Introduction

To deal with EM, we use 3 types of co-ordinate systems.

1. cartesian co-ordinate system
2. cylindrical co-ordinate system
3. spherical co-ordinate system.

In CCS, any point 'P' in the space can be represented as (x, y, z)

$$-\infty < x < \infty, \quad -\infty < y < \infty, \quad -\infty < z < \infty$$

and a vector, $\vec{A}$ can be written as,

$$\vec{A} = A_x a_x + A_y a_y + A_z a_z$$

a_x, a_y, a_z are unit vectors along the $x, y,$ and z directions.

In cylindrical co-ordinate system any point 'P' in the space can be represented as, (ρ, ϕ, z)

$0 \leq \rho < \infty$ $\qquad$ ρ - radius

$0 \leq \phi \leq 360 \ (2\pi)$ $\qquad$ ϕ - Azimuth angle

$-\infty < z < \infty$ $\qquad$ z - z axis.

$$\vec{A} = A_\rho a_\rho + A_\phi a_\phi + A_z \hat{a}_z.$$

Spherical co-ordinate system, a point 'p' can be represented as, (r, θ, ϕ).

$0 \leq r \infty$ — r - radius of sphere

$0 \leq \theta \leq \pi \, (180^\circ)$ — θ - colatitude

$0 \leq \phi \leq 2\pi \, (360^\circ)$ — ϕ - azimuth angle.

$$\vec{A} = A_r a_r + A_\theta a_\theta + A_\phi a_\phi$$

Important points of Vector calculus

let a vector, $\vec{A} = A_x a_x + A_y a_y + A_z a_z$

the magnitude of vector, $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

the direction of vector, $\hat{a}_A = \dfrac{\vec{A}}{|\vec{A}|}$

$$= \frac{\vec{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Dot product

let two vectors $\vec{A}$ and $\vec{B}$, the dot product or simply scalar product $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}.$$

cross product

the cross product of two vectors $\vec{A}$ and $\vec{B}$ is

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta_{AB} \ \hat{a}_N.$$

cross product of two vectors is again a vector.

Del operator

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z.$$

$$\nabla = a_\rho \frac{\partial}{\partial \rho} + a_\phi \frac{1}{\rho}\frac{\partial}{\partial \phi} + a_z \frac{\partial}{\partial z}$$

$$\nabla = a_r \frac{\partial}{\partial r} + a_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + a_\phi \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi}$$

The curl of a vector, $\vec{A}$

$$\nabla \times \vec{A} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \vec{A} = \frac{1}{\rho} \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_r & r a_\theta & r\sin\theta\, a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \partial/\partial\phi \\ A_r & r A_\theta & r\sin\theta A_\phi \end{vmatrix}$$

Stokes theorem :

$$\oint_L \vec{A} \cdot dl = \int_S (\nabla \times \vec{A}) \cdot ds$$

Divergence theorem:

$$\oint_S \vec{A} \cdot ds = \int_V (\nabla \cdot \vec{A})\, dv$$

Laplacian operator : ∇^2

$$\nabla^2 \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla \times \nabla \times \vec{A}$$

$$\nabla \times \nabla \times \vec{A} = \nabla \cdot (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

UNIT-I

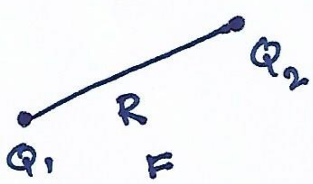
ELECTROSTATICS

Coulomb's law : Coulombs law deals with the force 'F' between two point charges Q_1 and Q_2.

According to coulombs law, the force between two point charges is,

1. Along the line joining them
2. Directly proportional to the product of charges, Q_1Q_2
3. Inversely proportional to the square of the distance, R between them.

Mathematically expressed as,



$$F \propto Q_1 Q_2$$

$$F \propto \frac{1}{R^2}$$

$$\therefore F \propto \frac{Q_1 Q_2}{R^2}$$

$$F = k \frac{Q_1 Q_2}{R^2}$$

where, k is proportionality constant and its value is, $k = \frac{1}{4\pi\varepsilon_0}$

ε_0 = permittivity of free space ($\frac{1}{36\pi \times 10^9}$ F/m)

$k = 9 \times 10^9$ m/F

∴ The force between the charges, $\boxed{F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2}}$ Newtons

Direction of force

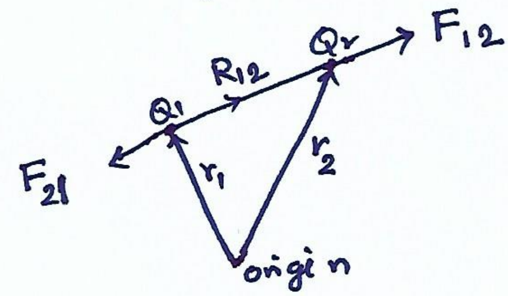
Since the force is vector quantity, its direction depends on the polarities of charges Q_1 and Q_2.

Case: i

when both Q_1 and Q_2 are like charges, repel each other

$Q_1 Q_2 > 0$

Force F_{12} on Q_2 due to Q_1



is $F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} \hat{a}_{R_{12}}$

$\hat{a}_{R_{12}}$ is unit vector indicates direction of force

$$\hat{a}_{R_{12}} = \frac{R_{12}}{|R_{12}|} = \frac{r_2 - r_1}{|r_2 - r_1|}$$

∴ The force, $\boxed{F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{(r_2 - r_1)}{|r_2 - r_1|^3}}$

Similarly, the force F_{21} on Q_1 due to Q_2 is

$$F_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \hat{a}_{R_{21}}$$

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Case ii

when both the charges are unlike charges, the Force will be attractive force.

$$Q_1 Q_2 < 0$$

If we have morethan two charges, the resultant force $\vec{F}$ on charge 'Q' is vector sum of forces on Q due to $Q_1, Q_2, Q_3 \ldots\ldots Q_N$.

$$\vec{F} = \frac{Q Q_1 (r - r_1)}{4\pi\varepsilon_0 |r - r_1|^3} + \frac{Q Q_2 (r - r_2)}{4\pi\varepsilon_0 |r - r_2|^3} + \cdots\cdots$$

$$+ \frac{Q Q_N (r - r_N)}{4\pi\varepsilon_0 |r - r_N|^3}$$

$$\boxed{\vec{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{Q_i (r - r_i)}{|r - r_i|^3}}$$

Problem : what is force between two 1Coulomb charges when they are separated by 1m distance.

