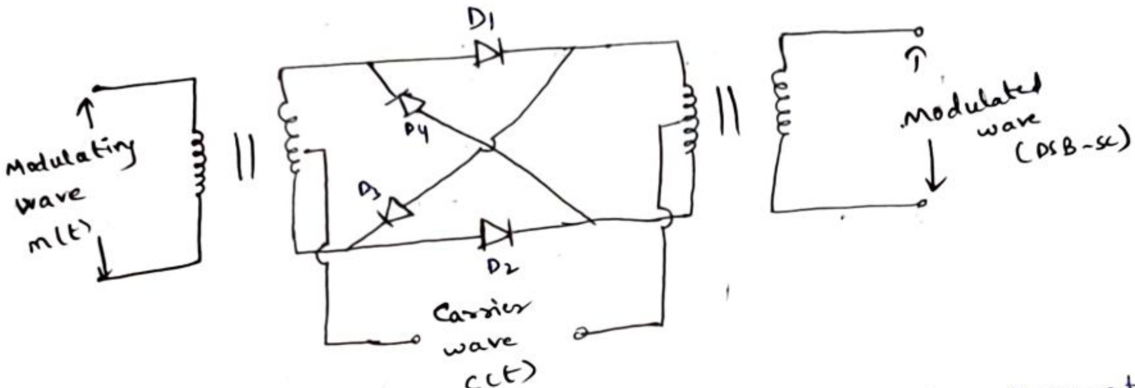
A bandpass filter is that type of filter which allows to pass a band of frequencies. Here, since the band pass filter is centered around $\pm\omega_c$, it will pass a narrow band of frequencies centered at $\pm\omega_c$ with a small bandwidth of '$2f_m$'

∴ The output of BPF centered around $\pm\omega_c$ is given by

$V_0 = \cancel{4} bR\, m(t) \cdot \cos\omega_c t$

$\boxed{V_0 = K\, m(t) \cdot \cos\omega_c t}$ → this represents the DSB-SC wave.

(iii) Ring Modulator :-



Most useful product modulators that is used for generating a DSBSC wave is ring modulator.

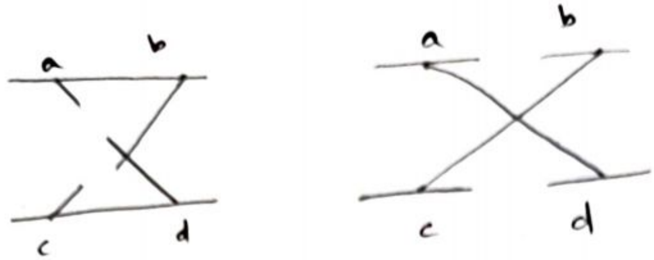
The four diodes form a ring in which they all point in the same way - hence, the name 'ring modulator'.

The diodes are controlled by a square-wave carrier c(t) of frequency 'fc', which is applied by means of two center-tapped transformers.

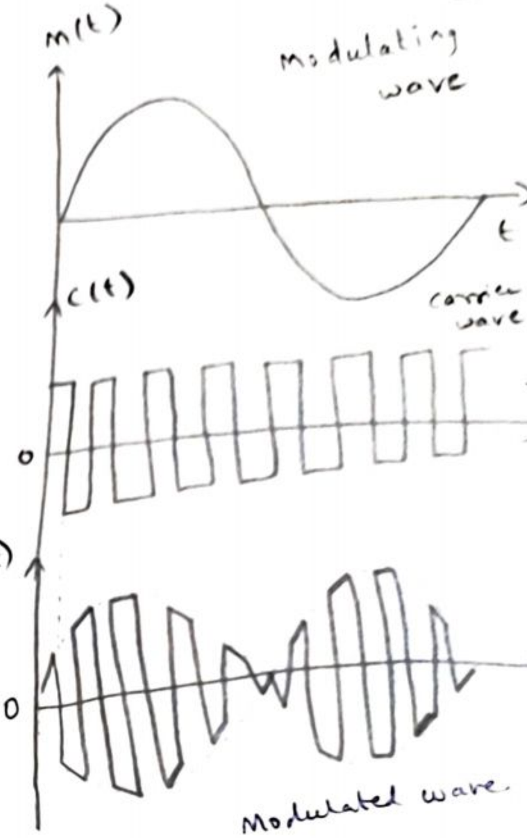
when carrier supply is positive, the outer diodes (D_1 & D_2) are 'ON' and D_3 & D_4 are 'off'. so that the modulator multiplies the baseband signal m(t) by +1. as shown in fig. below (a)

when the carrier supply is negative, the situation becomes reversal as shown in fig below (b) and modulator multiplies the baseband signal by (-1).

Thus the ring modulator, in its ideal form, is a product modulator for a square-wave carrier and the base band signal as shown in fig. below



(a)



The square-wave carrier c(t) can be represented in fourier series as:

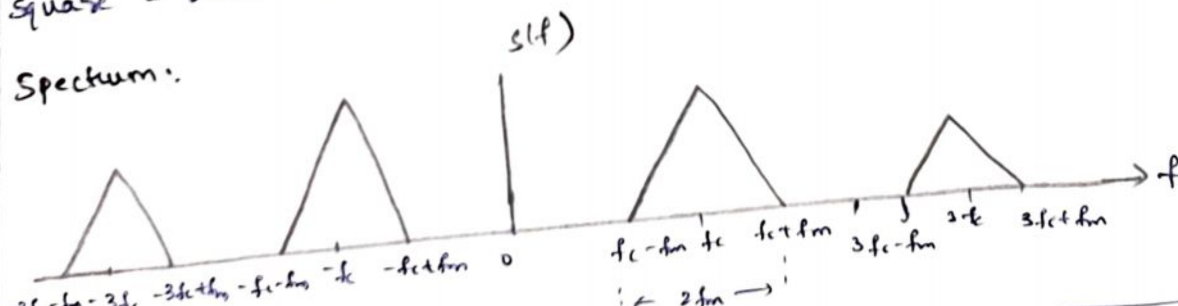
$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t(2n-1)\right]$$

Ring modulator output is

$$= \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t(2n-1)\right] \cdot m(t)$$

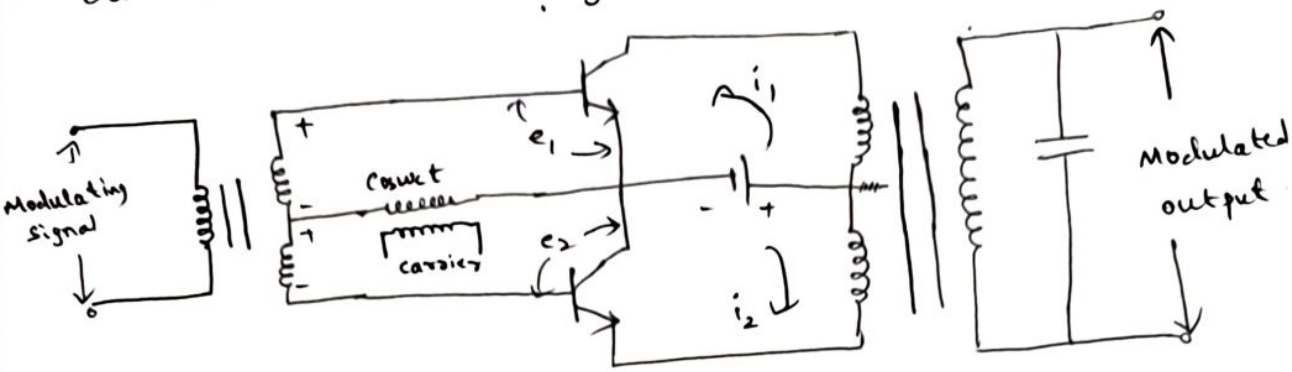
∴ The modulator o/p consists entirely of modulation products and not the carrier frequency. Thus ring modulator is referred to as "Double-balanced modulator", because it is balanced with respect to the base band signal as well as the square-wave carrier

Spectrum:



To prevent sideband overlap; we must choose $\boxed{f_c > f_m}$.

(iv) Balanced Modulator using transistors:-



Principle of operation of circuit is identical to the diode circuit. The transistors are operated in the non-linear region. The voltage & currents are shown in fig. which are same as that of diodes using balanced modulator.

The equations relating to voltages & currents are identical to the ones of diode balance modulator circuit. The circuit gives

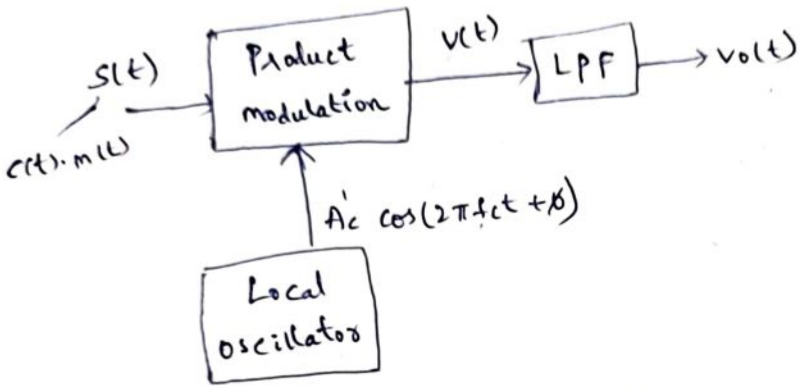
DSB-SC signal $V_0 = 2bR\, m(t) \cos\omega_c t$

$= k\, m(t). \cos\omega_c t.$

Transistors are amplifying devices. The circuit can be fabricated using other amplifying devices, like FET or electron tubes. The diode circuit is cheaper, where as amplifying devices Provide power gain.

(i) Coherent detection of DSB-SC modulated waves:-

The baseband signal $m(t)$ can be recovered from a DSB.SC wave $s(t)$ by multiplying $s(t)$ with a locally generated sine-wave and then low-pass filtering the product as shown in fig.



Local oscillator signal is exactly coherent or synchronized, in both frequency and phase with carrier wave $c(t)$ used in product modulator to generate $s(t)$. This method of demodulation is known as "coherent detection or synchronous detection".

Output of product modulator is :

$$v(t) = A_c' \cos(2\pi f_c t + \phi) \cdot s(t)$$

$$= A_c' \cdot \cos(2\pi f_c t + \phi) \cdot c(t) \cdot m(t)$$

$$= A_c' A_c \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \phi) \cdot m(t)$$

$$= \underbrace{\frac{A_c' A_c}{2} \cos(4\pi f_c t + \phi) \cdot m(t)}_{1} + \underbrace{\frac{A_c A_c'}{2} \cos\phi \cdot m(t)}_{2}$$

The first term (1) in above equation represents a DSB-SC wave with a carrier frequency '$2f_c$'.

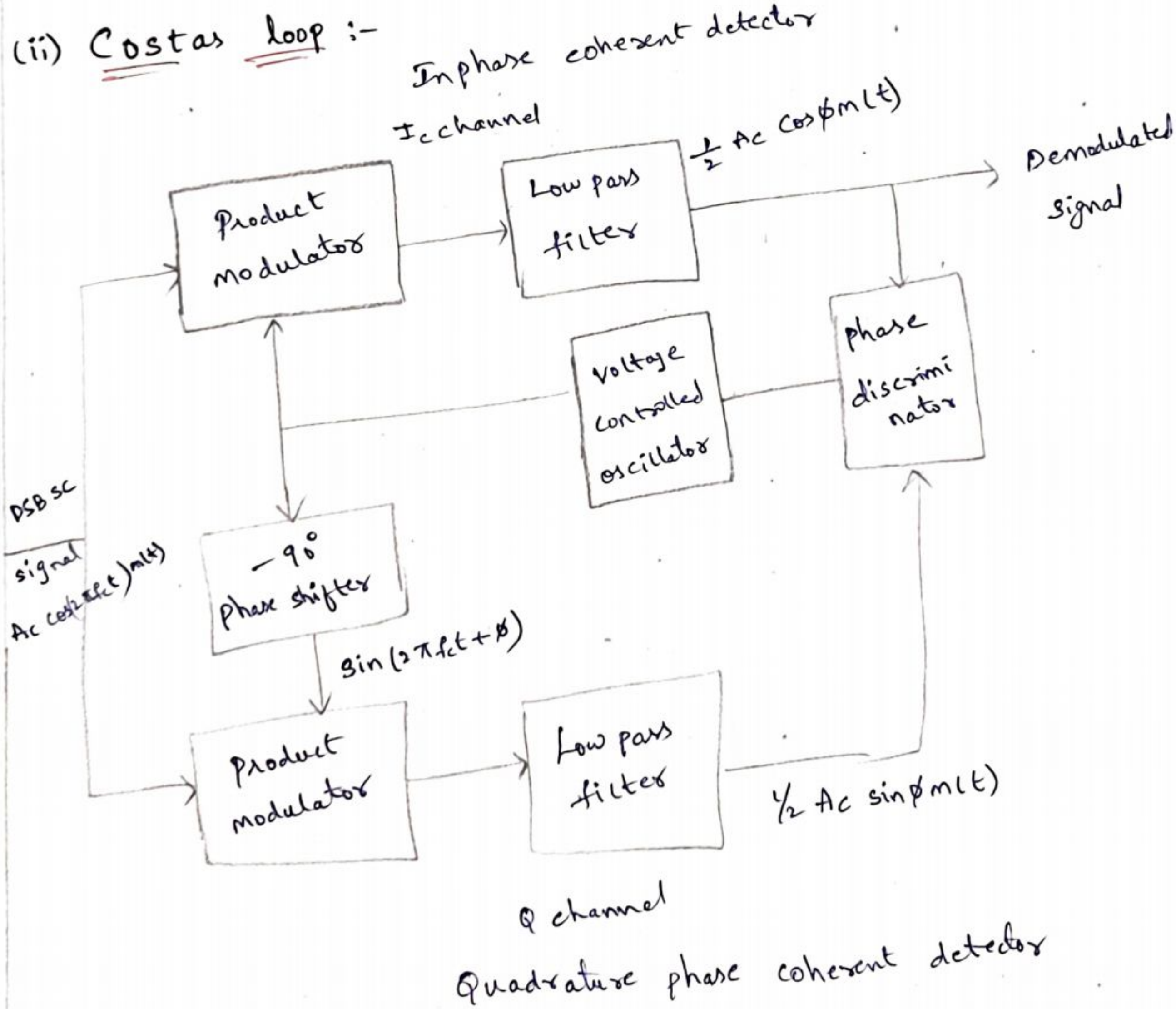
The second term (2) is proportional to the baseband signal $m(t)$, The v(t) is passed through LPF which have a cut-off frequency of 'f_m' and ① term is eliminated only ② terms is filtered at output side.

At the filter output we have

$$\boxed{V_0(t) = \tfrac{1}{2} A_c A_c' \cos\phi . m(t)}$$

The demodulated signal $v_0(t) \propto m(t)$ when phase error is constant. The amplitude of this demodulated signal is maximum when $\phi = 0$, and it is minimum (zero) when $\phi = \pm \pi/2$.

(ii) **Costas loop :-**



Problem:

1. Calculate the percent power saving for a DSB-SC signal for the percent modulation of a) 100% & b) 50%.

Sol: Total power in AM wave; $P_T = P_c\left[1 + \frac{m^2}{2}\right]$

a) $M = 100\% = 1$. $P_T = P_c\left[1 + \frac{1}{2}\right] = 1.5\,P_c$

$\therefore$ % power saving $= \frac{P_c}{P_T} \times 100 = \frac{P_c}{1.5\,P_c} = \frac{1}{1.5} = \frac{10}{15} = 66.66\%$.

b) $M = 50\% = 0.5$; $P_T = P_c\left[1 + \frac{0.5^2}{2}\right] = 1.125\,P_c$

$\therefore$ % power saving $= \frac{P_c}{P_T} \times 100 = \frac{P_c}{1.25\,P_c} \times 100 = 88.88\%$.

SSB Modulation :-

Amplitude Modulation and DSB-SC modulation are wasted of bandwidth because they both require a transmission bandwidth equal to twice the Message bandwidth ($2 f_m$).

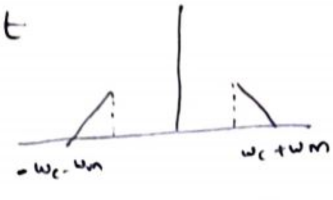
Communication channel needs to provide only the same bandwidth as the baseband signal this is happened in SSB modulation.

when only one side band is transmitted, the modulation system is referred to as a "single-side band (SSB) system".

The principal disadvantages of the SSB modulation system is its cost and complexity.

To get SSB-SC waveform, we have to eliminate one of the two side bands.

This represents a time-domain signal $\cos(\omega_c - \omega_m)t$



SSB-SC wave with LSB is expressed as

$$\cos(\omega_c - \omega_m)t = \cos\omega_m t \cos\omega_c t + \sin\omega_m t \sin\omega_c t \quad \text{.....} \; (1)$$

Similarly SSB-SC wave with USB is expressed as

$$\cos(\omega_c + \omega_m)t = \cos\omega_m t \cos\omega_c t - \sin\omega_c t \sin\omega_m t \quad \text{.....} \; (2)$$

(1) & (2) combined

$$s(t)_{SSB} = \cos\omega_m t \cos\omega_c t \pm \sin\omega_m t \cdot \sin\omega_c t .$$

$+$ ∴ LSB

$-$ ∴ USB.

$\sin\omega_c t = \cos[\omega_c t - \pi/2]$; $\sin\omega_m t = \cos(\omega_m t - \pi/2]$

Sin terms are obtained by using cosine terms by giving the phase shift of $(-\pi/2)$.

Expression for SSB-SC wave modulated by a general modulating signal s(t) is

$$s(t)_{SSB} = x(t)\cos\omega_c t \pm x_h(t)\sin\omega_c t$$

$x_h(t)$ is "hilbert transform" of $x(t)$ i.e., $x_h(t) = x(t - \pi/2)$

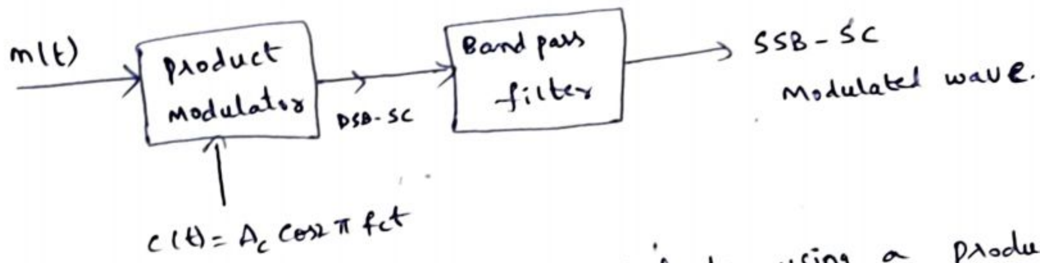
$$s(t)_{SSB} = m(t)\cos\omega_c t \pm \omega_h(t)\sin\omega_c t$$

Generation of SSB waves :-

SSB wave

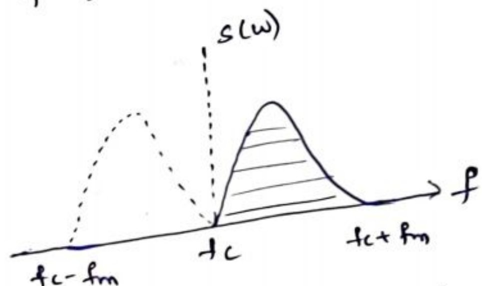
- Frequency discrimination method (frequency domain) (or) Filter method
- phase discrimination method (or) phase - shift method (or) Phasing method (Time domain)

1) Frequency discrimination Method :-

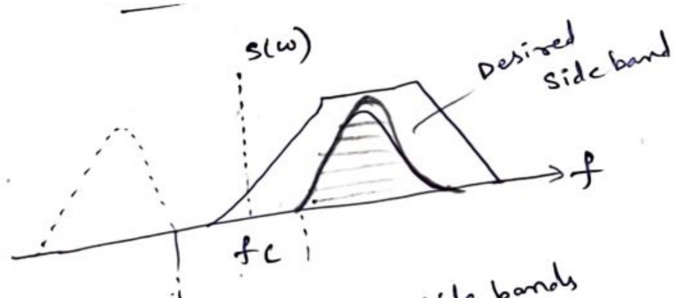


First a DSB-SC signal is generated by using a product modulator or a balanced modulator, and then one of the sidebands is filtered out by an appropriate bandpass filter.

Design of BPF is very critical and puts some limitations on the modulating and carrier frequencies.



overlapping sidebands

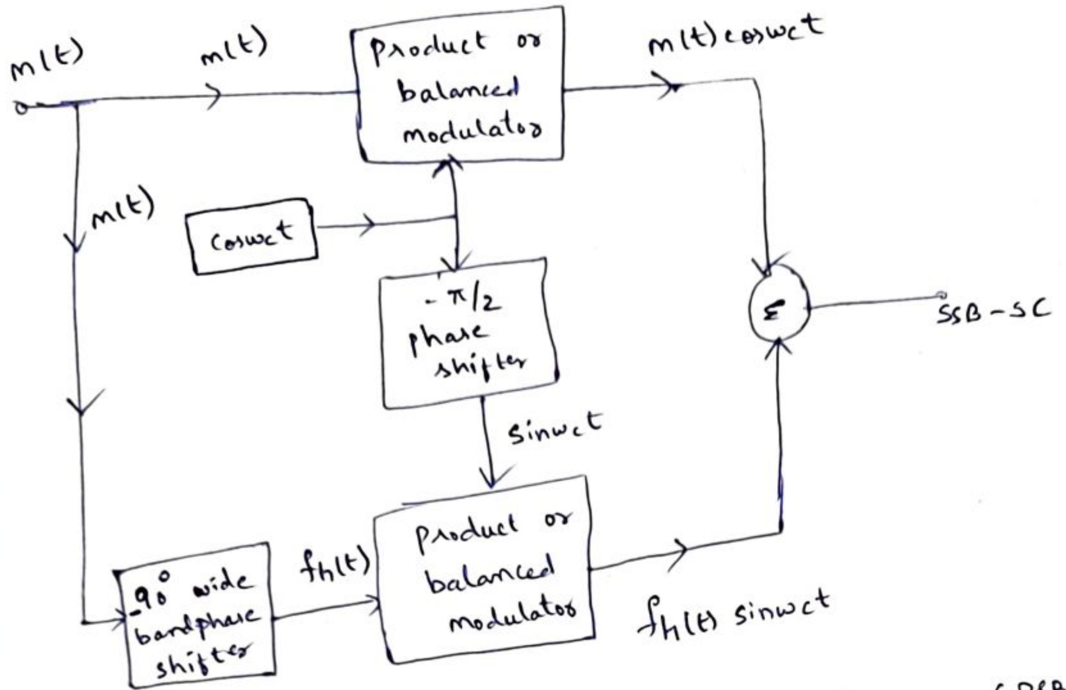


Non-overlapping side bands

2) phase discrimination Method (or phasing method :-

In this method, the time domain discription of SSB-SC wave with LSB SSB-SC signal is given by the expression:

$$s(t)_{SSB} = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$



The product term m(t) cosωct is an AM-SC (DSB-SC) signal which is generated by a simple product modulator, or a balanced modulator.

The product term mh(t) sinωct is generated by passing fh(t) & sinωct through another product or balanced modulator

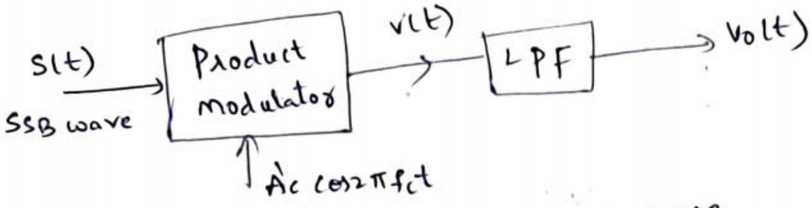
the function fh(t) is generated by passing m(t) through a wide band (-π/2) phase shifter.

A wide band phase-shifter is needed so that all the frequency components present in the baseband signal are shifted by 90°, keeping their amplitudes unchanged.

Signal sinωct is obtained by phasing cosωct through a simpler (-90°) phase shifter.

The product terms m(t) cosωct and mh(t) sinωct are added in an ~~odd~~ oder to generate the SSB-SC signal.

Demodulation of SSB-SC waves :-



Coherent detection of SSB-wave

Demodulation is accomplished by using coherent detection, which involves applying the SSB wave s(t) with a locally generated sine wave $A_c' \cos 2\pi f_c t$ to a product modulator and then low pass filtering the modulator output as shown in above fig

The output of product modulator for the case when USB only is transmitted

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$v(t) = A_c' \cos 2\pi f_c t \left(m(t) \cos 2\pi f_c t - m_h(t) \sin 2\pi f_c t\right]$$

$$= A_c' m(t) \cos^2 2\pi f_c t - A_c' m_h(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$\cos A \sin B = \tfrac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$$

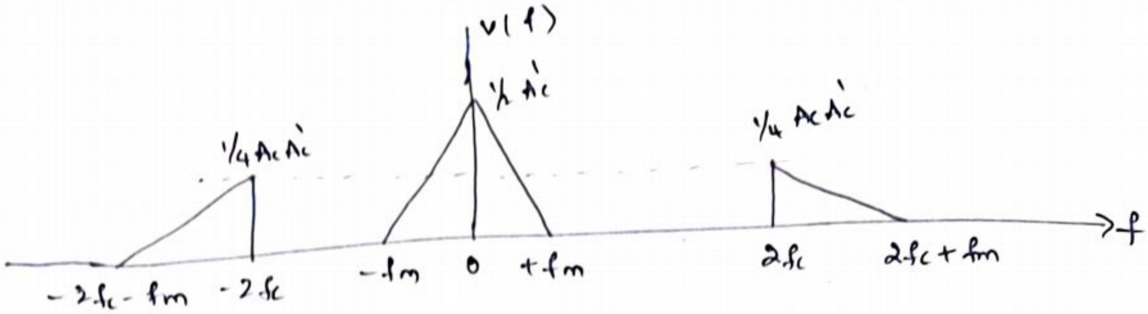
$$= A_c' m(t) \left(1 + \frac{\cos 4\pi f_c t}{2}\right] - A_c' m_h(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$= \tfrac{1}{2} A_c' m(t) + \tfrac{1}{2} A_c' m(t) \cos 4\pi f_c t - \tfrac{1}{2} A_c' m_h(t) \sin 4\pi f_c t$$

$$v(t) = \tfrac{1}{2} A_c' m(t) + \frac{1}{2} A_c' \left(m(t) \cos 4\pi f_c t - m_h(t) \sin 4\pi f_c t\right]$$

the first term is the desired demodulated signal, where as the second term represents an SSB wave corresponding to a carrier frequency $2f_c$. //

The frequency translations involved in above equation are shown in below fig. The high freq components are required by LPF there by yielding the desired baseband signal.



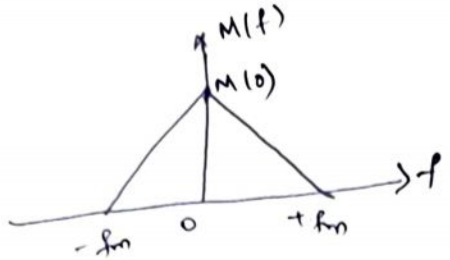
Spectrum of V(t)

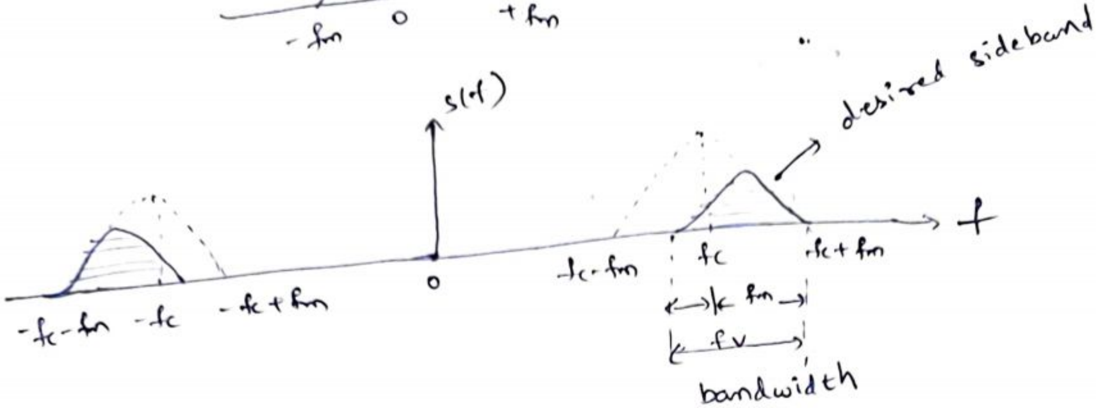
Vestigial side band modulation:-

Advantages of SSB-SC modulation is that it reduces the bandwidth requirement to half as compared to DSB-SC modulation.

But SSB-SC signals are relatively difficult to generate due to difficulty in isolating the desired sideband. The required filter must have a very sharp cut-off characteristic, particularly when the base-band signal contains extremely low frequencies (eg:- Television, and telegraphic signals). this difficulty is overcome by a scheme known as "vestigial side band" modulation, which is a compromise between SSB-SC and DSB-SC modulation.

In VSB modulation, the desired side band is allowed to pass completely, where as just a small portion (called trace or vestige) of the undesired sideband is also allowed as shown in fig. The transmitted vestige of undesired sideband compensates for the loss of the unwanted (upper) side band.





Spectrum of VSB wave:

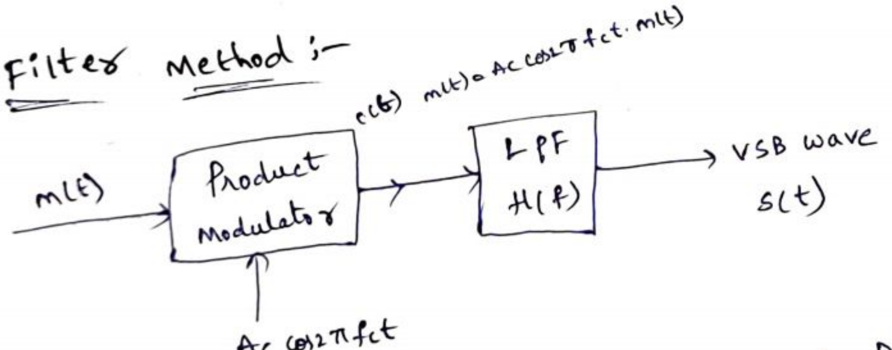
Transmission bandwidth of VSB is $B_T = f_m + f_v$

$f_m \rightarrow$ is message bandwidth,

$f_v \rightarrow$ width of vestigial side band

Generation of VSB wave :-

(i) Filter Method :-

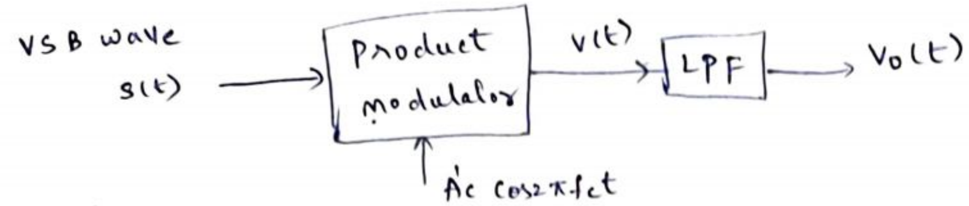


VSB modulation can be generated by passing a DSBSC wave through an appropriate filter of transfer function $H(f)$. The spectrum $S(f)$ of VSB wave is given by

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f) \rightarrow (1)$$

$m(t) \xleftrightarrow{F.T} M(f)$; $H(f)$ = Transfer function of filter

Detection of VSB wave :-



S(t) is multiplied by locally generated sine-wave $A'_c \cos 2\pi f_c t$ which is synchronous with the carrier wave $A_c \cos 2\pi f_c t$ in both frequency and phase.

$$v(t) = A'_c \cos 2\pi f_c t \; s(t)$$

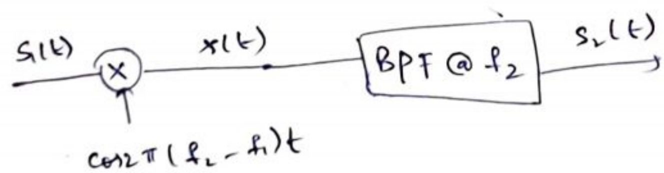
frequency domain

$$V(f) = \frac{A'_c}{2}\left[S(f-f_c) + S(f+f_c) \right]$$

⇒ frequency translation :-

→ suppose we have a modulated wave $s_1(t)$ whose spectrum is centered around frequency f_1 and we wish to move it upward in frequency, so that its spectrum is centered around f_2.

→ this can be accomplished by multiplying $s_1(t)$ by $\cos 2\pi(f_2-f_1)t$ and passing through a BPF.



$$X(f) = 0.5\, S_1(f-f_2+f_1) + 0.5\, S_1(f+f_2-f_1)$$

Downward frequency translation :-

→ we can also decrease the frequency of a modulated signal by multiplying $\cos 2\pi(f_2-f_1)$ and then filtering out the higher frequency (sum) component, and using the lower frequency (difference) component.

Method	s(t)	Power	Efficiency	Generation.
AM	$A_c(1+K_A\, m(t))\cos 2\pi f_c t$	$P_T = P_c\left(1+\frac{\mu^2}{2}\right)$	33.33%	→ square law → switching law → collector Demodulation: square law / envelope
DSB-SC	$m(t) \cdot c(t)$	$P_T = P_{USB} + P_{LSB}$		→ Balanced modulator → Balanced modulator using diode → " using transistor → Ring modulator Detection: coherent / Costas
SSB-SC	$m(t)\cos w_c t \pm m_h(t)\sin w_c t$	$P_T = P_{USB} \mid P_{LSB}$		→ frequency discrimination → phase discrimination Detection – coherent
VSB	$\frac{A_c}{2}\left[M(f-f_c)+M(f+f_c)\right]\cdot H(f)$	$P_T = P_{USB} + P_V$	→	① detection coherent detection.

Non. linearity effects in FM:-

In FM non-linear effects refers to distortion in modulated signal caused by non-linear components within the transmission system, leading to unwanted additional frequency in the spectrum or system, which can manifest as signal degradation and interferences without channels.

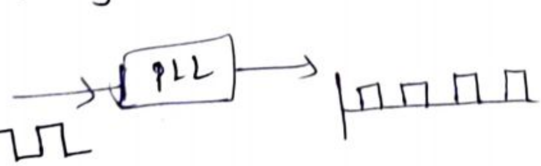
phase lock loop (PLL):-

Applications:-

1) used in Microprocessors.
2) used in synchronous and detector circuit.
3) used for clock & recovery.
4) To reduce noise in a system.
5) used in frequency synthesizers | tune generators.

Purpose:-

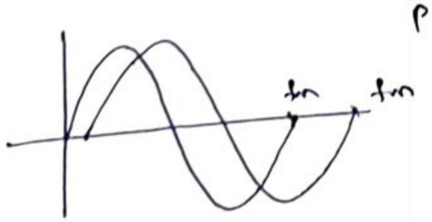
output frequency is multiple of i/p frequency.



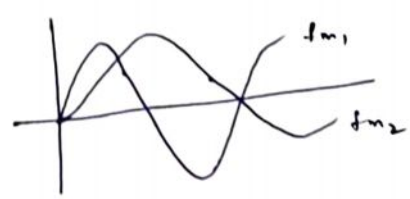
Phase diagram:-

phase is ~~same~~ different.

diff freq
same time

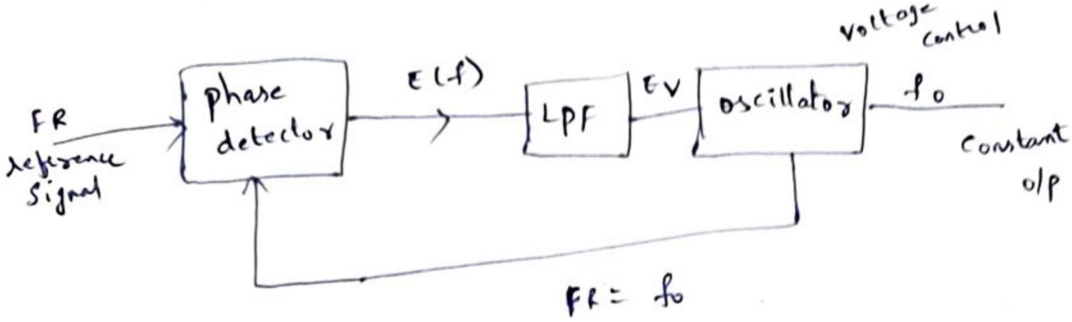


same freq
diff phase



same time
diff freq
phase is changing with time

* When for a system when i/p frequency = o/p frequency the system is in phase lock.



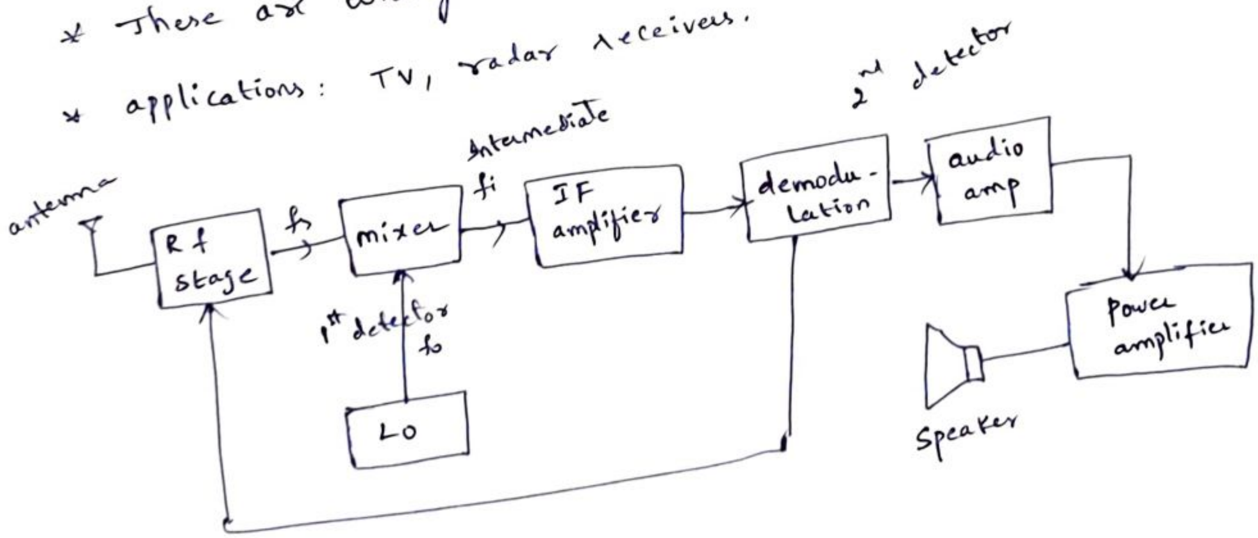
Superheterodyne receivers:-

hetero. mixing

Receivers are of two types:

1) Tuned radio receiver
2) superheterodyne receiver.

* It removes drawbacks of tuned radio receiver
* These are widely used
* applications: TV, radar receivers.



1) Antenna → accepting the EMWave
→ Conversion of EMwave into electrical wave

2) RF amplifier → 1) Select the desired frequency
(high frequency) 2) Also amplifying the signals.

(3) Mixer : converts the desired signal into medium low frequency signal.

$f_i = f_o - f_s$ → intermediate

(4) IF amplifier :- If amplifier features.

↑ selectivity , ↑ sensitivity , ↑ fedility → ↓ loss

(5) detector | demod :- detect the information (radio)

(6) Loud speaker:- convert electrical to sound.

Angle Modulation :-

Introduction :-

Angle modulation is the process in which angle of the carrier wave is varied according to the baseband signal. In this method amplitude of carrier wave is kept constant. Important feature of angle modulation is that it can provide better discrimination against noise and interference than AM. i.e., in Angle modulation interference due to noise is reduced.

Angle modulation

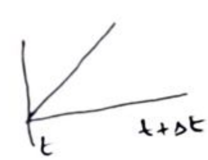
- frequency modulation
- phase modulation.

The general expression for Angle - Modulated wave is

$s(t) = A_c \cos[\theta_i(t)]$ where $\theta_i(t)$ denote the angle of a modulated sinusoidal carrier, which is a function of the message & A_c is the carrier amplitude.

If $\theta_i(t)$ increases with time, the average frequency in Hz over an interval from 't' to 't+Δt' is given by

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t}$$



Instantaneous frequency of angle - modulation wave $s(t)$ is

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \left(\frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi \Delta t} \right)$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\,\theta_i(t)}{dt}.$$

The angle modulated wave $s(t)$ has a rotating phasor of length A_c and angle $\theta_i(t)$. Angular velocity of such phasor is

$$\frac{d\theta_i(t)}{dt}.$$

For an unmodulated carrier, the angle $\theta_i(t)$ is

$$\theta_i(t) = 2\pi f_c t + \phi_c$$

and phasor rotates with an angular velocity equal to '$2\pi f_c$'.
The constant 'ϕ_c' is the value of $\theta_i(t)$ at $t=0$.

(1) "Phase Modulation (PM)": is that form of angle modulation in which the angle $\theta_i(t)$ is varied linearly with the base band signal $m(t)$, it is given by

$$\boxed{\theta_i(t) = 2\pi f_c t + K_p m(t).}$$

The term '$2\pi f_c t$' represents the angle of the unmodulated carrier, and constant 'K_p' represents the "phase sensitivity" of the modulator expressed in radians/volt.
The phase modulated wave $s(t)$ in time-domain is

$$s(t) = A_c \cos\theta_i(t)$$

$$\boxed{s(t) = A_c \cos[2\pi f_c t + K_p m(t)]}$$

(2) "Frequency Modulation" (FM) is that form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the base band signal $m(t)$.

$$f_i(t) = f_c + K_f m(t) \rightarrow (1)$$

$f_c \rightarrow$ frequency of unmodulated carrier

K_f - "frequency sensitivity" of modulator Hertz/Volt

Integrating eq (1) w.r.t time and multiplying the result by 2π, we get

$$2\pi \int f_i dt = 2\pi \int [f_c + k_f m(t)] dt$$

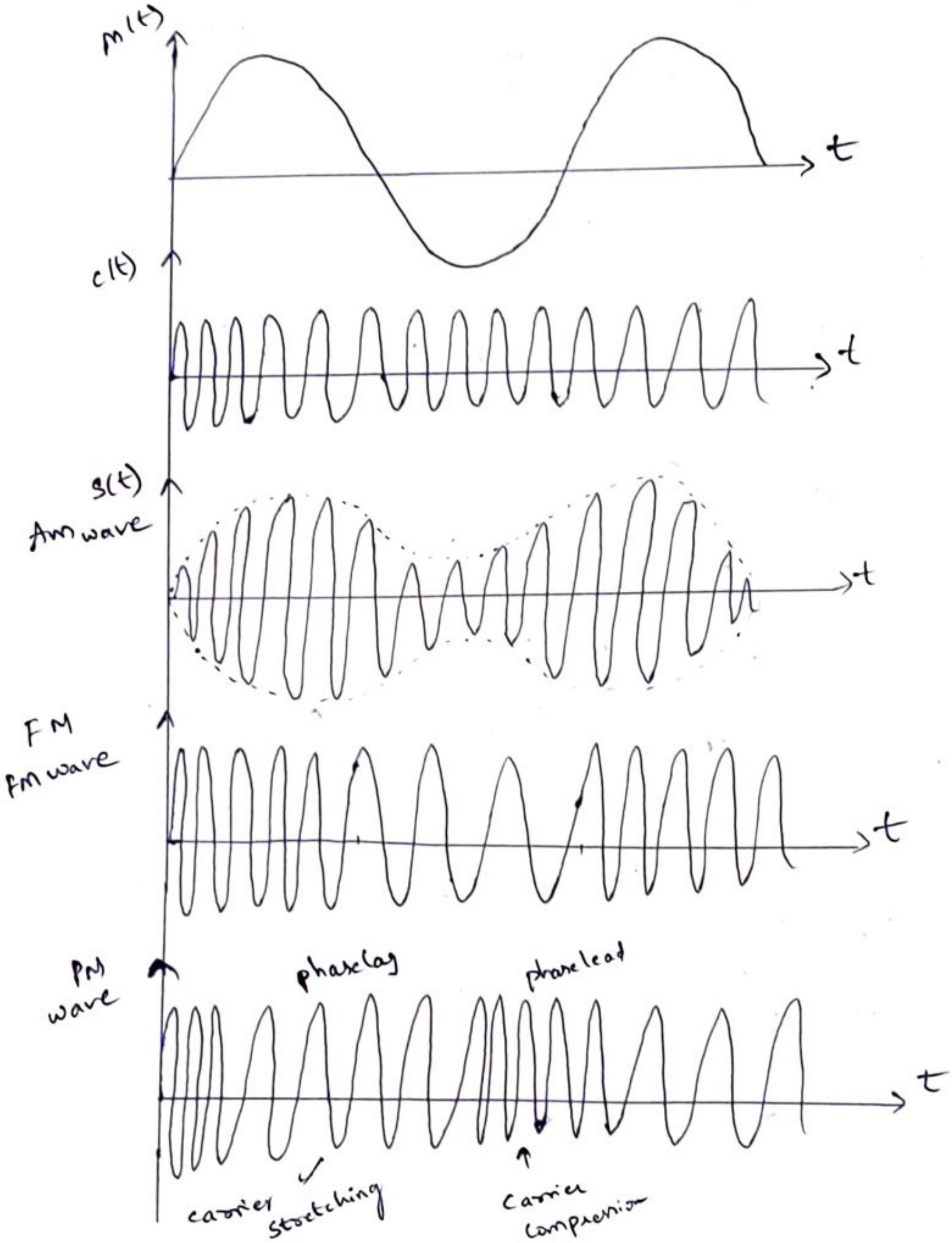
$$\theta_i(t) = 2\pi f_c t + \int_0^t 2\pi k_f m(t) dt$$

$$\therefore \boxed{\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt}$$

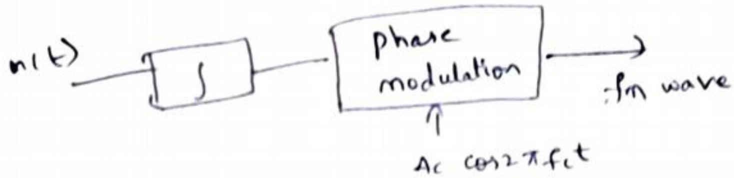
∴ The frequency-modulated wave in time domain is

$$s(t) = A_c \cos \theta_i(t).$$

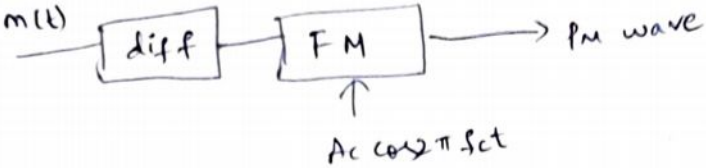
$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t) dt\right)$$



I) FM from PM:



II) PM from FM:



Single - Tone frequency Modulation:-

FM wave $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$ is a non-linear function of Modulating wave $m(t)$. Hence frequency modulation is a non linear modulation process.