Ans : 9×10^9 Newtons

Problem

A charge $Q_A = -20\,\mu C$ is located at $A(-6,4,7)$ and a charge $Q_B = 50\,\mu C$ is at $B(5,8,-2)$ in free space find $\vec{R}_{AB}$ and vector force exerted on Q_A by Q_B

Sol

Given $Q_A = -20\,\mu C$ $A(-6,4,7)$

$Q_B = 50\,\mu C$ $B(5,8,-2)$

$\vec{R}_{AB} = ?$ $\vec{F}_{BA} = ?$

$\vec{R}_{AB} = \vec{B} - \vec{A} = (5,8,-2) - (-6,4,7) = (11,4,-9)$

$\vec{R}_{AB} = 11a_x + 4a_y + 9\hat{a}_z$

$$\therefore \vec{F}_{BA} = \frac{Q_A Q_B (A-B)}{4\pi\varepsilon_0 |A-B|^3} = 9\times10^9 \frac{(-20\times10^{-6})(50\times10^{-6})(-6,4,7)-(5,8,-2)}{|(-6,4,7)-(5,8,-2)|^3}$$

$$= \frac{(-900\times10^{-3})(-11,-4,9)}{(11^2+4^2+9^2)\sqrt{11^2+4^2+9^2}}$$

$$\vec{F}_{BA} = 30.76\hat{a}_x + 11.184\hat{a}_y - 25.16\hat{a}_z$$

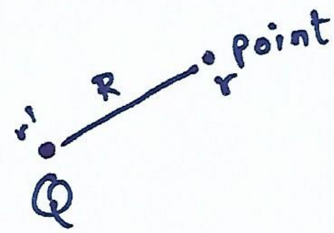
<u>Electric field intensity:</u>

The electric field intensity (or) electric field strength is the force per unit charge when place in an electric field.

Thus $$\vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{Q}$$

(or) Simply, $$\vec{E} = \frac{\vec{F}}{Q}$$ Newtons/Coulomb (or) volt/meter

The electric field intensity at point 'r' due to charge 'Q' is,



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\boxed{\vec{E} = \frac{Q\,(r - r')}{4\pi\varepsilon_0 |r - r'|^3}}$$

The direction of $\vec{E}$ is same as direction of $\vec{F}$ provided Q is positive (i.e $Q > 0$).

If there are 'N' number of point charges, Q_1, Q_2, Q_3 .... Q_N and are located at r_1, r_2, r_3 ..... r_N, then the electric field intensity at point 'r' is the vector sum of electric field intensities due to indivisual charges.

Mathematically,

$$\vec{E} = \frac{Q_1(r_1 - r_1)}{4\pi\varepsilon_0|r - r_1|^3} + \frac{Q_2(r - r_2)}{4\pi\varepsilon_0|r - r_2|^3} + \frac{Q_3(r - r_3)}{4\pi\varepsilon_0|r - r_3|^3}$$

$$+ \cdots\cdots + \frac{Q_N(r - r_N)}{4\pi\varepsilon_0|r - r_N|^3}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{Q_i(r - r_i)}{|r - r_i|^3}}$$

<u>Problem</u>: Two point charges 1mC and -2mC are located at (3,2,-1) and (-1,-1,4) respectively. Calculate electric force on a 10 nC charge located at (0,3,1) and the electric field intensity at that point.

Sol

Given point charges, $Q_1 = 1\,mC$ located at $\overset{r_1}{(3,2,-1)}$

$Q_2 = -2\,mC$ located at $\overset{r_2}{(-1,-1,4)}$

Q_3 (or) $Q = 10\,nC$ located at $\overset{r_0}{(0,3,1)}$

The force on 'Q' due to Q_1 and Q_2 is,

$$\vec{F} = \frac{Q_1 Q\,(r-r_1)}{4\pi\varepsilon_0\,|r-r_1|^3} + \frac{Q_2 Q\,(r-r_2)}{4\pi\varepsilon_0\,|r-r_2|^3}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{Q_1\,(r-r_1)}{|r-r_1|^3} + \frac{Q_2\,(r-r_2)}{4\pi\varepsilon_0\,|r-r_2|^3}\right]$$

$$= 10\times10^{-9}\,.\,9\times10^{9}\left[\frac{1\times10^{-3}\left[(0,3,1)-(3,2,-1)\right]}{|(0,3,1)-(3,2,-1)|^3}\right.$$

$$\left. + \frac{(-2)\times10^{-3}\left[(0,3,1)-(-1,-1,4)\right]}{|(0,3,1)-(-1,-1,4)|^3}\right]$$

$$= 9 \times 10^{-2} \left[\frac{(-3,1,2)}{14\sqrt{14}} + \frac{(-2,-8,6)}{26\sqrt{26}} \right]$$

$$\vec{F} = -6.507\, a_x - 3.817\, a_y + 7.506\, a_z \quad m \text{ Newtons}$$

At that point 'r' the electric field intensity is,

$$\vec{E} = \frac{\vec{F}}{Q}$$

$$= (-6.507, -3.817, 7.506) \cdot \frac{10^{-3}}{10 \times 10^{-9}}$$

$$\vec{E} = -650.7\, a_x - 381.7\, a_y + 750.6\, a_z \quad kv/m$$

Problem 2 : Two point charges 5 nC and -2 nC are located at (2, 0, 4) and (-3, 0, 5) respectively.

a) Determine the force on 1nC point charge located at (1, -3, 7)

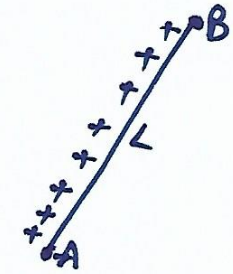
b) Find electric field at (1, -3, 7)

Answer: a) $-1.004\, a_x - 1.284\, a_y + 1.4\, a_z$ nN

b) $-1.004\, a_x - 1.284\, a_y + 1.4\, \hat{a}_z$ V/m.

Electric fields due to line charge

In the figure, we can see the line charge with uniform charge density (ρ_L), and is extend from A to B.



Line charge

Now consider small length 'dL' of line charge, the charge in length 'dL' is, dQ

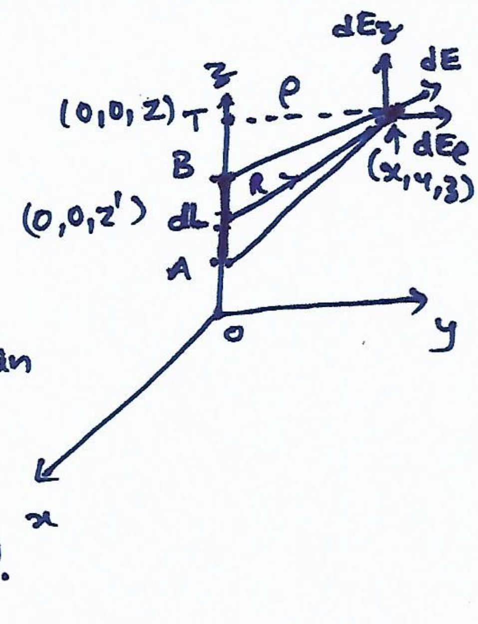
$$dQ = \rho_L \, dL$$

$$Q = \int_L \rho_L \, dL \qquad \text{total charge of line charge distribution.}$$

$$Q = \int_A^B \rho_L \, dL$$

place this line charge along the z-axis as shown in the figure.

So, total charge, $Q = \int_{z_A}^{z_B} \rho_L \, dz'$



The electric field intensity at an arbitary point P(x,y,z), when 'dL' points are denoted by (0,0,z').

$$\vec{E} = \int \frac{\rho_L \, dL}{4\pi\varepsilon_0 R^2} \, \hat{a}_R$$

$$E = \int_{z_A}^{z_B} \frac{\rho_L \, dz'}{4\pi\varepsilon_0 R^2} \cdot \frac{\vec{R}}{|\vec{R}|}$$

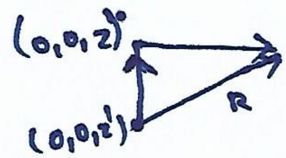
$\vec{R}$ = distance vector, from $(0,0,z')$ to (x,y,z)

$$\vec{R} = (x,y,z) - (0,0,z')$$

$$\vec{R} = x\,\hat{a}_x + y\,\hat{a}_y + (z-z')\,\hat{a}_z.$$

from figure,

$$\vec{R} = \rho\,\hat{a}_\rho + (z-z')\,\hat{a}_z.$$



$$\therefore R^2 = |\vec{R}|^2 = x^2 + y^2 + (z-z')^2 = \rho^2 + (z-z')^2$$

$$\frac{\hat{a}_R}{R^2} = \frac{\rho\,\hat{a}_\rho + (z-z')\,\hat{a}_z}{\left[\rho^2 + (z-z')^2\right]^{3/2}}$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int \frac{\rho\,\hat{a}_\rho + (z-z')\,\hat{a}_z}{\left[\rho^2 + (z-z')^2\right]^{3/2}}\, dz'$$

from the figure,

$$\cos\alpha = \frac{\rho}{\vec{R}} = \frac{\rho}{\sqrt{\rho^2 + (z-z')^2}}$$

$$\sqrt{\rho^2 + (z-z')^2} = \rho \sec\alpha$$

and $\tan\alpha = \dfrac{(z-z')}{\rho}$

$$\rho \tan\alpha = z - z'$$

$$z' = z - \rho \tan\alpha.$$

$$dz' = -\rho \sec^2\alpha \, d\alpha$$

$\vec{E}$ will become like,

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{\rho \sec^2\alpha \left[\cos\alpha \, \hat{a}_\rho + \sin\alpha \, \hat{a}_z\right] d\alpha}{\rho^2 \sec^2\alpha}$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0 \rho} \int_{\alpha_1}^{\alpha_2} \left[\cos\alpha \, \hat{a}_\rho + \sin\alpha \, \hat{a}_z\right] d\alpha$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0 \rho} \left[-(\sin\alpha_2 - \sin\alpha_1) \hat{a}_\rho + (\cos\alpha_2 - \cos\alpha_1) \hat{a}_z \right]$$

The above equation is electric field for finite line charge.

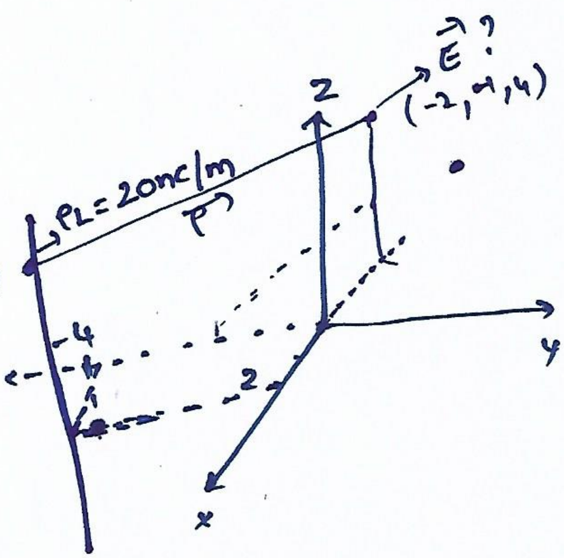
For infinite line charge, point B $(0,0,\infty)$ and A $(0,0,-\infty)$, $\alpha_1 = \pi/2$ and $\alpha_2 = -\pi/2$

then the electric field $\vec{E}$ become,

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\hat{a}_\rho}$$

Problem : uniform line charge with density, $\rho_L = 20\,nC/m$ is there on line $x = 2m$, $y = -4m$, Determin $\vec{E}$ at $(-2,-1,4)$.

Sol :



The electric field,

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\hat{a}_\rho$$

where,

$\rho =$ is perpendicular distance from the line charge to observation point $(-2,-1,4)$

$$\rho = (-2,-1,4) - (2,-4,4) = (-4,3,0)$$

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\frac{\vec{\rho}}{|\vec{\rho}|} = \frac{\rho_L}{2\pi\varepsilon_0 \rho^2}\vec{\rho}$$

$$\vec{E} = \frac{(20\times 10^{-9})}{\cancel{2\pi}\times\left(\frac{1}{\cancel{36\pi}\times 10^{9}}\right)\cdot(25)}(-4,3,0)$$