Consider sinusoidal modulating wave

$$m(t) = A_m \cos 2\pi f_m t$$

Instantaneous frequency of resulting FM wave is

$$f_i(t) = f_c + k_f\, m(t)$$

$$= f_c + k_f A_m \cos 2\pi f_m t$$

$$f_i(t) = f_c + \Delta f \cos 2\pi f_m t$$

where $\Delta f = k_f A_m$ is called 'frequency deviation'. which represents the maximum departure of instantaneous frequency of FM wave from the carrier frequency f_c.

$\Delta f \propto A_m$ (Amplitude of modulating wave)

Angle $\theta_i(t)$ of FM wave is

$$\theta_i(t) = 2\pi \int_0^t f_i(t)\, dt$$

$\int \cos ax \, dx = \frac{\sin ax}{a}$

$$= 2\pi \int_0^t \left[f_c + \Delta f \cos 2\pi f_m t \right] dt$$

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t$$

The ratio of frequency deviation Δf to the modulation frequency f_m is called the "Modulation Index", of FM wave.

It is denoted by β

$$\beta = \frac{\Delta f}{f_m}$$

$$\therefore \theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t$$

The parameter β represents the phase deviation of FM wave, i.e, the maximum departure of the angle $\theta_i(t)$ from the angle $2\pi f_c t$ of unmodulated carrier.

FM is given by

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right].$$

(i) If β is small then it is "Narrow band FM"

(ii) If β is large then it is "wide band FM".

(ii) Narrow-band frequency Modulation:-

FM wave is $s(t) = A_c \cos\left(2\pi f_c t + \beta \sin 2\pi f_m t\right)$

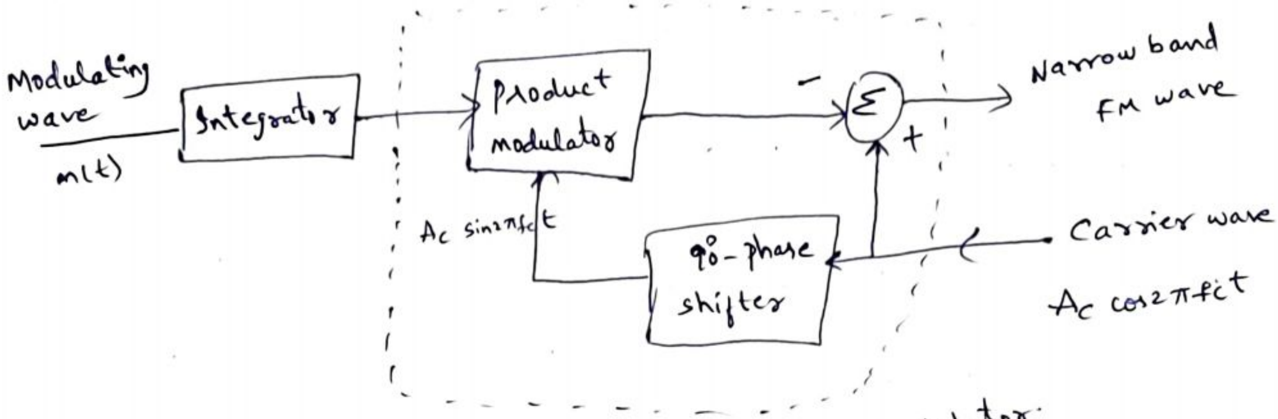
Expanding this we get

$$s(t) = A_c \cos 2\pi f_c t \cdot \cos(\beta \sin 2\pi f_m t) - A_c \sin 2\pi f_c t \sin(\beta \sin 2\pi f_m t)$$

β is small then $\cos(\beta \sin 2\pi f_m t) \simeq 1$ &

$$\sin(\beta \sin 2\pi f_m t) \simeq \beta \cdot \sin 2\pi f_m t$$

$$\therefore S(t) = A_c \cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$



Narrow- band phase Modulator.

Carson's Rule :-

Bandwidth of a single - Tone wideband FM is given by carson's rule.

According to this rule, the FM bandwidth is given by

$$BW = 2(\Delta f + f_m)$$

$$= 2 f_m \left(\frac{\Delta f}{f_m} + 1 \right) = 2 f_m (1+\beta)$$

(i) when $\Delta f << f_m$ (i.e, $\beta << 1$) narrow band FM

$BW = 2 f_m$ which is equivalent to AM.

(ii) when $\Delta f >> f_m$ (i.e, $\beta >> 1$) wide band FM

$BW = 2 \Delta f$

UNIT - II

SK. Khudsia Tabassum

Pulse Modulation :-

Introduction :- In pulse modulation systems, some parameter of Pulse is varied in accordance with instantaneous value of modulating signal.

These are two types of pulse modulation system :

(i) pulse Amplitude modulation (PAM)

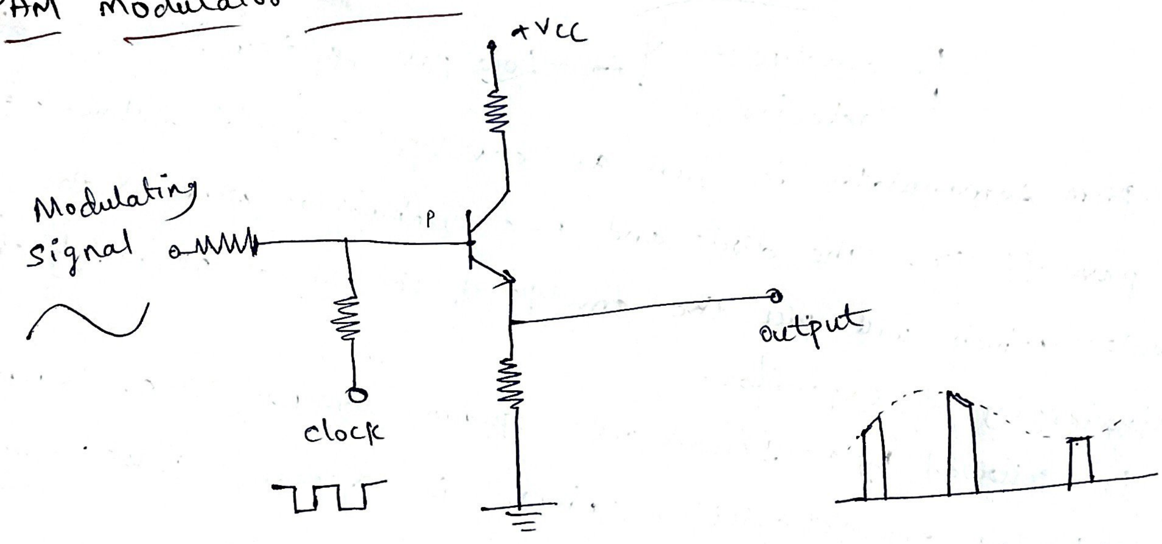
(ii) pulse Time modulation (PTM)

PTM → PWM, PPM

Pulse Amplitude Modulation (PAM) :-

In PAM, the amplitude of the pulse of carrier pulse train is varied in accordance with the modulating signal.

PAM modulator circuit :-



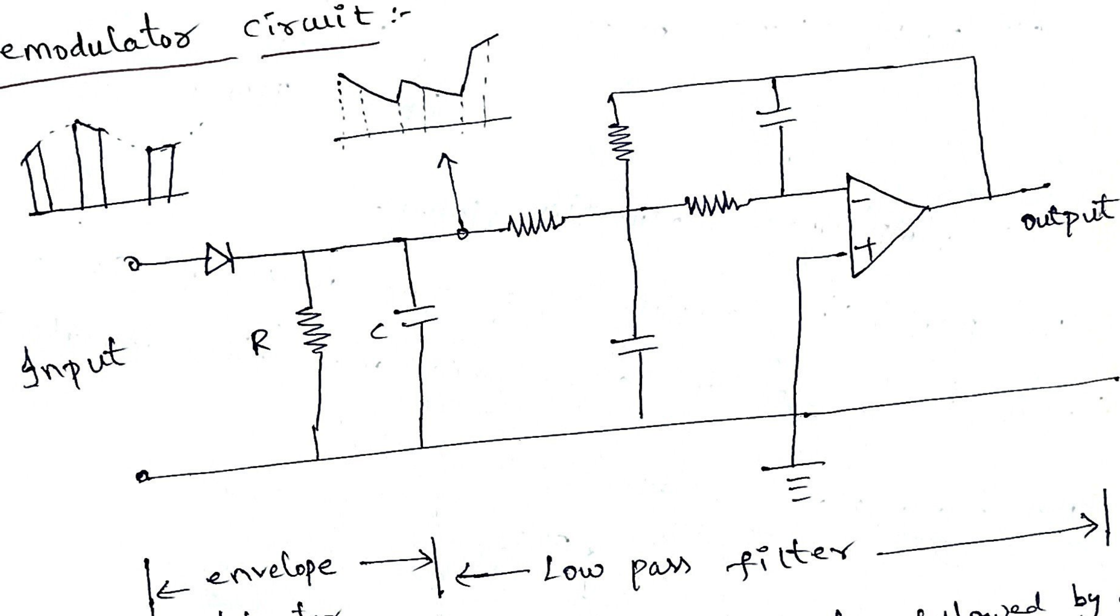
This circuit is a simple emitter follower. In the absence of the clock signal, the output follows the input. Modulating signal is applied as the input signal.

Another input to the base of the transistor is clock sign

when the clock signal is high, the circuit behaves as an emitter follower, and the output follows the input modulating signal.

when clock is low, the transistor is cut-off and the output is zero. Thus the output waveforms is desired pulse amplitude modulated signal.

PAM Demodulator circuit :-



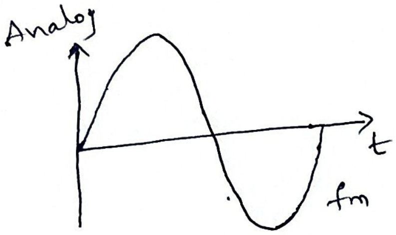
The PAM demodulator is just an envelope detector followed by a low pass filter. The diode and R-C combination work as the envelope detector which will pick the envelope of the signal accordingly charging & discharging of capacitor.

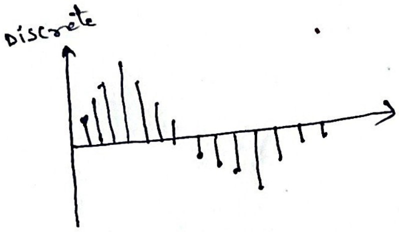
This is followed by a second order op-Amp low pass filter to have a good filtering characteristic. Thus, for received pulse amplitude modulated signal as the input signal, the desired demodulated signal i.e; base band signal is the output.

Sampling theorem (or) process:-

A continous band limited signal of highest frequency component 'f_m' can be converted into discrete with rate of frequency to be 'f_s' ; $f_s \geq 2 f_m$ (f_s = sampling frequency).

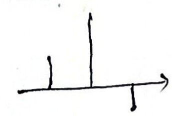
The recovered signal can be obtained from a low pass filter



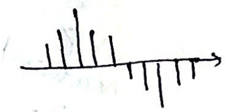


Sampling cases:-

1) under sampling $f_s > 2 f_m$



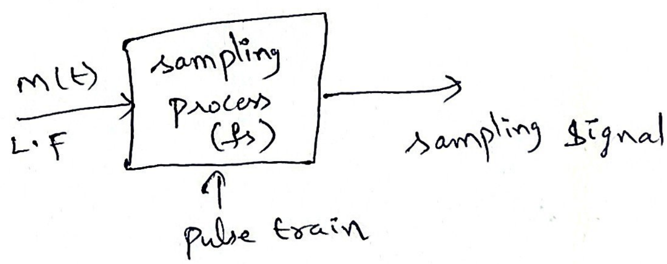
2) perfect sampling $f_s = 2 f_m$



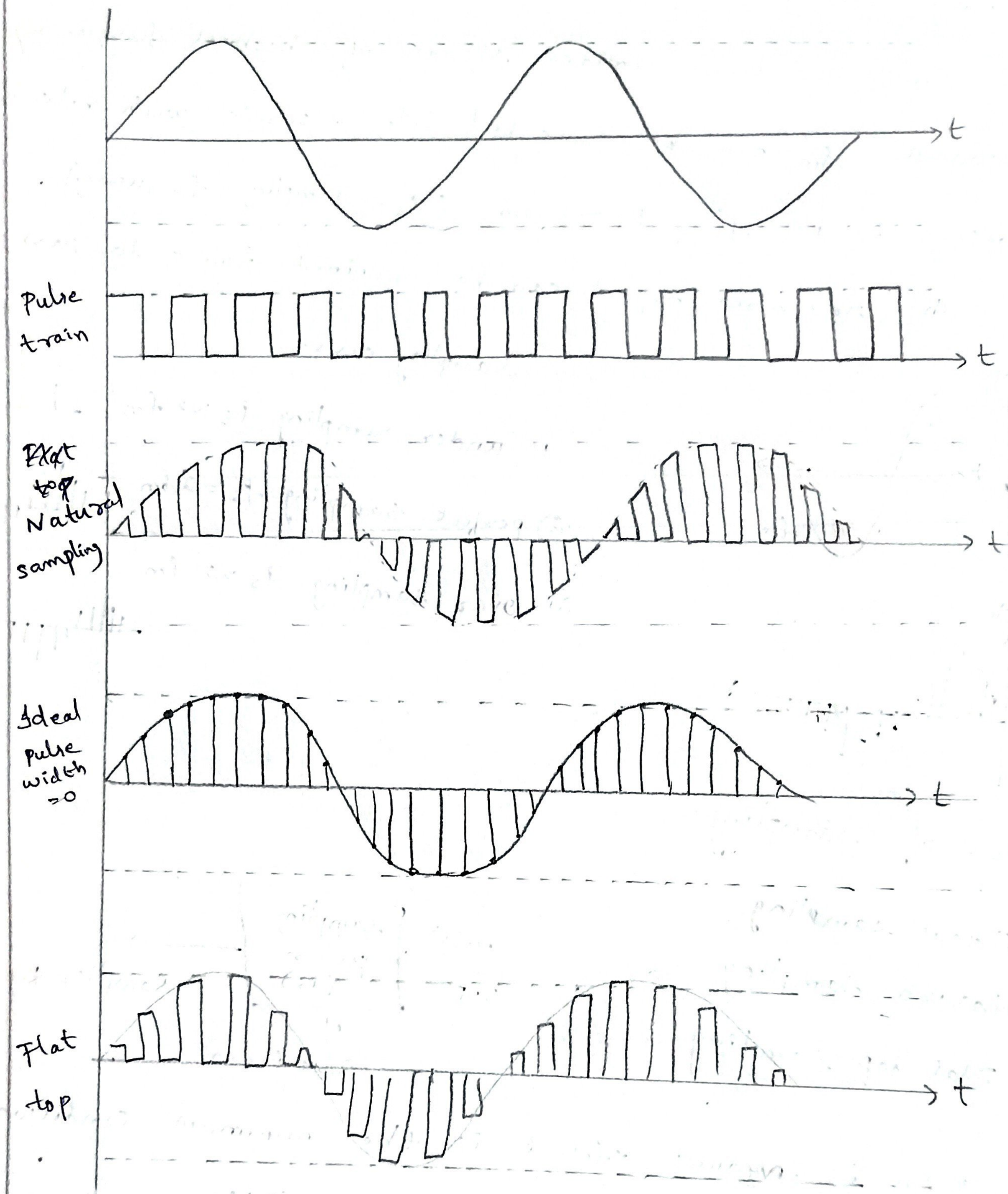
3) over sampling $f_s > 2 f_m$

Types of sampling:-

1) Ideal sampling
2) Natural sampling
3) Flat top sampling

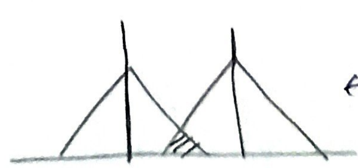


$f_s = 2 f_m$ is Nyquist effect is the minimum condition to get back the filters using lowpass filter.



Aliasing:

Aliasing is a phenomenon that a reconstructed signal from samples of the original signal contains low frequency components that are not present in the original one.



← under sampling

$f_s < 2 f_m$.

Time - Divison Multiplexing (TDM) :-

The sampling theorem provides the basis for transmitting the information contained in a band-limited message signal m(t) as a sequence of samples of m(t) taken uniformly at a rate slightly higher than the nyquist rate. An important feature of the sampling process is a conservation of time. That is, the transmission of the message samples engages the communication channel for only on a fraction of the sampling interval on a periodic basis, and in this way some of the time interval between adjacent samples is cleared for nsec by other independent message sources on a time-shared basis. we therefore obtain a time-division multiplexing (TDM) system, which enables the joint utilization of a common communication channel by a number of independent message sources without mutual interference among them.

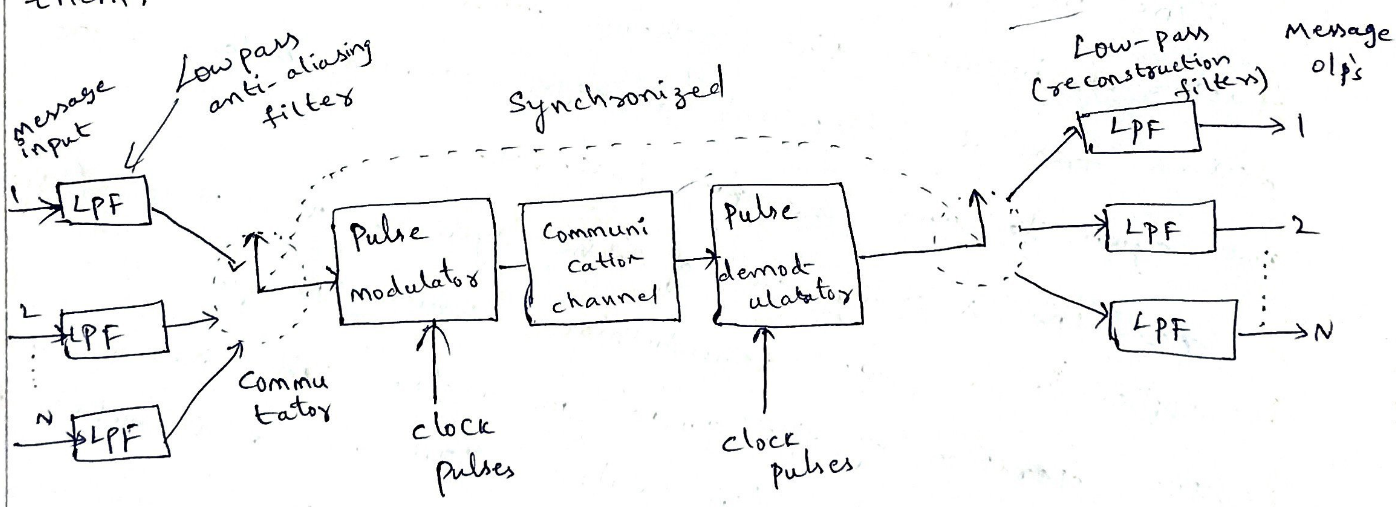


fig: Block diagram of TDM System.

The concept of TDM is shown in above fig. Each i/p message signal is first restricted in bandwidth by a low-pass anti-aliasing filter to remove the frequencies, that are nonessential to an adequate signal representation.

The low-pass filter outputs are then applied to a commutator. the function of the commutator is two fold:

(i) To take a narrow sample of each of the N i/p messages at a rate f_s that is slightly higher than $2W$, where W is the cut off frequency of the anti-aliasing filter, and

(ii) To sequentially interleave these N samples inside the sampling interval T.

inserting

Following the communication process, the multiplexed signal is applied to a pulse modulator, the purpose of which is to transform the multiplexed signal into a form suitable for transmission over the common channel.

The use of TDM introduces, a bandwidth expansion of factor N, because the scheme must squeeze N samples derived from N independent message sources into a time slot equal to one sampling interval.

At the receiving end of the system, the received signal is applied to a pulse demodulator, which performs the reverse operation of the pulse modulator. The narrow samples produced at the pulse demodulator output are distributed to the appropriate low-pass reconstruction filters by means of a decommutator, which operates in synchronism with the commutator in the transmitter

Band

Bandwidth noise Trade - off :

The capacity of channel is $C = B\log_2\left[1 + S/N\right]$

Capacity of noise less channel:

$C = B\log(1+SNR)$

when max power = 0

$\Rightarrow \boxed{SNR = \alpha}$

$= B\log(1+\alpha)$

$C = \alpha$

→ If ther is less noise then signal information will be more & hence SNR will be more

$\frac{S\uparrow}{N\downarrow} \rightarrow SNR\uparrow$

Capacity of infinite Bw channel:

As Bandwidth increses, noise power increses, SNR decreses
($N_o B$)

$\frac{S}{N} = SNR.$

As $B.W \rightarrow \alpha.$

$\frac{1}{x}\log(1+x)$

$= \log(1+x)^{1/x}$

$C = B\log_2\left(1+\frac{S}{N}\right)$

$= \frac{S}{N_o} \cdot \frac{N_o B}{S} \log_2\left(1+\frac{S}{N_o B}\right)$

$= \frac{S}{N_o} \log_2\left[1+\frac{S}{N_o B}\right]^{N_o B/S}$

$= \frac{S}{N_o} \log_2\left(1+\frac{S}{N_o B}\right)^{1/ S/N_o B}$

As $Bw \rightarrow \alpha$, $C_\alpha = \underset{B\rightarrow\alpha}{lt} \cdot C$

let $x = \frac{S}{N_o B}$ then as $B \rightarrow \alpha$, $x \rightarrow 0$

$C_\alpha = \frac{S}{N_o} \underset{x\rightarrow 0}{lt} \log_2(1+x)^{1/x}$

we know that $\ln x \log_2^e = \log_2^x$

$$C_\infty = \frac{S}{N_0} \lim_{x \to 0} \ln(1+x)^{1/x} \log_2^e$$

$$= \frac{S}{N_0} \lim_{x \to 0} \frac{1}{x} \ln(1+x) \log_2^e$$

$$= \frac{S}{N_0} \lim_{x \to 0} \frac{\ln(1+x)}{x} \log_2^e$$

$$= \frac{S}{N_0} \lim_{x \to 0} \frac{\frac{d}{dx}\ln(1+x)}{\frac{d}{dx}(x)} \log_2^e$$

$$= \frac{S}{N_0} \lim_{x \to 0} \frac{1/1+x}{1} \cdot \log_2^e$$

$$C_\infty = \frac{S}{N_0} \log_2^e. \qquad \log_2 = 1.44$$

$$\boxed{C_\infty = 1.44\, S/N_0}$$

As bandwidth noise trade off refers to the concept in many systems, increasing the bandwidth (range of frequencies) a system can handle will also increase the amount of noise capature, meaning a higher SNR becomes harder to achieve, escentially the more frequencies are allowed in more likely to pick up unwanted noise along the desired signal.

Quantization process :-

A continous signal, such as voice, has a continous range of amplitude and therefore its sample have a continous amplitude range i.e, with in the finite amplitude range of the signal, we find an infinite number of amplitude levels.

Quantization is defined as the process of transforming the sample amplitude $m(nT_s)$ of a message signal $m(t)$ at time $t=nT_s$ into a discrete amplitude $v(nT_s)$ taken from a finite set of possible amplitudes.

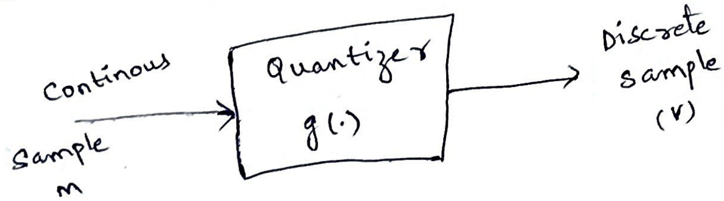


fig: quantizer

In the process of quantization, a signal $m_q(t)$ which is an approximation to $m(t)$ is generated.

Consider a baseband signal $m(t)$ varying between the lower limit V_L and the upper limit V_m as shown in fig, below

Now the base band signal is applied to a quantizer

The response of a quantizer is a stair-case waveform. The wave form $m_q(t)$ is having an advantage that it is free from additive noise.

The baseband signal $m(t)$ is divided into 'M' no. of levels and the separation b/w two successive levels be 'S'.

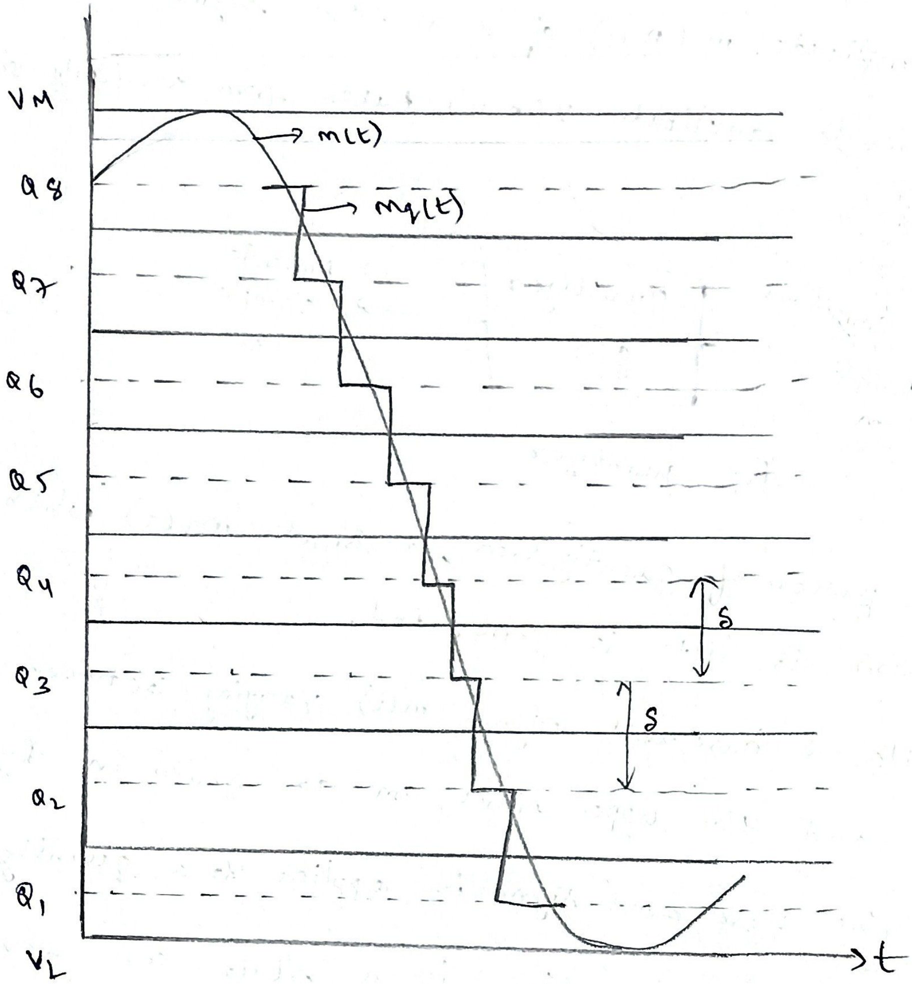
This 'δ' is referred to as step size or quantum.

$$q = M = \frac{V_H - V_L}{\delta}$$

If n = no. of bits in a word

$$\boxed{q = M = 2^n}$$ M = no. of levels.

Between every two successive levels, locate a dashed level. These levels are named as quantization levels. The o/p of the quantizer will always be at any one of quantization levels.



step size is constant → uniform quantization

step size is varied → Non uniform quantization

The quantizer o/p will make an abrupt transition from one quantization level to the other only at the point where the base band signal crosses the level represented by the solid pot.

Two successive q-levels are separated by the step size δ. The o/p of the quantizer is the quantization level to which the base band signal is closest.

Quantization can be two types:

1) Uniform quantization and 2) Non uniform quantization.

Uniform quantization :- In uniform quantization, the quantization step or difference between two quantization levels remains constant over the complete amplitude range. Depending upon the transfer characteristic there are three types of uniform quantizer

1) Midtread quantizer
2) Mid riser quantizer
3) Biased quantize.

Non- uniform quantization :- In non uniform quantization, the quantization step or difference between two quantization levels varied over amplitude range.

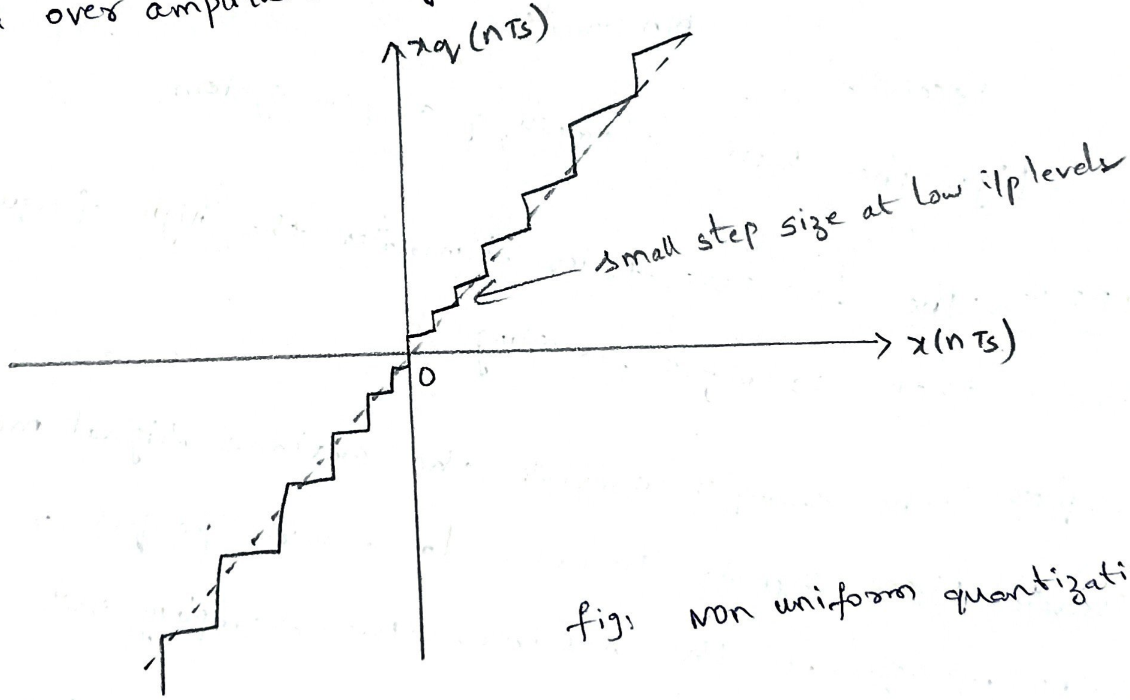
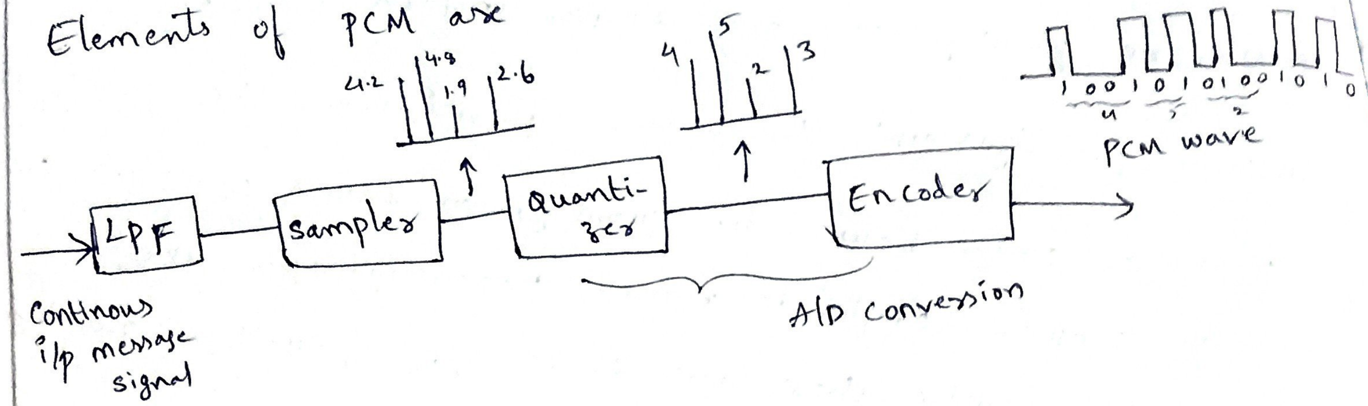
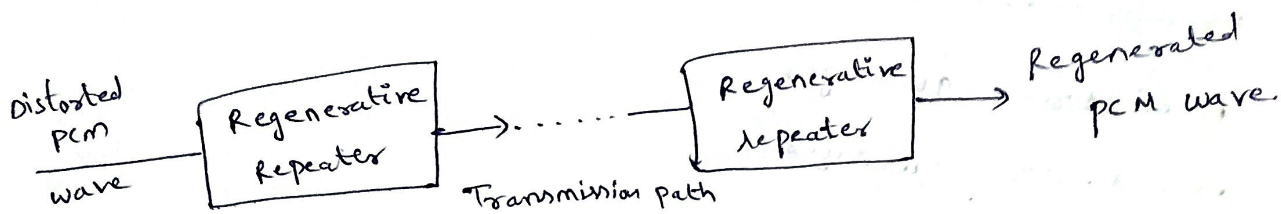


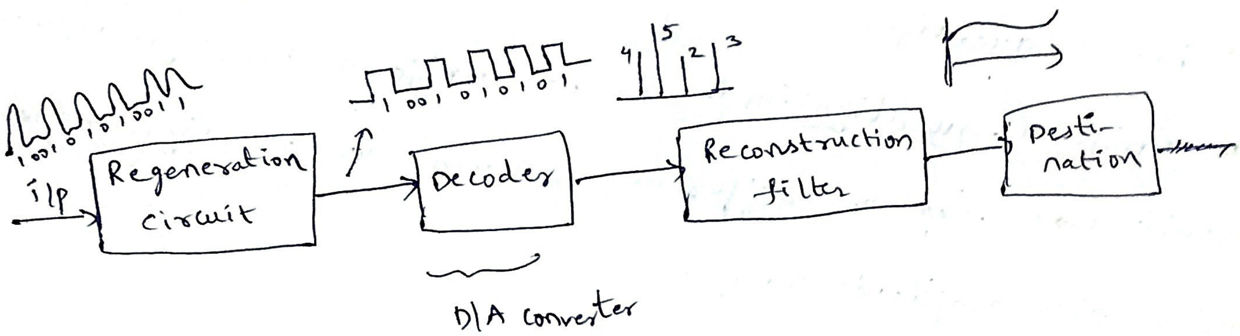
fig: Non uniform quantization

Pulse Code Modulation (PCM) :-

Elements of PCM are







Receiver

fig: The basic elements of a PCM system

LPF:- The low pass filter eliminates the high frequency greater than w before sampling

Sampler:- The sampler converts the continous signal m(t) into a discrete time signal. In order to ensure perfect reconstuction of the message at the receiver, the sampling rate must be greater than twice the highest frequency component, w of the message wave in accordance with the sampling theorem.

Basically sampling involves the multiplication of the base band signal with an impulse train which is periodic with period Ts.

The sampled signal at each sampling instant is equal to base signal at that instant. i.e, the instant at which the impulse occurs.

Quantization:- Quantization is the process which makes a digitial communication system to have superior noise performance than the analog communication system.

The effect of additive noise on the signal being transmitted in the channel is reduced by using quantization.

In the process of quantization, a signal being transmitted $m_q(t)$ which is an approximation to $m(t)$ is generated.

Encoding:-

The encoder converts the i/p discrete time signal to 'n' digits binary word. Thus the o/p of quantizer is converted to 'n' binary bits. The encoder is also called digitizer.

If 'n' is the no. of bits in a code word then the total no. of code words $= 2^n$.

Line coding:- 1) unipolar non return to zero (NRZ)

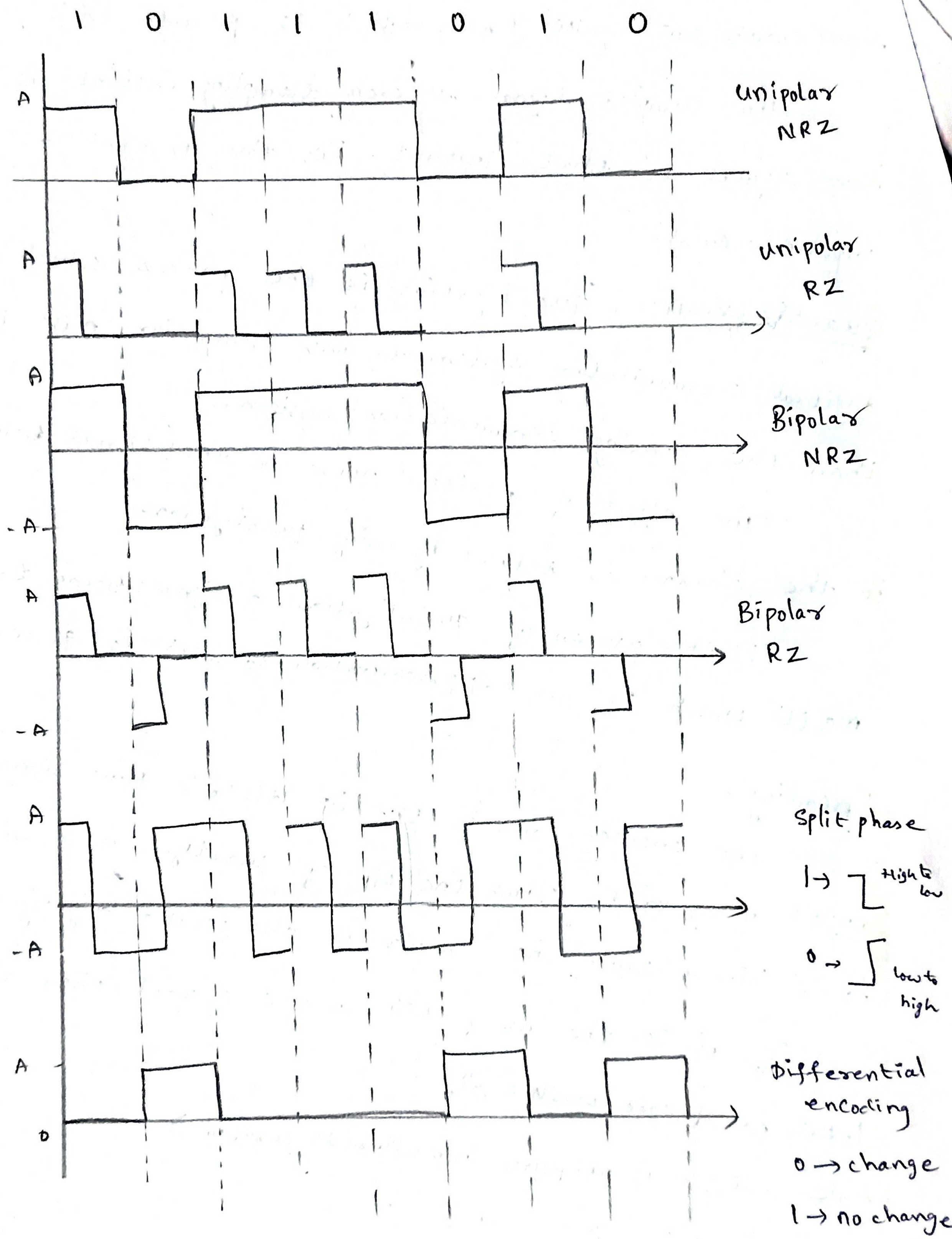
2) unipolar return to zero (RZ)

3) Bipolar non return to zero (NRZ)

4) Bipolar return to zero (RZ)

5) split phase manchester encoding

6) Differential encoding



1 0 1 1 1 0 1 0
A
Unipolar NRZ
A
Unipolar RZ
A
-A
Bipolar NRZ
A
-A
Bipolar RZ
A
-A
Split phase
1 → High to low
0 → low to high
A
0
Differential encoding
0 → change
1 → no change

Regeneration :- The most important feature of pcm system lies in the ability to contol the effects of distortion and noise produced by transmitting a pcm wave through a channel. This capability is accomplished by reconstucting the pcm wave by means of a chain of regenerative repeaters located at sufficiently close spacing along the transmission path.

Decoding :- The first operation in the receiver is to regenerate the received ~~path~~ pulses. These clean pulses are then regrouped into code words and decoded. The decoding process involves generating a pulse the amplitude of which is the linear sum of all the pulses in the code word, with each pulse weighted by its place value ($2^0, 2^1, 2^2, 2^3 ---$) in the code

Ex: $101 \rightarrow 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 4 + 0 + 1 = 5 \rightarrow$ pulse amplitude

Filtering :- The final operation in the receiver is to recover the signal wave by passing the decoder o/p through a LP reconstruction filter whose cut off freq is equal to same as message bandwidth W.

Regenerative repeater :-

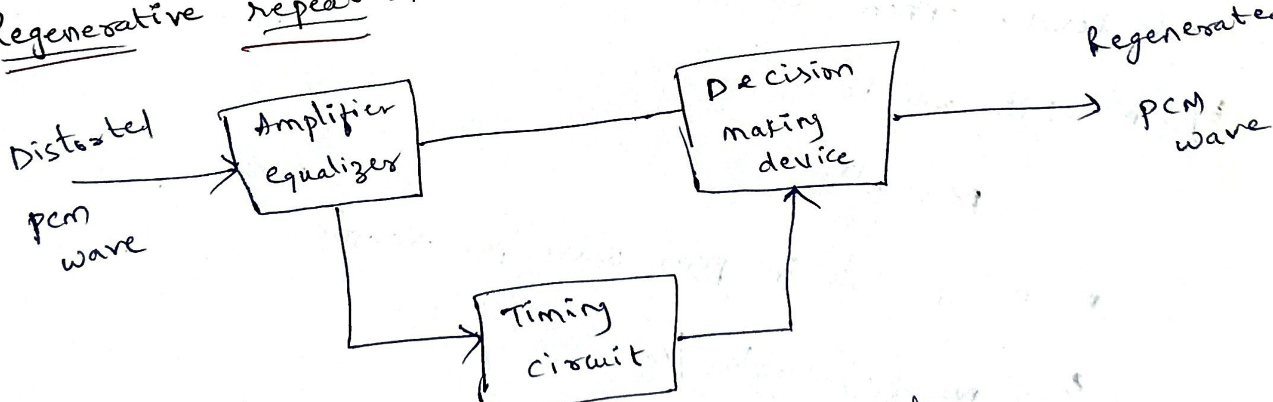


Fig: Block diagram of regenerative repeater

The equalizer shapes the received pulses so as to compensate for the effects of amplitude and phase distortions produced by the channel.

The timing circuity provides a periodic pulse train, derived from the received pulses, for sampling the equalized pulses at the instants of time where the signal-to-noise ratio is a maximum.

Each sample so extracted is compared to a predetermined threshold in the decision-making device as follows.

* If received signal $>$ Threshold $\Rightarrow$ symbol '1'

$\rightarrow$ A pulse of amplitude 'A' volts is transmitted to the next repeater.

* If received signal $<$ threshold $\Rightarrow$ symbol '0'. then no pulse is transmitted.

Transmission Bandwidth in PCM :-

Let the quantizer use 'n' number of binary digits to represent each level. Then the number of levels that can be represented by 'n' digits will be

$$M = 2^n$$

n = no. of bits in pcm wave per sample.

f_s = No. of samples /sec.

No. of bits per sec = (no. of bits / sample) × (No. of samples / sec)

$= n \times f_s$

Sampling rate (r) :- The no. of bits per second is also called signalling rate of PCM.

Sampling rate = bit rate (r) = $n f_s$ where $f_s > 2W$

$$\boxed{\text{Sampling rate } (r) = n f_s}$$

Bandwidth of PCM :- Bandwidth needed for PCM transmission will be given by half of the signaling rate i.e.,

$$B.W = B_T = \begin{cases} B_T \geq \frac{1}{2} r \\ B_T \geq \frac{1}{2} n f_s \\ B_T \geq n W \end{cases}$$

r = sampling rate ; n = no. of bits / sample

W = message bandwidth ; f_s = sampling frequency

SNR in PCM:

$\left(\frac{S}{N}\right)_{dB} = 1.8 + 6V$ → for sinusoidal signal

$\left(\frac{S}{N}\right)_{dB} = 4.8 + 6V$ → for general case

Noise considerations in PCM systems:-

In PCM system there are two major sources of noise are

1) channel noise :- It is introduced anywhere between the transmitter o/p and the receiver i/p. channel noise is always present, once the equipment is switched ON.

2) Quantization noise :- which is introduced in the transmitter and is carried all the way along to the receiver o/p. unlike channel noise, quantization noise is a signal dependent in the sense that it dissappears when the message signal is switched, off.