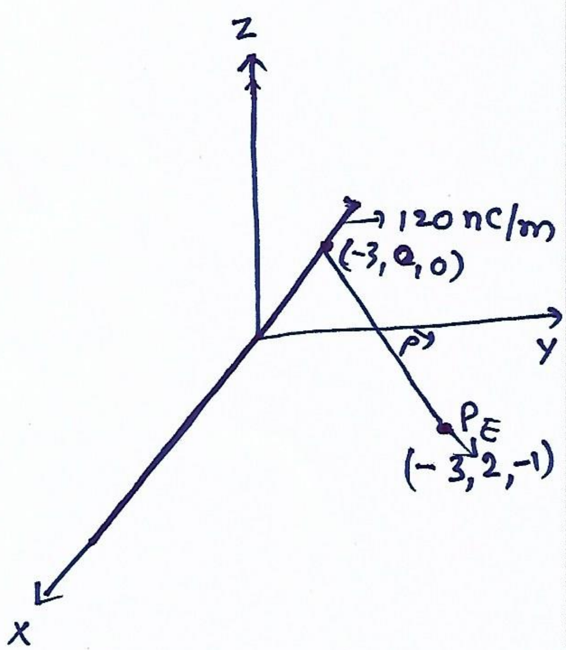
$$18$$

$$\vec{E} = \frac{20\times 10^{-9}\times 18\times 10^{9}}{25}(-4,3,0)$$

$$\vec{E} = 14.4\,(-4,3,0)$$

$$\vec{E} = (-57.6,\ 43.2,\ 0)\quad N/C$$

Problem 2 : uniform line charge of 120 nc/m lie along the entire x-axis. Find Electric field at $P(-3,2,-1)$.



sol $\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\hat{a}_\rho$

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho^2}\vec{\rho}$$

$$\vec{\rho} = (-3,2,-1)-(-3,0,0)$$

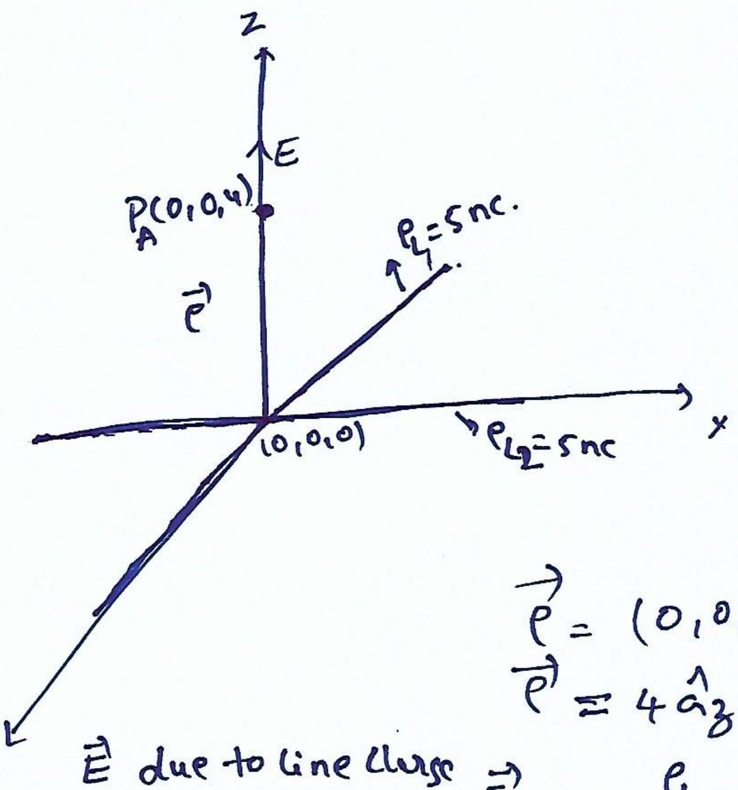
$$\vec{\rho} = (0,2,-1)$$

$$\vec{E} = (0,864,-432)\quad N/C.$$

Problem 3

Infinite uniform line charges of 5 nc lie along the (positive and negative) x and y axes in free space. Find $\vec{E}$ at a) $P_A(0,0,4)$ b) $P_B(0,3,4)$

Sol



$$\vec{\rho} = (0,0,4) - (0,0,0)$$

$$\vec{\rho} = 4\hat{a}_z \Rightarrow |\vec{\rho}| = 4$$

$\vec{E}$ due to line charge along x-axis, $\vec{E} = \dfrac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho^2}\vec{\rho}$$

$$\vec{E} = \frac{5\times\cancel{10^{-9}}\times18\times\cancel{10^{9}}}{16}(0,0,4)$$

$$\vec{E} = \frac{90}{4}\hat{a}_z$$

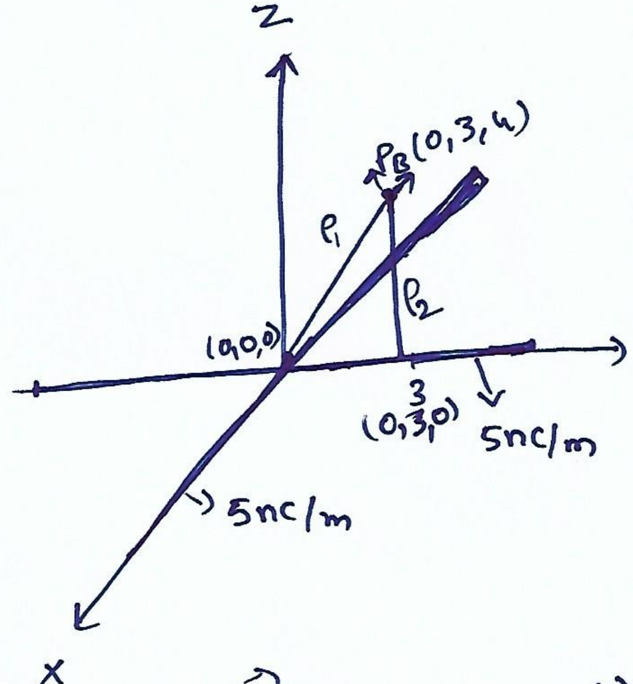
$\vec{E}$ due to line charge along y-axis, $\vec{E} = \frac{90}{4}\hat{a}_z$

net electric field intensity at point 'A' is

$$\vec{E} = \left(\frac{90}{4} + \frac{90}{4}\right) \hat{a}_z$$

$$\vec{E} = 45\hat{a}_z$$

b)



$P_B(0,3,4)$

$\rho_1 = (0,3,4) - (0,0,0)$

$|\vec{\rho}_1| = \sqrt{25}$

$\vec{E}$ due to line charge lie on x-axis,

$\vec{E} = \frac{18}{5}(3\hat{a}_y + 4\hat{a}_z)$ V/m

$$\vec{\rho}_2 = (0,3,4) - (0,3,0)$$

$$\vec{\rho}_2 = (0,0,4) = 4\hat{a}_z.$$

$$|\vec{\rho}_2| = 4$$

$\vec{E}$ due to line charge lie on Y-axis,

$$\vec{E} = \frac{(5\times\cancel{10^{-9}})(18\times\cancel{10^{9}})}{16}(0,0,4)$$

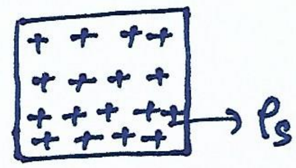
$$\vec{E} = \frac{90\times 4}{16}\hat{a}_z$$

net electric field, $\vec{E} = 10.8\hat{a}_y + 36.9\hat{a}_z$ V/m N/C

Electric Field due to Surface charge

Surface charge:-

charge is distributed uniformly over the surface with density ρ_s (C/m^2).