Delta Modulation :-

Operating Principle :-

Delta Modulation transmits only one bit per sample. That is the present sample value is compared with the previous sample value and the indication, whether the amplitude is increased or decreased is sent

→ I/p signal x(t) is approximated to step signal by the delta modulator. This step size is fixed. The difference between the input signal x(t) and staircase approximated signal confined to two levels i.e., $+\delta$ & $-\delta$

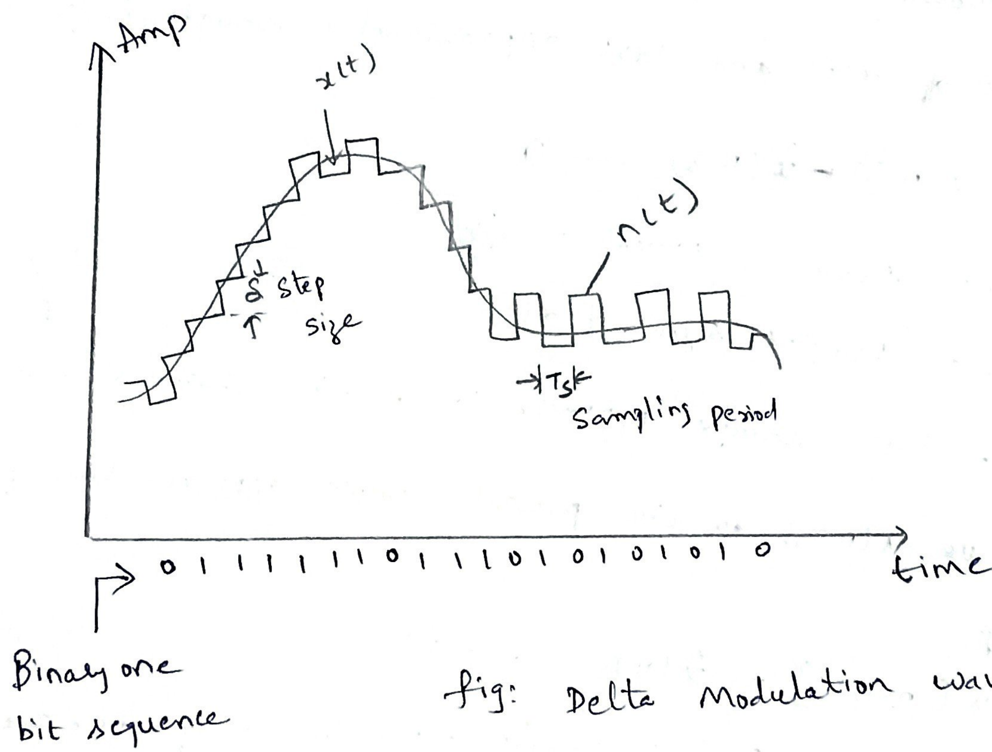


fig: Delta Modulation wave form.

→ If the difference is positive, then approximated signal increased by one step i.e. 'δ'. If the difference is negative, then approximated signal is reduced by 'δ'.

→ when the step is reduced, '0' is transmitted and if the step is increased '1' is transmitted. Thus for each sample, only one binary bit is transmitted. Fig above shows the analog signal $x(t)$ and its staircase approximated signal by the delta modulator.

Mathematical analysis :-

The principle of delta modulation can be explained by the following set of equations. The error between the sampled value of $x(t)$ and last approximated sample is given as

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \longrightarrow ①$$

Here $e(nT_s)$ = Error at present sample

$x(nT_s)$ = sampled signal of $x(t)$

$\hat{x}(nT_s)$ = Last sample approximation of the staircase waveform

we can call ~~us~~ $u(nT_s)$ as the present sample approximation of staircase o/p

$\hat{x}(nT_s)$ is step size

then $u[(n-1)T_s] = \hat{x}(nT_s) \longrightarrow ②$

step signal = Last sample approximation of stair case waveform

Let the quantity $b(nT_s)$ be defined as

$$b(nT_s) = \delta \; \text{sgn}[e(nT_s)] \longrightarrow ③$$

This is depending on the sign of error $e(nT_s)$ the sign of step size δ will be decided. In other words

$$b(nT_s) = +\delta \quad \text{if} \quad x(nT_s) \geq \hat{x}(nT_s)$$

$$= -\delta \quad \text{if} \quad x(nT_s) < \hat{x}(nT_s) \longrightarrow (4)$$

If $b(nT_s) = +\delta$; binary '1' is transmitted

$b(nT_s) = -\delta$ binary '0' is transmitted

T_s = sampling interval.

DM transmitter :-

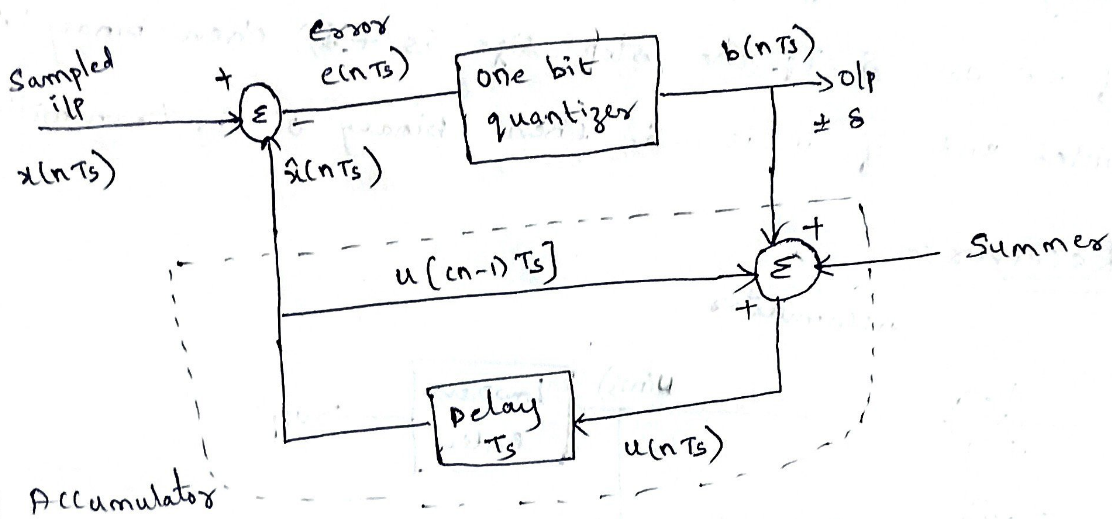


fig (a) DM transmitter

Fig (a) shows the transmitter the summer in the accumulator adds quantizer o/p $(\pm\delta)$ with the previous sample approximation. This gives present sample approximation i.e.,

$$u(nT_s) = u[nT_s - T_s] + [\pm\delta] \quad \text{or} \longrightarrow (5)$$

$$= u[(n-1)T_s] + b(nT_s)$$

The previous sample approximation $u[(n-1)T_s]$ is restored by delaying one sample period T_s. The sampled i/p signal $x(nT_s)$ and staircase approximation signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$.

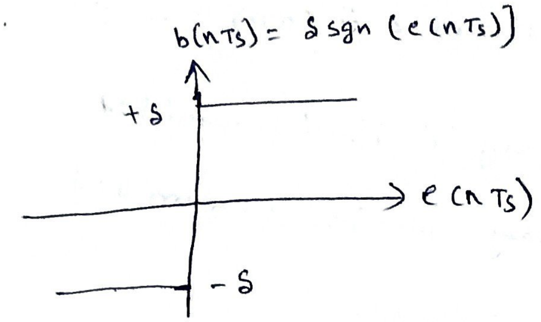


fig (b) : Input output characteristics of quantizer for DM system.

Depending on the sign of $e(nT_s)$ one bit quantizer produces an o/p step of $+\delta$ or $-\delta$. If the step size is $+\delta$, then binary '1' is transmitted and if it is $-\delta$, then binary '0' is transmitted.

DM Receiver:-

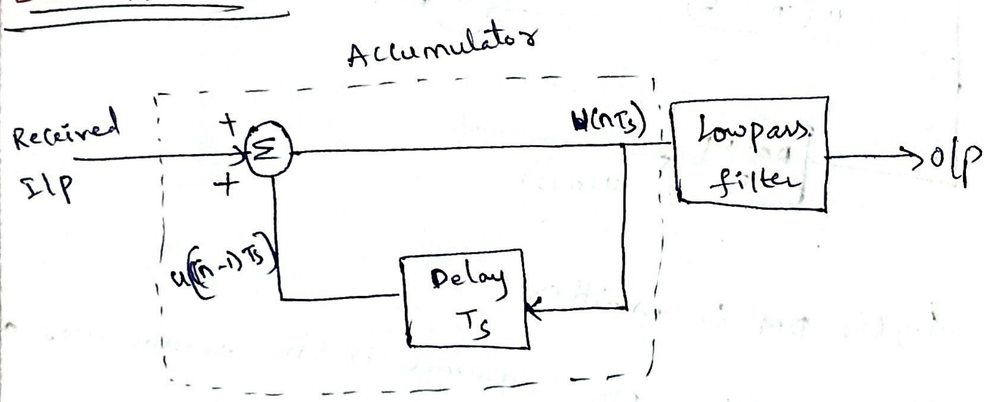


fig (c) Delta Modulator receiver

DM receiver is shown in fig (c). The accumulator generates the staircase approximated signal o/p and is delayed by one sampling period T_s. It is then added to the i/p signal.

If i/p is binary '1' then it adds +S step to the previous o/p (which is delayed). If i/p is binary '0' then one step 'S' is subtracted from the delayed signal as shown in Eqn (5)

The Low-pass filter has the cutoff frequency equal to highest frequency in x(t). This filter smoothens the stair case signal to reconstruct x(t).

Advantages:-

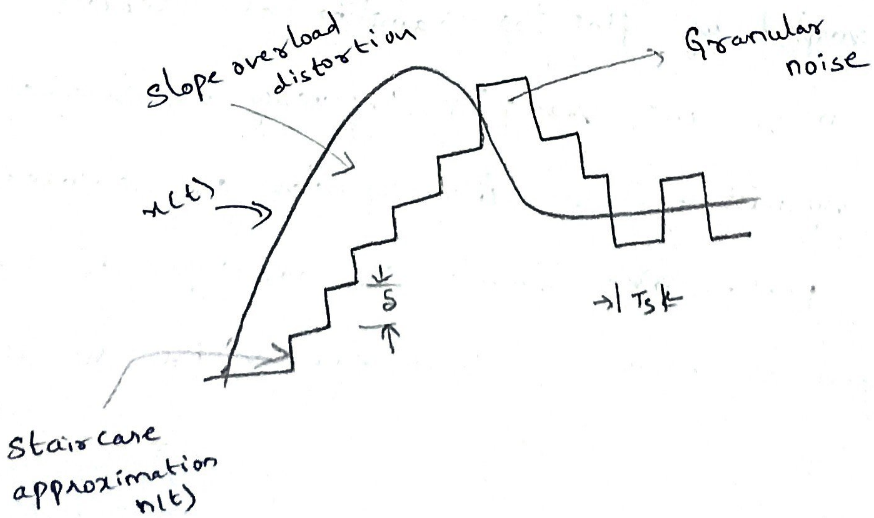
1) DM transmits only one bit for one sample. Thus the transmission channel bandwidth is quite small for delta modulation.

2) The transmitter and receiver implementation is very much simple for DM. There is no analog to digital converter involved in DM.

Disadvantages:-

1) slope overload distortion (startup error)

2) granular noise (hunting).



Differential pulse Code Modulation :-

If the sampling frequency is selected to be higher, than nyquist rate, then the samples of a signal are highly correletad with each other. This is because any signal does not change fast. That is its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with little difference. when these samples are encoded by standard PCM system, the resultant encoded signal contains redundant information.

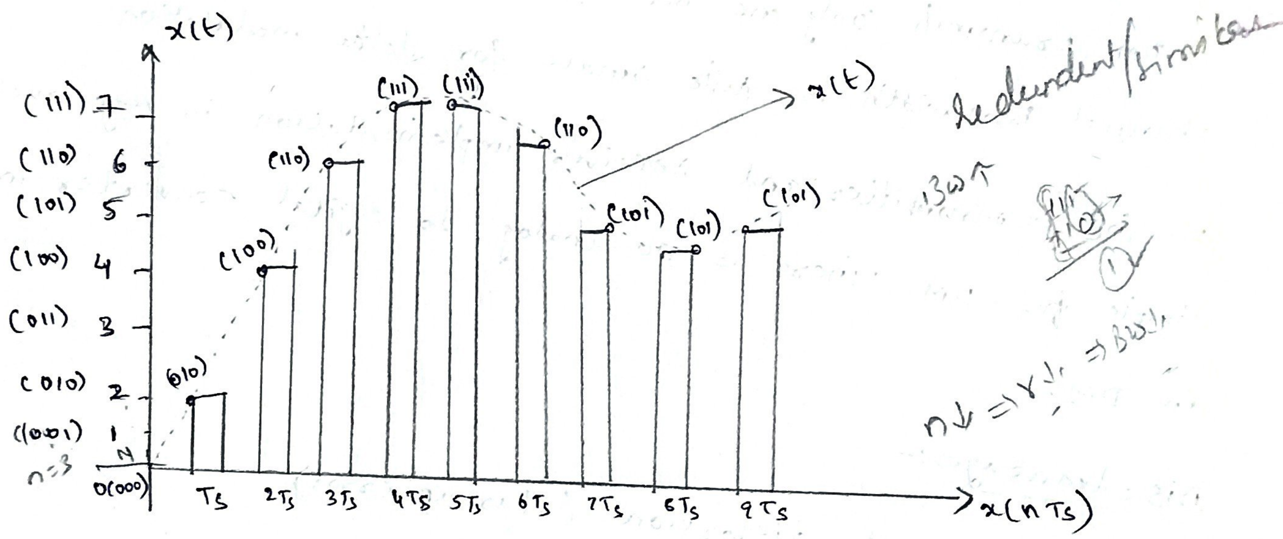


fig: Redundant information in PCM

Figure above shows a continuous time signal x(t) by dotted line. This signal is sampled by flat top sampling at intervals Ts, 2Ts, 3Ts ..... nTs. The sampling frequency is selected to be higher than nyquist rate. The samples are encoded by using 3 bit level PCM. The sample is quantized to the nearest digital level as shown by small circles in the diagram

The encoded binary value of each sample is written on the top of the samples. we can see from fig. that the samples taken at $4T_s$, $5T_s$ and $6T_s$ are encoded to same value of (110). This information can be carried only by one sample. But three samples are carrying the same information means it is redundant.

Consider another example of samples taken at $9T$ and $10T_s$. The difference between these samples is only due to last bit and first two bits are redundant, since they do not change.

Principle of DPCM :-

If this redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced. This type of digital pulse modulation scheme is called Differential pulse code modulation (DPCM).

If n reduces, r → reduces ⇒ B.W is reduce
Sampling rate

DPCM Transmitter :-

The DPCM works on the principle of prediction. The value of the present sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual sample value.

Figure below shows the transmitter of DPCM system. The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator finds out the difference between $x(nT_s)$ and $\hat{x}(nT_s)$. This is called error and it is denoted by $e(nT_s)$.

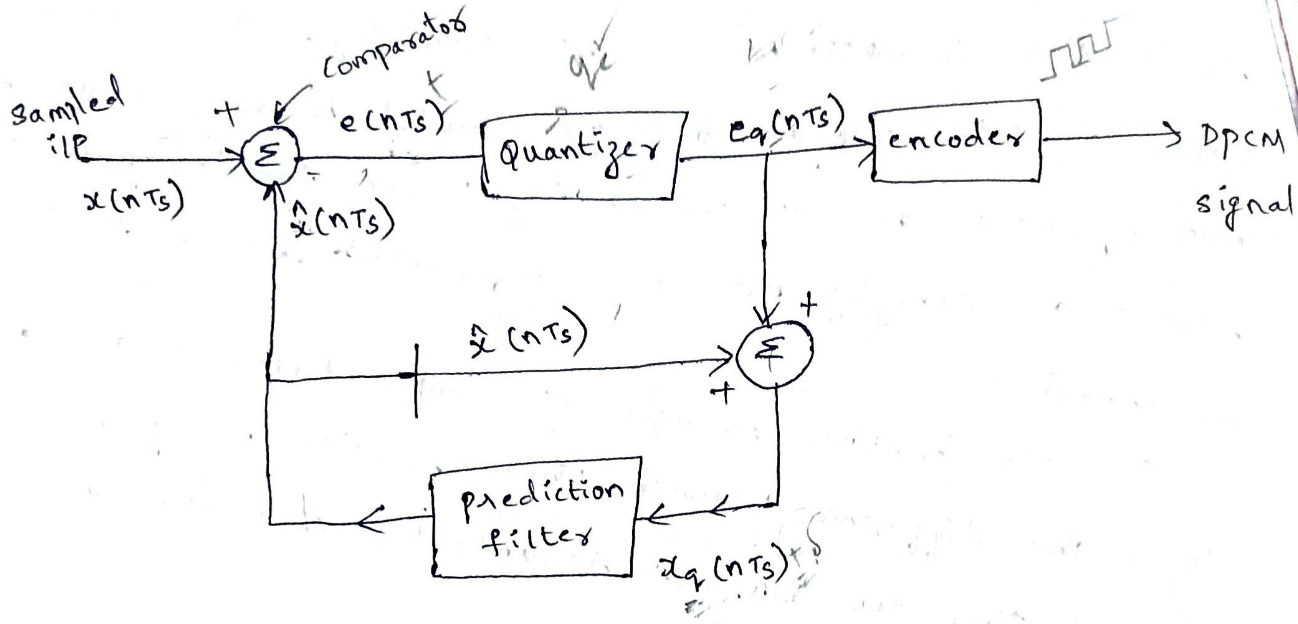


fig: DPCM transmitter

→ The error is given by

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \longrightarrow (1)$$

The prediction value is produced by using a prediction filter. The quantizer o/p signal $e_q(nT_s)$ are previous prediction is added and given as i/p to the prediction filter.

This signal is called $x_q(nT_s)$. This makes the prediction more and more lose to the actual sampled signal. we can see that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small number of bits. Thus number of bits per sample are reduced in DPCM.

The quantizer output can be written as,

$$e_q(nT_s) = e(nT_s) + q(nT_s) \longrightarrow (2)$$

Here $q(nT_s) \rightarrow$ quantization error.

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \rightarrow (3)$$

By substituting eqn (2) in eq (3).

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \rightarrow (4)$$

$x_q(nT_s) \rightarrow$ quantized version of original signal.

from eqn (1)

$$e(nT_s) + \hat{x}(nT_s) = x(nT_s) \rightarrow (5)$$

Substitute eqn (5) in eq (4)

$$x_q(nT_s) = x(nT_s) + q(nT_s) \rightarrow (6)$$

Thus the quantized version of the signal $x_q(nT_s)$ is the sum of original sample value and quantization error, $q(nT_s)$. The quantization error can be +ve or -ve.

Reconstruction of DPCM signal:

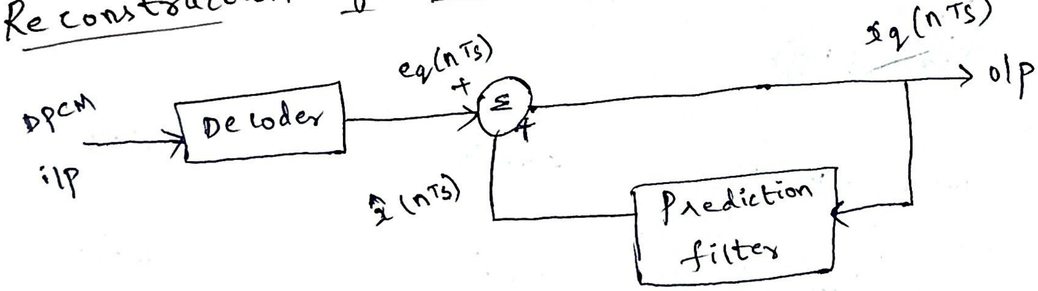


fig: DPCM receiver

Figure shows the block diagram of DPCM receiver. The decoder first reconstructs the quantized error signal $e_q(nT_s)$ from incoming binary signal. The prediction filter o/p and quantized error signals are summed up to give the quantized version of the original signal. Thus the signal at the receiver differs the original signal.

from actual signal by quantization error $q(nT_s)$, which is introduced permanently in the reconstructed signal.

Comparision of Digital pulse Modulation methods:

Parameters	PCM	DM	DPCM
1. No. of bits	'n' bits (2, 4, 6, 8 ......)	only one bit	In between 'n' and one. More than 1, less than n.
2. Levels step size	$m = 2^n$ step size fixed	two levels either 1 or 0 step size is fixed.	Fixed no. of levels
3. Quantization error and distortion	Quantization error depends on no. of levels used	two types 1. slope - over 2. hunting	two types 1. slope - over 2. Hunting & quantization noise
4. Bandwidth	highest b.w $B.W = \frac{1}{2} n f_s$	Low bandwidth	Moderate
5. Feedback	No feed back	exists	exists
6. Complexity	Complex	simple	simple
7. S/N	Good	POOR	fair
8) Area of application	Audio & video telephony	Speech & images	speech & videos.

UNIT-II

BASEBAND PULSE TRANSMISSION

Introduction:

There are basically two types of transmission of digital signals. They are

① **Baseband data transmission:-**

The digital data is transmitted over the channel directly. There is no carrier or any modulation. This is suitable for transmission over short distance. Draw back of this is Intersymbol Interference (ISI).

② **Passband data transmission:-**

The digital data modulates high frequency sinusoidal carrier. Hence it is also called digital CW modulation. It is suitable ~~for~~ for transmission over long distances.

In this chapter we study the transmission of digital data over a baseband channel. Digital data have a broad spectrum with a significant low-frequency content. Baseband transmission of digital data therefore requires the use of a low-pass channel with a bandwidth large enough to accommodate the essential frequency content of the data stream. Typically, however, the channel is dispersive in that its frequency response deviates from that of an ideal low-pass filter. The result of data transmission over such a channel is that each received pulse is affected somewhat by adjacent pulses, thereby

- giving rise to a common form of interference called Intersymbol Interference (ISI). ISI is a major source of bit errors in the reconstructed data stream at the receiver o/p.

Another source of bit errors in a baseband data transmission system is channel Noise. Naturally, noise and ISI arise in the system simultaneously.

Now we deal with the detection of a pulse signal of known waveform that is immersed in additive white noise. The device for the optimum detection of such a pulse involves the use of a linear - time - invariant filter known as a matched filter. It is called matched filter because its impulse response is matched to the pulse signal.

Matched Filter

Matched filter is a linear - time - invariant filter, whose ~~impulse~~ impulse response is matched to the pulse signal.

A basic problem that often arises in the study of communication systems is that of detecting a pulse transmitted over a channel that is corrupted by channel noise (i.e., additive noise at the front end of the receiver).

Consider the receiver model shown in figure below, involving a linear time - invariant filter of impulse response $h(t)$. The filter input $x(t)$ consists of a pulse signal $g(t)$ corrupted by

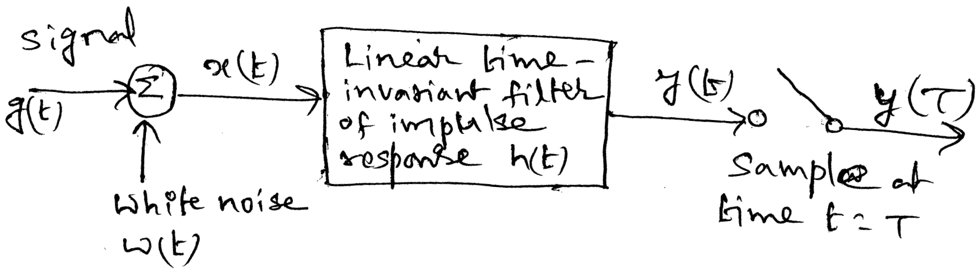


fig: Linear receiver

- additive channel noise $w(t)$, as shown by

$$x(t) = g(t) + w(t), \quad 0 \le t \le T \longrightarrow (1)$$

where T is an arbitrary observation interval. The pulse signal $g(t)$ may represent a binary symbol 1 & 0 in a digital communication system. $w(t)$ is the sample function of a white noise process of zero mean and power spectral density $N_0/2$. It is assumed that the receiver has knowledge of the waveform of the pulse signal $g(t)$. The function of the receiver is to detect the pulse signal $g(t)$ in an optimum manner, given the received signal $x(t)$. To satisfy this requirement we have to optimize the design of the filter so as to minimize the effects of noise at the filter o/p in some statistical sense, and thereby enhance the detection of the pulse signal $g(t)$.

Since the filter is linear, the resulting o/p $y(t)$ may be expressed as

$$y(t) = g_0(t) + n(t) \longrightarrow (2)$$

where $g_0(t)$ is o/p produced by the signal $g(t)$ and $n(t)$ is o/p produced by the ~~s~~ noise ~~w~~ $w(t)$. ~~To make $g_0(t) > n(t)$ we have to maximiz~~

$g_0(t)$ can be made greater than $n(t)$ by maximiz the peak pulse signal-to-noise ratio, defined by

$$\eta = \frac{|g_0(T)|^2}{E[n^2(t)]} \longrightarrow (3)$$

where $|g_0(T)|^2$ is the instantaneous power in the o/p signal at $t = T$. , $E \to$ expectation operator.

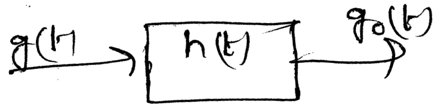
$E[n^2(t)]$ is a measure of the average o/p noise power.

The requirement is to specify the impulse response $h(t)$ of the filter such that the o/p signal-to-noise ratio in eq (3) is maximized.

Let $g(t) \xrightarrow{FT} G(f)$

$h(t) \xrightarrow{\text{Freq. resp.}} H(f)$

Then the ~~o/p~~ Fourier transform of the o/p signal $g_0(t)$ is equal to $H(f)\,G(f)$

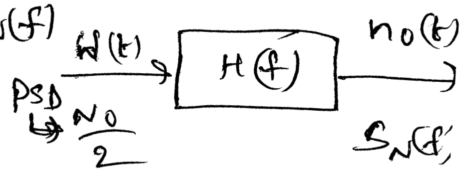


i.e $G_0(f) = H(f)\,G(f)$

$$\Rightarrow g_0(t) = IFT\{G_0(f)\} = \int_{-\infty}^{\infty} H(f)\,G(f)\,e^{j2\pi ft}\,df \longrightarrow (4)$$

Consider next the effect ~~of~~ on the filter o/p due to the noise $w(t)$ acting alone.

The power spectral density ~~of~~ $S_N(f)$ of the o/p noise $n(t)$ is



$$S_N(f) = \frac{N_0}{2}\,|H(f)|^2 \longrightarrow (5)$$

where $\frac{N_0}{2} \to$ PSD of white noise $w(t)$

The average power of the o/p noise $n(t)$ is

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)\,df$$

$$\Rightarrow E[n^2(t)] = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df \rightarrow (6)$$

From eq (4) when the filter o/p is sampled at $t = T$,

$$|g_0(T)|^2 = \left| \int_{-\infty}^{\infty} H(f) G(f) e^{(j2\pi f T)} df \right|^2 \rightarrow (7)$$

By substituting eq (6) & (7) into eq (5), we may write the peak pulse signal-to-noise ratio as

$$\eta = \frac{\left| \int_{-\infty}^{\infty} H(f) G(f) e^{j2\pi f T} df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df} \rightarrow (8)$$

our problem is to find, for a given $G(f)$, the particular form of the frequency response $H(f)$ of the filter that makes η a maximum.

By applying Schwarz' inequality to above eq

consider two complex functions $\phi_1(x)$ and $\phi_2(x)$ in real variable x, satisfying the conditions

$$\int_{-\infty}^{\infty} |\phi_1(x)|^2 dx < \infty$$

and $$\int_{-\infty}^{\infty} |\phi_2(x)|^2 dx < \infty$$

then we may write

$$\left| \int_{-\infty}^{\infty} \phi_1(x) \phi_2(x) dx \right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(x)|^2 dx \int_{-\infty}^{\infty} |\phi_2(x)|^2 dx \quad (9)$$

- Eq (9) holds if and only if, we have

$$\phi_1(x) = k \phi_2^*(x) \rightarrow (10)$$

where k is an arbitrary constant.

Now let $\phi_1(x) = H(f)$ and $\phi_2(x) = G(f)\, e^{j2\pi fT}$ and by using Schwarz's inequality in eq (9), the numerator of eq (8) may be rewritten as

$$\left| \int_{-\infty}^{\infty} H(f) G(f)\, e^{(j2\pi fT)} df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |G(f)|^2 df \rightarrow (11)$$

$$\left(\because \left|e^{j2\pi fT}\right| = 1\right)$$

From eq (8)

$$\eta \leq \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df \rightarrow (12)$$

$$\Rightarrow \eta_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df \rightarrow (13)$$

From eq (10) the optimum value of $H(f)$

$$\boxed{H_{opt}(f) = k\, G^*(f)\, e^{(-j2\pi fT)}} \rightarrow (14)$$

where $G^*(f)$ is complex conjugate of $G(f)$

→ The above equation specifies the optimum filter in the frequency domain.

In time domain

$$h_{opt}(t) = k \int_{-\infty}^{\infty} \left[G^*(f)\, e^{-j2\pi fT}\right] e^{+j2\pi ft} df$$

$$= k \int_{-\infty}^{\infty} G^*(f)\, e^{-j2\pi f(T-t)} df \rightarrow (15)$$

Since for a real signal $g(t)$ we have $G^*(f) = G(-$

$$h_{opt}(t) = k \int_{-\infty}^{\infty} G(-f)\, e^{-j2\pi f(T-t)} df$$

$$= k \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f(T-t)} df$$

$= k\, g(T-t)$

$$\therefore \boxed{h_{opt}(t) = k\, g(T-t)}$$ Impulse response of the optimum filter, except for the scaling factor k, is a time-reversed and delayed version of the i/p signal $g(t)$; i.e., it is "matched" to the i/p signal. A linear time-invariant filter defined in this way is called a "matched filter".

Properties of Matched Filter

1. The peak pulse signal-to-noise ratio of a matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter input.

proof:- Consider a filter matched to a known signal $g(t)$. The Fourier transform of the resulting matched filter o/p $g_0(t)$ is

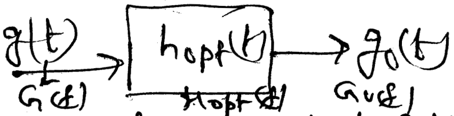


fig: Matched filter

$$G_0(f) = H_{opt}(f)\, G(f)$$

$$= k\, G^*(f)\, G(f)\, e^{-j2\pi fT}$$

$$= k\, |G(f)|^2 e^{-j2\pi fT} \longrightarrow (1)$$

By applying inverse fourier transform of above equation, we find the matched filter o/p $g_0(t)$

$$g_0(t) = IFT\{G_0(f)\} = \int_{-\infty}^{\infty} G_0(f)\, e^{j2\pi ft}\, df$$

at $t=T$, $$g_0(T) = \int_{-\infty}^{\infty} G_0(f)\, e^{j2\pi fT}\, df \longrightarrow (2)$$

By substituting eq (1) in eq (2)

$$g_0(T) = \cancel{\int_{-\infty}^{\infty} G_0(f)\, e} \int_{-\infty}^{\infty} k\, |G(f)|^2 e^{-j2\pi fT} \cdot e^{j2\pi fT}$$

$$g_0(T) = k\int_{-\infty}^{\infty}|G(f)|^2 df$$

According to Rayleigh's energy theorem,

$$E = \int_{-\infty}^{\infty} g^2(t)\,dt = \int_{-\infty}^{\infty}|G(f)|^2 df$$

Hence $g_0(T) = kE$

$$|g_0(T)|^2 = (kE)^2 \rightarrow (3)$$

We know that average Power of the o/p noise $n(t)$ is

$$E[n^2(t)] = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df$$

$$= \frac{N_0}{2}\int_{-\infty}^{\infty}|H_{opt}(f)|^2 df$$

but $H_{opt}(f) = k\,G^*(f)\,e^{-j2\pi fT}$

$$\Rightarrow E[n^2(t)] = \frac{N_0}{2}\int_{-\infty}^{\infty}\left|k\,G^*(f)\,e^{-j2\pi fT}\right|^2 df$$

$$= \frac{N_0}{2}k^2\int_{-\infty}^{\infty}|G(f)|^2 df \quad \left(\because \left|e^{-j2\pi fT}\right| = 1\right)$$

$$= \frac{k^2 N_0}{2}E \rightarrow (4)$$

$\therefore$ The peak pulse signal-to-noise ratio has the maximum value

$$\eta_{max} = \frac{|g_0(T)|^2}{E[n^2(t)]} = \frac{(kE)^2}{k^2\frac{N_0}{2}E} = \frac{2E}{N_0}$$

$$\Rightarrow \boxed{\eta_{max} = \frac{E}{N_0/2}} = \frac{\text{signal energy}}{\text{Noise PSD}}$$

2. For a matched filter the maximum signal component occurs at sampling instant $t = T$ and it has magnitude E, i.e. energy of the signal $g(t)$.

Proof :- Consider a filter matched to a known signal $g(t)$. The fourier transform of the resulting matched filter o/p $g_o(t)$ is

$g(t) \longrightarrow$ $h_{opt}(t)$ $\longrightarrow g_o(t)$

$G(f)$ $H_{opt}(f)$ $G_o(f)$

$$G_o(f) = H_{opt}(f)\, G(f)$$

but $H_{opt}(f) = k G^*(f)\, e^{-j2\pi fT}$

$$\Rightarrow G_o(f) = k\, G^*(f)\, e^{-j2\pi fT} \cdot G(f)$$

$$= k\,|G(f)|^2 e^{-j2\pi fT} \rightarrow (1)$$

The matched filter o/p $g_o(t)$ can be obtained by taking inverse fourier transform of above equation

i.e $g_o(t) = IFT\{G_o(f)\} = \int_{-\infty}^{\infty} G_o(f)\, e^{j2\pi ft}\, df$

By substituting eq (1) in above equation.

$$g_o(t) = \int_{-\infty}^{\infty} \left[k\,|G(f)|^2 e^{-j2\pi fT}\right] e^{j2\pi ft}\, df$$

$$= k\int_{-\infty}^{\infty} |G(f)|^2 e^{j2\pi f(t-T)}\, df \rightarrow (2)$$

at $t = T \Rightarrow g_o(T) = k\int_{-\infty}^{\infty} |G(f)|^2 e^{j2\pi f(T-T)}\, df$

$$= k\int_{-\infty}^{\infty} |G(f)|^2 e^0\, df$$

$$g_o(T) = k\int_{-\infty}^{\infty} |G(f)|^2 \rightarrow (3)$$

According to Rayleigh's energy theorem we know the

$\int_{-\infty}^{\infty} |G(f)|^2 df = \int g^2(t) = E$ = Energy of the signal g

$\Rightarrow g_0(T) = kE$

Maximum value of $g_0(T)$ will result when $k=$

$\Rightarrow \boxed{g_0(T) = E}$

<u>Prop 3:-</u> The o/p signal of a matched filter is proportional to a shifted version of the auto-correlation function of the i/p signal to which the filter is matched.

<u>Proof:-</u> Consider a filter matched to a known signal $g(t)$. The Fourier transform of the resulting matched filter o/p $g_0(t)$ is

$$G_0(f) = H_{opt}(f) \cdot G(f)$$

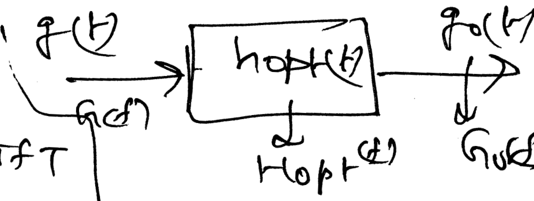


fig: Matched filte

but $H_{opt}(f) = k G^*(f) e^{-j2\pi fT}$

$$\Rightarrow G_0(f) = k G^*(f) e^{-j2\pi fT} \cdot G(f)$$

$$= k G^*(f) \cdot G(f) e^{-j2\pi fT}$$

$$G_0(f) = k |G(f)|^2 e^{-j2\pi fT} \rightarrow (1)$$

The matched filter o/p $g_0(t)$ can be obtained by taking inverse fourier transform of above equati

i.e. $g_0(t) = IFT\{G_0(f)\} = \int_{-\infty}^{\infty} G_0(f) e^{j2\pi ft} df$

$$\rightarrow g_0(t) = \int_{-\infty}^{\infty} \left[k |G(f)|^2 e^{-j2\pi fT}\right] e^{j2\pi ft} df$$

$$g_0(t) = k \int_{-\infty}^{\infty} |G(f)|^2 e^{j2\pi f(t-T)} df \rightarrow (2)$$

Here, $\psi(f) = |G(f)|^2$ = energy spectral density (ESD) 2.6

$$\Rightarrow \quad g_0(t) = k \int_{-\infty}^{\infty} \psi(f)\, e^{j2\pi f(t-T)}\, df$$

Autocorrelation function $R(\tau)$ and energy spectral density $\psi(f)$ form a fourier transform pair. i.e

$$R(\tau) \xleftrightarrow{F.T} \psi(f)$$

i.e. $\psi(f) = \int_{-\infty}^{\infty} R(\tau)\, e^{-j2\pi f\tau}\, d\tau$

and $R(\tau) = \int_{-\infty}^{\infty} \psi(f)\, e^{j2\pi f\tau}\, df$

$$\Rightarrow \boxed{g_0(t) = k\, R(t-T)}$$

i.e $g_0(t)$ is proportional to auto-correlation function of $g(t)$ shifted to right by T.

Error Rate Due to Noise (Prob. of error of Matched filter) 2.9

Consider a binary PCM system based on polar non-return-to-zero (NRZ) signaling. In this, symbol 1 and ~~zer~~ 0 are represented by +ve and -ve rectangular pulses of equal amplitude and equal durati[on]. The channel noise is modeled as additive white Gaussian noise $w(t)$ of zero mean and PSD of $N_0/2$. The signaling interval is $0 \le t \le T_b$, then the received signal $x(t)$

$$x(t) = \begin{cases} +A + w(t), & \text{symbol 1 was sent} \\ -A + w(t), & \text{symbol 0 was sent} \end{cases} \quad \rightarrow (1)$$

where T_b is the bit duration, A is pulse amplitud[e]. ~~The~~ It is assumed that the receiver has prior knowledge of the pulse shape, but not its polarity. Given the noisy signal $x(t)$, the receiver is required to make a decision in each signaling interval as to whether the transmitted symbol is a 1 or 0.

The structure of the receiver used to perform this decision-making process is shown in fig. below

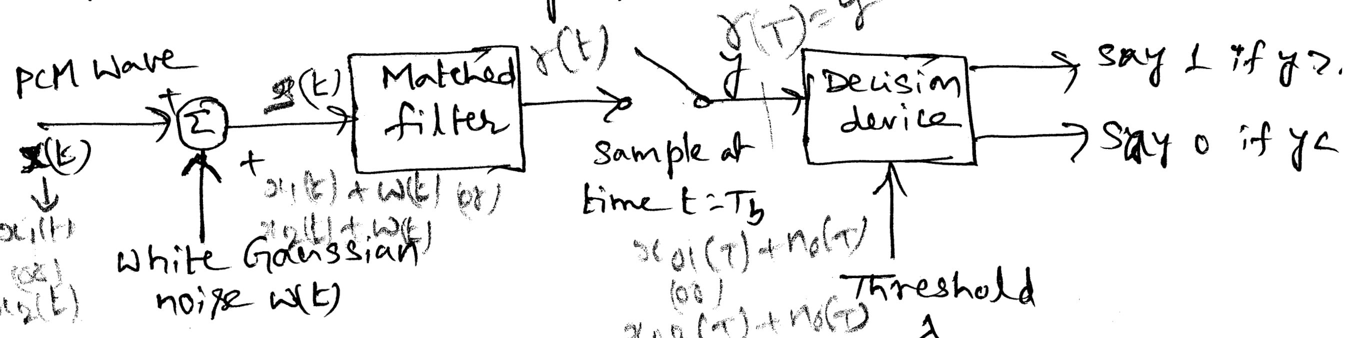


fig: Receiver for baseband transmission of binary-encoded PCM wave using polar NRZ signaling

It consists of ~~the~~ a matched filter followed by a sampler and then finally a decision device. Th

resulting matched filter o/p is sampled at the e
of each signaling interval. The presence of cha
noise $w(t)$ adds randomness to the matched
filter o/p.

Let y denote the sample value obtained at
the end of a signaling interval. The sample value
is compared to a preset threshold λ in the decisi
device. If the ~~threshold is~~ sample value y is
exceeded, the receiver makes a decision in favor
of symbol '1', if not, a decision is made in
favor of symbol '0'. When the sample value y is
exactly equal to the threshold λ, the receiver
just makes a guess as to which symbol was
transmitted; such a decision is the same as
that obtained by flipping a fair coin, the outcom
of which will not alter the average probability
of error.

There are two possible ~~kn~~kinds of error
to be considered.

1. Symbol 1 is chosen when a 0 was actually
transmitted; error of first kind
2. symbol 0 is chosen when a 1 was actually
transmitted; error of the second kind.

To determine the average probability of error
we consider these two situations separately.

Suppose that symbol 0 was sent. Then,
according to equation ① the received signal is

Thus in the absence of noise, decisions are taken clearly. But if noise is ~~added~~ present then, select $x_1(t)$ if $r(T)$ is closer to $x_{01}(T)$ than $x_{02}(T)$ and select $x_2(t)$ if $r(T)$ is closer to $x_{02}(T)$ than $x_{01}(T)$. Therefore the decision boundary will be midway b/w $x_{01}(T)$ and $x_{02}(T)$. i.e.

$$\text{Decision boundary} = \frac{x_{01}(T) + x_{02}(T)}{2}$$

~~Probability of Error of optimum Filter~~ :-

Error Conditions:-

Suppose that $x_2(t)$ was transmitted, but $x_{01}(T)$ is greater than $x_{02}(T)$. If noise $n_o(T)$ is positive an larger in magnitude than the voltage difference $\frac{1}{2}\left[x_{01}(T) + x_{02}(T)\right] - x_{02}(T)$, then incorrect decision will be taken. i.e. error will be generated if,

$$n_o(T) \geq \frac{x_{01}(T) + x_{02}(T)}{2} - x_{02}(T)$$

$r(T):$ $\frac{x_{02}(T) + x_{01}(T)}{2}$

$$\therefore \quad n_o(T) \geq \frac{x_{01}(T) - x_{02}(T)}{2} \rightarrow (1)$$

$= \frac{-5+6}{2}$

$= +ve$

The prob. density function (PDF) for $n_o(t)$ is

$$f_x(n_o(t)) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-[n_o(t)]^2/2\sigma^2} \rightarrow (2)$$

Evaluation of error Probability :-

Thus to obtain the probability of error, we should integrate the area under the PDF curve from $n_o(T) \geq \frac{x_{01}(T) - x_{02}(T)}{2}$.

~~Transmission Limitations~~ 2018

1. Baseband Transmission of Binary Data :-

Binary data can be transmitted in baseband or pass-band. In passband transmission the binary data modulates some carrier and the modulated carrier is transmitted over the channel. It is suitable for long distance. In baseband ~~trans~~ transmission, there is no modulation of high frequency carrier. The digital data is transmitted over the channel directly. This is suitable for transmission over the short distances.

One of the baseband system for transmission of digital data is discrete pulse amplitude modulation (PAM). In this discrete PAM, the amplitude of the pulses varies in discrete manner according to the i/p binary data.

The discrete PAM can have only two amplitude levels corresponding to binary '1' and '0'. Successive binary digits can be combined into symbols. There can be multiple amplitude levels corresponding to these symbols. They generate discrete PAM signals.

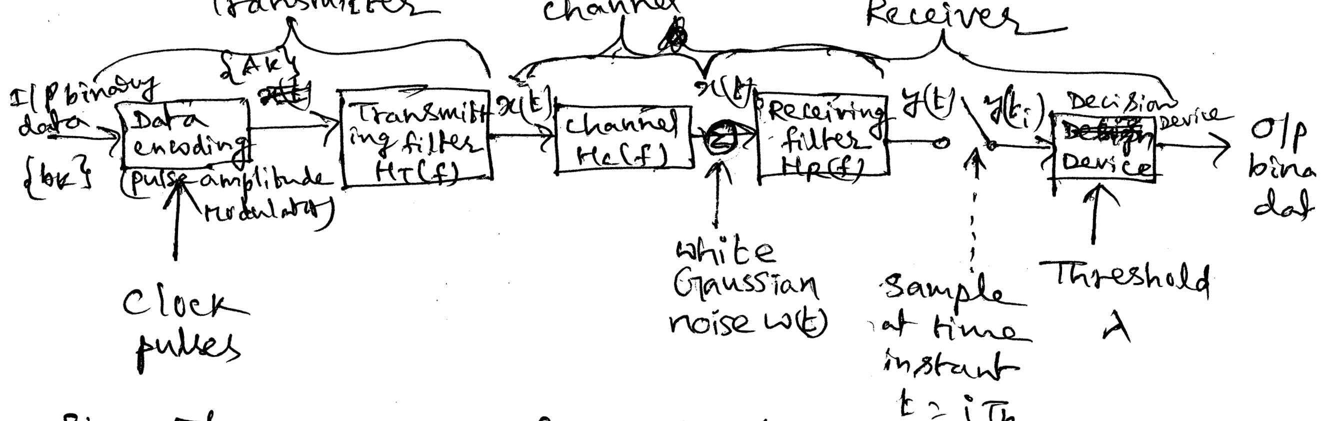


fig: Block diagram of baseband binary transmission system

These signals are transmitted (w/o any modulation), over the channel in baseband transmission. Figure above shows the block diagram of such baseband transmiss system. The binary data is applied to the encoder. The data encoder generates the pulse waveform x(t). This waveform can be represented mathematically as,

$$x(t) = \sum_{k=-\infty}^{\infty} A_k\, g(t - kT_b)$$

Here T_b = Duration of each i/p binary bit.

$g(t)$ is the shaping pulse.

And $$A_k = \begin{cases} +a & \text{if } b_k = 1 \\ -a & \text{if } b_k = 0 \end{cases}$$

The signal $x(t)$ is then passed through the transmittin filter. The transmitting filter combines all the necessar transmitting ckts and systems. The combined transf function of the transmitting filter is $H_T(f)$. The sign is then passed through the channel having the transfe function $H_c(f)$. The channel delivers the signal to th receiving filter. It consists of all the necessary receiving circuits and systems. The combined trans. function of the receiving filter is $H_R(f)$. The o/p of the receiving filter is $y(t)$. This $y(t)$ is noisy repl of the transmitted signal $x(t)$.

The signal $y(t)$ is sampled synchronously with the transmitter. The sampling instants are $t = iT$. These sampling instants are synchronous to the clock pulses at the transmitter. The sampled signal $y(t_i)$ is then given to the decision device. 2.

The decision device compares the i/p signal with threshold 'λ'. Then the decision is taken as follows.

If $y(t_i) > \lambda$ select symbol '1'.

If $y(t_i) \le \lambda$ select symbol '0'.