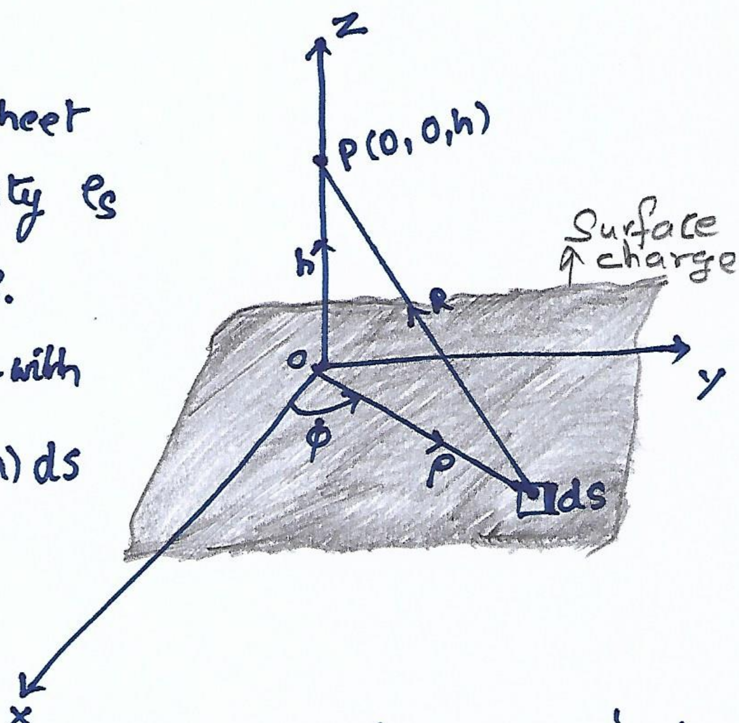


Surface charge

Now Consider a small area 'ds' on the surface, the charge in that small area is 'dQ' then,

$$dQ = \rho_s \cdot ds \text{ —— } ①$$

consider, an infinite sheet of charge with density ρ_s is placed in xy-plane. the charge associated with elemental area (small area) ds is,



$$dQ = \rho_s \, ds$$

The electric field intensity at point 'p' due to 'dQ' is,

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 R^2} \hat{a}_R \text{ —— } ②$$

From the figure, $\vec{R} = \rho(-\hat{a}_\rho) + h\,\hat{a}_z$

$$\vec{R} = -\rho\,\hat{a}_\rho + h\,\hat{a}_z \quad \text{— (3)}$$

$$|\vec{R}| = \sqrt{(-\rho)^2 + h^2} = \sqrt{\rho^2 + h^2}$$

$$R^2 = |\vec{R}|^2 = \rho^2 + h^2 \quad \text{— (4)}$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{-\rho\,\hat{a}_\rho + h\,\hat{a}_z}{\sqrt{\rho^2 + h^2}} \quad \text{— (5)}$$

$$ds = \rho\, d\phi\, d\rho \quad \text{— (6)}$$

Substitute eqn (4) & (5), (6) in (2)

$$d\vec{E} = \frac{\rho_s \cdot \rho\, d\phi\, d\rho\,[-\rho\,\hat{a}_\rho + h\,\hat{a}_z]}{4\pi\varepsilon_0\,[\rho^2 + h^2]^{3/2}} \quad \text{— (7)}$$

The total (net) electric field due to infinite Surface charge is,

$$E = \int_S dE$$

Because of Symmetry, $\vec{E}$ in $\hat{a}_\rho$ direction is zero, the electric field has only z-component.

$$E = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{\infty} \frac{\rho_s\, h\rho\, d\rho\, d\phi}{4\pi\varepsilon_0\,[\rho^2 + h^2]^{3/2}}\, \hat{a}_z.$$

$$= \frac{\rho_s h}{4\pi \varepsilon_0} \; 2\pi \cdot \int_0^{\infty} \left[\rho^2 + h^2\right]^{-3/2} \rho \cdot d\rho \; \hat{a}_z$$

$$d(\rho^2) = 2\rho \, d\rho$$

$$\Rightarrow \rho \, d\rho = \frac{1}{2} \, d(\rho^2) \quad \text{— ⑧}$$

⑧ in $\vec{E}$,

$$\vec{E} = \frac{\rho_s h}{\cancel{4\pi}_{2} \varepsilon_0} \cancel{2\pi} \int_{\rho=0}^{\infty} \left[\rho^2 + h^2\right]^{-3/2} \frac{1}{2} \, d\rho^2 \; \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0} \left\{ \left[\rho^2 + h^2\right]^{-1/2} \right\}_0^{\infty} \hat{a}_z$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_z.}$$

$\vec{E}$ has only one component in z-direction.

if the observation point is at $(0,0,-h)$

i.e $-$ve z-axis, then,

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0} (-\hat{a}_z)$$

that mean, the $\vec{E}$ due to surface charge will be normal to the surface.

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_n$$

[$\because \hat{a}_n$ = unit normal vector]

Likewise, we can also write the $\vec{E}$ expression for the parallel plate capacitor.

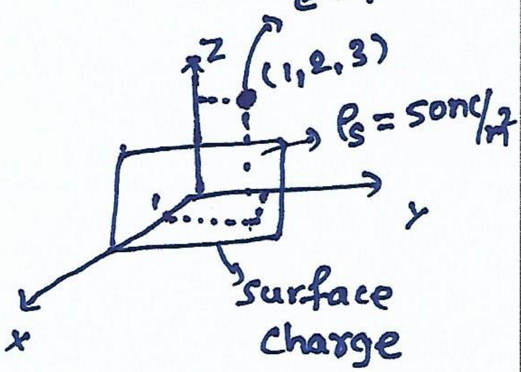
$$\vec{E} = \underbrace{\frac{\rho_s}{2\varepsilon_0}\hat{a}_n}_{(+\text{ve charge})} + \underbrace{\frac{-\rho_s}{2\varepsilon_0}(-\hat{a}_n)}_{(-\text{ve charge})}$$

$$\therefore \boxed{\vec{E} = \frac{\rho_s}{\varepsilon_0}\hat{a}_n}$$

Problem 1 A surface charge of density 50 nC/m² is lie on xy plane. Find the electric field intensity at (1, 2, 3). Also find the force on 1 μC charge located at (1, 2, 3).