2. Intersymbol Interference (ISI) problem

Consider the o/p $y(t)$ of the receiving filter in above fig(a). $y(t)$ can be given interms of A_k as,

$$y(t) = \mu \sum_{k=-\infty}^{\infty} A_k p(t - kT_b) + n(t) \rightarrow (1)$$

Here μ is the scaling factor

$p(t)$ is the shape different from that $g(t)$.

In above fig(a) observe that $A_k g(t)$ is the signal applied to the i/p of cascade of transmitting filter, channel and receiving filter. The o/p of this cascad connection ~~is $\mu A_k p(t)$~~ in the absence of noise is $\mu A_k p($

Let the fourier transform of $g(t)$ be $G(f)$ and that of $p(t)$ ~~is~~ be $P(f)$. Then in frequency domain we can write,

$A_k g(t) \rightarrow$ i/p

$\mu A_k p(t) \rightarrow$ o/p.

$$\mu A_k P(f) = H(f) A_k G(f) \rightarrow (2)$$

Here $H(f)$ is the combined transfer function of transmitting filter, channel and receiving filter. It is given for the cascade connection as,

$$H(f) = H_T(f) H_C(f) H_R(f) \rightarrow (3)$$

Hence eq ② ~~Can be~~ becomes

$$\mu P(f) = H_T(f)\, H_C(f)\, H_R(f)\, G(f) \rightarrow ④$$

The receiving filter o/p is sampled at $t_i = iT_b$.

From eq ①, at $t = iT_b$ we can write,

$$y(t_i) = \mu \sum_{K=-\infty}^{\infty} A_K\, p(iT_b - KT_b) + n(t_i)$$

$$= \mu \sum_{K=-\infty}^{\infty} A_K\, P[(i-K)T_b] + n(t_i) \rightarrow ⑤$$

$$y(t_i) = \mu A_i\, p(0) + \mu \sum_{\substack{K=-\infty \\ K \neq i}}^{\infty} A_K\, P[(i-K)T_b] + n(t_i) \rightarrow ⑥$$

In the absence of channel noise and ~~if~~ $p(t)$ is normalized such that $p(0) = 1$. Hence above equation becomes

$$y(t_i) = \mu A_i + \mu \sum_{\substack{K=-\infty \\ K \neq i}}^{\infty} A_K\, p[(i-K)T_b] \ \cancel{+n(t_i)} \rightarrow ⑦$$

where $i = 0, \pm 1, \pm 2, \pm 3, \ldots$

Comments :-

(i) The first term in above equation is μA_i. It is the contribution of the ith transmitted bit.

(ii) The second term represents the residual effect of all other bits transmitted before and after the sampling instant t_i.

Definition of ISI :-

The presence of o/ps due to other bits (sym interfere with the o/p of required bit (symbol). This effect is called Intersymbol Interference (IS

→ ~~Here we have not considered the effect of channel noise. Actually, channel noise and ISI both interfere the transmitted signal.~~

If the ~~intersymbol~~ intersymbol Interference is absent then the second term will not be present in above eq. i.e.

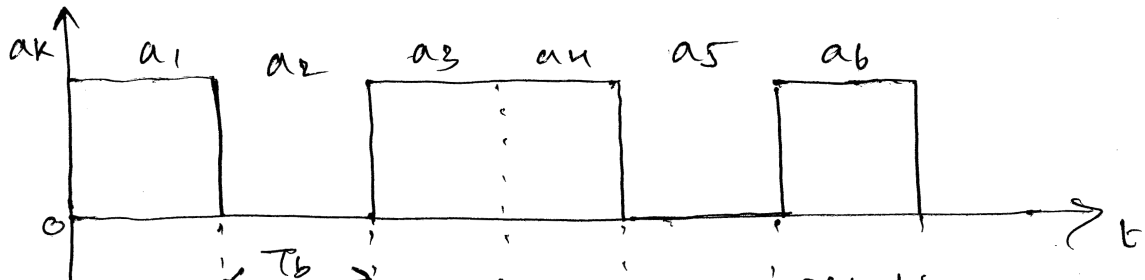
$$y(t_i) = \mu A_i \rightarrow ⑧$$

At $t = iT_b$, the correct bit is A_i. observe that it is decoded correctly in absence of ISI. It is not possible to eliminate the second term of ~~ab~~ eq ⑦ (ISI) totally. The ISI can be reduced by proper design of pulse spectrum $G(f)$, transmit filter $H_T(f)$ receive filter $H_R(f)$ and the channel $H_C(f)$.

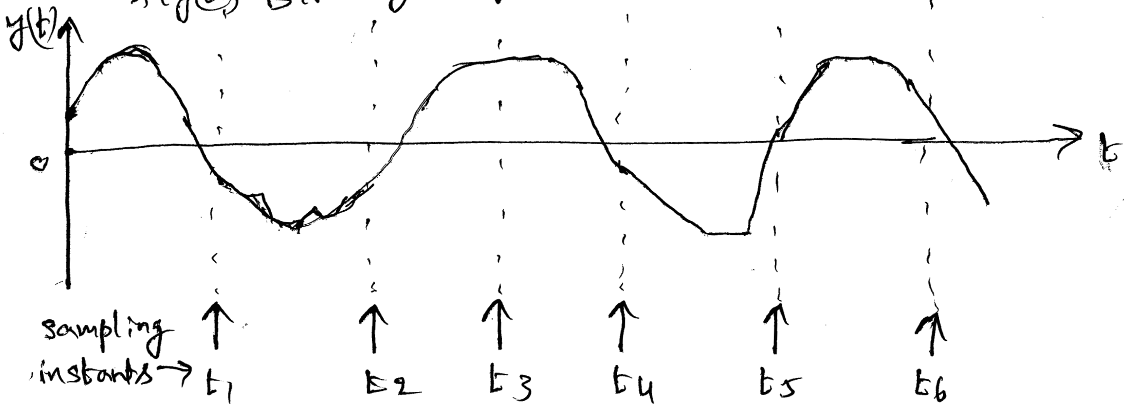
3. Eye patterns (Diagrams)

The Eye pattern is used to study the effect of ISI in baseband digital transmission.

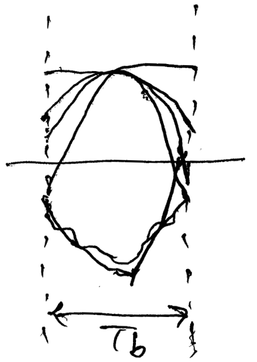
When the sequence is transmitted over a baseband binary data transmission system ~~of fig~~ the signal obtained at the o/p i.e $y(t)$ is a continuous time signal as shown in fig. below. Ideally this signal should go high and low depending on the symbol that was transmitted. But because of the nature of transmission channel, the signal becomes continuous with increasing and decreasing amplitudes. Fig(a) below shows the binary sequence that is transmitted and fig(b) shows the signal y(t) obtained at the o/p. Fig(b) also shows various sampling instants t_1, t_2, t_3, ... etc. Thus based on the signal obtained over the period T_b b/w two sampling instants, decision is taken by the decision device. If we cut the signal y(t) shown in fig(b) in each interval (T_b) and place it over one another, then we obtain the diagram as shown in fig(c). This diagram is called Eye pattern of the signal y(t).



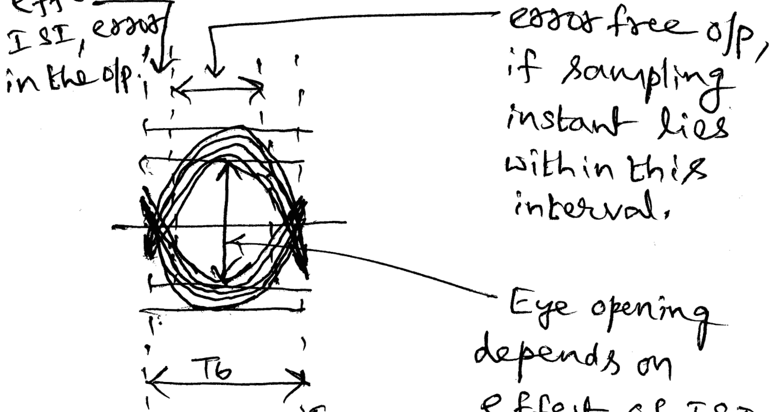
fig(a) Binary sequence transmitted



fig(b) Received signal by base band transmission system



fig(c) Eye pattern of signal in (b)



fig(d) ~~Error~~ Eye pattern for large no. of bits in waveform y(t)

→ The name 'eye' is given because it looks like an eye. This pattern can also be obtained on CRO if we apply y(t) to one of the i/p channels and apply an external trigger ~~of~~ signal of $1/T_b$ Hz. This makes one sweep of beam equal to 'T_b' seconds. Therefore the pattern shown in fig (c) will be obtained. When there are large no. of bits of the ~~to~~ sequence, then eye pattern will be as shown in fig (d).

4. Performance of the Data Transmission system using Eye pattern

Various important conclusions can be derived from eye pattern. Fig. below shows various points related to eye pattern.

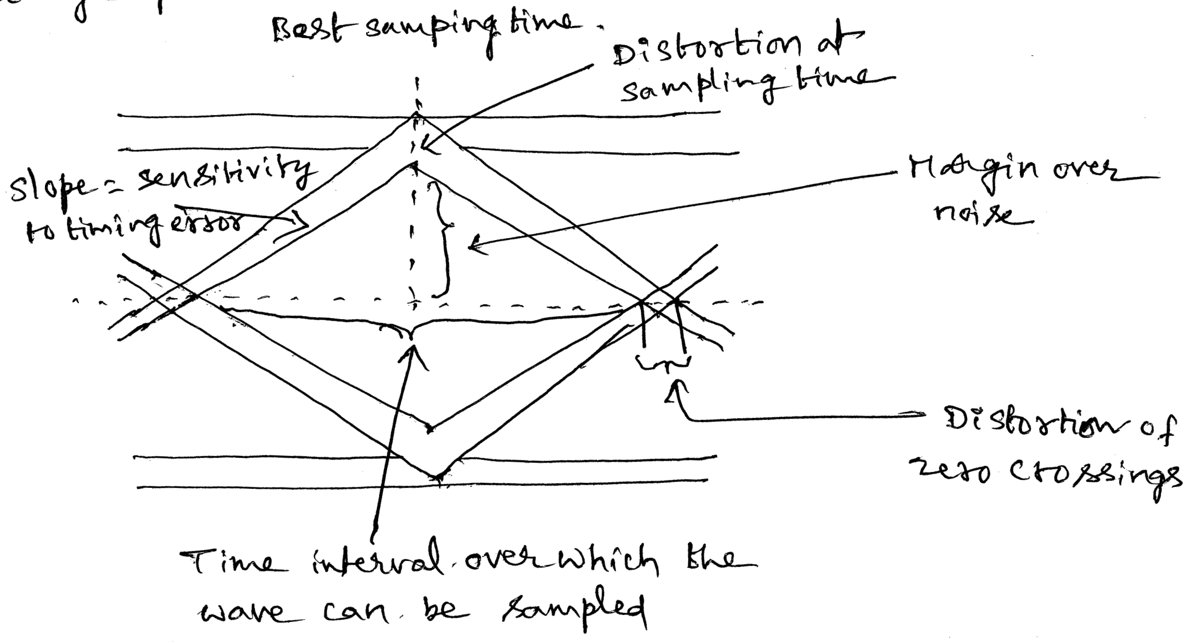


fig: Interpretation of the eye pattern

i) The width of the eye opening defines the interval over which the received wave can be sampled w/o error from intersymbol interference. It is preferable to sample the instant at which eye is open widest. The instant is shown as best sampling time in above fig

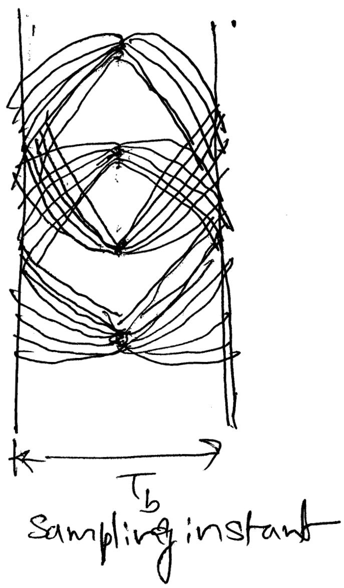
ii) The Sensitivity of the system to timing error is determined by the rate of closure of the eye ~~at~~ as the sampling time is varied.

iii) The height of the eye opening, at the specified sampling time is called margin over the noise.

As the effect of inter symbol interference increases, the eye opening reduces. If the eye is closed complete then it is not possible to avoid errors in the o/p.

All the above description is for two level (binary) system. If there are M-levels (M-ary System), then eye pattern contains (M-1) eye openings, stack vertically one upon the other.

Figure below shows the eye diagram for 4 level (M=4) system. ~~Therefore there are~~ 3 eye openings.



Correlative Level Coding

→ Correlative coding allows the signaling rate of $2B_o$ in the channel of bandwidth B_o. This is made physically possible by allowing ISI in the transmitted signal in controlled manner. This ISI is known to the receiver. Hence effects of ISI are eliminated at the receiver. The correlative coding is implemented by duobinary signaling and modified duobinary signaling.

signaling rate $= f_b = \frac{1}{T_b} = 2B_o$

Duobinary Encoding (or) Duobinary Signaling Scheme

Duobinary encoding reduces the maximum frequency of the base band signal. The word 'duo' means to double the transmission capacity of the binary system.

Consider the i/p sequence $\{b_k\}$ which contains binary symbols 1 and 0. By use of level shifter this sequence is converted to polar NRZ sequence $\{a_k\}$ i.e.

$$\left.\begin{array}{ll} a_k = +1 & \text{if } b_k = 1 \\ \text{and } a_k = -1 & \text{if } b_k = 0 \end{array}\right\} \longrightarrow (1)$$

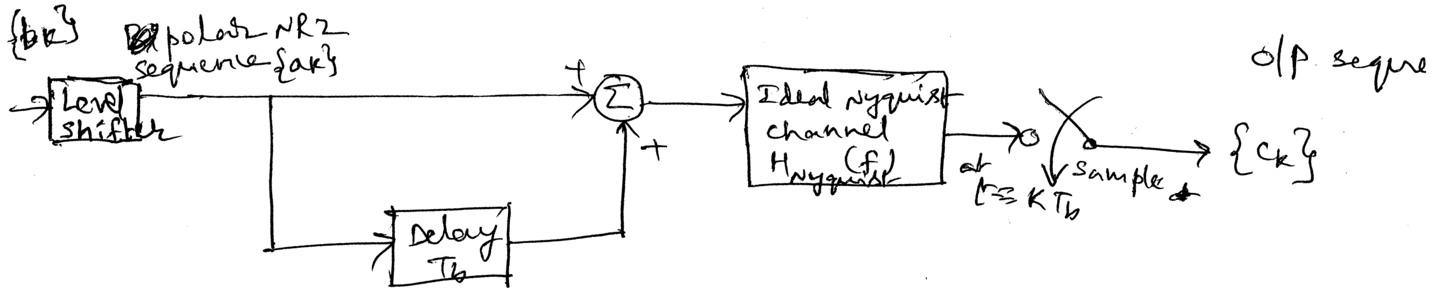


fig: Simple duobinary encoder without precoder

Encoder:–

Figure above shows the block diagram of duobinary encoder. This encoder accepts the sequence $\{a_k\}$ and

- Converts it to three level signal, i.e. -2, 0 and +2.

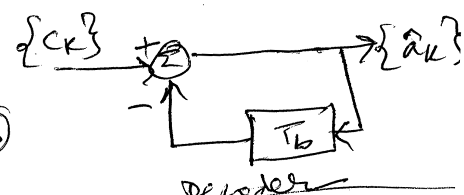
The o/p of duobinary encoder can be expressed as,

$$c_K = a_K + a_{K-1} \longrightarrow (2)$$

Because this encoder converts the uncorrelated sequence $\{a_K\}$ of two level pulses into correlated sequence of three level pulses, it introduces intersymb interference in the signal in the artificial manner to reduce bandwidth.

Reconstruction :-

Let $\hat{a}_K$ represents the estimate of a_K. Then w can obtain $\hat{a}_K$ as,



$$\hat{a}_K = c_K - \hat{a}_{K-1} \longrightarrow (3)$$

This shows that if c_K is received with error, then $\hat{a}$ will have error. This error will propagate in the o/p sequence.

Drawback :- Error propagation takes place in the decode

Frequency response :-

In above figure observe that there is a delay eleme 'T_b'. The frequency response of the delay element is $e^{-j2\pi f T_b}$. Hence frequency response of the delay line filter will be $1 + e^{-j2\pi f T_b}$.

The delay line filter is connected in cascade with ideal nyquist channel. Hence overall frequency respon of the scheme will be,

$$H(f) = H_{nyquist}(f)\left[1 + e^{-j2\pi f T_b}\right]$$

By rearranging

$$H(f) = H_{Nyquist}(f)\left[e^{-j\pi f T_b}\cdot e^{j\pi f T_b} + e^{-j\pi f T_b}\cdot e^{-j\pi f T_b}\right]$$ 2.2

$$= H_{Nyquist}(f)\left[e^{j\pi f T_b} + e^{-j\pi f T_b}\right] e^{-j\pi f T_b}$$

$$= H_{Nyquist}(f)\cdot 2\cos(\pi f T_b)\cdot e^{-j2\pi f T_b} \longrightarrow (4)$$

Here $H_{Nyquist}(f)$ is the spectrum of sinc pulse. It is given by

$$B_0 = \frac{1}{2T_b}$$
$$f_b = 2B_0$$
$$T_b = \frac{1}{f_b} = \frac{1}{2B_0}$$

$$H_{Nyquist}(f) = \begin{cases} \frac{1}{2B_0} & \text{for } -B_0 < f \le B_0 \\ 0 & \text{elsewhere} \end{cases}$$

$$\Rightarrow H(f) = \begin{cases} \frac{1}{B_0}\cos(\pi f T_b)\, e^{-j\pi f T_b} & |f| \le B_0 \\ 0 & \text{elsewhere} \end{cases}$$

Here $|H(f)| = \frac{1}{B_0}\cos(\pi f T_b)$ for $|f| \le B_0$

and $\angle H(f) = -\pi f T_b$.

Impulse response

$$\Rightarrow h(t) = IFT\{H(f)\} = \frac{T_b^2 \sin(2\pi \ldots}{\pi t(T_b - \ldots}$$

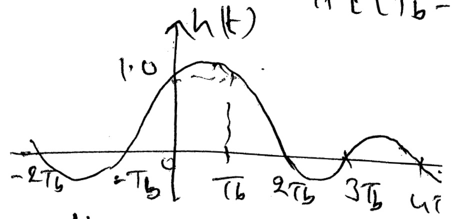


fig: Impulse resp of duobinary conversi filte

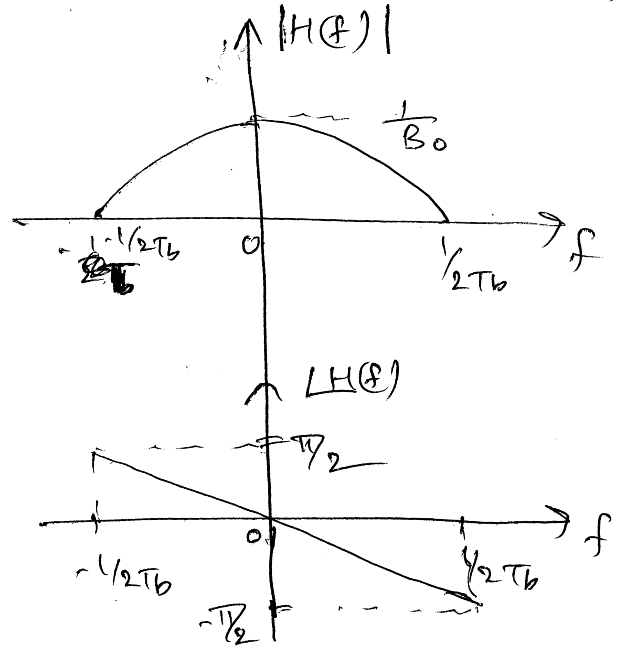


fig: Frequency response of duobinary encoding scheme.

P1 :- The binary data stream 0 0 1 1 0 1 0 0 1 is applied to the i/p of a duobinary system. Construct the duobinary encoder o/p and corresponding receiver o/p, w/o precoder.

Sol:- Table below shows the o/p of duobinary system & receiver o/p.

Sequences	k=0	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Binary seq. b_k	0	0	1	1	0	1	0	0	1
Polar representation a_k	-1	-1	+1	+1	-1	+1	-1	-1	+1
$c_k = a_k + a_{k-1}$	$a_{k-1}=0$ -1	-2	0	+2	0	0	0	-2	0
Estimated polar o/p $\hat{a}_k = c_k - \hat{a}_{k-1}$	$\hat{a}_{k-1}=0$ -1	-1	+1	+1	-1	+1	-1	-1	+1
Estimated binary o/p b_k	0	0	1	1	0	1	0	0	1

Thus the i/p & o/p sequences are same.

Duobinary Encoder with precoder (Differential Encoder):

Encoder :-

Precoder is used in duobinary encoder to avoid error propagation.

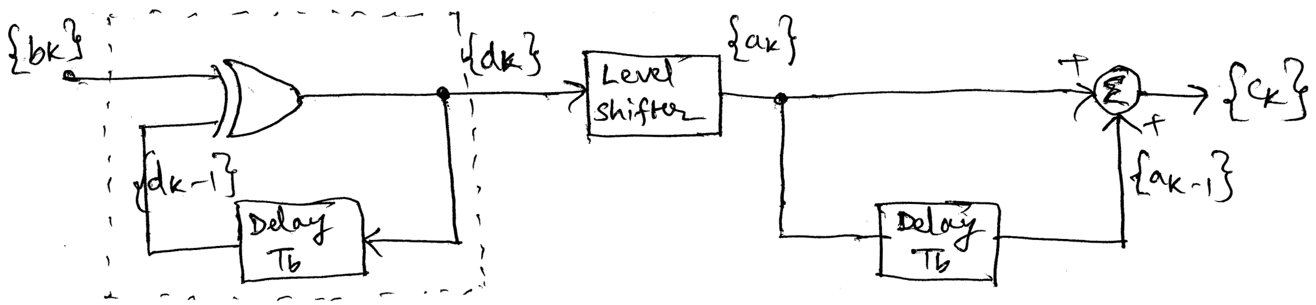


fig (a) : Block diagram of duobinary encoder with precoder.

Figure above shows the block diagram of duobinary 2.2 encoder with precoder. Precoder is nothing but a differen[tial] encoder. The o/p of the precoder is,

$$\boxed{d_k = b_k \oplus d_{k-1}} \rightarrow (1) \qquad d_k = 1 \text{ if either } b_k = 1 \text{ or } d_{k-1} = 1 \text{ but not both}$$

The sequence $\{d_k\}$ is applied to level shifter. ~~At~~ At the o/p of level shifter, the sequence $\{a_k\}$ is bipolar i.e.

$$\left.\begin{array}{l} \text{if } d_k = 1,\ a_k = +1 \\ \text{and} \\ \text{if } d_k = 0,\ a_k = -1 \end{array}\right\} \rightarrow (2)$$

The sequence $\{a_k\}$ is then applied to duobinary encoder. The o/p c_k is,

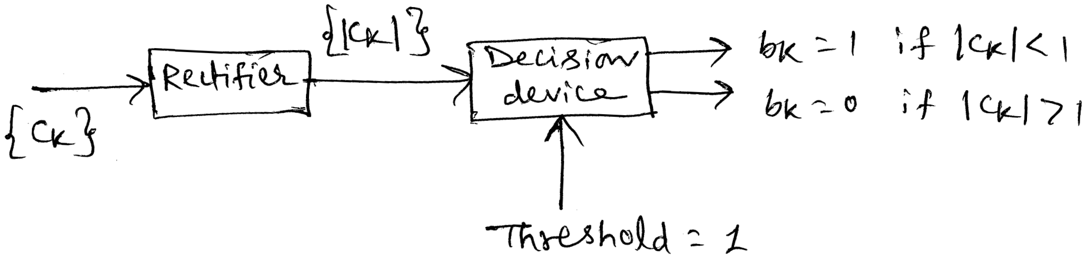
$$c_k = a_k + a_{k-1} \rightarrow (3)$$

The value of c_k will be,

$$\left.\begin{array}{l} c_k = 0 \quad \text{if } b_k \text{ is } 1 \\ \text{and} \\ \quad\ = \pm 2 \text{ if } b_k \text{ is '0'} \end{array}\right\} \rightarrow (4)$$

<u>Decoder</u> :-

Figure below shows the block diagram of duobinary detector.



fig(b): Detector of duobinary encoder

From eq (4) the detector of fig(b) is produced. The magnit[ude] of c_k is taken. It is compared with '1'. Then the decisions are taken as follows,

$$\left.\begin{array}{l} \text{if } |c_k| < 1 \text{ taken } b_k = 1 \\ \text{and if } |c_k| > 1 \text{ take } b_k = 0 \end{array}\right\} \rightarrow (5)$$

Here note that the o/p 'b_k' depends upon the present value of 'c_k'. Previous value of the o/p is not required. This shows that there will be no propagation of error in this system, as in case of duobinary encoding w/o precoder.

P:- The binary data stream 0010110 is applied to the i/p of a duobinary system. Construct duobinary code o/p and corresponding receiver o/p. Assume that there is precoder at the i/p.

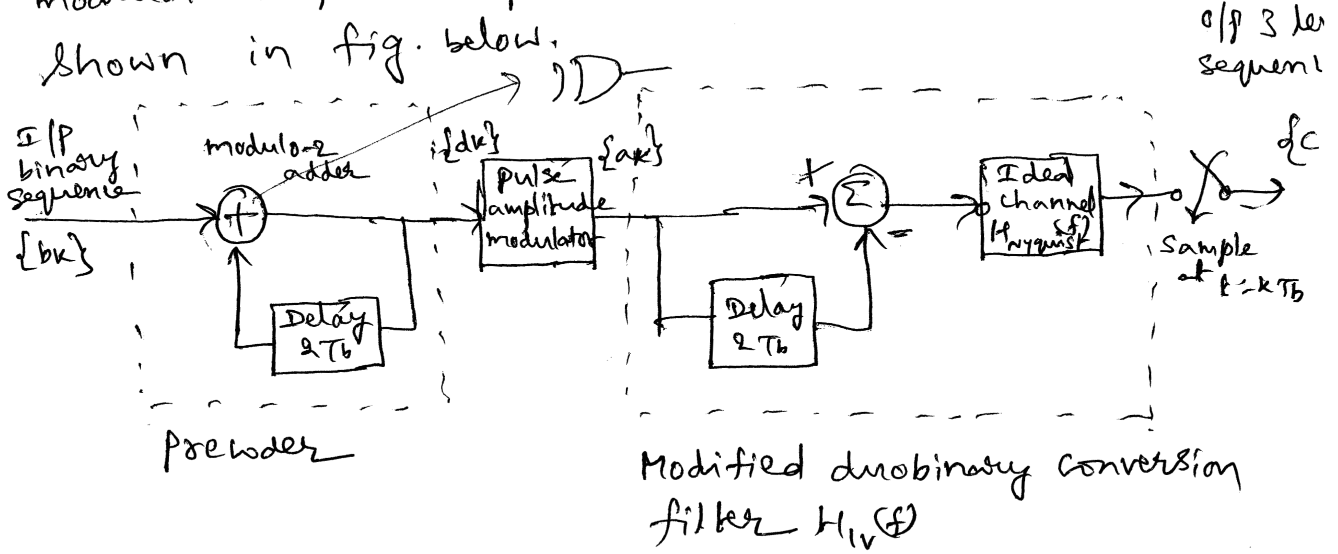
Sol:- To start with, we need to assume an extra bit at the o/p of precoder i.e. we should assume d_{k-1}. Let this bit be '1'. i.e $d_{-1} = 1$.

Sequence	k = -1	k = 0	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6
Binary seq. $\{b_k\}$		0	0	1	0	1	1	0
precoded sequence $\{d_{k-1}\}$		1	1	1	0	0	1	0
precoded sequence $\{d_k\}$	1	1	1	0	0	1	0	0
Two level seq. $\{a_k\}$	+1	+1	+1	-1	-1	+1	-1	-1
Two level seq. $\{a_{k-1}\}$		+1	+1	+1	-1	-1	+1	-1
Duobinary encoder o/p $c_k = a_k + a_{k-1}$		+2	+2	0	-2	0	0	-2
Magnitude of $\{c_k\}$ i.e $\{\|c_k\|\}$		+2	+2	0	+2	0	0	+2
o/p of decoder ($\{c_k\}$ using eq (5) to $\{c_k\}$ above)		0	0	1	0	1	1	0

The above table shows, in the absence of noise the decoder o/p is same ... sequence.

Modified Duobinary signaling (class IV partial response scheme) 2.2

In the duobinary signaling technique the frequency response $H(f)$, and consequently the power spectral density of the transmitted pulse, is nonzero at the origin. This is considered to be an undesirable feature in some applications since many communication channels cannot transmit a DC component. This can be corrected by using the class IV partial response or modified duobinary technique, which involves a correlation span of two binary digits. This special form of correlation is achieved by subtracting amplitude modulated pulse spaced $2T_b$ seconds apart, as shown in fig. below.



fig(a) Modified duobinary signaling scheme

The precoder involves a delay of $2T_b$ seconds. The o/p of the modified duobinary conversion filter is related to the i/p two-level sequence $\{a_k\}$ at the pulse-amplitude modulator o/p as follows:

$$c_k = a_k - a_{k-2} \longrightarrow (1)$$

Here, again, we find that a three-level signal generated. With $a_k = \pm 1$, we find that c_k tal on one of three levels : +2, 0 and -2.

The overall frequency response of the delay-line filter connected in cascade with an ideal Nyquist channel is given by

$$H_{IV}(f) = H_{Nyquist}(f)\left[1 - e^{-j4\pi f T_b}\right]$$

$$= 2j\, H_{Nyquist}(f) \cdot \sin(2\pi f T_b)\, e^{-j2\pi f T_b} \rightarrow (2)$$

where the subscript IV in $H_{IV}(f)$ indicates the pertinent class of partial response and $H_{Nyquist}(f)$ is

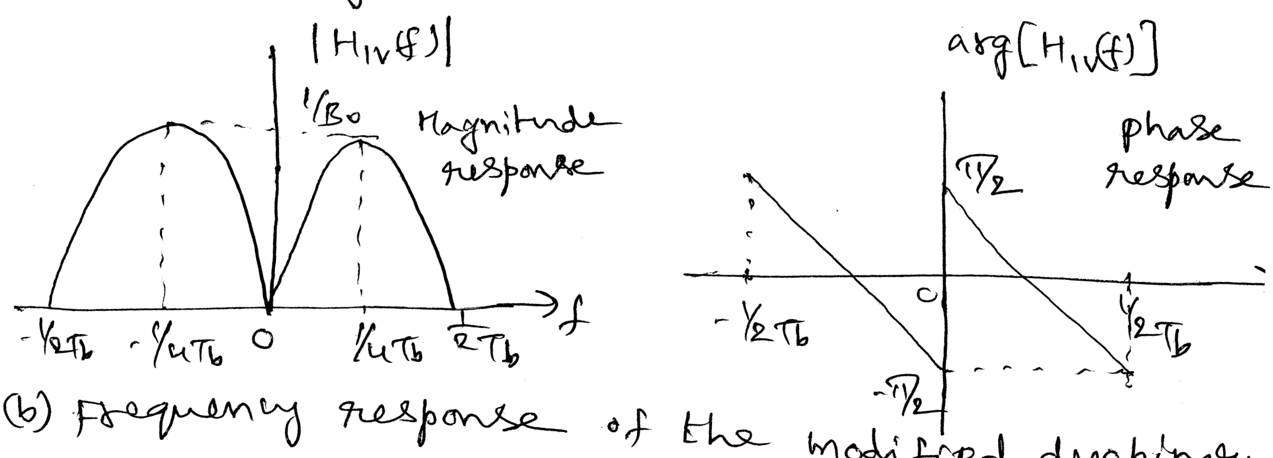
$$H_{Nyquist}(f) = \begin{cases} \frac{1}{2B_0} & \text{for } -B_0 \le f \le B_0 \\ 0 & \text{elsewhere.} \end{cases} \rightarrow (3)$$

~~Thef Theef~~ $h_{Nyquist}(t) = \text{sinc}(2B$

~~Theref~~ Therefore the overall frequency response in the form of a half-cycle sine function, as shown by

$$H_{IV}(f) = \begin{cases} \frac{j}{B_0} \sin(2\pi f T_b)\, e^{-j2\pi f T_b}, & |f| \le B_0 \\ 0 & \text{elsewhere} \end{cases} \rightarrow (4)$$

The corresponding magnitude response and phase response of the modified duobinary coder ~~as~~ are shown in fig. below.



fig(b) Frequency response of the modified duobinary conversion filter

A useful feature of the modified duobinary coder is the fact that its o/p has no DC component.

From the first line of eq ② and the definition of $H_{Nyquist}(f)$, we find that the impulse response of the modified duobinary coder consists of two sinc (Nyquist) pulses that are time - ~~delayed~~ displaced by $2T_b$ seconds w.r.t. each other as shown by

$$h_{IV}(t) = IFT\{H_{IV}(f)\}$$

$$= IFT\{H_{Nyquist}(f)\left[1-e^{-j4\pi f T_b}\right]\}$$

$$= IFT\{H_{Nyquist}(f)\} - IFT\{H_{Nyquist}(f)\, e^{-j4\pi f T_b}\}$$

$$= \text{sinc}(2B_0t) - \text{sinc}\left[2B_0(t-2T_b)\right]$$

$$= \frac{\sin(2\pi B_0 t)}{2\pi B_0 t} - \frac{\sin\left[2\pi B_0(t-2T_b)\right]}{2\pi B_0(t-2T_b)}$$

$$= \frac{\sin(2\pi B_0 t)}{2\pi B_0 t} - \frac{\sin\left[2\pi B_0 t - 2\pi\right]}{\cancel{\sin}\, 2\pi B_0(t-2T_b)}$$

$$= \frac{\sin(2\pi B_0 t)}{2\pi B_0 t} - \frac{\sin(2\pi B_0 t)}{2\pi B_0(t-2T_b)}$$

[sin (A-B) = sinA cosB − cosA sinB]

$$\boxed{h_{IV}(t) = \frac{2T_b^2 \sin(2\pi B_0 t)}{\pi t(2T_b - t)}} \longrightarrow ⑤$$

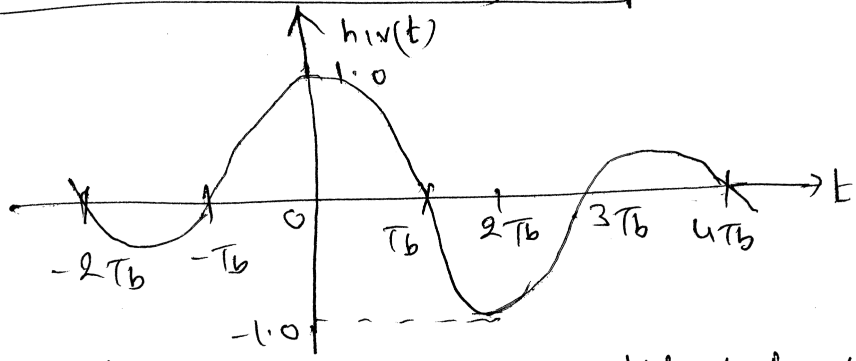


fig (c) Impulse response of the modified duobinary conversion filter

To eliminate the possibility of error ~~propagation~~ in the modified duobinary system, we use a precoding procedure similar to that used for the duobinary case. ~~specially~~ specifically, prior to the generation of the modified duobinary signal, a modulo-2 logic addition is used on signal $2T_b$ seconds apart as shown in fig(a).

$$d_k = b_k \oplus d_{k-2}$$

$$= \begin{cases} \text{symbol } 1 & \text{if either symbol } b_k \text{ or symbol } d_{k-2} \\ & \text{(but not both) is } 1 \\ \text{symbol } 0 & \text{otherwise.} \end{cases} \rightarrow (b)$$

where $\{b_k\}$ is the incoming binary data sequence and $\{d_k\}$ is the sequence at the precoder o/p. The precoded sequence $\{d_k\}$ is then applied to a pulse-amplitude modulator and then to the modified duobinary conversion filter.

In fig(a), the o/p digit c_k equals $-2, 0,$ or $+2$ assuming that the pulse-amplitude modulator uses a polar representation for the precoded sequence $\{d_k\}$

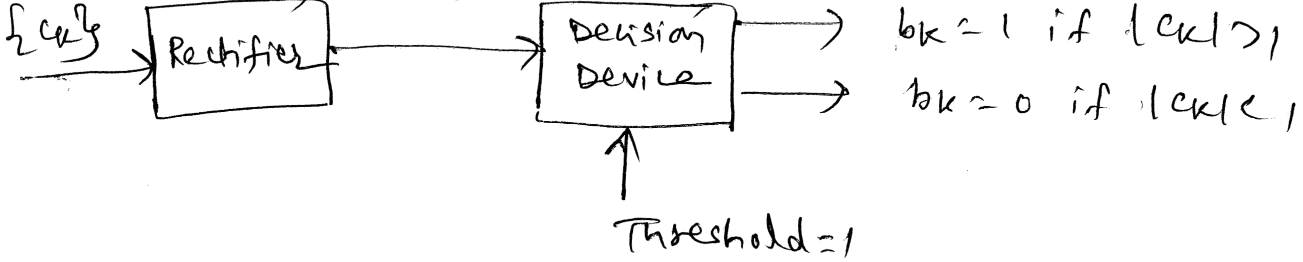


fig: Receiver

The detected digit $\hat{b}_k$ at the receiver o/p may be extracted from c_k by discarding the polarity of c_k. Specifically, we may formulate the

the following decision rule:

If $|c_k| > 1$, say symbol $b_k = 1$

If $|c_k| < 1$, say symbol $b_k = 0$ → (7)

when $|c_k| = 1$, the receiver makes a random guess in favor of symbol 1 & 0. As with the duobinary signaling, we may note the following.

→ In the absence of channel noise, the detected binary sequence $\{\hat{b}_k\}$ is exactly the same as the original binary sequence $\{b_k\}$ at the transmitter i/p.

→ The use of eq (6) requires the addition of two extra bits to the precoded sequence $\{a_k$ The composition of the decoded sequence $\{\hat{b}_k\}$ using equation (7) is invariant to the selection made for these two bits.

P :- The binary data 011100101 are applied to the i/p of a modified duobinary system.

(i) Construct the modified duobinary coder o/p and corresponding receiver o/p w/o a precoder.

(ii) Suppose that due to error in transmission, the level produced by the third digit is reduced to zero. Construct a new receiver o/p.

Sol:-

Sequences	k = -2	-1	0	1	2	3	4	5	6	7	8
Binary seq. $\{b_k\}$			0	1	1	1	0	0	1	0	1
polar seq. $\{a_k\}$	+1	+1	-1	+1	+1	+1	-1	-1	+1	-1	+1
$\{a_{k-2}\}$			+1	+1	-1	+1	+1	+1	-1	-1	+1
$c_k = a_k - a_{k-2}$			-2	0	+2	0	-2	-2	+2	0	0
$\hat{a}_{k-2}$			+1	+1	-1	+1	+1	+1	-1	~~+1~~ -1	+1
$\hat{a}_k = c_k + \hat{a}_{k-2}$	+1	+1	-1	+1	+1	+1	-1	-1	+1	-1	+1
Binary seq. $\{\hat{b}_k\}$			0	1	1	1	0	0	1	0	1

observe that the o/p and i/p sequences ~~are~~ exact same w/o noise interference.

Table below shows the results of (ii). The first row of the table represents the sequence $\{c_k\}$ w/o error. This sequence is taken from above table. Now because of error in the transmission the level produced by 3rd digit is reduced to zero.

Sequences k	-2	-1	0	1	2	3	4	5	6	7	8
$\{c_k\}$ w/o error			-2	0	+2	0	-2	-2	+2	0	0
$\{c_k\}$ error in 3rd digit			-2	0	0	0	-2	-2	+2	0	0
$\hat{a}_{k-2}$			+1	+1	-1	+1	-1	+1	-3	-1	-1
$\hat{a}_k = c_k + \hat{a}_{k-2}$	+1	+1	-1	+1	-1	+1	-3	-1	-1	-1	-1
Binary sequence $\{\hat{b}_k\}$			0	1	0	1	0	0	0	0	0

The reconstructed sequence shown in the last row of above table contains errors upto last digit. This shows that error propagates in this system. Hence precoder must be used to avoid error propagation.

Partial - Response signaling
(Generalized form of correlative - level coding)

The duobinary and modified duobinary techniq[ues] have correlation spans of 1 binary digit and 2 binary digits, respectively. The generalization of these two techniques to other schemes, ~~which~~ are known ~~as~~ collectively as generalized correlative - level coding & partial - response signaling schemes. This ~~generat~~ generalization

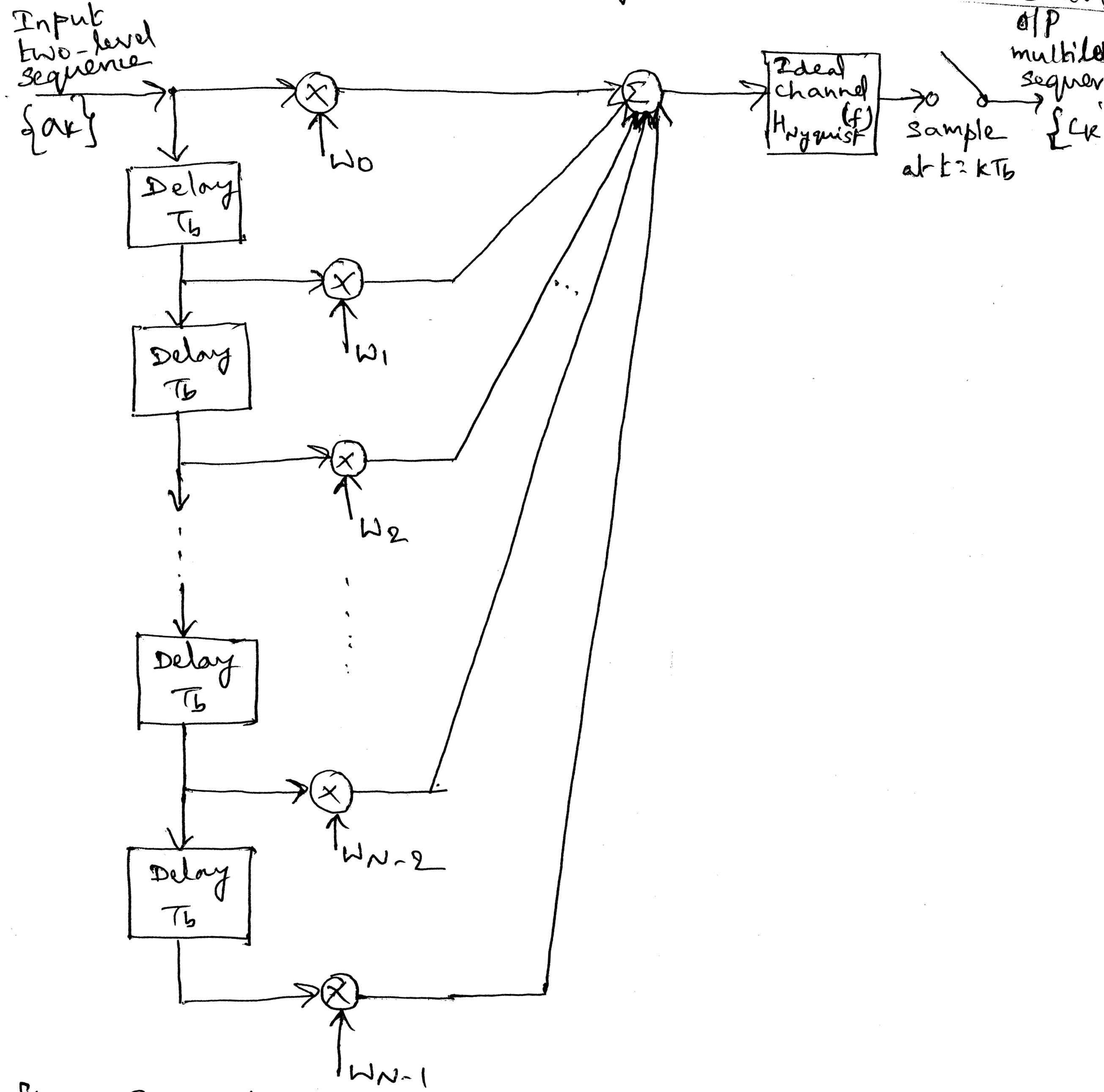


fig: Generalized correlative coding scheme.

a generalization is shown in above figure, where $H_{Nyquist}(f)$ is given by

$$H_{Nyquist}(f) = \begin{cases} \frac{1}{2B_0} & -B_0 \leq f \leq B_0 \\ 0 & \text{elsewhere} \end{cases} \longrightarrow (1)$$

It involves the use of a tapped-delay-line filter with tap-weights $w_0, w_1, w_2, \ldots, w_{N-1}$. Specifically, different classes of partial-response signaling schemes may be achieved by using a weighted linear combination of N ideal Nyquist (sinc) pulses, as shown by

$$h(t) = \sum_{n=0}^{N-1} w_n \operatorname{sinc}\left(\frac{t}{T_b} - n\right) \longrightarrow (2)$$

~~Table~~

Type of Class	N	w_0	w_1	w_2	w_3	w_4	Comments
I	2	1	1				Duobinary coding
II	3	1	2	1			
III	3	2	1	-1			
IV	3	1	0	-1			Modified duobinary coding
V	5	-1	0	2	0	-1	

Table: Different classes of partial-response signaling schemes referring to fig. above.

An appropriate choice of the tap-weights in equation (2) results in a variety of spectral shapes designed to ~~su~~ suit individual applications.
Table above presents the specific details of five different classes of partial-response signaling

- schemes. For example, in the duobinary case (class I partial response), we have

$w_0 = +1$

$w_1 = +1$

and $w_n = 0$ for $n \geq 2$.

In the modified duobinary case (class IV partial response), we have

$w_0 = +1$

$w_1 = 0$

$w_2 = -1$

and $w_n = 0$ for $n \geq 3$.

The useful characteristics of partial-response signali schemes may now be summarized as follows.

→ Binary data transmission over a physically baseba channel can be accomplished at a rate close to the Nyquist rate, using realizable filters with gradual cutoff characteristics.