$\vec{F} = ?$

$\vec{E} = ?$



Sol

$\rho_s = 50\ nC/m^2$

The electric field intensity due to the given Surface is,

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_z$$

$$= \frac{50\times10^{-9}}{2\times\left(\frac{1}{36\pi\times10^{9}}\right)}\hat{a}_z$$

$$\vec{E} = 2.82\times10^{3}\hat{a}_z \text{ V/m (or) N/C}$$

Since the surface charge is place in xy plane, $\vec{E}$ is present in z-direction ($\hat{a}_z$). i.e normal to the Surface.

Now, the force on $1 \mu C$ located at $(1,2,3)$ is,

$$\vec{F} = q\vec{E} \qquad \left[\because \vec{E} = \frac{\vec{F}}{Q}\right]$$

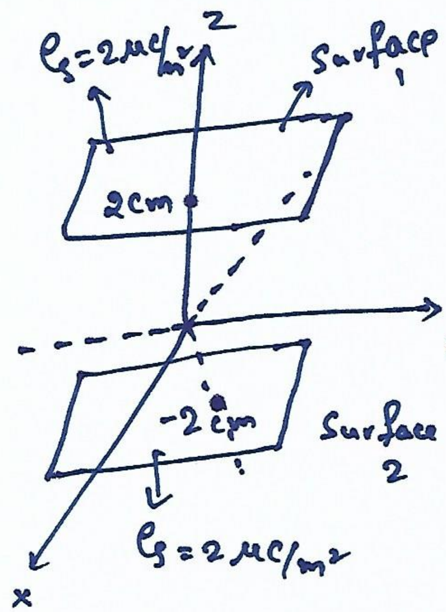
$$\vec{F} = 1 \times 10^{-6} \times 2.82 \times 10^{3} \, \hat{a}_z \text{ N}$$

$$\vec{F} = 2.82 \times 10^{-3} \, \hat{a}_z \text{ N}$$

Both $\vec{E}$ and $\vec{F}$ are in the same direction.

Problem 2 : Surface charges of density $2 \mu C/m^2$ are placed in the free space at $z = \pm 2$ cm. Find the force/m^2 between the two surface charges.

Sol



Given two surface charges,

$$\rho_s = 2 \mu C / m^2$$

two surface charges are placed on z-axis at ($z = \pm 2$ cm).

The force b/w sheets of charges,

$$\vec{F} = q\vec{E}$$

force/m^2 (or) Force/unit area,

$$\frac{\vec{F}}{A} = \frac{q}{A}\vec{E} \qquad \left[\because \frac{q}{A} = \rho_s\right]$$

$$\frac{\vec{F}}{A} = \rho_{s_1} \left(\frac{\rho_{s_2}}{2\varepsilon_0} \right) \hat{a}_z .$$

$\Rightarrow$ Force on surface charge1 due to surface charge2.

$$\vec{F}/A = (2 \times 10^{-6}) \cdot \frac{(2\times 10^{-6})}{2 \times \left(\frac{1}{36\pi \times 10^{9}} \right)} \hat{a}_z$$

$$\vec{F}/A = 0.225 \quad N/m^2$$

Electric field intensity due to volume charge :

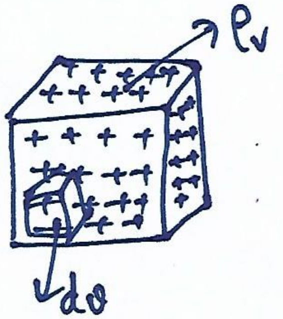


Fig: volume charge

The charge is distributed uniformly in this volume with density ρ_v.

ρ_v - volume charge density (C/m^3)

Now, letus consider a small (elemental) volume dv, the charge associated with dv is dQ.

$$dQ = \rho_v \cdot dv .$$

The total charge in the volume is, $Q = \int_v dQ$

$$Q = \int_V \rho_v \cdot dv.$$

The electric field intensity $(\vec{E})$ at point (r, θ, ϕ) due to the volume charge is,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r \qquad V/m$$

where
Q = Total charge of volume

The equation of electric field intensity due to volume charge is same as the electric field intensity due to point charge 'Q'.

<u>Table</u> : Different types of charge distribution.

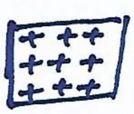
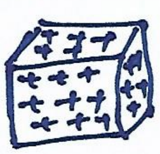
Charge distribution	Charge Density	total charge (Q)
+ + + + + + Line charge	ρ_L (C/m)	$Q = \int_L \rho_L \cdot dL$
 Surface charge	ρ_S (C/m^2)	$Q = \int_S \rho_s \cdot ds = \iint \rho_s\, ds$
 volume Charge	ρ_v (C/m^3)	$Q = \int_V \rho_v \cdot dv = \iiint \rho_v \cdot dv$

Table: Summary of Coulomb's law

charge distribution	parameter	Mathematical equation	Remarks
1. Two point charges (Q_1 and Q_2)	Force ($\vec{F}$)	$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\hat{a}_R$ (N)	Coulomb's law
2. Point charge 'Q_1'	Electric field Intensity ($\vec{E}$) $\vec{E} = \frac{\vec{F}}{Q}$	$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0 R^2}\hat{a}_R$ (V/m) (N/C)	Force per unit charge. $\vec{E} = \vec{F}/Q$
3. Infinite line charge with density 'ρ_L'	Electric field intensity ($\vec{E}$)	$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\hat{a}_\rho$ (V/m)	$\vec{\rho}$ - radial distance b/w line charge and observation point 'p'.
4. Infinite Surface Charge with density 'ρ_S'	Electric field intensity ($\vec{E}$)	$\vec{E} = \frac{\rho_S}{2\varepsilon_0}\hat{a}_n$	$\hat{a}_n$ - unit normal vector. if sheet is in xy plane $\vec{E}$ will be in z-direction ($\hat{a}_z$)
5. Volume charge with density 'ρ_v'	Electric field intensity ($\vec{E}$)	$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$	'Q' is total charge of volume $Q = \iiint \rho_v \cdot dv$.

Electric flux density

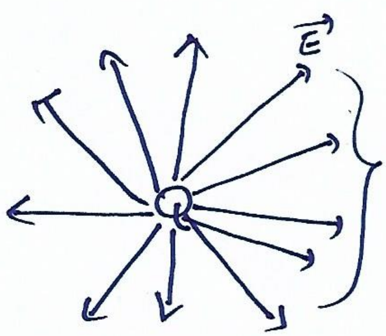
Electric flux (ψ) : Rate of flow of electric field lines

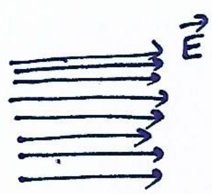
Electric flux density (D) : Rate of flow of electric field lines per unit area.

(or)

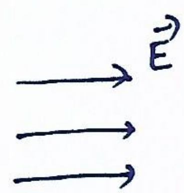
Electric flux per unit area

Let a charge 'Q', there will be electric field lines around it, the flow of electric field lines is called Electric flux.





Large flux value.



Small flux value

The flux (ψ) is proportional to the charge 'Q'

$$\psi = Q \quad \text{Coulombs}$$

if there is 1C chare there will be 1 electric filed line.

2C → 2 lines

nC → n lines

$\therefore \psi = Q$

The flux density = flux per unit spherical area

$$\vec{B} = \frac{\psi}{4\pi R^2} = \frac{Q}{4\pi R^2}$$

$$\vec{D} = \frac{Q}{4\pi R^2}$$

$4\pi R^2$ = Surface area of the sphere

$$\boxed{\vec{D} = \epsilon_0 \vec{E}} \qquad C/m^2$$

we know,

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}$$

Total flux, $\psi = \int_S \vec{D} \cdot ds$

Gauss's law

Gauss law states that the total flux through any closed surface is equal to the total charge enclosed by that surface.

$$\therefore \psi = Q_{enclosed} \quad \text{——— (1)}$$

that is, $\psi = \oint_S d\psi = \oint_S \vec{D} \cdot ds$ ——— (2)

$d\psi$ - Flux due to elemental surface.

the total charge enclosed by the surface is,

$$Q = \int_V \rho_v \, dv \quad \text{——— (3)}$$

(2) and (3) in (1)

$$\oint_S \vec{D}\cdot ds = \int_V \rho_v \, dv \quad \text{--- (4)}$$

Apply divergence theorem to the left term in the above equation,

$$\int_V \nabla\cdot\vec{D}\cdot dv = \int_V \rho_v \, dv$$

compare the two integrations,

$$\boxed{\nabla\cdot\vec{D} = \rho_v} \quad \text{--- (5)}$$

eqn (5) is the maxwell's first equation

" volume charge density is equal to (Same as) the divergence of electric flux density."

Problem : Determine '$\vec{D}$' at (4,0,3) if there is a point charge -5π mC at (4,0,0) and line charge 3π mC/m along y-axis.

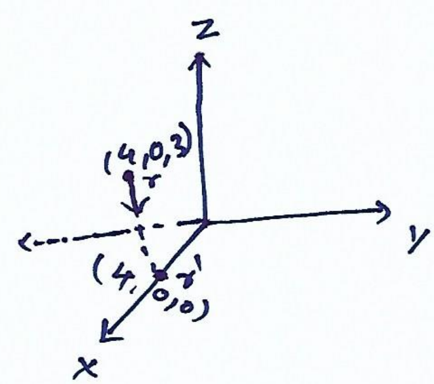
Sol

Electric field intensity at (4,0,3)

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 \vec{R}} \hat{a}_R$$

flux density $(\vec{D}) = \varepsilon_0 \vec{E}$

$$= \frac{Q}{4\pi} \frac{(r-r')}{|r-r'|^3}$$



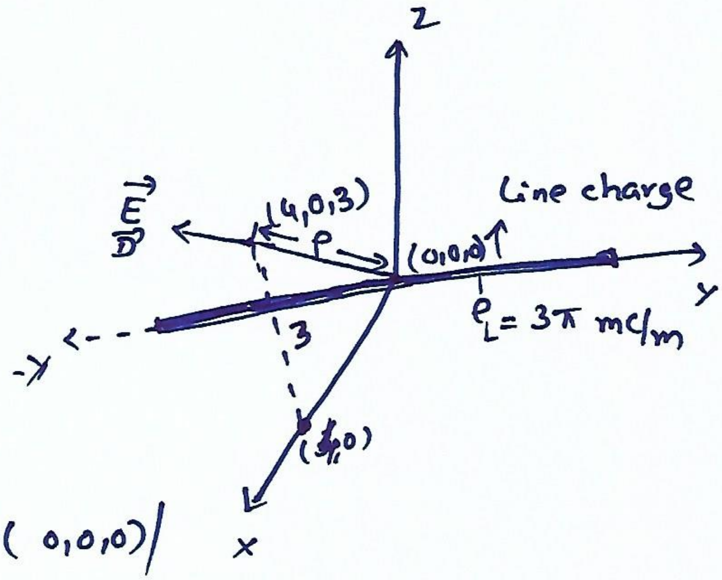
where $r - r' = (4,0,3) - (4,0,0) = (0,0,3)$

$$\vec{D}_Q = \frac{-5\pi \cdot \times 10^{-3} \quad (0,0,3)}{4\pi \left(\sqrt{0^2+0^2+3^2}\right)^3}$$

$$\vec{D}_Q = -0.138 \ \hat{a}_z \qquad mC/m^2$$

Also,

$$\vec{D}_L = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho$$



$$\hat{a}_\rho = \frac{\vec{\rho}}{|\vec{\rho}|}$$

$$\rho = |\vec{\rho}| = |(4,0,3) - (0,0,0)|$$

$$\rho = 5$$

$$\hat{a}_\rho = \frac{(4,0,3)}{5}$$

$$\vec{D}_L = \frac{(3\cancel{\pi} \times 10^{-3})}{2\cancel{\pi} \times 5} \times \frac{(4,0,3)}{5}$$

$$\vec{D}_L = 0.24 \ \hat{a}_x + 0.18 \ \hat{a}_z \qquad mC/m^2$$

Total $\vec{D}$,

$$\vec{D} = \underset{\text{Point charge}}{\vec{D}_Q} + \underset{\text{line charge}}{\vec{D}_L} = 0.24 \ \hat{a}_x + 0.042 \ \hat{a}_z \qquad mC/m^2$$