→ Different spectral shapes can be produced, appropriate for the application at hand.

However, these desirable characteristics are achiev at a price: A large SNR is required to yield t same avg. prob. of symbol error in the presence of noise as in the corresponding binary PAM system because of an increase in the number of sign levels used.

Baseband M-ary PAM Transmission

In the baseband binary PAM system, the pulse-amplitude modulator produces binary pulses, i.e., pulses with one of two possible amplitude levels. on the otherhand, in a baseband M-ary PAM system, the pulse-amplitude modulator produces one of M possible amplitude levels with $M > 2$. This form of pulse modulation is ~~illum~~ illustrated in figure below ~~for~~ the case of a quaternary $(M=4)$ system and the binary sequence 0010110111. This is gray coding waveform.

Dibit	Gray code	Amplitude
0 0	0 0	-3
0 1	0 1	-1
1 0	1 1	+3
1 1	1 0	+1

fig (a) Representation of the 4-possible dibits, based on Gray ~~coding~~ encoding.

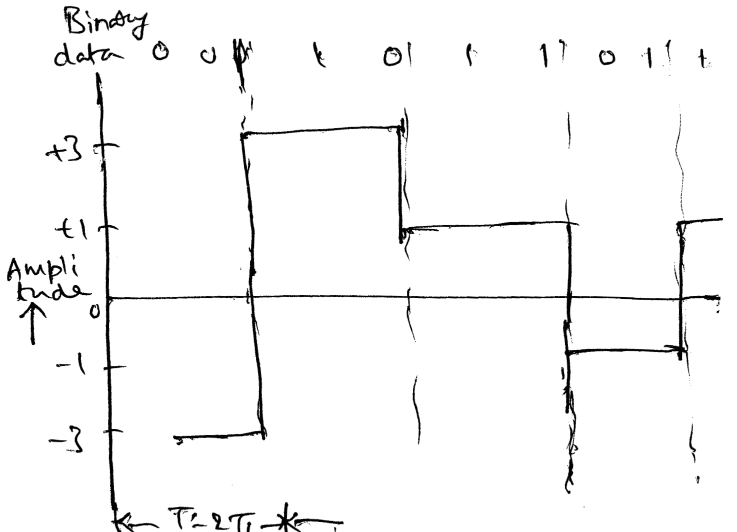


fig (b) waveform

fig: o/p of a quaternary system

In an M-ary system, the information source emit a sequence of symbols from an alphabet that consist of M symbols. Each amplitude level at the pulse-amplitude modulator o/p ~~pro~~ corresponds to a distinct symbol, so that there are M distinct amplitude levels to be transmitted. If the number of i/p s to be grouped is n the $N = \log_2 M$ or $M = 2^N$.

The symbol duration T of the M-ary PAM system is related to the bit duration T_b of the equivalent binary PAM system as

$$\boxed{T = T_b \log_2 M}$$

r_b = bit rate
r_s = symbol rate
T_b = bit duration
T_s = symbol duration

baud = symbol ~~signaling~~ rate $r_s = \frac{1}{T} = \frac{1}{T_b \log_2 M}$
(symbols/sec)

$$\boxed{r_s = \frac{r_b}{\log_2 M}}$$

$r_b \rightarrow$ signaling rate ~~in binary PAM~~, (or) bit rate

bit rate $= r_b = r_s \log_2 M$

Therefore, in a given channel bandwidth, we find that by using an M-ary PAM system, we are able to transmit information at a rate that is $\log_2 M$ faster than the corresponding binary PAM system. However, to realize the same average probability of symbol error, an M-ary PAM system requires more transmitted power. Specifically, we find that for M much larger than 2 and an average probability of symbol error small compared to 1, the transmitted power must be increased by the factor $M^2/\log_2 M$, compared to a binary PAM system.

In a baseband M-ary system, first of all, the sequence of symbols emitted by the information source is converted into an M-level PAM pulse train by a pulse-amplitude modulator at the transmitter i/p. Next, as with the binary PAM system, this pulse train is shaped by a transmit filter and then transmitted over the communication channe

(Baseband M-ary PAM Transmission)

→ which corrupts the signal waveform with both noise and distortion. The received signal is passed through a receive filter and then sampled at an appropriate rate in synchronism with the transmitter. Each sample is compared with preset threshold value and a decision is made as to which symbol was transmitted. We therefore find that the designs of the pulse-amplitude modulator and the decision-making device in an M-ary PAM are more complex than those in a binary PAM system. ISI, noise and imperfect synchronization cause errors to appear at the receiver o/p. The transmit and receive filters are designed to minimize these errors.

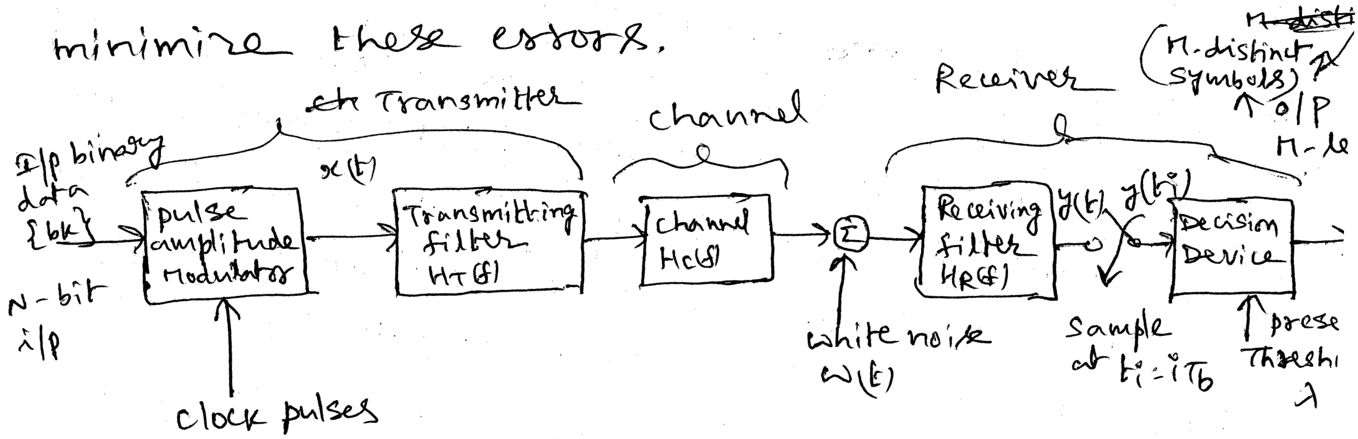


fig: Block diagram of baseband M-ary PAM Transmission.

~~BASEBAND DIGITAL TRANSMISSION-II~~

Bandlimited ~~digital PAM systems~~ :-

When the signal is transmitted over the channel, w/o any modulation, it is called baseband transmission

→ one of the major problem occured in baseband transmission is intersymbol interference. This interference takes place due to dispersive nature of the channel.

→ Nyquist criterion gives a condition for distortionless baseband transmission. It is possible to reduce the effect of intersymbol interference with the help of raised cosine spectrum. Correlative level coding is also used to minimize effects of intersymbol interference. Equalizers are used to compensate for distortion introduced in the channel.

Nyquist's criterion for Distortionless Baseband Binary Transmission

(Nyquist pulse shaping) :-

Time domain criterion :-

We know that o/p of receiver in binary data transmission system is

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} A_k p[(i-k)T_b]$$

$$y(t_i) = \mu A_i + \underbrace{\mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k p[(i-k)T_b]}_{ISI} \longrightarrow (1)$$

$i = 0, \pm 1, \pm 2, \pm 3 \ldots$

In above eq. if the ISI is absent, then the second term will not be present. i.e. the second term (summation) must be zero to eliminate effect of ISI. This is possible if the received pulse $p(t)$ is controlled such that,

$$p[(i-k)T_b] = \begin{cases} 1 & \text{for } i=k \\ 0 & \text{for } i\neq k \end{cases} \longrightarrow ②$$

If $p(t)$ satisfies the above condition, then we get a signal which is free from ISI. i.e.

$$y(t_i) = \mu A_i \longrightarrow ③$$

Hence eq ② gives the condition for perfect recept in absence of noise. It is the condition in time domain. This condition gives more useful criterio in frequency domain.

Criterion in Frequency Domain :-

Let $p(nT_b)$ represent the impulses at which $p(t)$ is sampled for decision. These samples are take at the rate of T_b. Fourier spectrum of these impulses is given as

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - nf_b) \longrightarrow ④$$

This means the spectrums of $p(t)$ are periodic with period ~~T_b (samp~~ $T_b \left(=\frac{1}{f_b}\right)$.

$f_b \rightarrow$ sampling freq.

$P_\delta(f) \rightarrow$ spectrum of $p(nT_b)$ and

$P(f) \rightarrow$ spectrum of $p(t)$.

We can think of $P_\delta(t)$ ~~$p(nT_b)$~~ as the infinite length of impulses with peri T_b, which are weighted with amplitudes of $p(t)$.

$$P_\delta(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\,\delta(t - nT_b) \longrightarrow ⑤$$

Fourier transform of $P_\delta(t)$ becomes,

$$P_\delta(f) = \int_{-\infty}^{\infty} P_\delta(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} p(nT_b)\, \delta(t-nT_b)\right] e^{-j2\pi ft}\, dt$$

Let $n = i-k$ in above eq.

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} p\left[(i-k)T_b\right] \delta\left[t-(i-k)T_b\right] e^{-j2\pi ft}\, dt$$

From eq ②

$$P_\delta(f) = \begin{cases} \int_{-\infty}^{\infty} p(0)\, \delta(t)\, e^{-j2\pi ft}\, dt & \text{for } i=k \\ \int_{-\infty}^{\infty} 0 \cdot \delta(t)\, e^{-j2\pi ft}\, dt & \text{for } i\neq k \end{cases}$$

$$\therefore P_\delta(f) = \int_{-\infty}^{\infty} p(0)\, \delta(t)\, e^{-j2\pi ft}\, dt \quad \text{for } i=k$$

$$= p(0) \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\, dt$$

$$= p(0) \times 1 \qquad \left(\because F.T\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\, dt = 1\right.$$

$$= p(0)$$

$$\therefore P_\delta(f) = p(0) \text{ for } i=k \longrightarrow ⑥$$

$$= 1 \quad \text{by normalization of } p(0)$$

Hence eq ④ becomes

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f-nf_b) = 1$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f-nf_b) = \frac{1}{f_b} = T_b$$

$$\therefore \boxed{\sum_{n=-\infty}^{\infty} P(f-nf_b) = T_b}$$

This is the freq. dom condition for zero ISI which is called Nyquist pulse shaping criterion for baseband transmission.

PASS BAND DATA TRANSMISSION

Introduction:-

There are basically two types of transmission of digital signals:

1). Base band data transmission:-

The digital data ~~modulates~~ is transmitted over the channel directly. There is no carrier or any modulation. This is suitable for transmission over short distances.

2). passband data transmission:-

The digital data modulates high frequency sinusoidal carrier. Hence it is also called digital CW modulation. It is suitable for transmission over long distances.

Types of passband Modulation:-

a). phase shift keying (PSK):-

In this technique, the digital data modulates phase of the carrier.

b). Frequency shift keying (FSK):-

In this technique, the digital data modulates frequency of the carrier.

c). Amplitude shift keying (ASK):-

In this technique, the digital data modulates amplitude of the carrier.

Types of Reception for passband Transmission

a). coherent (synchronous) detection:- In this method the local carrier generated at the receiver is phase locked with the carrier at the transmitter. Hence it is also called synchronous detection.

b). Non coherent (Envelope) detection :- In this method, the receiver carrier need not be phase locked with transmitter carrier. Hence it is also called envelope detection. Non coherent detection is simple but it has higher probability of error.

Requirements of pass band Transmission scheme :-

1. Maximum data transmission rate.
2. Minimum probability of symbol error
3. Minimum transmitted power.
4. Minimum channel bandwidth.
5. Maximum resistance to interfering signals
6. Minimum circuit complexity.

Advantages of passband Transmission over Baseband Transmission :-

1. Long distance transmission.
2. Analog channels, can be used for transmission.
3. Multiplexing techniques can be used for bandwidth conservation.
4. problems such as ISI and crosstalk are absent
5. passband transmission can take place over wireless channels also.
6. Large no. of modulation techniques are available.

Drawbacks of Passband Modulation

1. Modulation and demodulation equipments, transmitting/receiving antennas, interference problem make the system complex.
2. It is not suitable for short distance communication.

UNIT-IV

PASSBAND DATA TRANSMISSION

Passband Transmission Model :—

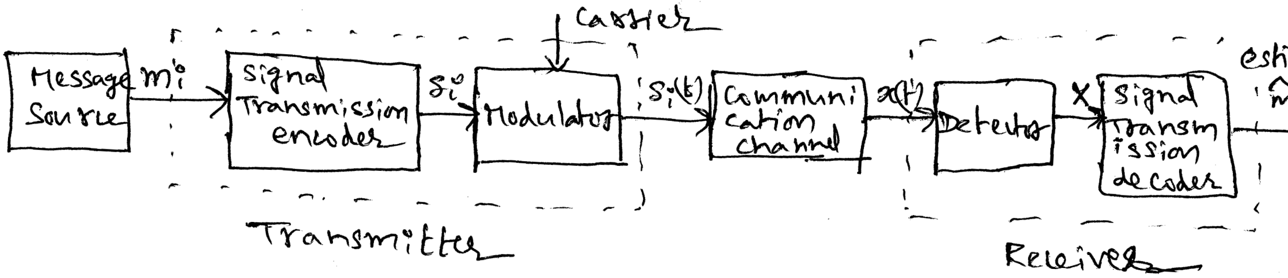


fig: Functional model of Passband data Transmission system

The Passband data transmission system is shown in figure above. The message source emits one symbol for every T seconds, with the symbols belonging to an alphabet of M symbols, denoted by $m_1, m_2, \ldots, m_M$. Let the probabilities of these symbols be $p(m_1)$, $p(m_2), \ldots, p(m_M)$. When the M symbols are equally likely, we write

$$p_i = p(m_i) = \frac{1}{M} \text{ for all } i \longrightarrow (1)$$

The M-ary o/p of the message source is given to a signal transmission encoder, producing a corresponding vector s_i made up of N real elements, where $N \leq M$. With the vector s_i as i/p, the modulator then constructs a distinct signal $s_i(t)$ of duration T seconds as the representation of the symbol m_i generated by the message source. The signal $s_i(t)$ is necessarily an energy signal, as shown by

$$E_i = \int_0^T s_i^2(t)\,dt, \quad i=1,2,\ldots M$$

$s_i(t)$ is real valued signal. One such signal is —

- transmitted every T seconds. With a sinusoidal carrier the modulator distingues one of signal from another by a step change in the amplitude (ASK), frequency (FSK) or ~~ph~~ phase (PSK) of the carrier.

The bandpass communication channel, coupling the transmitter to the receiver, is assumed to have two characteristics.

1. The channel is linear, with a bandwidth that is wide enough to accommodate the transmission of the modulated signal $s_i(t)$ with neglisible or no distortion.
2. The channel noise $w(t)$ is the sample function of a white Gaussian noise process of zero mean and PSD $N_0/2$.

The receiver, which consists of a detector followed by a signal transmission decoder, perform two functions:

1. It reverses the operations performed in the transmitter.
2. It minimizes the effect of channel noise on the estimate $\hat{m}$ computed for the transmitted symbol m_i.

Gram - Schmidth orthogonalization procedure

Gram - Schmidt Procedure

We know that any signal vector can be represented interms of orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$. Gram - schmidt orthogonalization procedure is the tool to obtain the orthonormal basis functions $\phi_i(t)$.

To derive an Expression for $\phi_i(t)$:-

We have the set of 'M' energy signals $s_1(t), s_2(t) \ldots s_M(t)$, for $s_1(t)$ the basis function is defined by

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \longrightarrow (1)$$

where E_1 is the energy of $s_1(t)$.

$$\Rightarrow s_1(t) = \sqrt{E_1}\, \phi_1(t) \longrightarrow (2)$$

~~From eq (1)~~ for $N=1$ we can write,

$$s_1(t) = s_{11}\phi_1(t) \longrightarrow (3) \qquad \left(\because s_i(t) = \sum_{j=1}^{N} s_{i1}\phi_j(t)\right)$$

By comparing eq (2) & (3)

$$s_{11} = \sqrt{E_1}$$

~~s_{21}~~ We know that $s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt$

from above eq,

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt \longrightarrow (4)$$

Let $g_2(t)$ be a new function which is given as,

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \longrightarrow (5)$$

The function is orthogonal to $\phi_1(t)$ over the interval 0 to T.

The second basis function is defined as,

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g2}}} \longrightarrow (6) \quad \text{here } E_{g2} = \int_0^T g_2^2(t)\,dt = \text{Energy of } g_2$$

To prove that $\phi_2(t)$ has unity energy :-

Energy of $\phi_2(t)$ will be

$$\int_0^T \phi_2^2(t)\,dt = \int_0^T \left[\frac{g_2(t)}{\sqrt{E_{g2}}}\right]^2 dt$$

$$= \frac{1}{E_{g2}} \int_0^T g_2^2(t)\,dt$$

$$= \frac{1}{E_{g2}} \cdot E_{g2} = 1$$

Thus $\phi_2(t)$ has unity energy.

To prove that $\phi_1(t)$ and $\phi_2(t)$ are ~~orthogonal~~ orthonormal :-

Consider the integration $\int_0^T \phi_1(t)\,\phi_2(t)\,dt$, By putting for $\phi_1(t)$ and $\phi_2(t)$,

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = \int_0^T \frac{s_1(t)}{\sqrt{E_1}} \cdot \frac{g_2(t)}{\sqrt{E_{g2}}}\,dt$$

$$= \frac{1}{\sqrt{E_1 E_{g2}}} \int_0^T s_1(t)\,g_2(t)\,dt$$

putting $g_2(t) = s_2(t) - s_{21}\phi_1(t)$ in above eq.

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = \frac{1}{\sqrt{E_1 E_{g2}}} \int_0^T s_1(t)\left[s_2(t) - s_{21}\phi_1(t)\right] dt$$

$$= \frac{1}{\sqrt{E_1 E_{g2}}} \left[\int_0^T s_1(t)\,s_2(t)\,dt - \int_0^T s_{21}\,s_1(t)\,\phi_1(t)\,dt\right]$$

Putting $s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt \rightarrow$ (from eq (ii))

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = \frac{1}{\sqrt{E_1 E_{g2}}} \left[\int_0^T s_1(t)\,s_2(t)\,dt - \int_0^T \int_0^T s_2(t)\,\phi_1(t)\,s_1(t)\,\phi_1(t)\,d\right.$$

In above eq. observe that there is a product of two terms $s_1(t)$ and $s_2(t)$. We know that the two symbols a

not present at a time. Hence the product $s_1(t) \cdot s_2(t)$
Therefore the integration terms will be zero and hence complete RHS will be zero. i.e

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = 0$$

Thus the two basis functions ~~for~~ are orthogonal.

Generalized equations for orthonormal basis functions

We know that

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2}}}$$

From above equation we can write the generaliz equation for orthonormal basis function $\phi_i(t)$ as

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{E_{g_i}}}, \quad i = 1, 2, \ldots N$$

where E_{g_i} = Energy of signal $g_i(t)$

$$= \int_0^T g_i^2(t)\,dt$$

where $g_i(t)$ is given by

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\,\phi_j(t) \qquad (\because g_2(t) = s_2(t) - s_{21}\phi_1$$

and the coeffs s_{ij} are given by

$$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt \quad j = 1, 2, \ldots i-1.$$

Geometric Representation of signals

The essence of geometric representation of signals is to represent any set of M energy signals $\{s_i(t)\}$ as linear combination of N orthogonal basis functions, where $N \leq M$. That is given a set of real-valued energy signals $s_1(t), s_2(t), \ldots, s_M(t)$, each of duration T seconds, we write

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t), \quad \begin{cases} 0 \leq t \leq T \\ i = 1, 2, \ldots, M \end{cases} \longrightarrow (1)$$

where the coefficients of the expansion are defined by

$$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt \quad \begin{cases} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{cases} \longrightarrow (2)$$

The real-valued basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ are orthonormal, i.e

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \longrightarrow (3)$$

where δ_{ij} is the Kronecker delta. The first condition of eq (3) states that each basis function is normalized to have unit energy. The second condition states that the basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ are orthogonal with respect to each other over the interval $0 \leq t \leq T$.

The set of coefficients $\{s_{ij}\}_{j=1}^{N}$ may naturally be viewed as an N-dimensional vector, denoted by s_i.

Given the N elements of the vectors s_i (i.e., $s_{i1}, s_{i2}, \ldots$, operating as i/p, we may use the scheme shown in fig (a) below to generate the signal $s_i(t)$, which follow directly from eq (1). It consists of a bank of N multipli

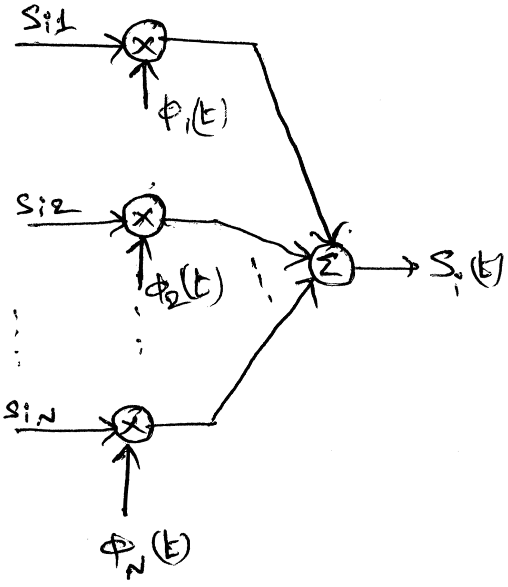
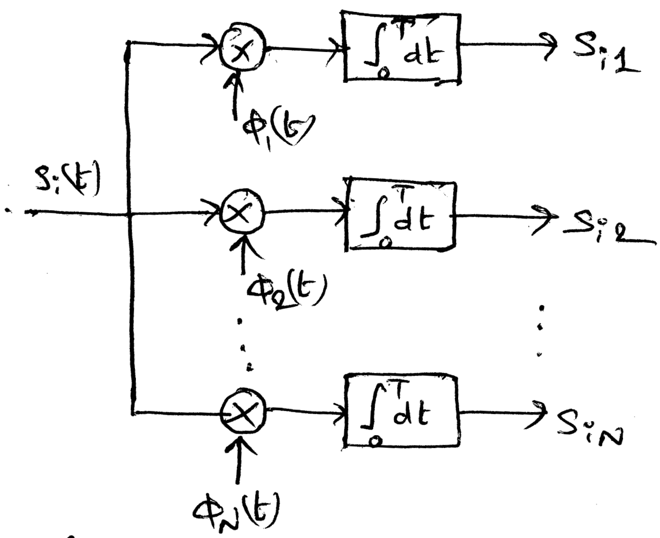


fig (a): synthesizer for generating the signal $s_i(t)$.



fig(b) Analyzer for generating the set of signal vectors $\{s_i\}$

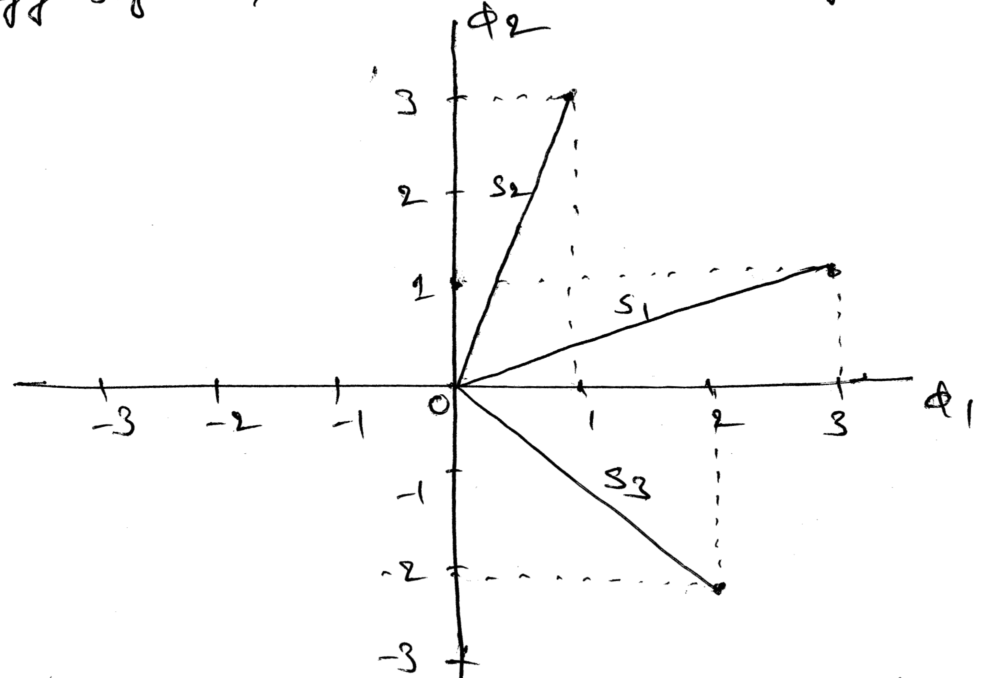
- with each multiplier having its own basis function, followe by a summer. This scheme may be viewed as a synthesi synthesizer.

→ Conversely, given the signals $s_i(t)$, $i = 1, 2, \ldots, M$, operatin as i/p, we may use the scheme shown in fig(b) above to calculate the coefficients $s_{i1}, s_{i2}, \ldots, s_{iN}$ which follow directly from eq ②. This second scheme consists of a ba of N product-integrators or correlators with a commo i/p and with each one of them supplied with its own basis function. The scheme of fig(b) above may be viewed as an analyzer.

Accordingly, we may state that each signal in th set $\{s_i(t)\}$ is completely determined by the vector of its coefficients

$$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}, \quad i = 1, 2, \ldots, M \longrightarrow ④$$

The vector s_i is called a signal vector. If we concept extend the conventional notation of two-dimensional and three dimensional Euclidean spaces to an N-dimensional Euclidean space, we may visualize the set of signal vectors $\{s_i \mid i=1,2,\ldots,M\}$ as defining a corresponding set of M-points in an N-dimensional Euclidean space, with N mutually perpendicular axes labeled $\phi_1, \phi_2, \ldots, \phi_N$. This N-dimensional Euclidean space is called the signal space. This provides the mathematical basis for the geometric representation of energy signals, which is shown in fig. below.



fig(c) Illustrating the geometric representation of Signals for the case when N=2 and M=3.

The squared-length of any signal vector s_i is defined to be the inner product or dot product of s with itself, as shown by

$$\|s_i\|^2 = s_i^T s_i = \sum_{j=1}^{N} s_{ij}^2, \quad i=1,2,\ldots,M \quad \rightarrow (5)$$

Where $\|s_i\| \rightarrow$ length & absolute value & norm of a signal vector s_i

s_{ij} is the jth element of s_i, $T \rightarrow$ Matrix transposition.

relationship b/w energy of a signal and its vector represen

The energy of a signal $s_i(t)$ of duration T secon is

$$E_i = \int_0^T s_i^2(t)\,dt \longrightarrow (6)$$

Substituting eq (1) into eq (6)

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\,\phi_j(t)\right]\left[\sum_{k=1}^{N} s_{ik}\,\phi_k(t)\right] dt$$

Interchanging the order of summation and integration, and then rearranging terms, we get

$$E_i = \sum_{j=1}^{N}\sum_{k=1}^{N} s_{ij}\, s_{ik} \int_0^T \phi_j(t)\,\phi_k(t)\,dt \longrightarrow (7)$$

But since the $\phi_j(t)$ form an orthogonal set, from eq(above equation reduces to

$$E_i = \sum_{j=1}^{N} s_{ij}^2 \qquad (j=k)$$

$$= \|s_i\|^2 \longrightarrow (8)$$

i.e energy of a signal $s_i(t)$ is equal to the squared length of the signal vector $s_i(t)$.

In case of a pair of signals $s_i(t)$ and $s_k(t)$, represented by the signal vectors s_i and s_k, respectively, we may also show that

$$\int_0^T s_i(t)\, s_k(t)\,dt = s_i^T s_k \longrightarrow (9)$$

i.e the inner product of the signals $s_i(t)$ and $s_k(t)$ ov the interval $[0,T]$, using their time-domain representati is equal to the inner product of their respective vector representations s_i and s_k.

Another useful relation involving the vector represent.

of the signals $s_i(t)$ and $s_k(t)$ is described by

$$\|s_i - s_k\|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2$$

$$= \int_0^T (s_i(t) - s_k(t))^2 dt \longrightarrow (10)$$

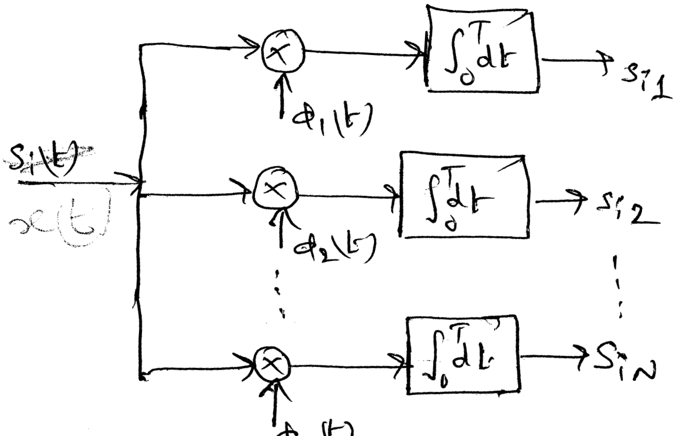
where $\|s_i - s_k\|$ is the Euclidean distance, d_{ik}, b/w the points represented by the signal vectors s_i and s_k.

The cosine of the angle θ_{ik} is

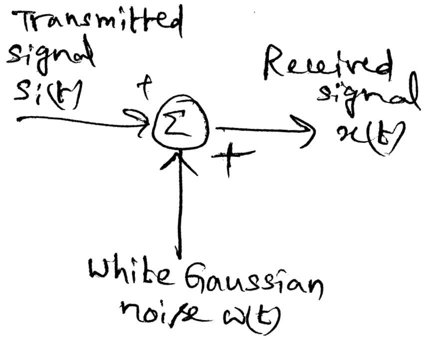
$$\cos \theta_{ik} = \frac{s_i^T s_k}{\|s_i\| \, \|s_k\|} \longrightarrow (11)$$

Thus two vectors s_i and s_k are orthogonal or perpendicular to each other if their inner product $s_i^T s_k$ is zero, in which case $\theta_{ik} = 90°$.

Conversion of the Continuous AWGN channel into a vector channel



fig(a) bank of N correlators



fig(b) AWGN model of a channel.

Suppose that the i/p to the bank of N product integrators or correlators in fig(a) is not the transmitted signal $s_i(t)$ but rather the received signal $x(t)$ defined in accordance with the idealized AWGN channel of fig(b). That is

$$x(t) = s_i(t) + w(t), \quad \begin{cases} 0 \le t \le T \\ i = 1, 2, 3, \ldots, M. \end{cases} \longrightarrow (1)$$

where $w(t)$ is a sample function of a white gaussian noise process $W(t)$ of zero mean and power spectral density $N_0/2$. Correspondingly, we find that the o/p of correlator j, say, is the sample value of a random variable X_j, as shown by

$$x_j = \int_0^T x(t)\phi_j(t)\,dt = \int_0^T [s_i(t) + w(t)]\phi_j(t)\,dt = \int_0^T s_i(t)\phi_j(t)\,dt + \int_0^T w(t)\phi_j(t)\,dt \longrightarrow (2)$$

$$= s_{ij} + w_j, \quad j = 1, 2, \ldots, N$$

where s_{ij} is a deterministic quantity contributed by the transmitted signal $s_i(t)$; it is defined by

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)\,dt \longrightarrow (3)$$

and w_j is the sample value of a random ~~vari~~ $\rightarrow$

– variable W_j that arises because of the presence of the channel noise $w(t)$; it is defined by

$$W_j = \int_0^T w(t)\,\phi_j(t)\,dt \longrightarrow (4)$$

Consider next a new random process $x'(t)$ whose sample function $x'(t)$ is related to the received signal $x(t)$ as follows:

$$x'(t) = x(t) - \sum_{j=1}^{N} x_j\,\phi_j(t) \longrightarrow (5)$$

Substituting eq ① & eq ② into eq ⑤

$$x'(t) = s_i(t) + w(t) - \sum_{j=1}^{N} (s_{ij} + w_j)\,\phi_j(t)$$

$$= s_i(t) + w(t) - \sum_{j=1}^{N} s_{ij}\,\phi_j(t) - \sum_{j=1}^{N} w_j\,\phi_j(t)$$

$$= s_i(t) + w(t) - s_i(t) - \sum_{j=1}^{N} w_j\,\phi_j(t) \quad \left(\because s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j\right)$$

$$= w(t) - \sum_{j=1}^{N} w_j\,\phi_j(t)$$

$$= w'(t) \longrightarrow (6)$$

Therefore the sample function $x'(t)$ depends solely on the channel noise $w(t)$. On the basis of equation ⑤ and ⑥, we may express the received signal as

$$x(t) = \sum_{j=1}^{N} x_j\,\phi_j(t) + x'(t) \longrightarrow (7)$$

$$= \sum_{j=1}^{N} x_j\,\phi_j(t) + w'(t)$$

Accordingly, we may view $w'(t)$ as a sort of remainder term that must be included on the right to preserve the equality in eq ⑦.

Likelihood Functions

The conditional probability density functions $f_X(x|m_i)$, $i=1,2,\ldots,M$, are the very characteriza of an AWGN channel. Their derivation leads to a functional dependence on the observation vector x, gi the transmitted ~~vector~~ message symbol m_i. However at the receiver we have the exact opposite situation: we are given the observation vector x and the requirement is to estimate the message symbol m_i that is responsible for generating x. To emphasize this, we introduce the idea of a likelihood function, denoted by $L(m_i)$ and defined by

$$L(m_i) = f_X(x/m_i), \quad i=1,2,\ldots,M \longrightarrow (1)$$

It is important however to recognize that although the $L(m_i)$ and $f_X(x|m_i)$ have exactly the same mathematical form, their individu meanings are different.

In practice, we find it more convenient to work with the log-likelihood function, denotec by $l(m_i)$ and defined by

$$l(m_i) = \log L(m_i), \quad i=1,2,\ldots,M \longrightarrow (2)$$

The log-likelihood function bears a one-to-on relationship to the likelihood function for two reasons:

$\rightarrow$

1. By definition, a probability density function is always nonnegative. It follows therefore that the likelihood function is likewise a nonnegative quantity.

2. The logarithmic function is a ~~mom monotonic~~ monotonically increasing function of its argument.

The conditional probability density function for AWGN is given by

$$f_X(x/m_i) = (\pi N_0)^{-N/2} e^{\left[-\frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{ij})^2\right]}, \quad i = 1, 2, \dots, M \rightarrow ③$$

where $x_j \rightarrow$ random variable

$s_{ij} \rightarrow$ Mean value

$N_0/2 \rightarrow$ variance

The use of eq ③ in eq ② yields the log-likelihood functions for an AWGN channel as

$$l(m_i) = -\frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{ij})^2, \quad i = 1, 2, \dots, M \rightarrow ($$

$$l(m_i) = \log_e\left[(\pi N_0)^{-N/2} e^{\left[-1/N_0 \sum_{j=1}^{N}(x_j - s_{ij})^2\right]}\right]$$

$$= \cancel{l_0} - \frac{N}{2}\log(\pi N_0) - \frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{ij})^2$$

$\log ab = \log a$ -

$\log_e e^x$ =

Where we have ignored the constant term $-\left(\frac{N}{2}\right)\log(\pi N_0)$ as it bears no relation whatsoever to the message symbol m_i. Note that the s_{ij}, $j = 1, 2, \dots, N$, are the elements of the signal vector s_i representing the message symbol m_i.

Coherent Detection of Signals in Noise: Maximum Likelihood Decoding

Suppose that in each time slot of duration T seconds, one of the M possible signals $s_1(t), s_2(t), \ldots, s_M(t)$ is transmitted with equal probability, $1/M$. For geometric signal representation, the signal $s_i(t)$, $i=1,2,\ldots,M$, is applied to a bank of correlators, with a common i/p and supplied with an appropriate set of N orthogonal basis functions. The resulting correlator o/ps define the signal vector s_i. We may represent $s_i(t)$ by a point in a Euclidean space of dimension $N \leq M$. We refer to this point as the transmitted signal point or message point. The set of message points corresponding to the set of transmitted signals $\{s_i(t)\}_{i=1}^{M}$ is called a signal costellation.

However, the representation of the received signal $x(t)$ is complicated by the presence of additive noise $w(t)$. The received signal $x(t)$ is applied to the bank of N correlators, the correlator o/ps define the observation vector x. The vectors differs from the signal vector s_i by the noise vector w as follows.

$$x = s_i + w, \quad i=1,2,\ldots,M. \longrightarrow (1)$$

The noise vector w is completely characterized b the noise $w(t)$;

Now, based on the observation vector x, we may represent the received signal $x(t)$ by a point in the same Euclidean space used to represent the transmitted signal. We refer to this second point as the received signal point. The received signal point wanders about the message point in a completely random fashion, in the sense that it may lie anywhere inside a Gaussian-distributed "cloud" centered on the message point as shown in fig (a) below. For a particular realization of the noise vector W, the relationship b/w the observation vector x and the signal vector s_i is as illustrated in fig (b).

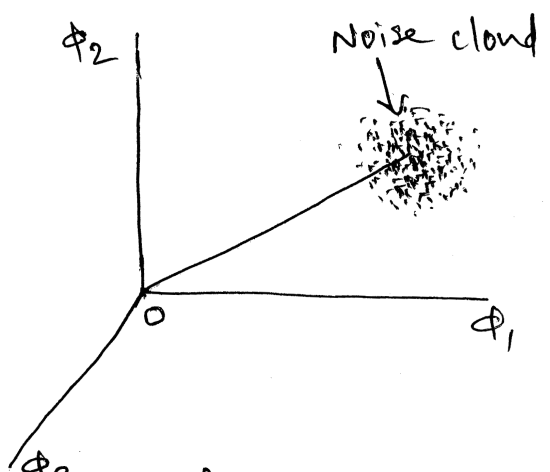


fig (a)

effect of noise perturbation

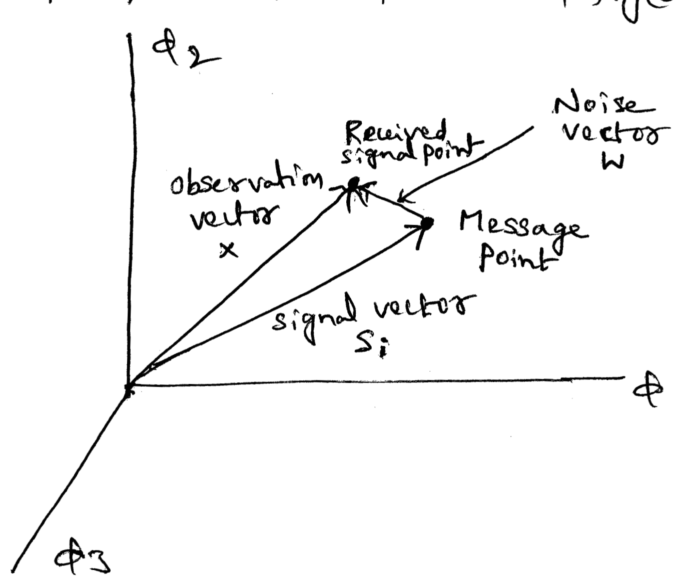


fig (b) The location of the received signal point

Now the signal detection problem is as follows.

Given the observation vector x, perform a mapping from x to an estimate $\hat{m}$ of the transmitted symbol, m_i, in a way that would minimize the probability of error in the decision-making process.

Suppose that, given the observation vector x, we make the decision $\hat{m} = m_i$. The probability of error

- this decision, which we denote by $P_e(m_i|x)$, is simply 3.1

$$P_e(m_i|x) = P(m_i \text{ not sent}|x) = 1 - P(m_i \text{ sent}|x) \longrightarrow (2)$$

The decision-making criterion is to minimize the probability of error in mapping each given observation vector x into a decision. on the basis of eq (2), we may therefore state the optimum decision rule:

$$\text{set } \hat{m} = m_i \text{ if}$$
$$P(m_i \text{ sent}|x) \geq P(m_k \text{ sent}|x) \text{ for all } k \neq i \longrightarrow (3)$$

where $k = 1, 2, \ldots, M$. The decision rule is referred to as the maximum a posteriori probability (MAP) rule.

The condition in eq (3) may be expressed more ~~exp~~ explicitly in terms of the a priori probabilities of the transmitted signals and in terms of the likelyhood functions. Using Baye's rule in eq (3), we may restate the MAP rule as follows:

$$\text{Set } \hat{m} = m_i \text{ if}$$

$$\frac{p_k f_x(x/m_k)}{f_x(x)} \text{ is maximum for } k = i \longrightarrow (4)$$

where p_k is the a priori probability of transmitting symbol m_k, $f_x(x/m_k)$ is the conditional probability density function of the random observation vector x gi the transmission of symbol m_k, and $f_x(x)$ is the unconditional probability density function of x. In eq (

→ The denominator term $f_x(x)$ is independent of the transmitted symbol.

→ The a priori probability, $p_k = p_i$ when all the source symbols are transmitted with equal probabili[ty]

→ The conditional probability density function $f_X(x|m_k)$ bears a one-to-one relationship to the log-likelihood function $l(m_k)$.

Accordingly, we may restate the decision rule of eq ④ in terms of $l(m)$ simply as follows:

$$\text{Set } \hat{m} = m_i \text{ if}$$
$$l(m_k) \text{ is maximum for } k = i \qquad \longrightarrow ⑤$$
$$\text{where } l(m_k) = \log\left[f_X(x|m_k)\right]$$

This decision rule is referred to as the maximum likelyhood rule, and the device for its implementation is correspondingly referred to as maximum likelyhood decoder. According to equation ⑤, a maximum likelyhood decoder computes the log-likelihood functions as metrics for all the M possible message symbols, compares them, and then decides in favor of the maximum. Thus the maximum likelihood decoder differs from the maximum a posteriori decoder in that it assumes equally likely message symbols.

It is useful to have a graphical interpretatio[n] of the maximum likelihood decision rule. Let Z den[ote] the N-dimensional space of all possible observation vectors x. We refer to this space as the observation space. Because we have assumed that the decision rule must say $\hat{m} = m_i$, where $i = 1, 2, \ldots, M$, the tot[al] observation space Z is correspondingly partitioned into

→ M-decision regions, denoted by $Z_1, Z_2, \cdots, Z_M$. 3

Accordingly, we may restate the decision rule of eq ⑤ as follows:

$$\text{observation vector } x \text{ lies in region } Z_i \text{ if } l(m_k) \text{ is maximum for } k = i \quad \rightarrow ⑥$$

Aside from the boundaries b/w the decision regions $Z_1, Z_2, \cdots, Z_M$, it is clear that this set of regions covers the entire space of possible observation vector x. We adopt the convention that all ties are resolved at random; that is, the receiver simply makes a guess. Specifically, if the observation vector x falls on the boundary b/w any two decision regions, Z_i and Z_k say, the choice b/w the two possible decisions $\hat{m} = m_i$ and $\hat{m} = m_k$ is resolved a priori by the flip of a fair coin.

The maximum likelihood decision rule of eq ⑤ or its geometric counterpart described in eq ⑥ is of a generic kind, with the channel noise $w(t)$ being additive as the only restriction imposed on it. We next specialize this rule for the case when w(t) is both white and Gaussian.

The log-likelihood function for an AWGN channel is

$$l(m_k) = -\frac{1}{N_0} \sum_{j=1}^{N} (x_j - s_{kj})^2, \quad k = 1, 2, \cdots, M$$

We ~~know~~ note that $l(m_k)$ attains its maximum value when the summation term

$$\sum_{j=1}^{N} (x_j - s_{kj})^2$$ is ~~minimum~~ minimized by

→ the choice $k = i$. Accordingly, we may formulate the maximum likelihood decision rule for an AWGN channel as

observation vector x lies in region Z_i if

$$\sum_{j=1}^{N} (x_j - s_{kj})^2 \text{ is minimum for } k = i \longrightarrow (7)$$

and we know that

$$\sum_{j=1}^{N} (x_j - s_{kj})^2 = \|x - s_k\|^2 \longrightarrow (8)$$

where $\|x - s_k\|$ is the Euclidean distance b/w the received signal point and message point, represented by the vectors x and s_k, respectively. Accordingly, we may restate the decision rule of eq (7) as follows;

observation vector x lies in ~~the~~ region Z_i if the Euclidean distance $\|x - s_k\|$ is minimum for $k = i$ → (9)

Eq (9) states that the maximum likelihood decision ru is simply to choose the message point closest to the received signal point, which is intuitively satisfying.

In practice, the need for squares in the decision rule of eq (9) is avoided by recognizing that

$$\sum_{j=1}^{N} (x_j - s_{kj})^2 = \sum_{j=1}^{N} x_j^2 - 2\sum_{j=1}^{N} x_j s_{kj} + \sum_{j=1}^{N} s_{kj}^2 \quad (10)$$

The first summation term of this expansion is independent of the index k and may therefore be ignored. The second summation term is the inner product of the observation vector x and signal vector s_k. The third summation term is the energy of the transmitted signal $s_k(t)$. Accordingly, we may formulate a decision rule equivalent to that of eq ⑨ as follows:

observation vector x lies in region Z_i if

$$\sum_{j=1}^{N} x_j s_{kj} - \frac{1}{2} E_k \text{ is maximum for } k = i \longrightarrow (11)$$

where E_k is the energy of the transmitted signal $s_k(t)$:

$$E_k = \sum_{j=1}^{N} s_{kj}^2 \longrightarrow (12)$$

From eq (11) we deduce that, for an AWGN channel the decision regions are regions of the N-dimensional observation ~~channel~~ space Z, bounded by linear ~~[(N-1)~~ [(N-1)-dimensional hyperplane] boundaries. Figure below shows the example of decision regions for $M=4$ signals and $N=2$ dimensions, assuming that the signals are transmitted with equal energy, E, and equal probability.

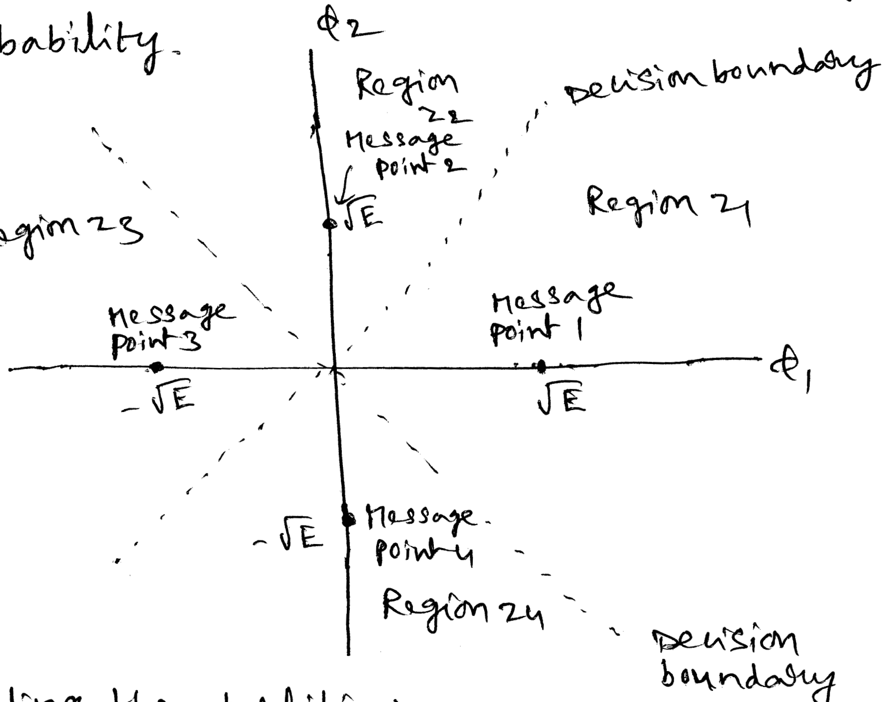
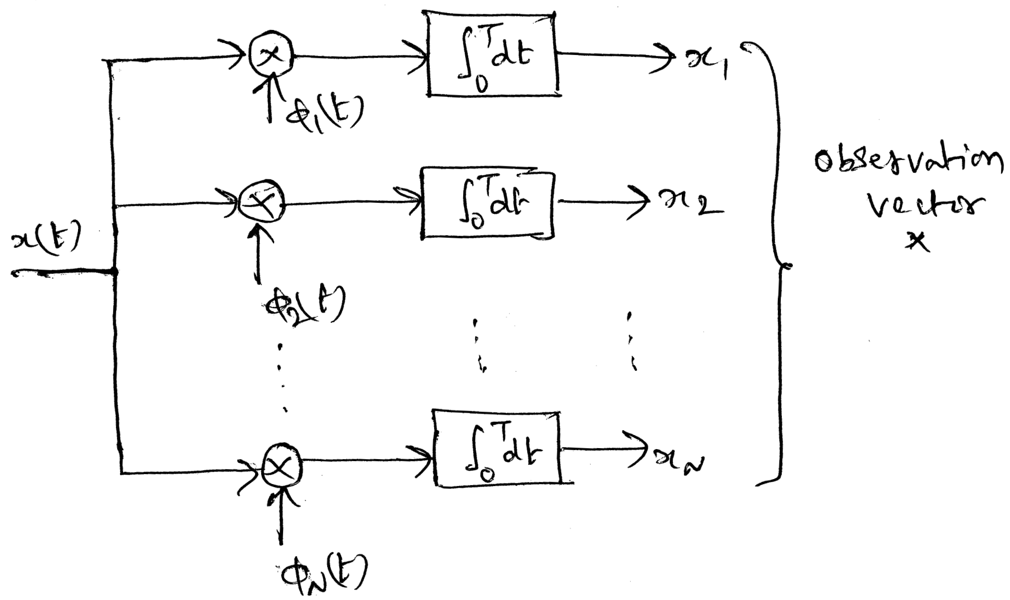


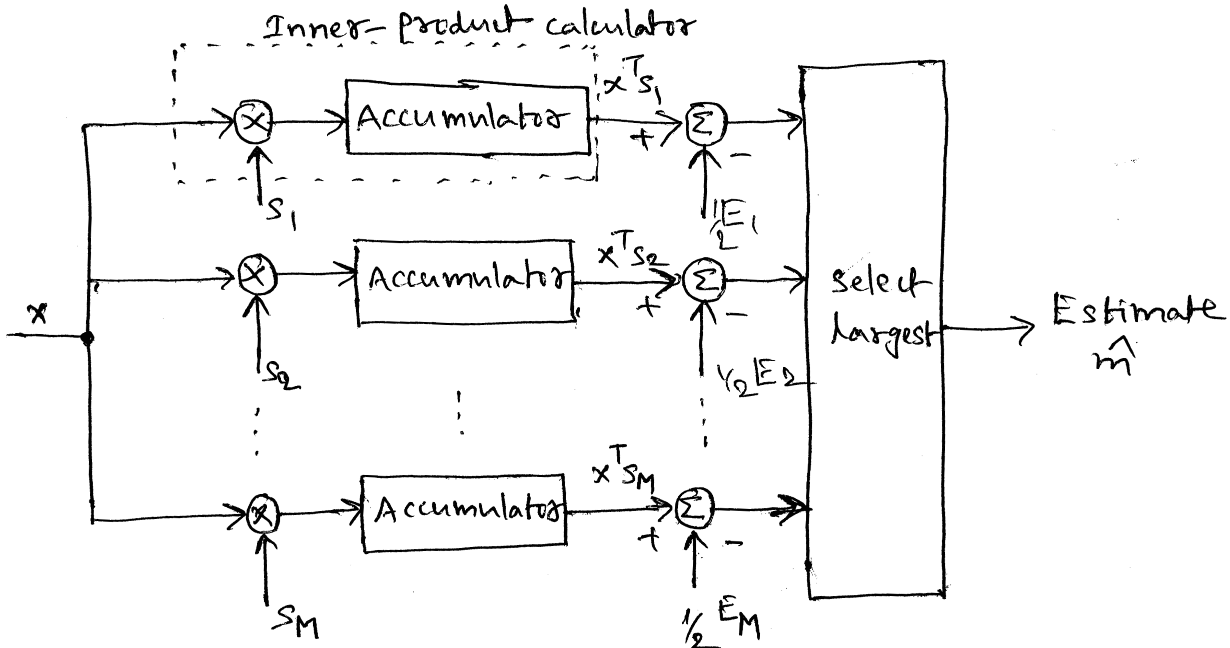
fig: Illustrating the partitioning of the observation space in decision regions for the case when $N=2$ and $M=4$; it is assumed that the M transmitted symbols are equally likely.

Correlation Receiver

For an AWGN channel and for the case when the transmitted signals $s_1(t), s_2(t), \ldots, s_M(t)$ are equally likely, the optimum receiver consists of two subsystems, as shown in figure below. wh[ich] is called as a correlation receiver.



fig(a) Detector or demodulator



fig(b): signal transmission decoder

The detector part of the receiver is shown in fig(a). It consists of a bank of M product-integrators or

— Correlators, supplied with a corresponding set of coherent reference signals or orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ that are generated locally. This bank of correlators operates on the received signal $x(t)$, $0 \le t \le T$, to produce the observation vector $\mathbf{x}$.

The second part of the receiver, namely, the signal transmission decoder is shown in fig (b). It is implemented in the form of a maximum-likelihood decoder that operates on the observation vector $\mathbf{x}$ to produce an estimate, $\hat{m}$, of the transmitted symbol m_i, $i = 1, 2, \ldots, M$, in a way that would minimize the average probability of symbol error. We know tha the decision rule as follows.

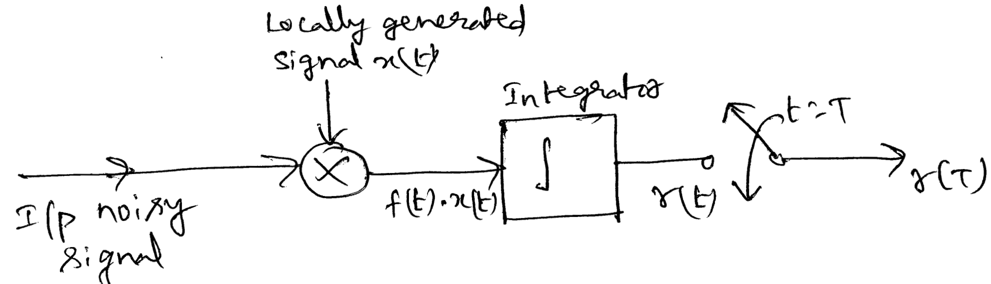
observation vector $\mathbf{x}$ lies in region Z_i if

$$\sum_{j=1}^{N} x_j s_{kj} - \frac{1}{2} E_k \text{ is maximum for } k = i$$

In accordance with with above equation, the N elements of the observation vector $\mathbf{x}$ are first multiplie by the corresponding N elements of each of the M signal vectors $s_1, s_2, \ldots, s_M$, and the resulting products are successively summed in accumulators to form the corresponding set of inner products $\{\mathbf{x}^T s_k \mid k = 1, 2, \ldots, M\}$. Next, the inner products are correct for the fact that the transmitted signal energies may be unequal. Finally, the largest in the resultin set of numbers is selected, and an appropriate decisio on the transmitted message is made.

Equivalence of Correlation and Matched filter Receiver

Correlation (Correlator) Receiver



$f(t) = x(t) + \cancel{n(t)} + w(t)$

fig: Block diagram of the correlator

Fig. above shows the block diagram of the correlator. Here $f(t)$ represents i/p noisy signal, i.e., $f(t) = x(t) + n$

The signal $f(t)$ is multiplied ~~to~~ by the locally generated replica of i/p signal $x(t)$. This result of multiplication $f(t) \cdot x(t)$ is integrated.

The o/p of the integrator is sampled at $t = T$. Then based on this sampled value, decision is made. This is how the correlator works. It is called correlator since it correlates the received signal $f(t)$ with a stored replica of the known signal $x(t)$. In the block diagram of above figure, the product $f(t)\,x(t)$ is integrated over one symbol period (T). Hence o/p r(t) can be written as,

$$r(t) = \int_0^T f(t)\,x(t)\,dt$$

At $t = T$

$$\text{o/p of correlator} = r(T) = \int_0^T f(t)\,x(t)\,dt \rightarrow (1)$$

Now let us consider the matched filter as shown below.

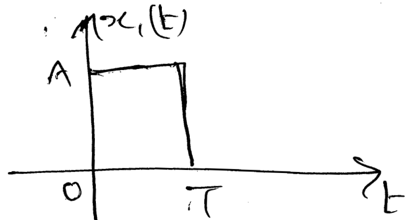
P4:- A polar NRZ w/f is to be received by a matched 2.9 filter. Binary 1 is represented by a rectangular positive pulse and binary zero is represented by a rectangular negative pulse. Find out the impulse ~~step~~ response of the matched filter and sketch it.

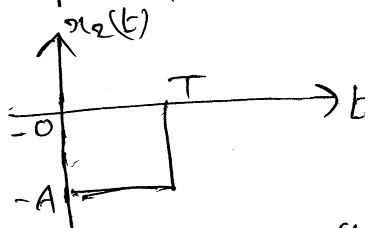
Sol:- Let $x_1(t)$ represent the positive rectangular pulse whose duration is 'T' and $x_2(t)$ represent a negative ~~step~~ rectangular pulse whose duration is also T.

i.e

$$x_1(t) = +A \quad \text{for} \quad 0 \le t \le T$$

and $x_2(t) = -A$ for $0 \le t \le T$



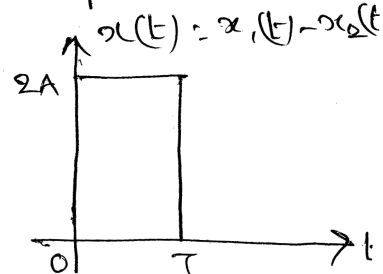


then

$$x(t) = x_1(t) - x_2(t) \quad \text{for} \quad 0 \le t \le T$$

$$= A - (-A)$$

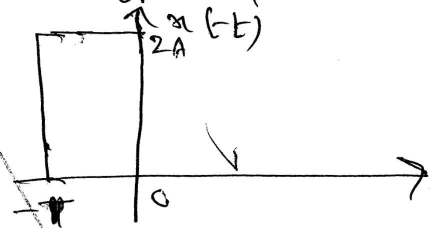
$$x(t) = 2A \quad \text{for} \quad 0 \le t \le T$$



$$\Rightarrow x(-t) = 2A \quad \text{for} \quad -T \le t \le 0$$

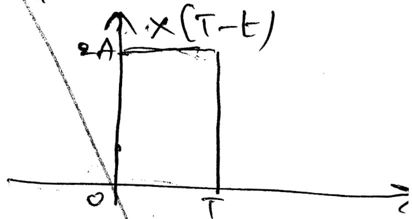
Now delay $x(t)$ by 'T' seconds,

i.e $x(T-t) = 2A$ for $0 \le t \le T$



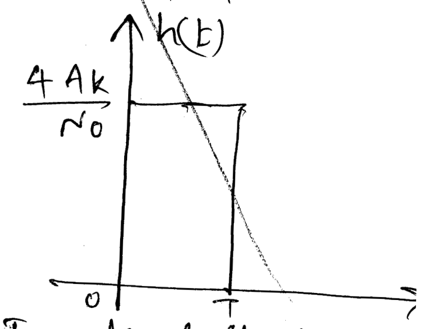
Impulse response of a matched filter is given by

$$h(t) = \frac{2K}{N_0} x(T-t)$$



$$= \frac{2K}{N_0} \cdot 2A \quad (\because x(T-t) = 2A)$$

$$= \frac{4AK}{N_0}.$$



Impulse resp. of match filter for rect. pulse

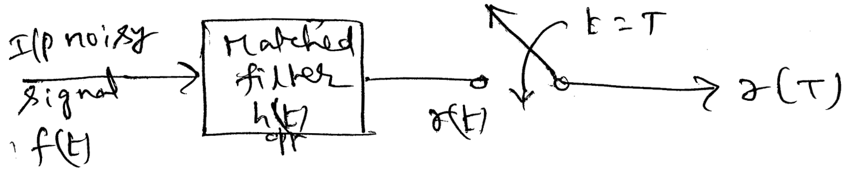


fig (b) : Block diagram of a matched filter receiver

In the above block diagram observe that the matched filter does not need locally generated replica of i/p signal $x(t)$. The o/p of the matched filter can be obtained by convolution of i/p $f(t)$ and its impulse response $h_{opt}(t)$. i.e.

$$r(t) = f(t) * h_{opt}(t)$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot h_{opt}(t-\tau)\, d\tau$$

We know that impulse response $h_{opt}(t)$ of the matched filter for the i/p $x(t)$ is given as,

$$h_{opt}(t) = K x(T-t)$$

$$\Rightarrow h_{opt}(t-\tau) = \cancel{K x(T-t+\tau)}\ K\, x(T-t+\tau)$$

$$\therefore r(t) = \int_{-\infty}^{\infty} f(\tau)\, K\, x(T-t+\tau)\, d\tau$$

Since the integration is performed over one bit period, we can change integration limits from 0 to T.

i.e.

$$r(t) = K \int_0^T f(\tau)\, x(T-t+\tau)\, d\tau$$

At $t = T$, the above eq. will be

$$r(T) = K \int_0^T f(\tau)\, x(T-T+\tau)\, d\tau = K \int_0^T f(\tau)\, x(\tau)\, d\tau$$

Let us put $\tau = t$ just for convenience of notation.

$\cancel{r(t)}$

→

→ O/P of matched filter = $r(T) = k\int_0^T f(t)\,x(t)\,dt \rightarrow (2)$

observe that eq (1) & eq (2) are identical. In eq (2) the constant k is present which can be normalized to 1. The similarity b/w eq (1) & eq (2) shows that the matched filter and correlator gives same o/p. Therefore we can state,

The matched filter and correlator are two distinct, independent techniques which give the same result. These two techniques are used to synthesize the optimum filter.

Probability of Error:-

Suppose that the observation space Z is partitioned in accordance with the maximum ~~likely~~ likelihood decision rule, into a set of M regions $\{Z_i\}_{i=1}^{M}$. Suppose also that symbol m_i (or, ~~equal~~ equivalently signal vector s_i) is transmitted, and an observation vector x is received. Then an error occurs ~~whene~~ whenever the received signal point represented by x does not fall inside region Z_i associated with the message point represented by s_i.

Averaging over all possible transmitted symbols, we readily see that the average probability of symbol error, P_e is

$$\cancel{P_e = \sum_{i=1}^{M} p_i\, P(x}$$

$$P_e = \sum_{i=1}^{M} p_i\, P(x \text{ does not lie in } Z_i \mid m_i \text{ sent})$$

$$= \frac{1}{M} \sum_{i=1}^{M} P(x \text{ does not lie in } Z_i \mid m_i \text{ sent}) \longrightarrow (1)$$

$$= 1 - \frac{1}{M} \sum_{i=1}^{M} P(x \text{ lies in } Z_i \mid m_i \text{ sent})$$

Since x is the sample value of random vector X, we may rewrite above equation in terms of the likelihood function (when m_i is sent) as follows:

$$P_e = 1 - \frac{1}{M} \sum_{i=1}^{M} \int_{Z_i} f_X(x \mid m_i)\, dx \longrightarrow (2)$$

For an N-dimensional observation vector, the integral in above equation is likewise N-dimensional.

Amplitude shift keying (ASK) or ON-OFF keying (OOK)

ASK is the simplest digital modulation technique. In this method, there is only one unity energy carrier and it is switched on or off depending upon the i/p binary sequence. The ASK w/f can be represented as,

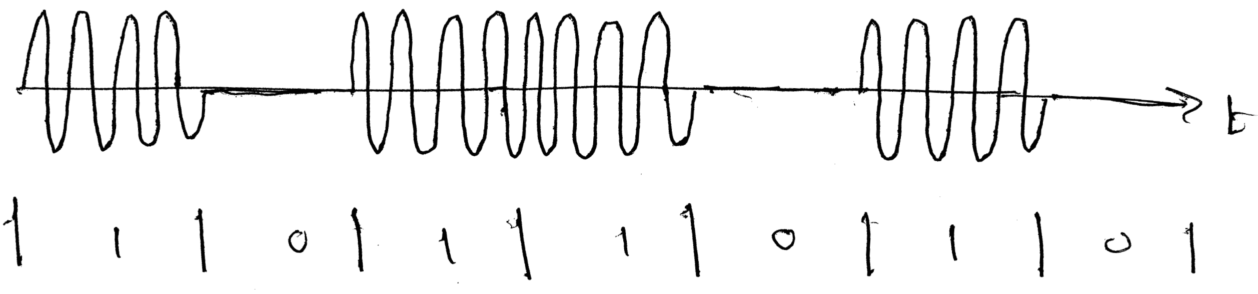
$$s(t) = \sqrt{2P_s} \cos(2\pi f_0 t) \longrightarrow (1)$$

To transmit symbol '0', the signal $s(t) = 0$. That is no signal is transmitted. $s(t)$ contains some complete cycles of carrier freq 'f'. Thus

symbol '1' ⇒ pulse is transmitted,

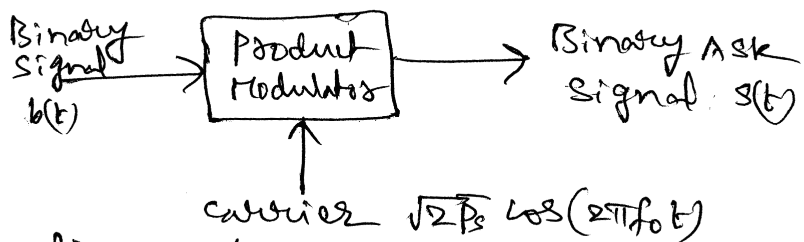
symbol '0' ⇒ no pulse is transmitted.

Thus the ASK w/f looks like an ON-OFF of the signal. Hence it is also called ON-OFF keying (OOK).



fig(a) ASK w/f

ASK Generator :-



fig(b) Block diagram of ASK generator

Fig above shows the ASK generator. The i/p binary sequence is applied to the product modulator. The product modulator amplitude modulates the sinusoidal carrier. It passes the carrier when i/p bit is '1'. It blocks the carrier (i.e. zero o/p) when i/p bit is '0'. The w/f of ASK is shown in fig(a) above.

Coherent ASK Detector :-

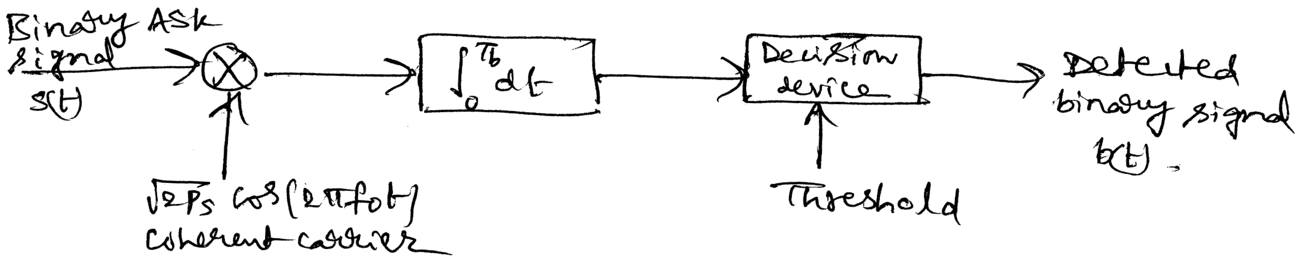


fig (c) Block diagram of coherent ASK detector

Fig (c) above shows the block diagram of coherent ASK detector. The ASK signal is applied to the ~~carrier~~ correlator consisting of multiplier and integrator. The locally generated coherent carrier is applied to the multiplier. The o/p of multiplier is integrated over one bit period. The decision device takes the decision at the end of ~~the~~ every bit period. It compares the o/p of integrator with the threshold. Decision is taken in favour of '1' when threshold is exceeded. Decision is taken ~~to~~ as '0' if threshold is not exceeded.

Noncoherent ASK Reception :-

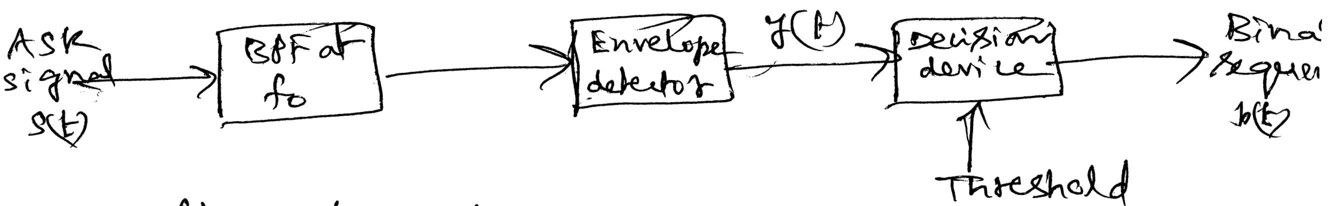


fig: Block diagram of non-coherent ASK receiver

Fig. above shows the block diagram of non-coherent ASK receiver. The received ASK signal is given to BPF. This BPF ~~filt~~ passes only carrier frequency, fo

Coherent Binary systems

Binary phase shift keying (BPSK) :-

In BPSK, binary symbol '1' and '0' modulate the phase of the carrier. Let the carrier be

$$\therefore s(t) = A\cos(2\pi f_0 t)$$

The power dissipated $= P = \left(\frac{A}{\sqrt{2}}\right)^2 = (V_{rms})^2 = \frac{A^2}{2}$

$$\Rightarrow \quad A = \sqrt{2P}$$

When the symbol is changed, then the phase of the carrier is changed by 180 degrees (π radians)

Consider for example,

$$\text{symbol '1'} \Rightarrow s_1(t) = \sqrt{2P}\cos(2\pi f_0 t)$$

if next symbol is '0' then

$$\text{Symbol '0'} \Rightarrow s_2(t) = \sqrt{2P}\cos(2\pi f_0 t + \pi)$$

$$= -\sqrt{2P}\cos(2\pi f_0 t) \quad (\because \cos(\theta+\pi) = -\cos\theta)$$

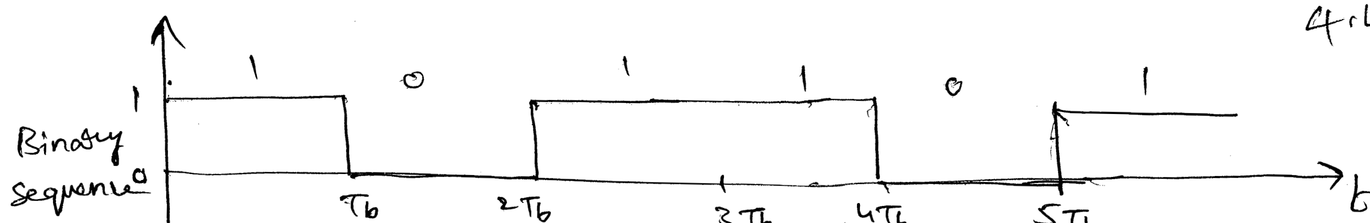
With the above eq. we can define BPSK signal combinely as

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

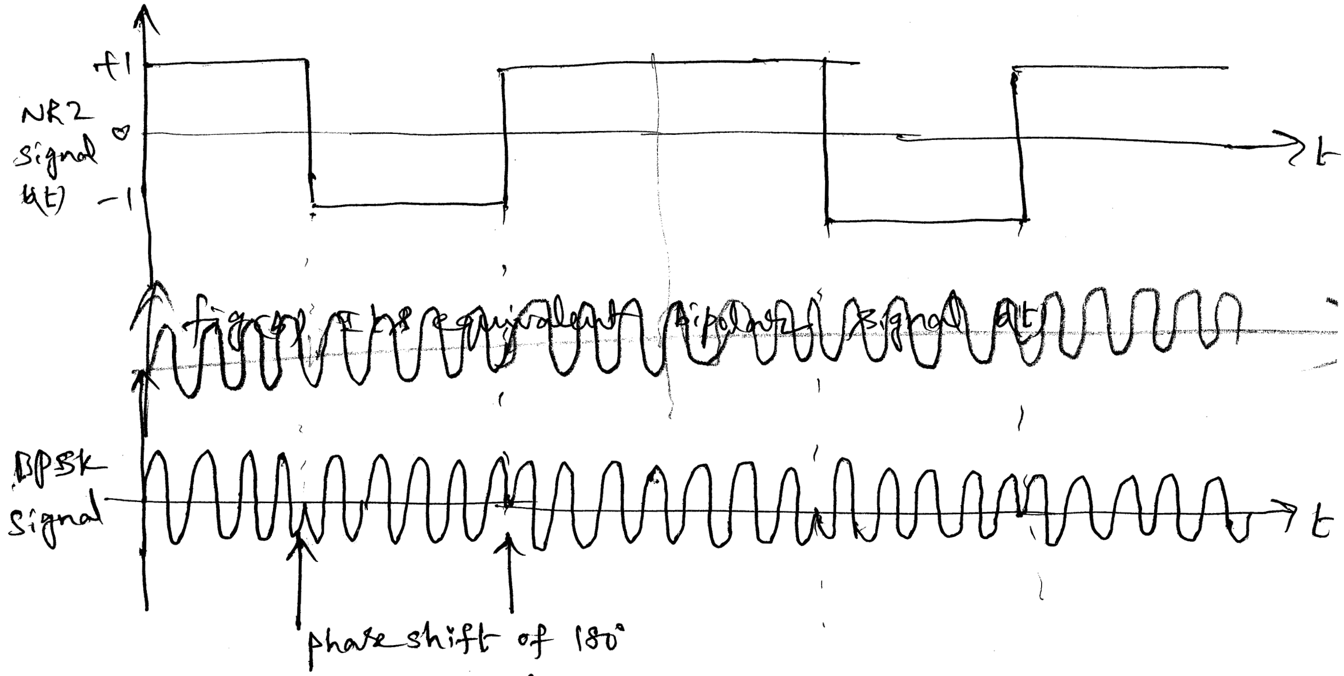
here $b(t) = +1$ when binary '1' is to be transmitted

$= -1$ when binary '0' is to be transmitted

The fig(a) below shows binary sequence and fig(c) shows the BPSK signal.



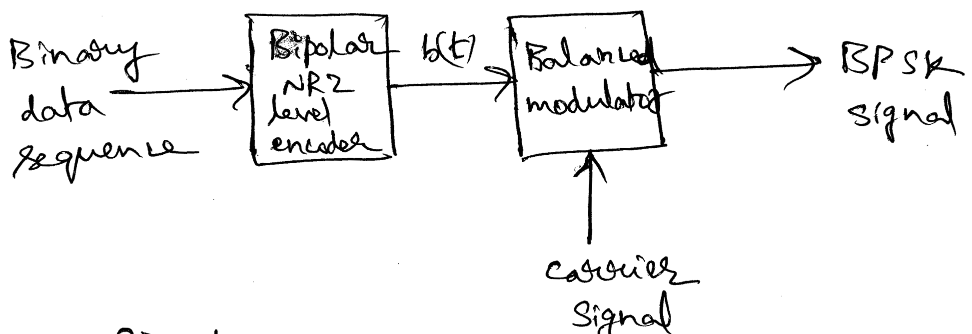
fig(a) Binary sequence



fig(b) is equivalent bipolar signal b(t)

fig(c) BPSK signal.

Generation of BPSK signal :-



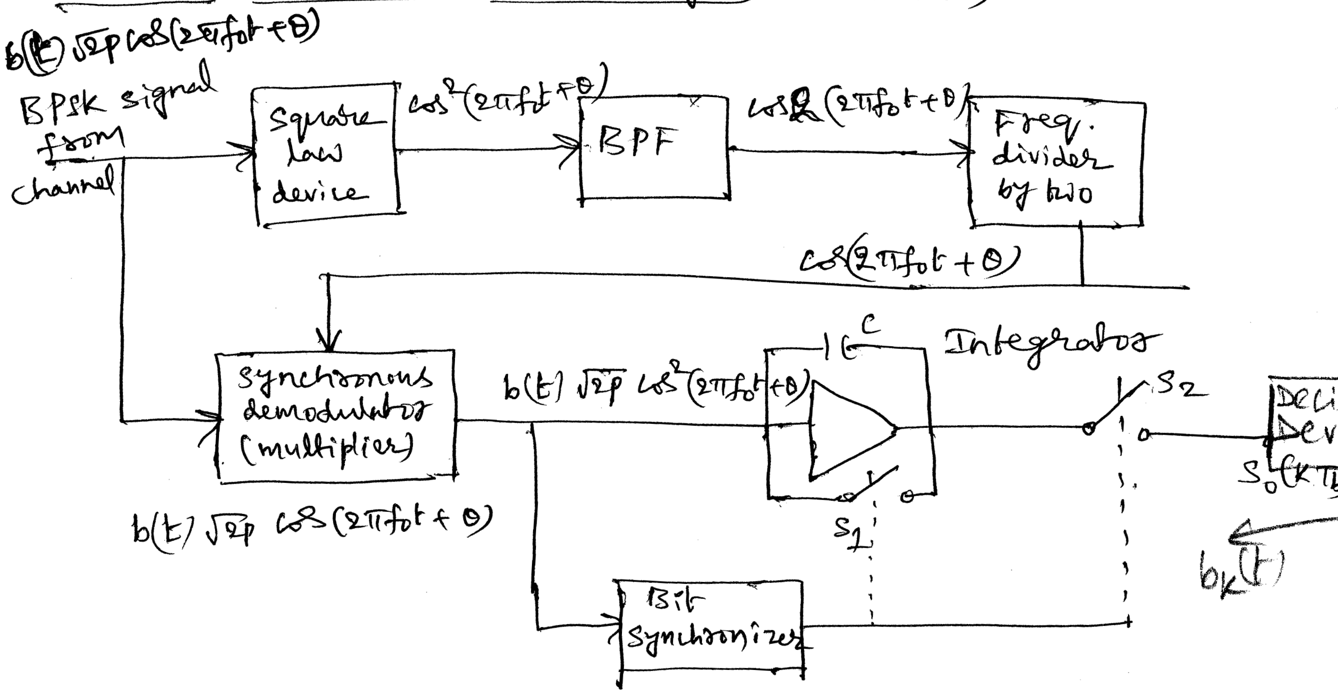
fig(d) BPSK generation scheme

The BPSK signal can be generated by applying carrier sig to the balanced modulator.

The baseband signal b(t) is applied as a modulating sign to the balanced modulator as shown in above fig.

The NRZ level encoder converts the binary data sequence into bipolar NRZ signal.

Coherent Reception of BPSK signal (Detection)



fig(e): Reception BPSK scheme

Fig(e) above shows the block diagram of the scheme to recover baseband signal from BPSK signal. The transmi -tted BPSK signal is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

Operation of the receiver:

The signal undergoes a phase shift of θ. $\theta = 180°$ Therefore the signal at the i/p of the receiver is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta)$$

From this signal, a carrier is separated since this is coherent detection. The received signal is passed through a square law device. At the o/p of the square law device the sign will be,

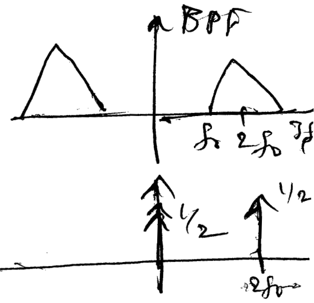
$$\cos^2(2\pi f_0 t+\theta)$$

Here we have neglected the amplitude, because we are only interested in phase of the carrier of the signal.

$$\cos^2(2\pi f_0 t + \theta) = \frac{1+\cos 2(2\pi f_0 t+\theta)}{2}$$ 4.5

$$= \underbrace{\tfrac{1}{2}}_{\text{DC level.}} + \tfrac{1}{2}\cos 2(2\pi f_0 t+\theta)$$

This signal is then passed through a BPF whose passband is centered around $2f_0$. BPF removes the DC level of $\frac{1}{2}$ and at its o/p we get



$$\cos 2(2\pi f_0 t+\theta) = \cos[2\pi(2f_0)t + 2\theta]$$

This signal has freq. of $2f_0$.

This signal is passed through a freq. divider by two. Therefore at the o/p of freq. divider we get a carrier signal whose freq. is f_0, i.e. $\cos(2\pi f_0 t+\theta)$

The synchronous (coherent) demodulator multiplies the i/p signal and the recovered carrier. Therefore at the o/p of multiplier we get,

$$b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta)\times\cos(2\pi f_0 t+\theta)$$

$$= b(t)\sqrt{2P}\cos^2(2\pi f_0 t+\theta)$$

$$= b(t)\sqrt{2P}\times\frac{1}{2}[1+\cos 2(2\pi f_0 t+\theta)]$$

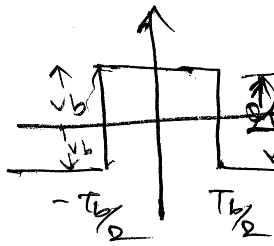
$$= b(t)\sqrt{P/2}\,[1+\cos 2(2\pi f_0 t+\theta)]$$

The above signal is then applied to the bit synchronizer and integrator. The integrator integrates the signal over one bit period. The bit synchronizer takes care of starting and ending times of a bit.

At the end of bit duration T_b, the bit synchronizer closes switch S_2 temporarily. This connects the o/p of a integrator to the decision device. It is equivalent to sampling the o/p of integrator. Then the ~~device~~ decision device will take decision in favour binary '0' or '1'

The synchronizer then opens switch S_2 and switch S_1 is closed temporarily. This resets the integrator voltage to zero. The integrator then integrates next bit.

Spectrum of BPSK signals



We know that the w/f $b(t)$ is NRZ bipolar w/f as shown in fig(a). In this w/f there are rectangular pulses of amplitude $\pm V_b$.

The F.T. of this type of pulse is given as,

$$X(f) = V_b T_b \frac{\sin(\pi f T_b)}{(\pi f T_b)} \longrightarrow (1)$$

For large no. of such positive & negative pulses the PSD $S(f)$ is given as

$$S(f) = \frac{|\overline{X(f)}|^2}{T_s} \longrightarrow (2)$$

where $\overline{X(f)}$ is the avg. value of $X(f)$ due to all the pulses in $b(t)$ and

T_s is symbol duration.

from (1) & (2)

$$\Rightarrow S(f) = \frac{V_b^2 T_b^2}{T_s} \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

For BPSK signal only one bit is transmitted at a time symbol and bit durations are same. i.e. $T_b = T_s$. Then above eq. becomes

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \longrightarrow (3)$$

$S(f)$ = PSD of baseband signal $b(t)$

The BPSK signal is generated by modulating a carrier by the baseband signal $b(t)$. Because of modulation of the carrier of freq. f_0, the spectral components.

translated from f to $f-f_0$ and $f+f_0$. The magnitude of each of these components is divided by half.

$$\therefore S_{BPSK}(f) = V_b^2 T_b \left\{ \frac{1}{2}\left[\frac{\sin\pi(f-f_0)T_b}{\pi(f-f_0)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin\pi(f+f_0)T_b}{\pi(f+f_0)T_b}\right]^2 \right\}$$

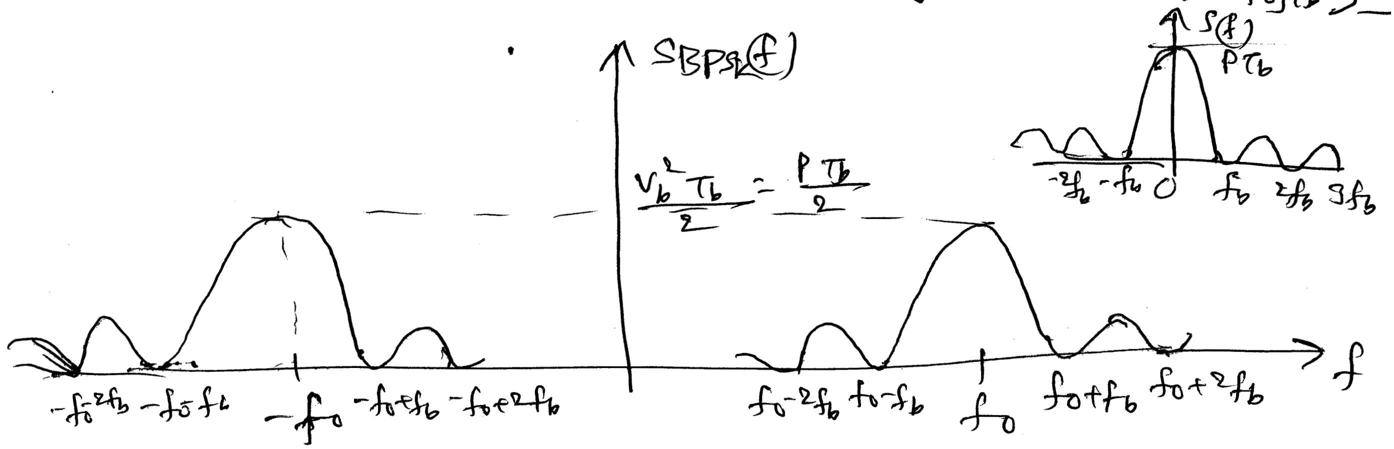


fig (b) PSD of BPSK signal.

Bandwidth of BPSK signal :-

The spectrum of the BPSK signal is centered around the carrier freq. f_0.

If $f_b = \frac{1}{T_b}$, then for BPSK the max. freq. in the baseband signal will be f_b. From the fig (b) above the main lobe is centered around carrier freq. f_0 and extends from f_0-f_b to f_0+f_b. Therefore bandwidth of BPSK signal is,

B.W = Highest freq - Lowest freq. in the main lobe

$= f_0+f_b - (f_0-f_b)$

$= 2f_b$.

$\therefore$ $\boxed{B.W = 2f_b}$

Thus the minimum bandwidth of BPSK signal is equal to twice that of highest freq. contained in baseband signal (f_b).

P:— Determine the minimum bandwidth for a BPSK modulator with a carrier freq. of 40 MHz and an i/p bit rate of 500 kbps.

Sol:— $r = \frac{1}{T_b} = f_b = 500\,KHz$

The i/p bit rate indicates highest freq. of the baseband signal.

$B.W = 2 f_b = 2 \times 500\,kHz = 1\,MHz.$

Differentially coherent PSK (Non-coherent phase shift keying) or Differential phase shift keying (DPSK) :—

DPSK does not need a synchronous (coherent) carrier at the demodulator. The i/p sequence of binary bits is modified such that the next bit depends upon the previous bit. Therefore in the receiver the previous received bits are used to detect the present bit.

Transmitter / Generation of DPSK :—

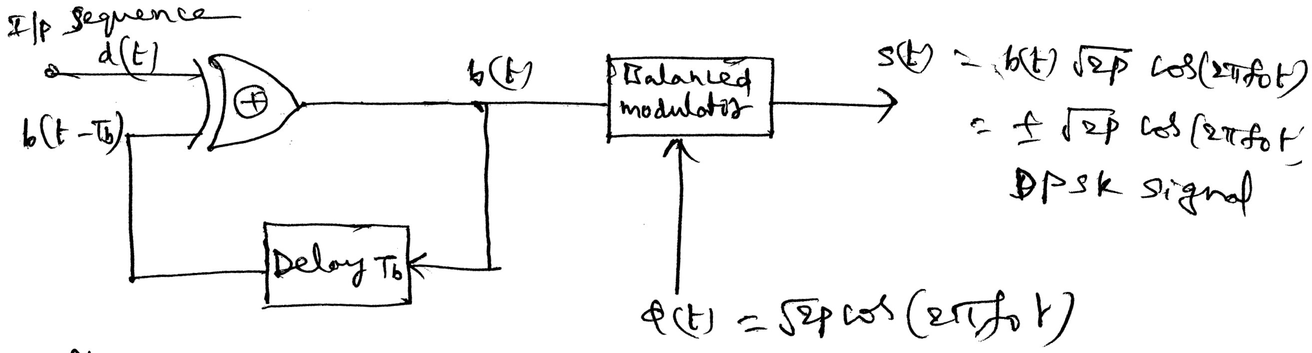


fig (a): Block diagram of DPSK generator or transmitter

The i/p sequence is $d(t)$, o/p sequence is $b(t)$ and $b(t-T_b)$ is the previous o/p delayed by one bit period. Depending upon values of $d(t)$ and $b(t-T_b)$, exclusive OR gate generates the o/p sequence $b(t)$. An arbitrary sequence $d(t)$ is taken. Depending on this ~~seq~~ sequence, $b(t)$ and $b(t-T_b)$ are found.

In BPSK phase of the carrier changes on both the symbol '1' and '0'. Whereas in DPSK phase of the carrier changes only on symbol '1'.

DPSK Receiver :-

Always two successive bits of $d(t)$ are checked for any change of level. Hence one symbol has two bits. ~~Fig(b) represent~~

$\therefore$ symbol duration (T) = Duration of two bits $(2T_b)$

$$\boxed{T = 2T_b}$$

DPSK Receiver :-

Figure ~~blow~~ below shows the method to recover the binary sequence from DPSK signal. Fig(a) & fig(b) are equivalent to each other. Fig(b) represents DPSK receiver

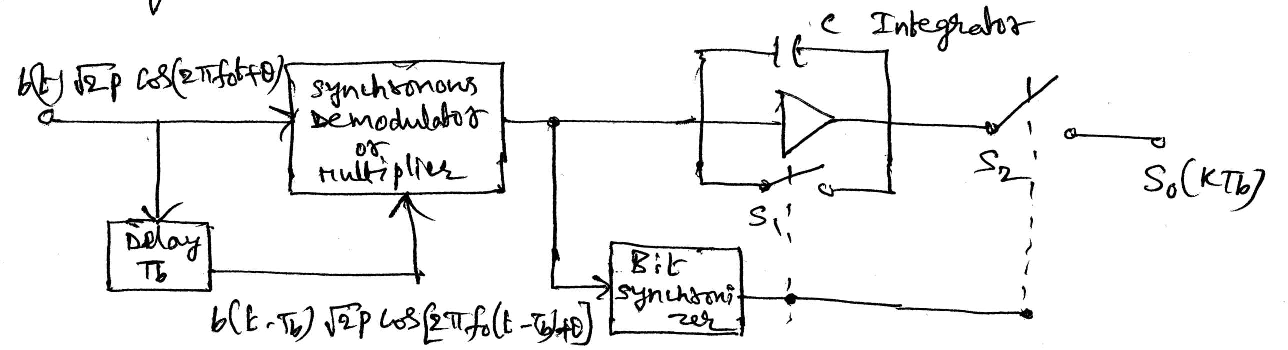


fig (a) DPSK receiver

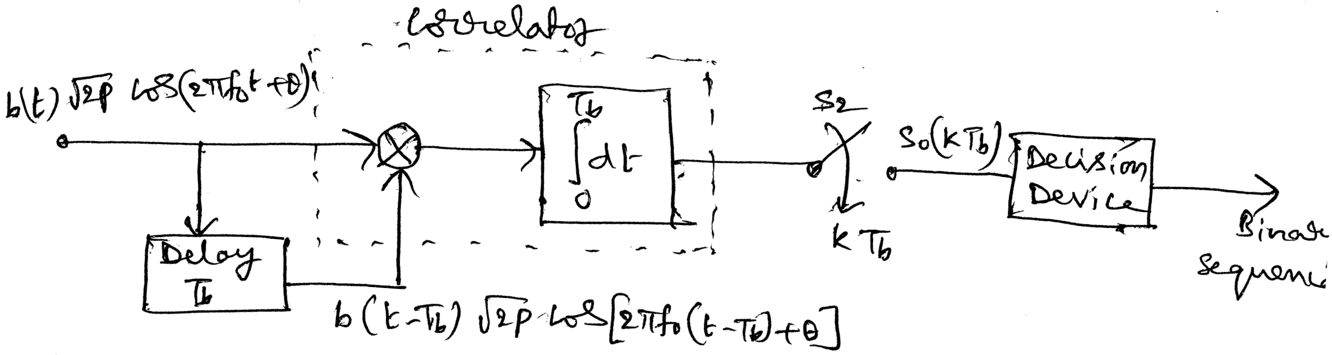


fig (b) Equivalent diagram of DPSK receiver using correlator

d(t)	b(t − Tb)	b(t)
0	0	0
0	1	1
1	0	1
1	1	0

0 → −1V
1 → +1V

Truth table of ~~Ex~~ XOR operation

$b(t) = d(t) \oplus b(t-T_b)$

Initially $b(t-T_b)$ is not known. Therefore it is assumed to be zero.

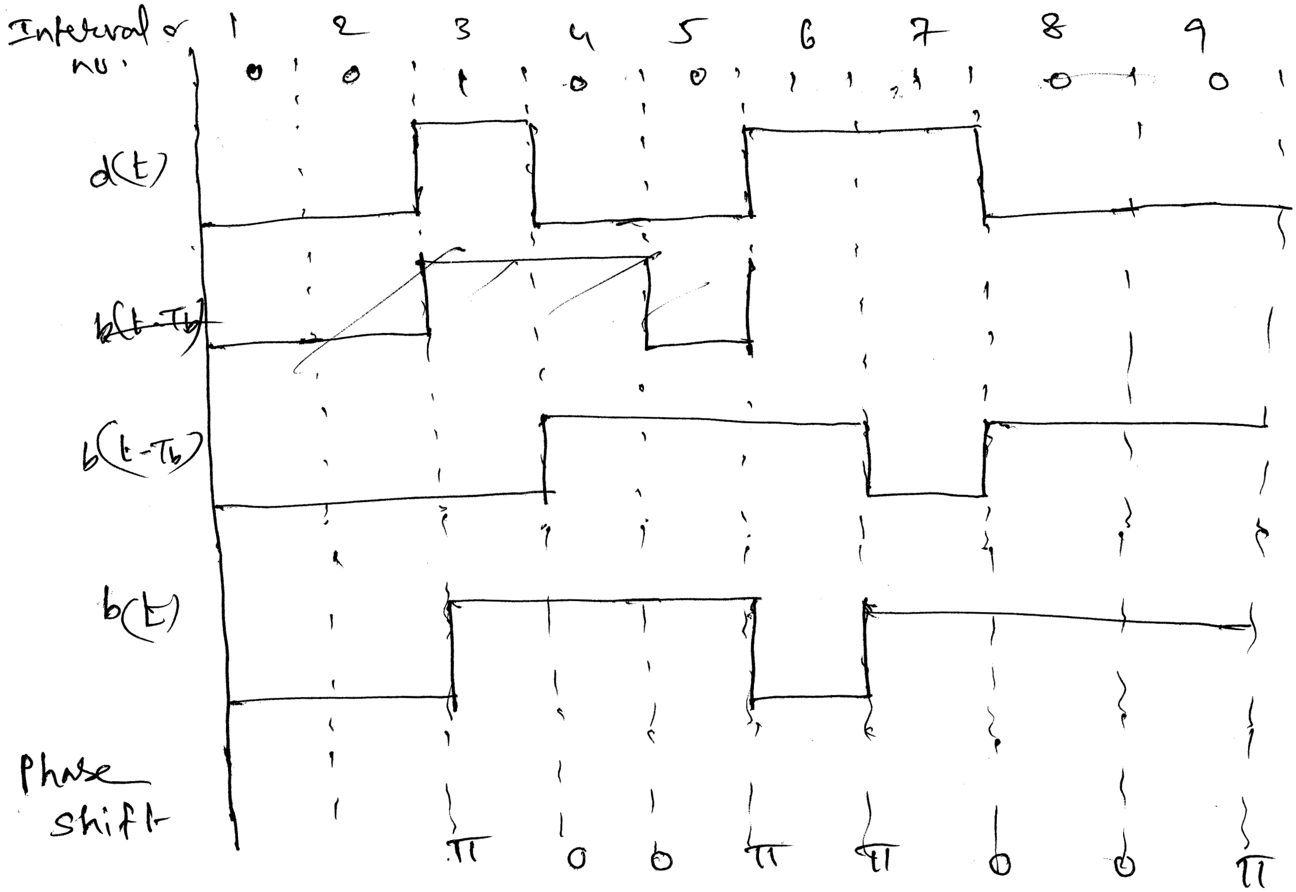


fig: DPSK waveforms.

-using correlator. Fig(a) shows multiplier and integrators 4.8 separately.

operation:-

During the transmission, the DPSK signal undergoes some phase shift θ. Therefore the signal received at the i/p of the receiver is,

$$\text{Received signal} = b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta) \rightarrow ①$$

The signal is multiplied with its delayed version by one bit. Therefore the o/p of the multiplier is,

$$\text{multiplier o/p} = b(t)\, b(t-T_b)\, (2P) \cos(2\pi f_0 t + \theta) \cos[2\pi f_0 (t - T_b) + \theta]$$

$$= b(t)\, b[t-T_b]\, P \left\{ \cos 2\pi f_0 T_b + \cos\left[4\pi f_0 \left(t - \frac{T_b}{2}\right) + 2\theta\right]\right\} \rightarrow ②$$

$(\because 2\cos A \cos B = \cos(A-B) + \cos(A$

where f_0 is the carrier freq. and T_b is one bit period. $\cancel{f_0}$ T_b contains integral no. of cycles of f_0. We know that,

$$f_b = \frac{1}{T_b}$$

If T_b contains 'n' cycles of f_0 then we can write,

$T_b = nT_0 = \frac{n}{f_0}$

$$\cancel{f_0 = nf_b} \Rightarrow f_0 = \frac{n}{T_b}$$

$$\therefore f_0 T_b = n$$

$T_b = nT_0$

$\cancel{f_b} = \frac{n}{f_0}$

$\Rightarrow f_0 T_b = n.$

Putting $f_0 T_b = n$ in first cosine term in eq ②

$$\text{Multiplier o/p} = b(t)\, b(t-T_b)\, P \left\{ \cos 2\pi n + \cos\left[4\pi f_0\left(t - \frac{T_b}{2}\right) + 2\theta\right]\right\}$$

Since $\cos 2\pi n = 1$, the above eq. will be

$$\text{Multiplier o/p} = b(t)\, b(t-T_b)\, P + b(t)\, b[t-T_b]\, P \cos\left[4\pi f_0\left(t - \frac{T_b}{2}\right) + \right.$$

$\rightarrow ($

The above signal is given to an integrator. In the kth bit interval, the integrator o/p can be written as

$$S_0(KT_b) = b(KT_b)\, b[(k-1)T_b]\, P \int_{(k-1)T_b}^{KT_b} dt$$

$$+ b(KT_b)\, b[(k-1)T_b]\, P \int_{(k-1)T_b}^{KT_b} \cos\left[4\pi f_0\left(t - \frac{T_b}{2}\right) + 2\theta\right] dt$$

The integration of the second term will be zero since it is integration of carrier over one bit duration. The carrier has integral no. of cycles over one bit period hence integration is zero. Therefore we can write,

$$S_0(KT_b) = b(KT_b)\, b[(k-1)T_b]\, P[KT_b - (k-1)T_b]$$

$$= b(KT_b)\, b[(k-1)T_b]\, PT_b \longrightarrow (4)$$

Here know that $PT_b = E_b$; i.e. energy of one bit. The product $b(KT_b)\, b[(k-1)T_b]$ decides the sign of PT_b.

The transmitted data bit $d(t)$ can be verified easil from product $b(KT_b)\, b[(k-1)T_b]$. From w/f s of DPSK when $b(t) = b(t-T_b)$, $d(t) = 0$. That is if both are +1V or -1V then $b(t)\, b(t-T_b) = 1$. Alternately we can write,

If $b(t)\, b(t-T_b) = 1V$ then $d(t) = 0$.

We know that $b(t) = \overline{b(t-T_b)}$ then $d(t) = 1$. That is, $b(t) = -1V$, $b(t-T_b) = +1V$ and vice versa. Therefore $b(t)\, b(t-T_b) = -1$. Alternately we can write,

If $b(t)\, b(t-T_b) = -1V$, then $d(t) = 1$.

The decision device is shown in fig(b).

$$S_0(KT_b) = b(KT_b)\, b[(k-1)T_b]\, PT_b$$

If $S_0(KT_b) = \begin{cases} -PT_b, & \text{then } d(t) = 1 \text{ and} \\ +PT_b, & \text{then } d(t) = 0. \end{cases}$

Bandwidth of DPSK signal

We know that one previous bit is used to decide the phase shift of next bit. Change in $b(t)$ occurs only if i/p bit is at level '1'. No change occurs if i/p bit is at level '0'.

Symbol duration $T = 2T_b$

B.W. is given as,

$$B.W = \frac{2}{T}$$

$$= \frac{2}{2T_b} = \frac{1}{T_b}$$

or $\boxed{B.W = f_b}$

Thus the minimum bandwidth in DPSK is equal to f_b; i.e. maximum baseband signal frequency.

Advantages & Disadvantages of DPSK:-

DPSK has some advantages over BPSK, but at the same time it has some drawbacks.

Advantages:-

1. DPSK does not need carrier at its receiver. Hence the complicated ckt for generation of local carrier is avoided.
2. The bandwidth requirement of DPSK is reduced compared to that of BPSK.

Disadvantages:-

1. The probability of error of DPSK is higher than that of BPSK.
2. Since DPSK uses two successive bits for its reception error in the first bit creates error in the second bit. Hence error propagation in DPSK is more. Whereas in PSK single bit can go in error since detection of eac[h] bit is independent.
3. Noise interference in DPSK is more.

Binary Frequency shift keying (BFSK)

In binary frequency shift keying, the freq. of the carrier is shifted according to the binary symbol. The phase of the carrier is unaffected. That is we have two different freq. signals according to binary symbols. Let there be a freq. shift by Ω. Then we can write following eqs.

Let the carrier be $s(t) = \sqrt{2P_s}\cos(2\pi f_0 t)$

$$\left.\begin{aligned} &\text{If } b(t) = \text{'1'}; \quad S_H(t) = \sqrt{2P_s}\cos(2\pi f_0 + \Omega)t \\ &\text{If } b(t) = \text{'0'}; \quad S_L(t) = \sqrt{2P_s}\cos(2\pi f_0 - \Omega)t \end{aligned}\right\} \rightarrow (1)$$

We can write the above eqs combinely as

$$s(t) = \sqrt{2P_s}\cos\left[(2\pi f_0 + d(t)\Omega)t\right] \longrightarrow (2)$$

$b(t)$ i/p	$d(t)$	$P_H(t)$	$P_L(t)$
1	+1V	+1V	0
0	-1V	0V	+1V

$P_H(t)$ & $P_L(t)$ are unipolar signals.

Conversion table for BFSK representation

Thus when symbol '1' is to be transmitted, the carrier frequency will be $f_0 + \left(\frac{\Omega}{2\pi}\right)$.

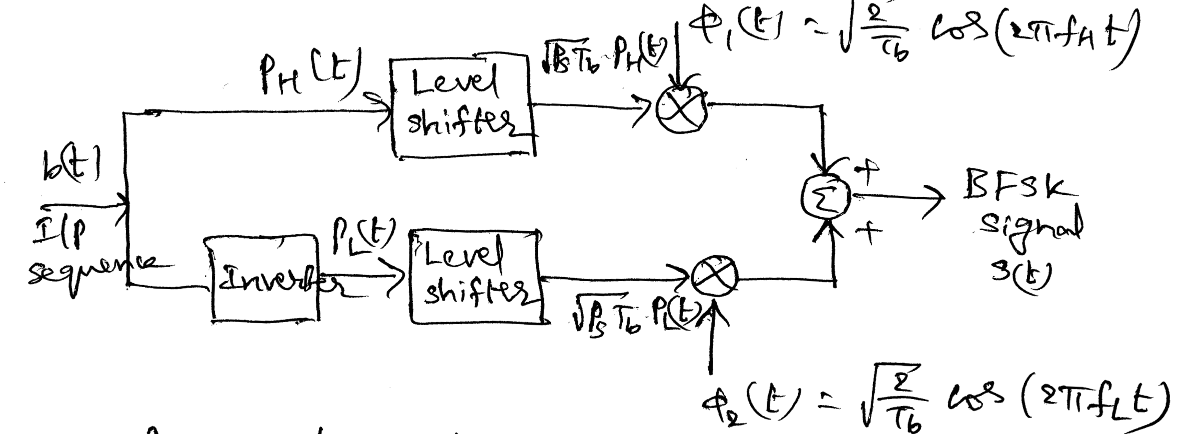
If symbol '0' is to be transmitted, the carrier freq. will be $f_0 - \left(\frac{\Omega}{2\pi}\right)$. i.e.

$$\left.\begin{aligned} f_H &= f_0 + \frac{\Omega}{2\pi} \quad \text{for symbol '1'} \\ f_L &= f_0 - \frac{\Omega}{2\pi} \quad \text{for symbol '0'}. \end{aligned}\right\} \rightarrow (3)$$

BFSK Transmitter :-

From the above table, we know that $P_H(t)$ is same as $b(t)$. $P_L(t)$ is inverted version of $b(t)$.

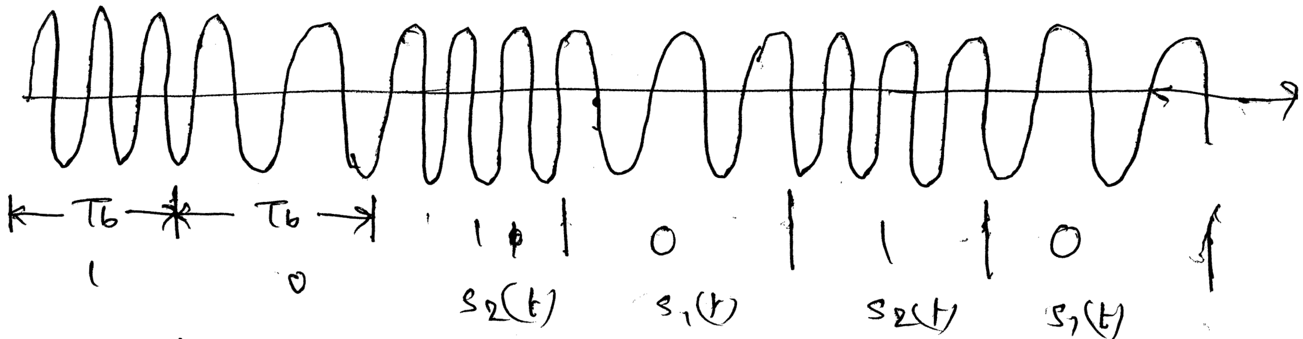
The block diagram of BFSK transmitter is shown in fig(a) below.



fig(a) Block diagram of BFSK transmitter

We know that i/p sequence b(t) is same as $P_H(t)$. An inverter is added after b(t) to get $P_L(t)$. $P_H(t)$ and $P_L(t)$ are unipolar signals. The level shifter converts the '+1' level to $\sqrt{P_s T_b}$. ~~=~~ zero level is unaffected. Thus the o/p of the level shifters will be either $\sqrt{P_s T_b}$ (if i/p or zero (if i/p is zero). Further there are product modulators after level shifter. The two carrier signals $\phi_1(t)$ and $\phi_2(t)$ are used. $\phi_1(t)$ and $\phi_2(t)$ are orthogonal to each other. In one bit period of i/p signal (T_b), ϕ_1 or $\phi_2(t)$ have integral no. of cycles.

Therefore the modulated signal has continuous phase. Such BFSK signal is shown in fig(b) below. →



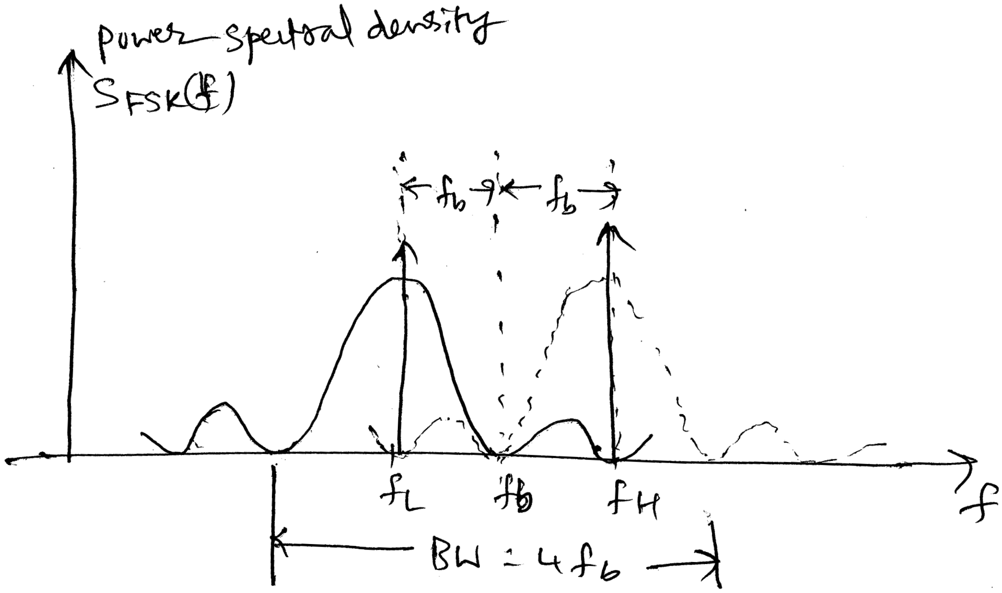
fig(b) BFSK signal.

The adder then adds the two signals. Here note the o/ps from both the multipliers are not possible at a time. This is because $P_H(t)$ and $P_L(t)$ are complementary to each other. Therefore if $P_H(t) = 1$, then o/p will be only due to upper modulator and lower modulator o/p will be zero (since $P_L(t) = 0$).

Spectrum of BFSK :-

We can write BFSK signal $s(t)$ as,

$$s(t) = \sqrt{2P_s}\, P_H(t) \cos(2\pi f_H t) + \sqrt{2P_s}\, P_L(t) \cdot \cos(2\pi f_L t) \rightarrow (4)$$



fig(c): Power spectral density of BFSK signal.

B.W of BFSK Signal :-

From above fig. it is clear that the width of one lobe is $2f_b$. The two main lobes due to f_H and f_L are place ~~such~~ such that the total width due to both main lobes is $4f_b$. i.e.

B.W of BFSK $= 2f_b + 2f_b = 4f_b$.

$\therefore$ BW (BFSK) = 2 × BW (BPSK).

Coherent BFSK Receiver

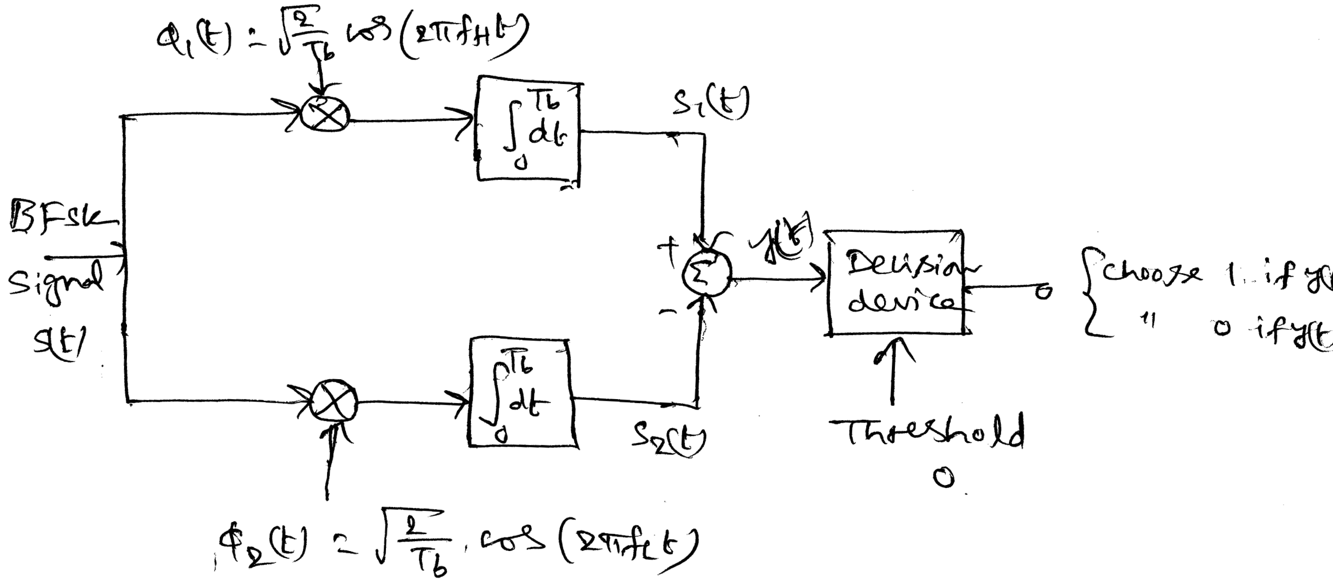


fig (a) Coherent BFSK receiver

Fig (a) above shows the block diagram of coherent BFSK receiver. There are two correlators for two frequencies of FSK signal. These correlators are supplied with locally generated carriers $\phi_1(t)$ and $\phi_2(t)$. If the transmitted frequency is f_H, then o/p $s_1(t)$ will be higher than $s_2(t)$. Hence $y(t)$ will be greater than zero. The decision device then decides in favour of binary '1'. If $s_2(t) > s_1(t)$, then $y(t) < 0$ and decision device decides in favour of '0'.

Non coherent BFSK Receiver :-

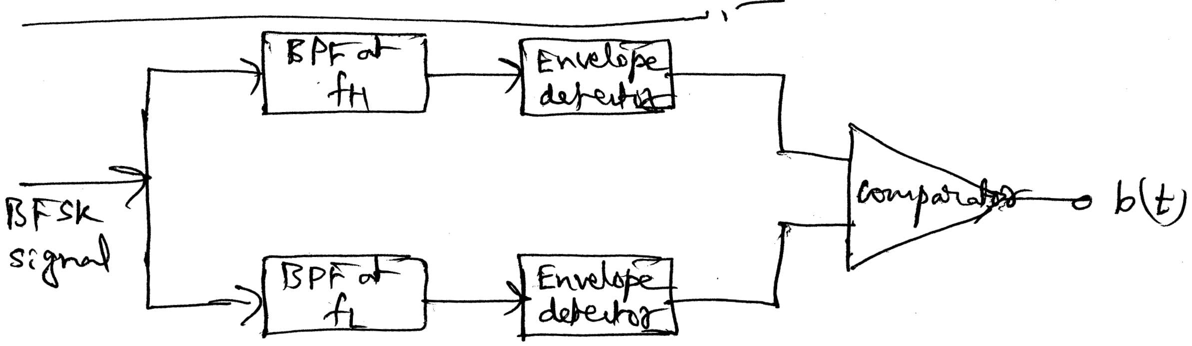


fig : Block diagram of BFSK receiver

Fig. above shows the block diagram of BFSK receiver. The receiver consists of two bandpass filters; one with center freq f_H and other with centre freq f_L. Since $f_H - f_L = 2f_b$, the o/p's of filters do not overlap. The Bandpass filters pass their corresponding main lobes w/o much distortion.

The o/ps of filters are applied to envelop detectors. The o/ps of detectors are compared by the comparator. If unipolar comparator is used, then the o/p of comparator is the bit sequence b(t).

FSK Detection using PLL :-

The error voltage in PLL is proportional to difference b/w the phase or freq. of the i/p signal and VCO frequency. Fig. shows the block diagram of FSK detection using PLL. The free running freq. of the voltage controlled oscillator (VCO) is kept in b/w

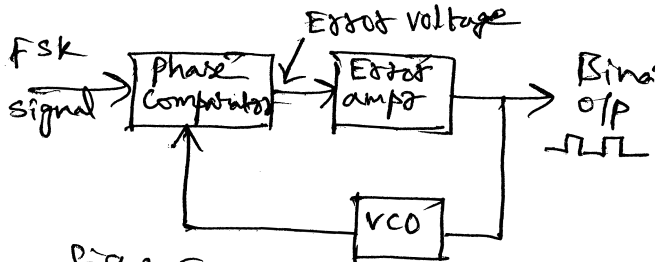


fig: FSK detection using PLL

f_H and f_L. When F_H is transmitted, the error voltage becomes positive, and hence binary o/p goes high. It remains high as long as F_H is transmitted for a bit period.

When F_L is transmitted, the error voltage becomes negative and hence binary goes low. It remains low as long as F_L is transmitted for a bit period.

The VCO generates free running freq. b/w F_H & F_L. Hence phase comparator detects the difference b/w VCO freq and F_H & F_L.

PLL detector is a non coherent type of detector.

BANDPASS DIGITAL TRANSMISSION - II

Quadrature carrier and M-ary systems

→ Quadrature carrier systems use two quadrature carriers. They have 90° phase shift b/w them.

→ The Quadrature phase shift keying (QPSK) and Quadrature Amplitude Modulation (QAM) are the two well known Quadrature carrier systems.

→ M-ary systems use M no. of (bits) symbols to be transmitted simultaneously.

→ The quadrature and M-ary system reduce the bandwidth of the transmission channel considerably.

→ These techniques are commonly used for digital transmission on telephone lines and other bandlimited channels.

Quadrature phase shift keying (QPSK).

Principle :-

If two or more bits are combined in some symbols, then the signaling rate is reduced. Therefore the frequency of the carrier required is also reduced. This reduces the transmission channel bandwidth. Thus because of grouping of bits in symbols, the transmission channel bandwidth is reduced.

In QPSK, two successive bits in the data sequence are grouped together. This reduces the bits rate ~~of~~ or signaling rate (i.e. f_b) and hence reduces the bandwidth of the channel.

→

QPSK Transmitter (Offset QPSK (OQPSK) or staggered QPSK)

Fig (a) below shows the block diagram of QPSK transmitter. The i/p binary sequence is first converted to a bipolar NRZ type of signal. This signal is called $b(t)$. It represents binary '1' by +1v and binary '0' by -1v.

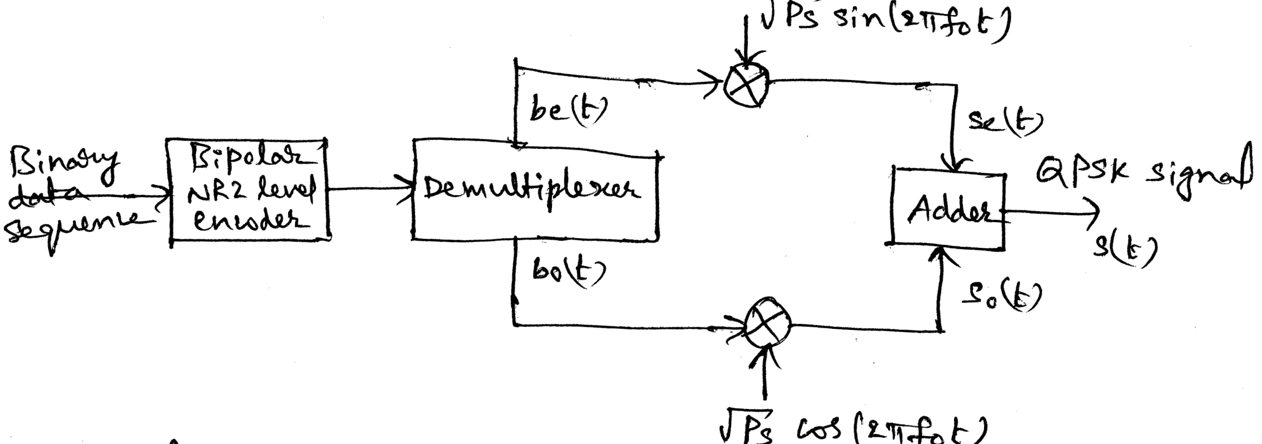


fig (a): QPSK transmitter

The demultiplexer divides $b(t)$ into two separate bit stream of the odd numbered and even numbered bits. i.e $b_e(t)$ & $b_o(t)$. The symbol duration of both of these odd and even number sequences is $2T_b$. Thus every symbol contains two bits. Fig and fig (c) shows the w/fs of $b_e(t)$ and $b_o(t)$.

Observe that the first even bit occurs after the first odd bit. Therefore even numbered bit sequence $b_e(t)$ starts with the delay of one bit period due to first odd bit. Thus first symbol of $b_e(t)$ is delayed by one bit period 'T_b' w.r.t. first symbol of $b_o(t)$. This delay of T_b is called offset. Hence the name offset QPSK.

The bit stream $b_o(t)$ modulates carrier $\sqrt{P_s}\cos(2\pi f_0 t)$ and $b_e(t)$ modulates $\sqrt{P_s}\sin(2\pi f_0 t)$. These modulators are balanced modulators. These carriers also called quadrature carriers. The two modulated signals are,

$$\left.\begin{aligned} s_e(t) &= b_e(t)\sqrt{P_s}\sin(2\pi f_0 t) \text{ and} \\ s_o(t) &= b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) \end{aligned}\right\} \rightarrow (1)$$

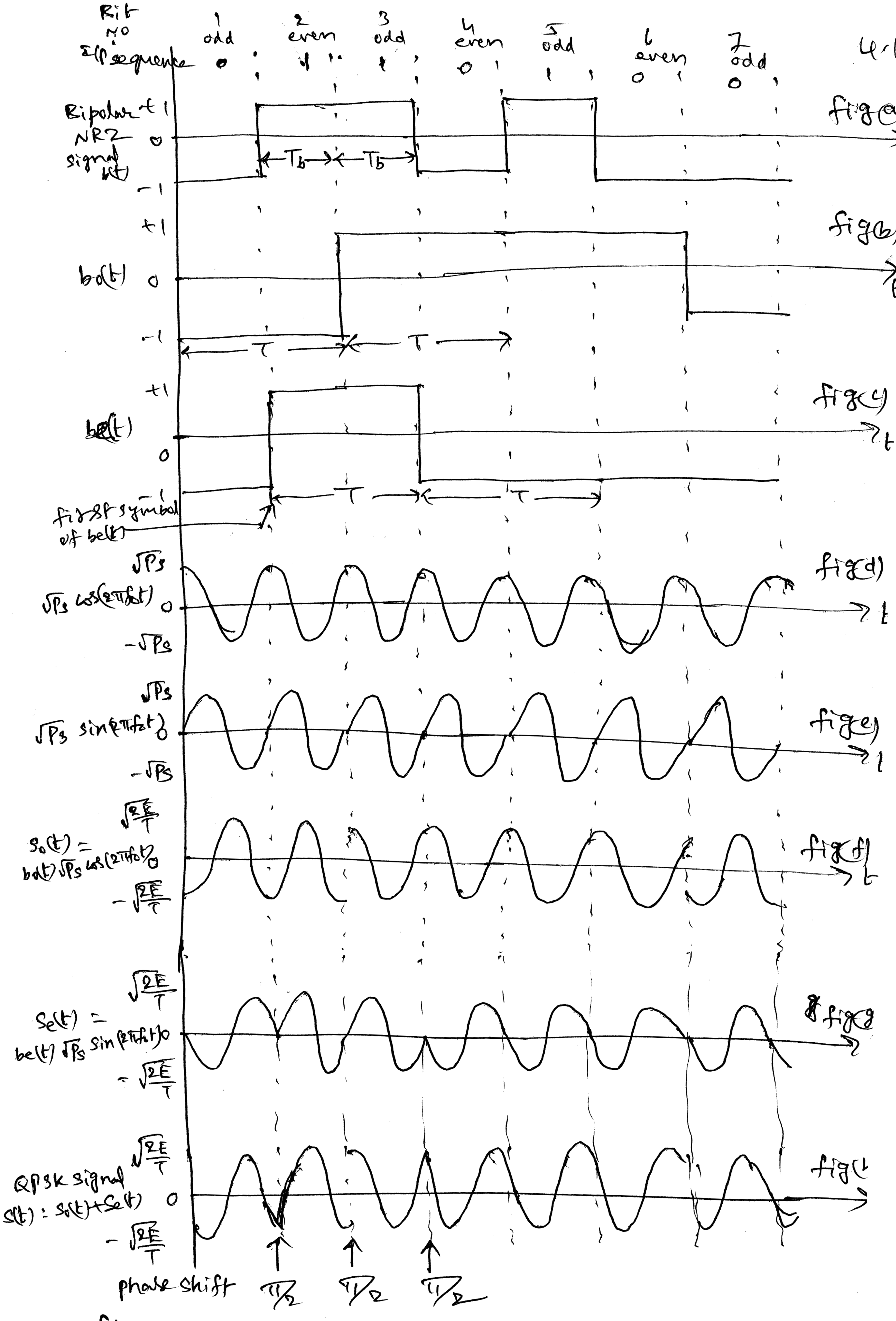


fig: Waveforms.

Thus $s_e(t)$ and $s_o(t)$ are basically BPSK signals and ~~they are similar~~ $T = 2T_b$. The values of $b_e(t)$ and $b_o(t)$ ~~are~~ will be +1V & -1V. Fig (f) and (g) shows the waveforms of $s_e(t)$ and $s_o(t)$.

The adder adds these two signals $s_e(t)$ and $s_o(t)$. The o/p of the adder is QPSK signal and it is given as,

$$s(t) = s_o(t) + s_e(t)$$
$$= b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

Fig (h) shows the QPSK signal represented by above equation. The phase change occurs at minimum duration of T_b. This is because the two signals $s_e(t)$ and $s_o(t)$ have an offset of 'T_b'. Because of this offset, the phase shift in QPSK is $\pi/2$. $b_e(t)$ and $b_o(t)$ can not change at the same time because of offset b/w them. Fig below shows the phasor diagram of QPSK signal.

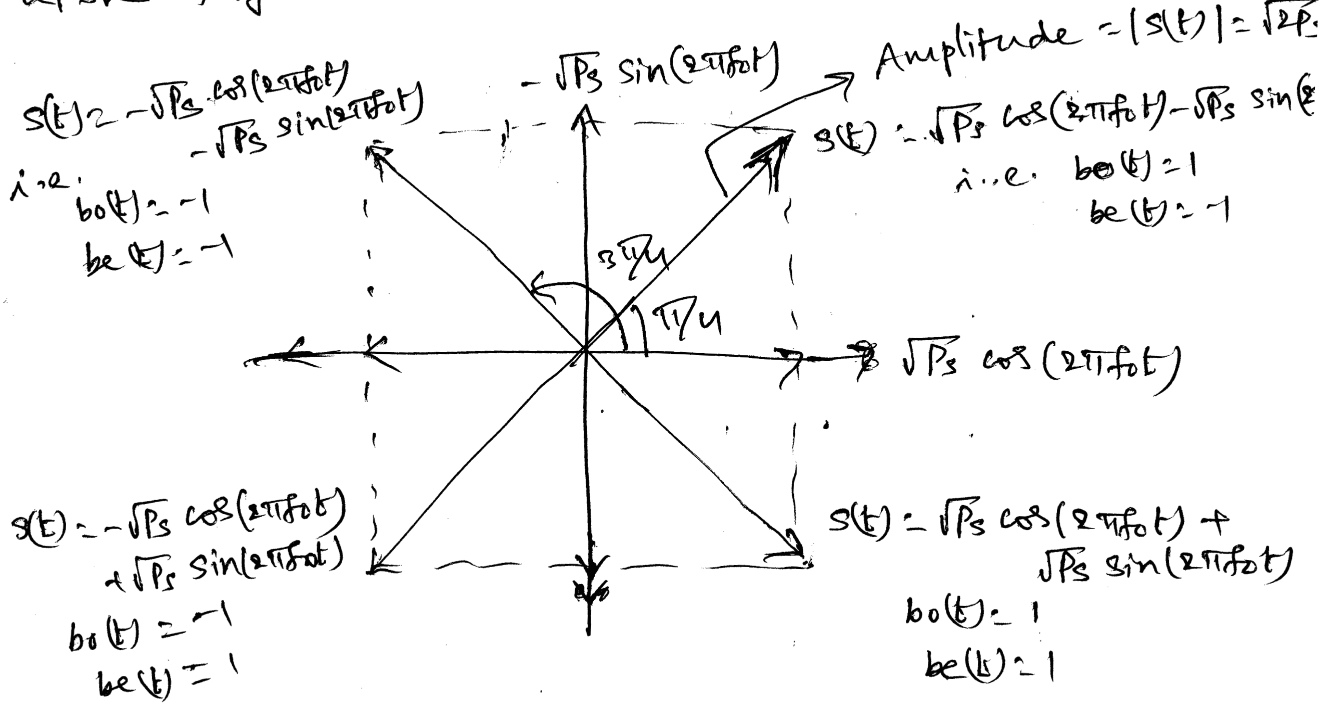


fig: phasor diagram of QPSK signal.

Note) Non-offset QPSK: We know that there is an offset of 'T_b' b/w $b_e(t)$ & $b_o(t)$. If we delay $b_e(t)$ by 'T_b' then there will be no offset. Then the sequences $b_e(t)$ and $b_o(t)$ will change at the same time. This change will occur

QPSK Receiver

Fig. below shows the QPSK receiver. This is synchronous reception. Therefore coherent carrier is to be recovered from the received signal s(t).

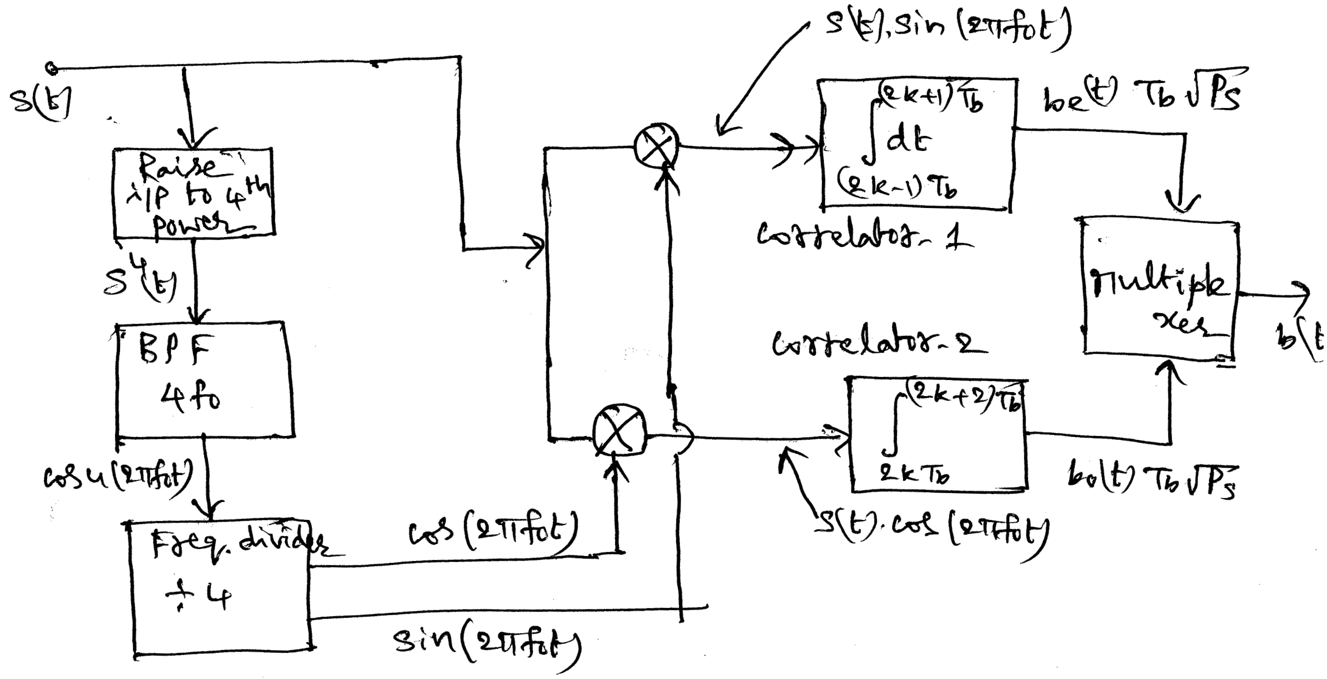


fig: QPSK Receiver

The received signal s(t) is first raised to its 4th power, i.e. $s^4(t)$. Then it is passed through a BPF center around 4fo. The o/p of the BPF is a coherent carrier of frequency 4fo. This is divided by 4 and it gives two coherent quadrature carriers cos (2πfot) and sin(2πfot). These coherent carriers are applied to two synchronous demodulators. These synchronous demodulators consist of multiplier and an integrator. The incoming signal is applied to both the multipliers. The integrator integrates the product signal over two bit interval (i.e. $T_s = 2T_b$). The o/p of integrator is sampled at the end of this period. The o/ps of the two integrators are sampled at the offset of one bit period, T_b. Hence the o/p of multiplexer is the signal b(t). That is, the odd and even sequences are combined by multiplexer.

Bandwidth of QPSK signal

We know that the bandwidth of BPSK signal is equal to $2f_b$. Here $T_b = \frac{1}{f_b}$ is the one bit period. In QPSK the two w/fs $b_e(t)$ and $b_o(t)$ ~~for~~ from the baseband signals one bit period for both of these signals is equal to $2T_b$.

Therefore bandwidth of QPSK signal is

$$\boxed{BW = 2 \times \frac{1}{2T_b} = \frac{1}{T_b} = f_b.}$$

Thus the bandwidth of QPSK signal is half of the bandwidth of BPSK signal.

Advantages :-

1. For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.
2. variation in QPSK amplitude is not much. Hence carrier power almost remains constant.

Signal space Representation of QPSK signals

The QPSK signal can be written as

$$s(t) = \sqrt{2P_s} \cos\left[2\pi f_0 t + (2m+1)\pi/4\right] \quad m = 0,1,2,3 \rightarrow ①$$

Here, the above eq. takes four values.

The above eq. can be expanded as, $(\cos(A+B) = \cos A \cos B - \sin A \sin B)$

$$s(t) = \sqrt{2P_s} \cos(2\pi f_0 t) \cos\left[(2m+1)\pi/4\right] - \sqrt{2P_s} \sin(2\pi f_0 t) \sin\left[(2m+1)\pi/4\right]$$

By rearranging the above eq.

$$s(t) = \left\{\sqrt{P_s T_s} \cos\left[(2m+1)\pi/4\right]\right\} \sqrt{\frac{2}{T_s}} \cos(2\pi f_0 t) - \left\{\sqrt{P_s T_s} \sin\left[(2m+1)\frac{\pi}{4}\right]\right\} \sqrt{\frac{2}{T_s}} \sin(2\pi f_0 t) \rightarrow ②$$

Let $\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_0 t)$ and $\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_0 t)$

The above two signals are called orthogonal signals and they are used as carriers in QPSK modulators.

Let $b_o(t) = \sqrt{2}\cos\left[(2m+1)\frac{\pi}{4}\right]$ and

$b_e(t) = -\sqrt{2}\sin\left[(2m+1)\frac{\pi}{4}\right]$

$$\Rightarrow s(t) = \sqrt{P_s T_s}\,\frac{1}{\sqrt{2}}\,b_o(t)\phi_1(t) + \sqrt{P_s T_s}\cdot\frac{1}{\sqrt{2}}\,b_e(t)\,\phi_2(t)$$

$$= \sqrt{P_s \cdot \frac{T_s}{2}}\,b_o(t)\,\phi_1(t) + \sqrt{P_s\cdot\frac{T_s}{2}}\,b_e(t)\,\phi_2(t)$$

Here T_s = symbol duration and $T_s = 2T_b$

$$\Rightarrow T_b = \frac{T_s}{2}.$$

Then the above eq. becomes,

$$s(t) = \sqrt{P_s T_b}\,b_o(t)\,\phi_1(t) + \sqrt{P_s T_b}\,b_e(t)\,\phi_2(t).$$

Since bit energy $E_b = P_s T_b$

$$s(t) = \sqrt{E_b}\,b_o(t)\,\phi_1(t) + \sqrt{E_b}\,b_e(t)\,\phi_2(t)$$

↳ ~~sign~~

The above eq. gives signal space representation of QPSK signal. The two orthogonal signals, $\phi_1(t)$ and $\phi_2(t)$ form the two axes of the signal space.

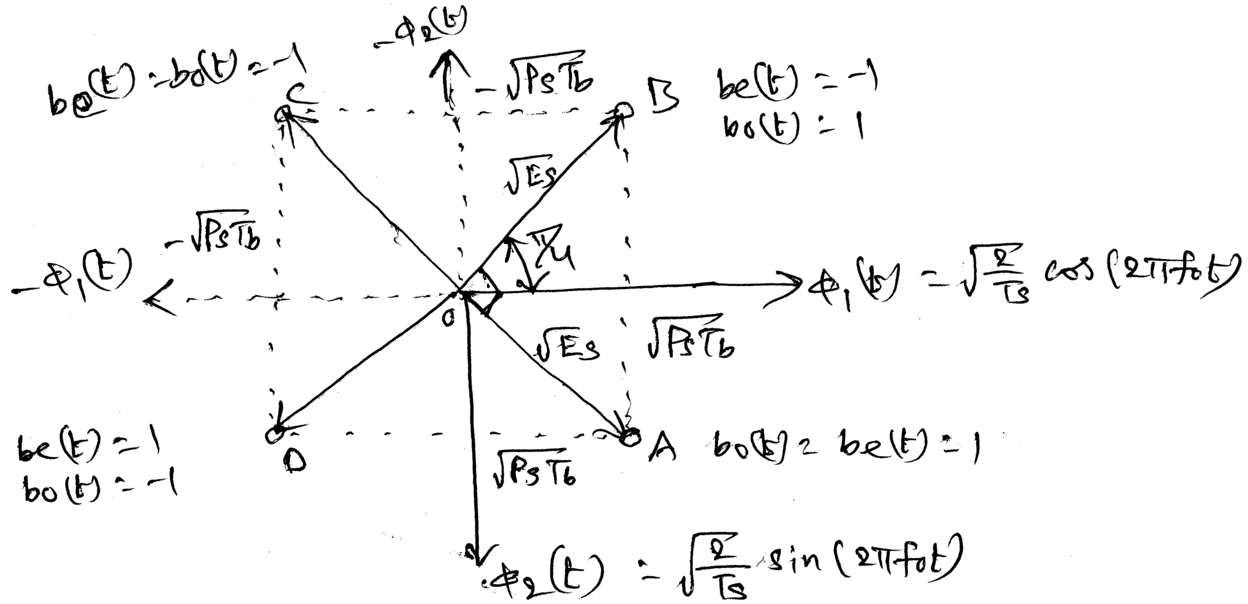


fig: signal space representation of QPSK signals

The ~~p~~ possible 4 signal points are shown by small circles on $\phi_1 \phi_2$ plane. From each signal point, we obtain two bits. For example from point 'A', we obtain two bits as (1,1) and from 'B' we obtain bits as (-1,1).

The distance of any signal point from origin given as,

$$\text{Distance } OB' = \sqrt{P_s T_b + P_s T_b}$$
$$= \sqrt{2 P_s T_b}$$
$$= \sqrt{P_s T_s} \qquad (\because 2T_b = T_s)$$
$$= \sqrt{E_s} \qquad (\because P_s T_s = E_s)$$

Spectrum of QPSK signal :-

The i/p sequence b(t) is of bit duration T_b. It is NRZ bipolar w/f. The PSD of such w/f is

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

and $V_b = \sqrt{P_s}$, then above equation becomes,

$$S(f) = P_s T_b \left[\frac{\sin(\pi f T_b)}{(\pi f T_b)}\right]^2 \rightarrow \text{PSD of signal } b(t)$$

This signal is divided into be(t) and bo(t) each of bit period $2T_b$. If we consider that symbols 1 and 0 are equal likely, then we can write PSDs of be(t) and bo(t) as,

$$S_e(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \text{ \& } S_o(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]$$

where T_s is the ~~bit~~ period of bit in be(t) and bo(t).

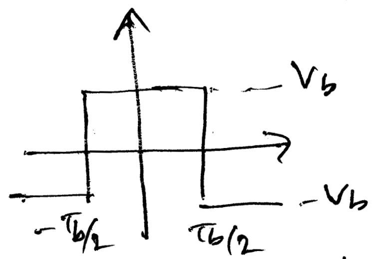
Since inphase & quadrature components [be(t) and bo(t)] are statistically independent, the PSD of QPSK signal equal to sum of the individual PSD's of be(t) and bo(t), i.e.

$$S_B(f) = S_e(f) + S_o(f)$$
$$= 2 P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2.$$

Upon modulation of carrier of f_0, the spectral density g by above eq. is shifted at $\pm f_0$. Thus plots of PSD of QPSK will be similar to that of BPSK.

Spectrum of QPSK signals

We know that the w/f ~~of~~ b(t) is NRZ bipolar w/f. In this w/f there are rectangular pulses of amplitude $\pm V_b$. If ~~the~~ each pulse is $\pm T_b/2$ around its center as shown in fig (a), then its F.T is



fig(a) NRZ pulse

$$x(f) = V_b T_b \frac{\sin(\pi f T_b)}{(\pi f T_b)} \longrightarrow (1)$$

PSD of NRZ pulse

For large number of such positive and negati pulses the PSD $S(f)$ is given as

$$S(f) = \frac{|\overline{X(f)}|^2}{T_s} \longrightarrow (2)$$

Here $\overline{X(f)}$ is avg value of $X(f)$ due to all the pulses in $b(t)$ and T_s is symbol duration

substituting eq (1) in eq (2)

$$S(f) = \frac{V_b^2 T_b^2}{T_s} \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

~~PSD of~~ but input sequence $b(t)$ is of bit durati T_b. ie $T_s = T_b$

$$\Rightarrow S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

and $P_s = V_b^2$ = Avg. Power of Pulse

$$\Rightarrow V_b = \sqrt{P_s}$$

$\longrightarrow$

$$\Rightarrow S(f) = P_s T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \longrightarrow (3)$$

$= PSD$ of signal $b(t)$

PSDs of even and odd numbered sequence

The signal $b(t)$ is divided into $b_e(t)$ and $b_o(t)$ each of bit period $2T_b$. If we consider that symbols 1 and 0 are equally likely, ~~tt~~ then we can write PSDs of $b_e(t)$ and $b_o(t)$ as,

$$\left.\begin{aligned} S_e(f) &= P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \\ \text{and } S_o(f) &= P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \end{aligned}\right\} \longrightarrow (4)$$

PSD of QPSK signal

Since inphase and quadrature components $[b_e(t)$ and $b_o(t)]$ are statistically independent, the baseband PSD ~~den~~ of QPSK signal equals the sum of the individual PSDs of $b_e(t)$ and $b_o(t)$

i.e

$$S_B(f) = S_e(f) + S_o(f)$$

$$= 2P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \longrightarrow (5)$$

$= PSD$ of QPSK signal.

The power spectral density of QPSK signal is shown in figure below.

$\longrightarrow$

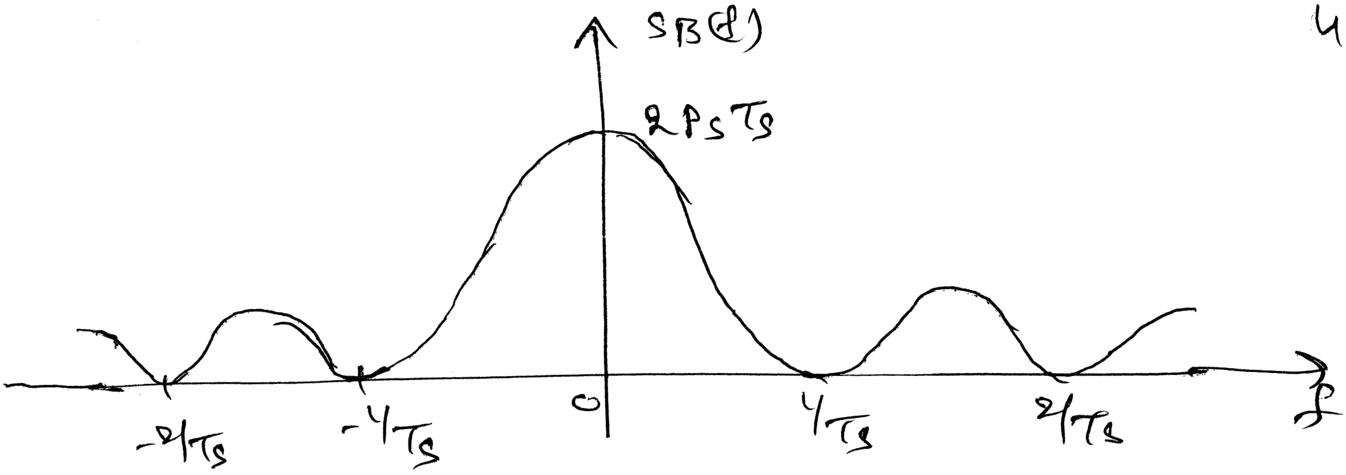


fig: Plot of PSD of QPSK signal

From above fig. the bandwidth of QPSK signal is

B.W = (Highest frequency - Lowest frequency) in main lobe

$$= f_H - f_L$$

$$= \frac{1}{T_s} - \left(-\frac{1}{T_s}\right)$$

$$= \frac{2}{T_s}$$

We know that $T_s = 2T_b$

$$\therefore BW = B_T = \frac{2}{2T_b} = \frac{1}{T_b} = f_b$$

$$\therefore \boxed{B_T = f_b}$$

Quadrature Amplitude Shift keying (QASK)

M-ary or M-ary

Quadrature Amplitude Modulation (QAM)

The correct detection of the signal depends upon the separa-tion b/w the signal points in the signal space. In case of PSK systems all points lie on the circumference of the ~~&~~ circle. This is because PSK signal has constant amplitude. If amplitude of the signal is also varied, then the points will lie inside the circle also in ~~on~~ the signal space diagram. This further increases the noise immunity of the system. Such system involves phase as well as amplitude shift keying. It is called QAM

~~Transmitter~~

The QAM signal ~~is represented~~ as,

~~$s(t) = k_1 a \phi_1(t)$~~

Geometrical Representation and Euclidean Distance of QASK signals (or signal space Representation or signal space constellation)

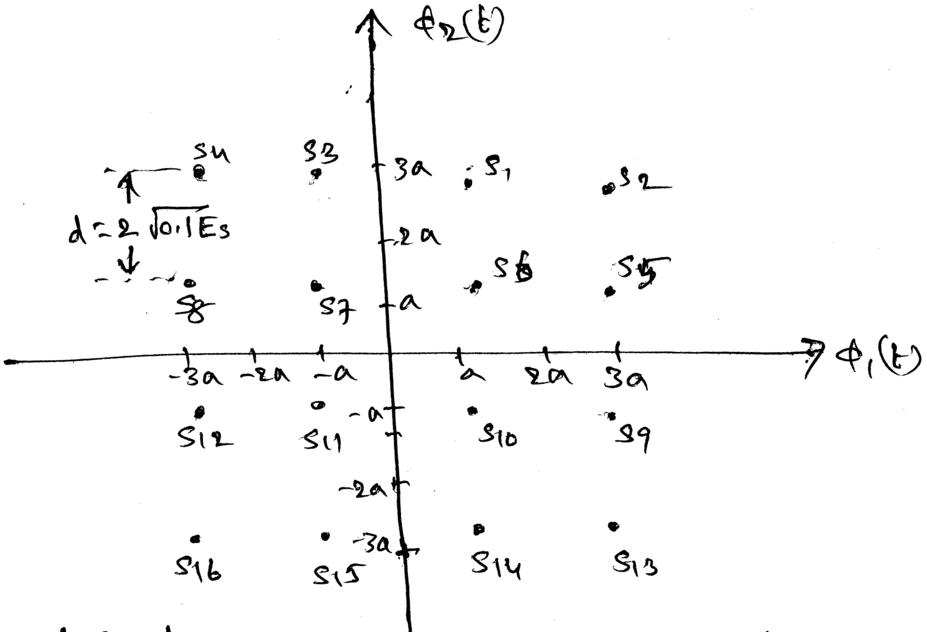


fig: Geometrical representation of 16 signals in QASK syste

Let us consider the case of 4-bit symbol. Then there will be $2^4 = 16$ possible symbols. In the QASK system, such 16 symbols are represented geometrically as shown in above fig. It shows geometrical representation of 16 QASK signals. The distance from the neighbouring points is $d = 2a$. Let the signals be equally likely. Then the avg. energy associated with the signal can be obtained as (considering first quadrant)

$$E_s = \frac{1}{4}\left[(a^2+a^2) + (9a^2+a^2) + (a^2+9a^2) + (9a^2+9a^2)\right]$$

$$= 10a^2$$

$$\Rightarrow \quad a = \sqrt{0.1 E_s}$$

Since $d = 2a$ we have,

$d = 2\sqrt{0.1 E_s} = \sqrt{0.4 E_s}$ = distance b/w two signal points in 16 QASK. In each symbol there are 4 bits. Hence bit energy and symbol energy are related as,

$$E_s = 4E_b.$$

$$\therefore \quad d = \sqrt{0.4 \times 4E_b} = \sqrt{1.6 E_b}$$

Transmitter

The signal in above fig. is represented as,

$$s(t) = k_1 a \phi_1(t) + k_2 a \cdot \phi_2(t) \longrightarrow (2)$$

Here k_1 and k_2 will take values of ± 1 or ± 3. $\phi_1(t)$ and $\phi_2(t)$ are orthogonal carriers having the values as follows

$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_0 t)$$

and $$\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_0 t)$$

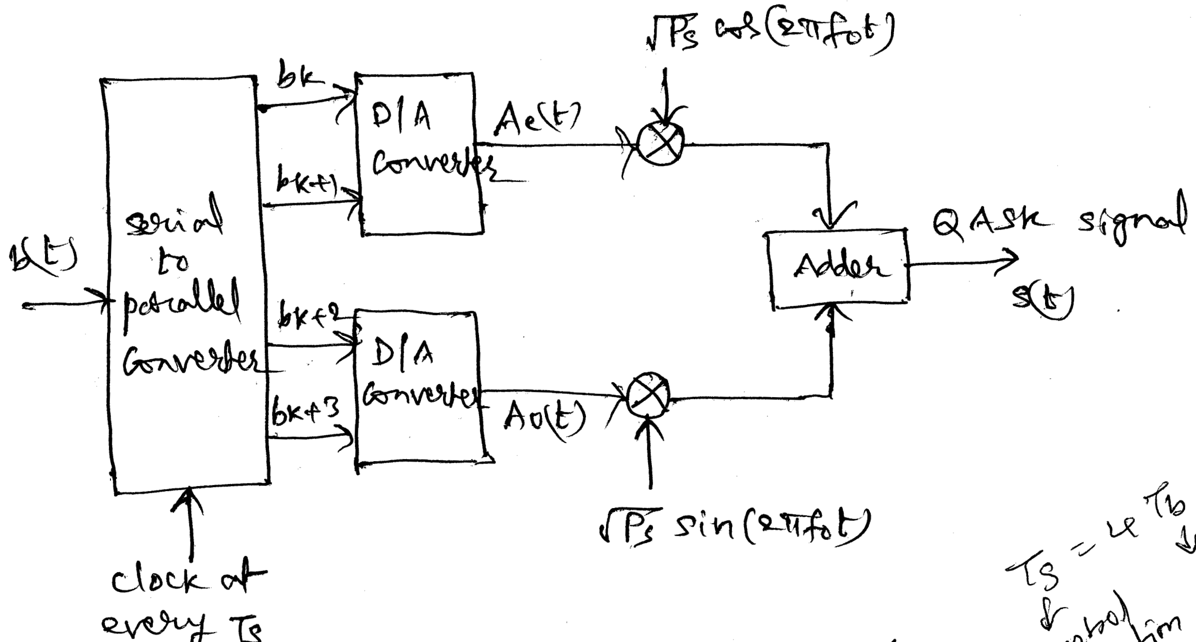
We know that $a = \sqrt{0.1 E_s}$.

$$\Rightarrow s(t) = k_1 \sqrt{0.2 \frac{E_s}{T_s}} \cos(2\pi f_0 t) + k_2 \sqrt{0.2 \frac{E_s}{T_s}} \sin(2\pi f_0 t)$$

We know that $E_s = P_s T_s \Rightarrow \frac{E_s}{T_s} = P_s$

$\Rightarrow \quad s(t) = k_1 \sqrt{0.2 P_s} \cos(2\pi f_0 t) + k_2 \sqrt{0.2 P_s} \sin(2\pi f_0 t) \rightarrow (3)$

The above eq. gives the QASK signal. Here k_1 and ... defined the amplitudes of the modulated signal.



$T_s = 4 T_b$ bit duration; T_s symbol duration

fig : Generation of QASK signal

Fig above shows the transmitter for 4-bit QASK (16-QASK) system. The i/p bit ~~ste~~ stream is applied to serial to parallel converter. Four successive bits are applied to the digital to analog converters. These bits are applied after every T_s seconds. T_s is the symbol period and $T_s = 4T_b$. Bits b_k and b_{k+1} are applied to upper D/A converter and b_{k+2} and b_{k+3} are applied to lower D/A converter. Depending upon two i/p bits, the o/p of D/A converter takes four o/p levels. Thus $A_e(t)$ and $A_o(t)$ takes 4 levels depending upon combination of two i/p bits. $A_e(t)$ modulates the carrier $\sqrt{P_s} \cos(2\pi f_0 t)$ and $A_o(t)$ modulates $\sqrt{P_s} \sin(2\pi f_0 t)$. The adder combines two signals to give QASK signal. It is given as,

$s(t) = A_e(t) \sqrt{P_s} \cos(2\pi f_0 t) + A_o(t) \sqrt{P_s} \sin(2\pi f_0 t) \rightarrow (4)$

If we compare eq ③ & ④ (∵ k_1 & k_2 will be ±1 or ±3)

$A_e(t)$ and $A_o(t) = \pm\sqrt{0.2}$ or $\pm 3\sqrt{0.2}$ ⟶ ⑦

(depending upon i/p to D/A converter)

Receiver

$s(t) = A_e(t)\sqrt{P_s}\cos(2\pi f_0 t) + A_o(t)\sqrt{P_s}\sin(2\pi f_0 t)$

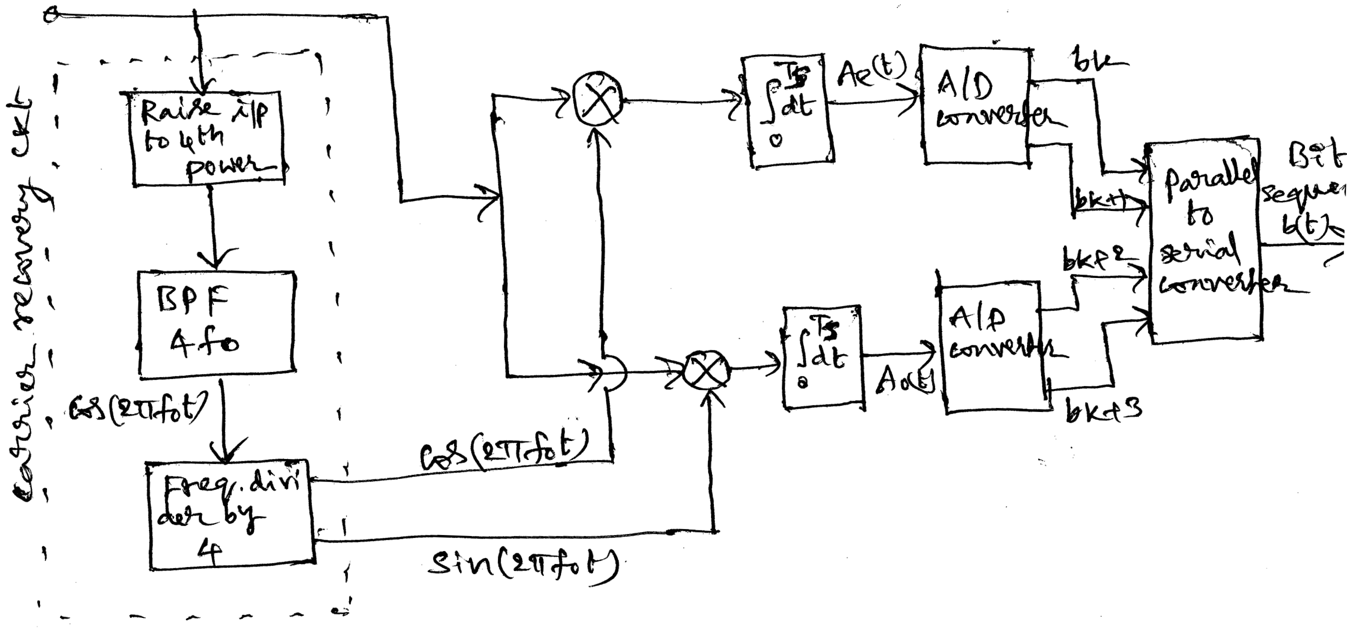


fig: 4-bit QASK receiver block diagram

Fig. above shows the receiver of 16-QASK (4-bit QASK) system. The i/p signal s(t) is raised to 4th power. It then passed through a BPF centered around the freq $4f_0$, then the signal freq is divided by 4. It gives a ~~cohe~~ coherent carrier, $\cos(2\pi f_0 t)$. Quadrature carrier $\sin(2\pi f_0 t)$ is produced by phase shifting of 90°. The inphase and quadrature coherent carriers are multiplied with QASK signal s(t).

Since the amplitude of $A_e(t)$ and $A_o(t)$ are bit constant and equal, let us check whether we can really recover the carrier correctly. The 4th power QASK signal is,

$$s^4(t) = P_s^2\left[A_e(t)\cos(2\pi f_0 t) + A_o(t)\sin(2\pi f_0 t)\right]^4 \longrightarrow ①$$

This signal is passed through a BPF of $4f_0$. Therefore we will consider only the frequencies of $4f_0$ i.e.

$$s^4(t) = \frac{P_s}{8}\left[A_e^4(t) + A_o^4(t) - 6A_e^2(t)A_o^2(t)\right]\cos 4(2\pi f_0 t) + \frac{P_s}{2}\left[A_e(t)A_o(t)\left\{A_e^2(t) - A_o^2(t)\right\}\right]\sin 4(2\pi f_0 t)$$

The avg value of second term will be zero, hence only first term is passed through a BPF centered at $4f_0$. This happens because all power of $A_e(t)$ and $A_o(t)$ in the first term are even. The integrators integrate the multiplied signals over one symbol period. The o/p of integrators at sampling period give $A_e(t)$ and $A_o(t)$. The A/D converters gives the four bits b_k, b_{k+1}, b_{k+2} and b_{k+3}. The parallel to serial converter then generates the bit sequence $b(t)$.

Power spectral Density :-

The Power spectral density of baseband QASK signal is

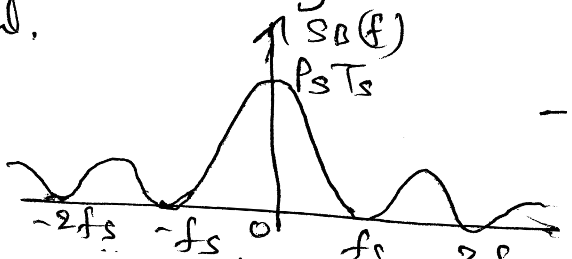
$$S(f) = P_s T_s\left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \longrightarrow (1)$$

The above eq. gives PSD of $A_e(t)$ and $A_o(t)$. When they modulate the carrier, the main lobe given by above eq. is shifted by carrier freq. f_0. i.e

$$S(f) = \frac{P_s T_s}{2}\left[\frac{\sin\pi(f-f_0)T_s}{\pi(f-f_0)T_s}\right]^2 + \frac{P_s T_s}{2}\left[\frac{\sin\pi(f+f_0)T_s}{\pi(f+f_0)T_s}\right]^2 \rightarrow (2)$$

Which is a PSD of QASK signal.

The plot of the main lobe of QASK signal given by eq (1) is shown in fig.



B.W of QASK signal:-

$$BW = f_s - (-f_s) = 2f_s = \frac{2}{T_s}$$

$$= \frac{2}{T_s} \quad (\because f_s = \frac{1}{T_s})$$

$$= \frac{2}{NT_b} \quad (\because T_s = NT_b)$$

$$= \frac{2f_b}{N} \quad (\because f_b = \frac{1}{T_b})$$

Comparision of QPSK and QASK

S.NO	parameter	QPSK	QASK
1.	Modulation	Quadrature phase	Quadrature amplitude and phase
2.	Location of signal points	All signal points placed on circumference of circle	signal points are replaced symmetrically about origin
3.	Distance b/w signal points	$2\sqrt{0.15 E_b}$ for 16 symbols and $2\sqrt{E_b}$ for 4 symbols	$2\sqrt{0.4 E_b}$ for 16 symbols
4.	complexity	Relatively simpler	Relatively complex
5.	Noise immunity	Better than QASK	Poor than QPSK
6.	Error probability	Less than QASK	Higher than QPSK
7.	Type of demodulation	Coherent	Coherent

P:- Assume that $R = 9600$ kbps. For a rectangular 4.l pulse calculate the second null-to-null bandwidth of BPSK, QPSK and QAM.

Sol:- bit rate, $R = \frac{1}{T_b} = f_b = 9600k\ Hz$

BPSK : $BW = 2f_b = 2 \times 9600\ Hz = 19200\ kHz = 19.2\ MH$

QPSK : $BW = f_b = 9600\ kHz = 9.6\ MHz$

QAM : $BW = \frac{2f_b}{N} = \frac{2 \times 9600\ kHz}{4} = 4.8\ MHz$

M-ary PSK

BPSK transmits one bit at a time and it has only two symbols. Hence whenever the symbol is changed, the phase shift is,

phase shift in BPSK $= \frac{2\pi}{\text{No. of symbols}} = \frac{2\pi}{2} = \pi$ & $180°$.

In QPSK two successive bits are combined to form 4 distinct symbols. Hence whenever symbol is changed,

phase shift $= \frac{2\pi}{\text{No. of symbols}} = \frac{2\pi}{4} = \pi/2$ or $90°$

This can be extended for 'N' bits. If we combine N successive bits, then there will be $M = 2^N$ possible symbols. then

phase shift in M-ary PSK $= \frac{2\pi}{M}$

The duration of each symbol will be NT_b thus,

$$\boxed{NT_b = T_s}$$

Since there are M-symbols, this method is called M-ary PSK. The transmitted waveform is represented in M-ary PSK as,

$$\boxed{s(t) = \sqrt{2P_s}\cos(2\pi f_0 t + \phi_m)} \quad \rightarrow (1)$$

$m = 0, 1, 2, \cdots M-1$

$$\boxed{\phi_m = (2m+1)\frac{\pi}{M}} \quad \text{symbol phase angle} \rightarrow (2)$$

Transmitter of M-ary PSK :-

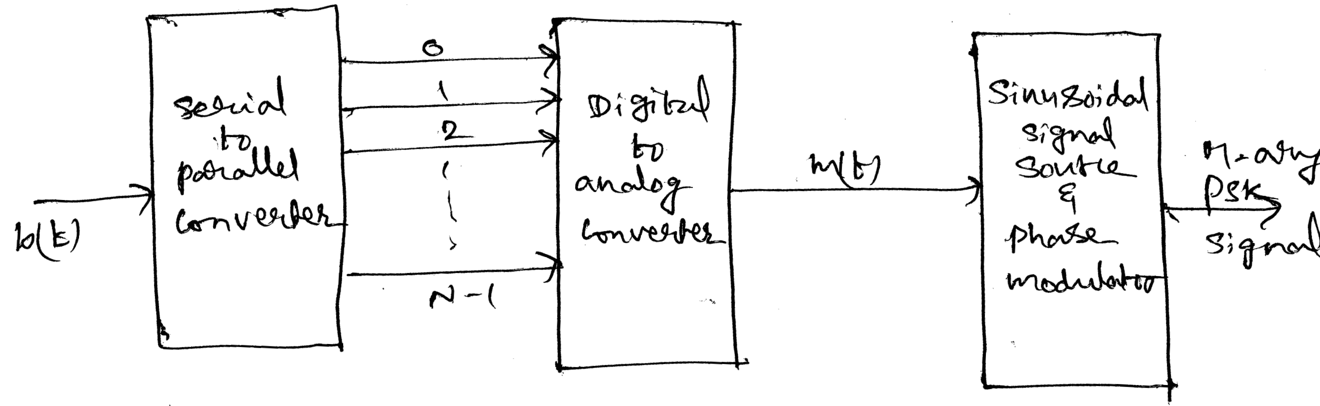


fig: Block diagram of Transmitter

Fig. above shows the M-ary PSK transmitter. The serial to parallel converter forms a symbol of 'N' successive bits. That is the o/p of serial to parallel converter is 'N' bit word.

The D/A converter o/p remains unchanged till last N bit is received. Then depending upon the i/p 'N' bits, the o/p of D/A converter is defined. This o/p is m(t). Again the serial to parallel converter starts taking bit for next 'N' word. The o/p of D/A converter remains unchanged till last bit is received. Thus m(t) is hold for the period of NT_b. m(t) takes $2^N = M$ different value depending upon the i/p bits. The voltage m(t) is applie to modulator. This modulator modulates the phase of sinusoidal carrier depending upon the amplitude of the symbol m(t).

The M-ary PSK signal is

$$s(t) = \sqrt{2P_s} \cos(2\pi f_0 t + \phi_m)$$

$$= \sqrt{2P_s} \cos\phi_m \cos(2\pi f_0 t) - \sqrt{2P_s} \sin\phi_m \sin(2\pi f_0 t)$$

Signal space Diagram for M-ary PSK

We know that the M-ary PSK is

$$s(t) = \sqrt{2P_s}\cos(2\pi f_0 t + \phi_m) \longrightarrow (1)$$

$m = 0, 1, 2, \ldots M-1,$

The symbol phase angle

$$\phi_m = (2m+1)\frac{\pi}{M}$$

Eq (1) can be expanded as

$$s(t) = \sqrt{2P_s}\cos\phi_m \cos(2\pi f_0 t) - \sqrt{2P_s}\sin\phi_m \sin(2\pi f_0 t) \rightarrow (2)$$

By rearranging the above eq.

$$s(t) = \sqrt{P_s T_s}\cdot\sqrt{\frac{2}{T_s}}\cos\phi_m \cos(2\pi f_0 t) - \sqrt{P_s T_s}\sqrt{\frac{2}{T_s}}\sin\phi_m \sin(2\pi f_0 t)$$

$$= \sqrt{P_s T_s}\cos\phi_m\, \phi_1(t) - \sqrt{P_s T_s}\sin\phi_m\, \phi_2(t) \rightarrow (3)$$

Where $\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t)$ and

$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t)$

The above two equations are orthonormal ~~wavefo~~ waveforms, Fig. below shows the signal space diagram based on eq (3).

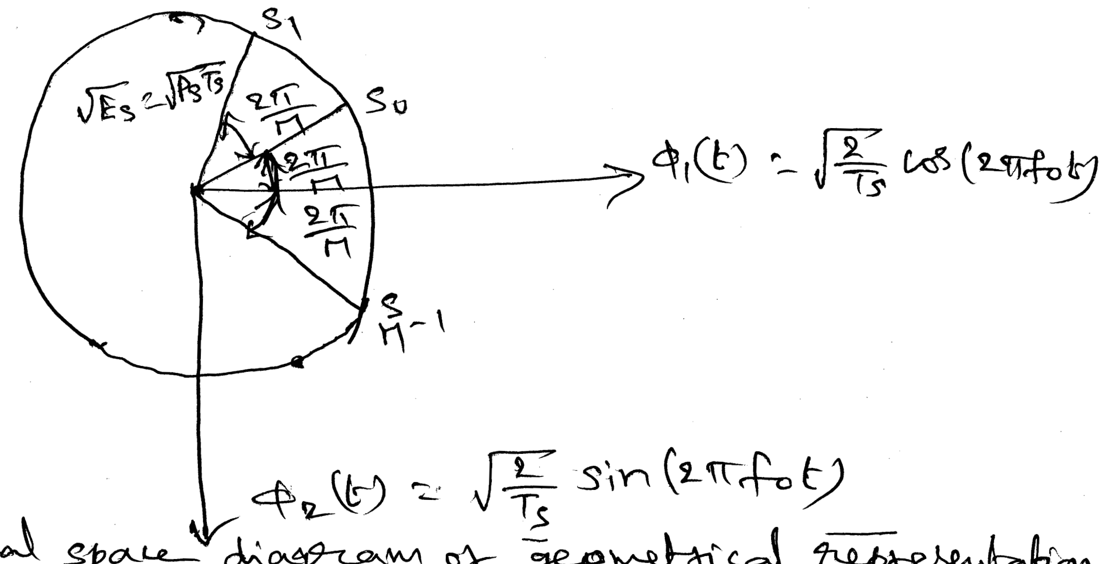


fig: signal space diagram or geometrical representation of M-ary PSK signals.

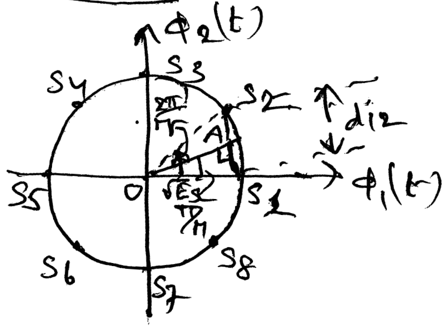
The orthonormal carriers $\phi_1(t)$ and $\phi_2(t)$ form two axes. The signal points $s_0, s_1, s_2, \ldots s_{M-1}$ are placed on the circumference of the ~~circle~~. circle. The signal points are equispaced with the phase shift of $\frac{2\pi}{M}$. The distance of each signal point from the origin is $\sqrt{P_s T_s}$.

Here $P_s T_s = E_s$ (symbol energy)

Thus we can say that QPSK is the special case of ~~M-ary~~ M-ary PSK with $M=4$.

<u>Distance b/w signal points (Euclidean Distance)</u>

Fig. shows signal space diagram of a M-ary PSK, with $M=8$. The distance b/w signal points s_1 and s_2 can be obtained by considering the triangle formed by $s_1 O A$. This distance b/w s_1 & s_2 is denoted by d_{12}



$\therefore$ Distance $s_1 A = s_2 A = \frac{d_{12}}{2}$

$\therefore \frac{\text{Distance } s_1 A}{\text{Distance } O s_1} = \sin\frac{\pi}{M}$

$\therefore \frac{d_{12}/2}{\sqrt{E_s}} = \sin\frac{\pi}{M} \rightarrow d_{12} = 2\sqrt{E_s}\sin\frac{\pi}{M}$

Using the same procedure we can easily establish the distance b/w s_1 and s_8. It is same as the distance b/w s_1 & s_2. i.e.

$$\boxed{d_{18} = d_{12} = 2\sqrt{E_s}\sin\frac{\pi}{M}}$$ Euclidean distance for generalized value of 'M'

For QPSK $M=4$

$$\Rightarrow \boxed{d = 2\sqrt{E_s}\sin\frac{\pi}{4}}$$ for QPSK.

Power spectral Density of M-ary PSK :-

BPSK & QPSK are the special cases of M-ary PSK. The symbol duration for M-ary PSK is given by

$$T_s = N T_b.$$

where N = no. of i/p successive bits combined.

The baseband PSD of QPSK is

$$S_B(QPSK)(f) = 2 P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

If we put $T_s = N T_b$ in above eq. we will get PSD of M-ary PSK. i.e.

$$S_B(f) = 2 P_s N T_b \left[\frac{\sin(\pi f N T_b)}{\pi f N T_b}\right]^2$$

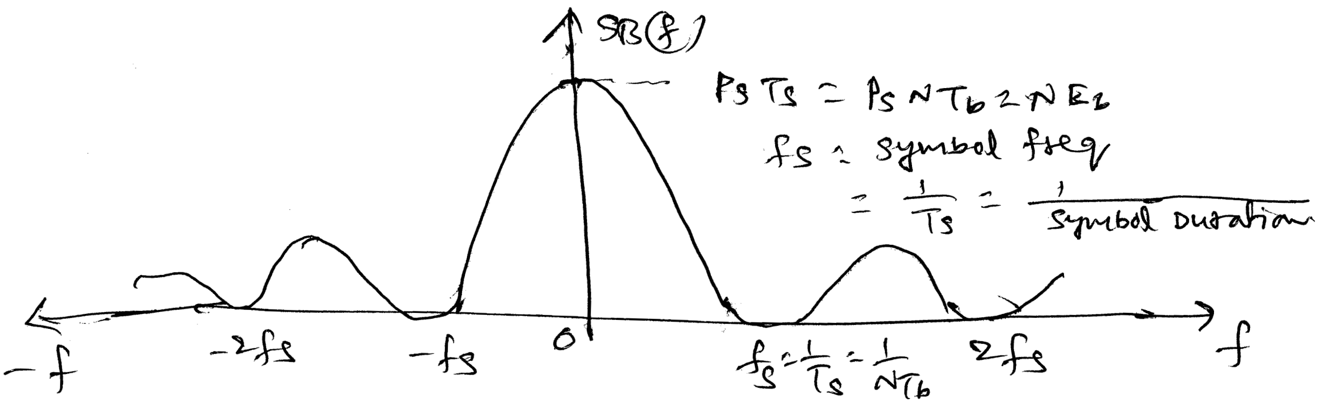


fig: plot of PSD of baseband M-ary PSK.

B.W. :-

The B.W. required by the system is equal to the width of the main lobe i.e.

$$BW = f_s - (-f_s)$$
$$= 2f_s$$
$$= \frac{2}{T_s}$$
$$= \frac{2}{N T_b} \quad (\because T_s = N T_b)$$
$$= \frac{2 f_b}{N} \quad (\because f_b = \frac{1}{T_b})$$

⇒ As No. of successive bits (N) per symbol are increased, the B. reduces.

M-ary PSK Receiver :-

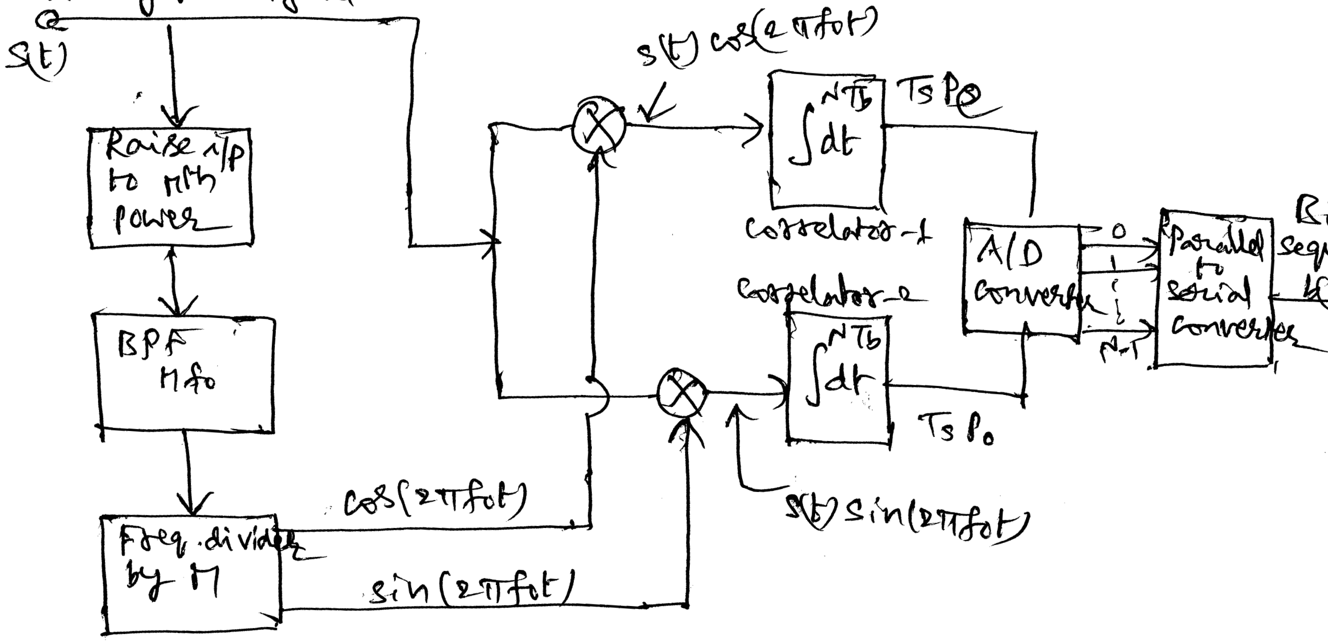


fig: M-ary PSK receiver

Fig. above shows the receiver of M-ary PSK. It is similar to QPSK receiver. The i/p signal s(t) is raised to Mth power. The BPF extracts the freq component Mfo. This freq is divided by 'M' to obtain carrier freq. fo. The coherent carriers are thus generated and applied to the two multipliers.

The o/ps of the multipliers are given to the integrators. The integrators integrate over the period of $T_s = NT_b$. The o/ps of integrators are sampled after the period T_s in every cycle, and applied to the analog to digital converter. The integrator o/ps are proportional to $T_s P_e$ and $T_s P_o$. These voltages are applied to A/D converter, which reconstructs 'N' bit symbol. This 'N' bit symbol is given to the parallel to serial converter. It then generates the bit sequence b(t).

M-ary FSK

The BFSK is for two symbols. This principle can be extended further to 'N' successive bits. These 'N' bits form $M = 2^N$ different symbols. Every symbol uses separate frequency for transmission. Such system is called M-ary FSK system. The principle of transmission and reception of M-ary FSK is different than BFSK.

The fig. below shows the M-ary FSK transmitter. The 'N' successive bits are presented in parallel to digital to analog converter. These 'N' bits forms a symbol at the o/p of digital to analog converter. There will be total $M = 2^N$ possible symbols.

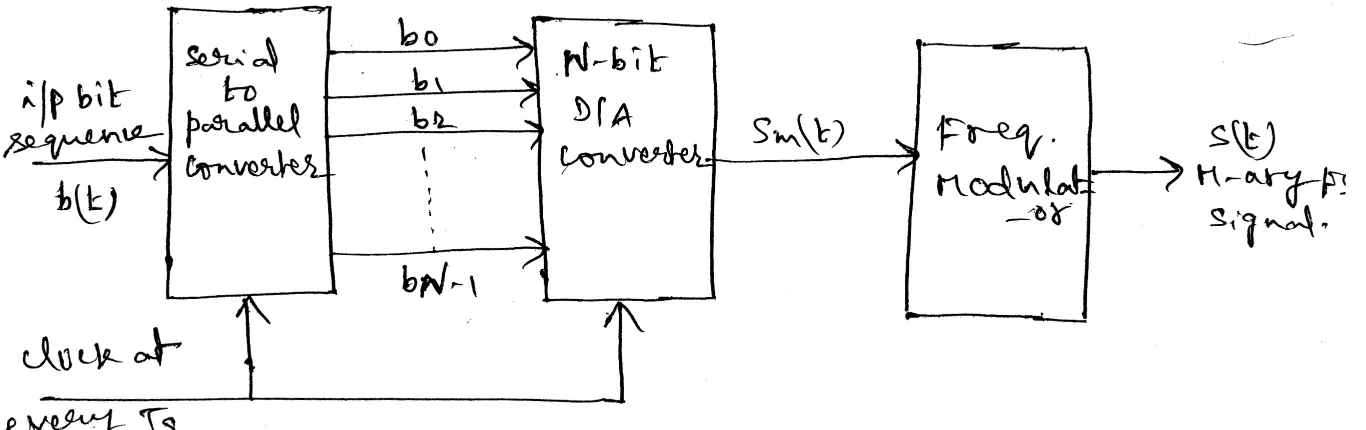


fig: M-ary FSK transmitter

The symbol is presented every $T_s = NT_b$ period. The o/p of D/A converter is given to a frequency modulator. Thus depending upon the value of symbol, the frequency modulator generates the o/p frequency. For every symbol, the frequency modulator produces different freq. ~~o/ps.~~ o/p. This particular freq. signal remains at the o/p for one symbol duration

Thus for 'M' symbols, there are 'M' frequency signals at the o/p of modulator. Thus the transmitted freqs. are $f_0, f_1, f_2, \ldots f_{M-1}$ depending upon the i/p symbol to to the modulator.

<u>M-ary FSK Receiver</u>:

The fig. below show the block diagram of M-ary FSK receiver. It is the extension of BFSK receiver. The M-ary FSK signal is given to the set of 'M' bandpass filters. The centre frequencies of these filters are $f_0, f_1, f_2, \ldots f_{M-1}$. These filters pass their particular frequency and alternate others. The envelope detectors

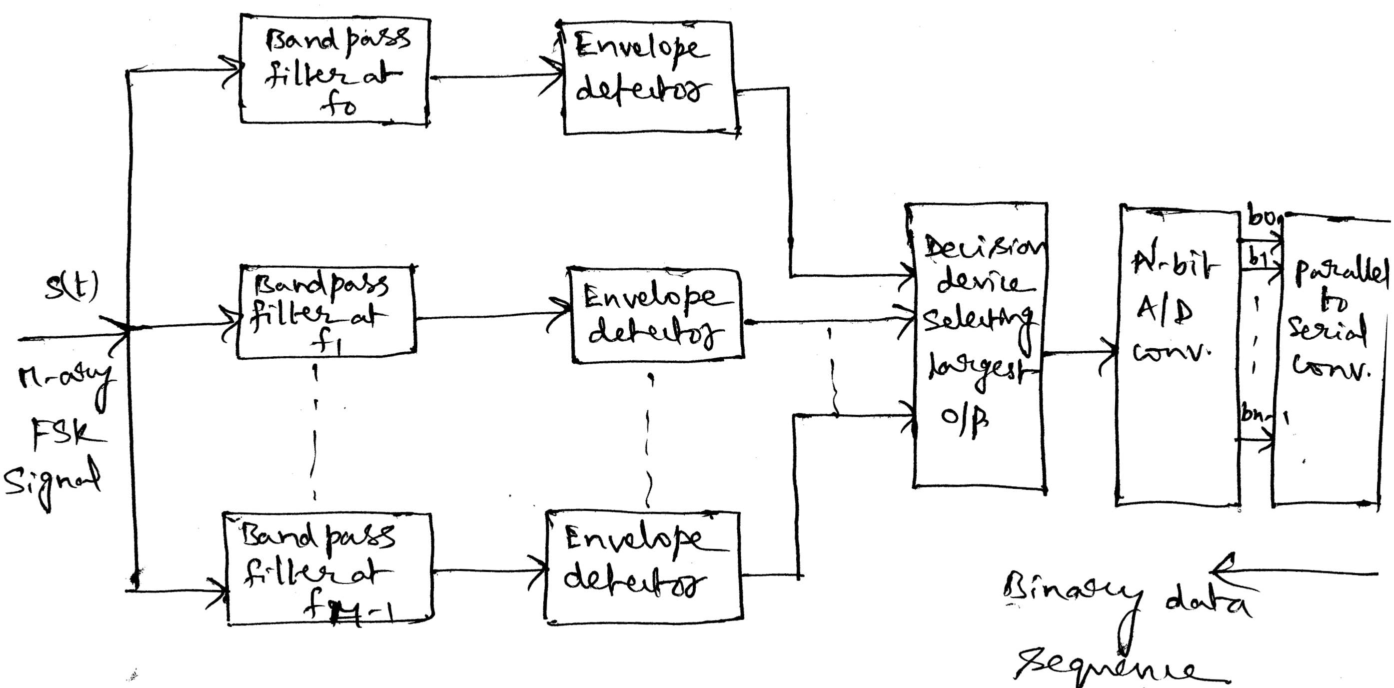


fig: Block diagram of M-ary FSK receiver system.

o/ps are applied to a decision device. The decision device produces its o/p depending upon the highest i/p. Depending upon the particular symbol, only one evelope detector will have ~~highest~~ higher output. The o/ps of others detectors will be very low. The o/p of the decision device is given to N-bit A/D convert

~~The~~ The analog to digital converter o/p is the 'N' bit symbol in parallel. These bits are then converted to serial bit stream by parallel to serial converter.

In some cases the bits appear in parallel. Then there is no need to use serial to parallel and parallel to serial converters.

Bandwidth of M-ary FSK:-

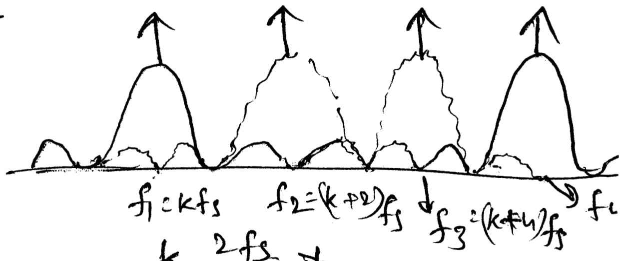


fig: PSD of M-ary FSK

$$B.W = M \times (2f_s)$$

$$= 2Mf_s$$

where $M = 2^N$ & $f_s = \frac{f_b}{N}$.

$$BW = 2 \cdot 2^N \cdot f_b/N$$

$$\boxed{BW = \frac{2^{N+1} f_b}{N}} = \frac{2^{N+1} f_b}{N}$$

~~on comparati~~

On comparision of M-ary PSK bandwidth it is clear that M-ary FSK needs comparatively large bandwidth

Note:- M-ary QAM

M-ary QAM is same as QASK. ~~with~~ Replace ~~4~~-bit with N & 16 with M.