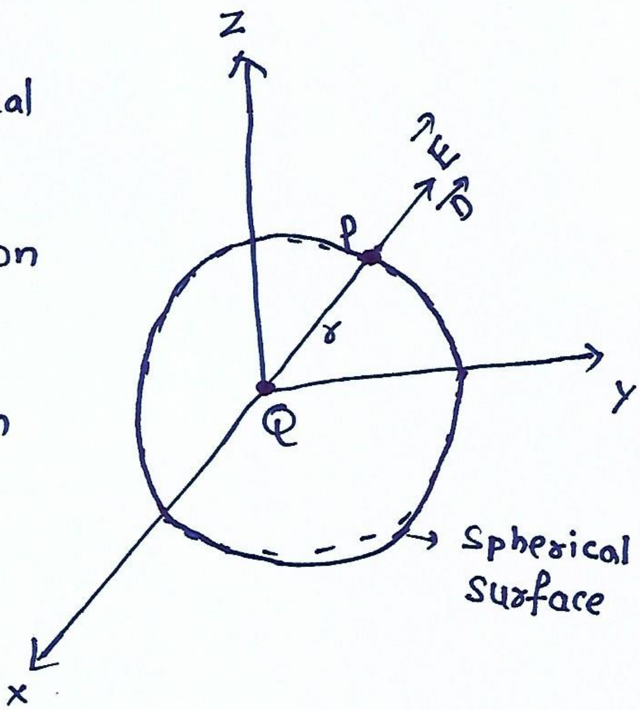
Applications of Gauss's Law

like coulomb's law, Gauss' Law can also be used to calculate the electricfield intensity, $\vec{E}$. But, we can apply Gauss's law only when charge distribution is symmetry.

A. Point charge

Point charge 'Q' is located at origin. To determine $\vec{D}$ at point 'p', choose a spherical surface containing 'p' and centered at origin as shown in the figure.

The surface of the choosen sphere is called Gaussian Surface.



Now, due to charge 'Q', there will be flux, that flux ($\vec{D}$) is everywhere normal to the Gaussian surface, that is $\vec{D} = D_r \hat{a}_r$

$\therefore$ total flux through the surface, $\psi = \oint_S \vec{D} \cdot ds = D_r \oint_S ds$

$$\psi = D_r \cdot 4\pi r^2$$

where, $\oint ds = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi = 4\pi r^2$ surface area of the sphere (Gaussian surface).

According to Gauss's law.

$$\psi = Q_{enclosed}.$$

$$D_r 4\pi r^2 = Q$$

$$D_r = \frac{Q}{4\pi r^2}$$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2} \cdot \hat{a}_r}$$ Electric flux density due to point charge

$$\boxed{\vec{E} = \frac{1}{\varepsilon_0} \frac{Q}{4\pi r^2} \hat{a}_r. = \frac{Q}{4\pi \varepsilon_0 r^2} \hat{a}_r}$$ Electric field intensity.

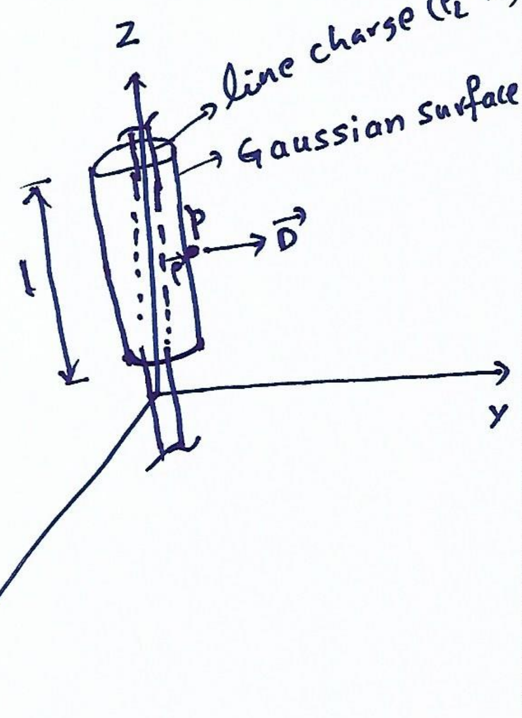
B. Infinite line charge

We have infinite length uniform line charge, of density ρ_L (C/m) along z-axis.

To determine $\vec{D}$ at point 'p' choose cylindrical surface containing 'p' to satisfy symmetry condition.

$\vec{D}$ is constant on cylindrical surface and ~~an~~ normal to the surface.

$$\vec{D} = D_\rho \, \vec{a}_\rho$$



Total charge of line is, $Q = \rho_L l$

Total flux (ψ) through the gaussian surface is,

$$\psi = \oint_S \vec{D} \cdot ds = D_\rho \oint_S ds = D_\rho \, 2\pi\rho l$$

where, $\oint_S ds = 2\pi\rho l$ is surface area of gaussian surface (cylindrical surface).

According Gauss's law, $\psi = Q$

$$D_\rho \, 2\pi\rho l = \rho_L l$$

$$D_\rho = \frac{\rho_L \cancel{l}}{2\pi\rho \cancel{l}} = \frac{\rho_L}{2\pi\rho}$$

$$\boxed{\vec{D} = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho}$$ Electric flux density due to line charge.

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho} \hat{a}_\rho}$$ Electric field intensity due to line charge.

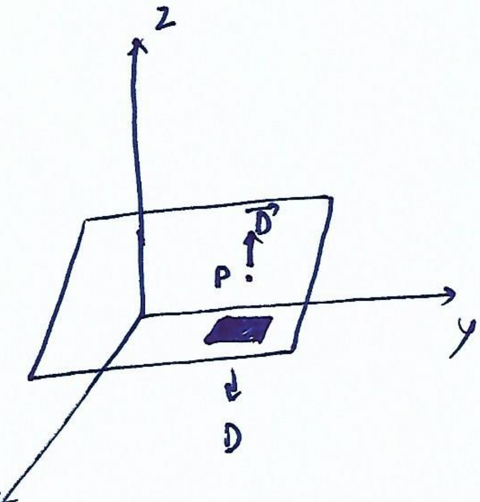
C. Infinite sheet of charge

sheet charge in xy plane,

To determine $\vec{D}$ at point 'P' choose rectangular box, that is cut symmetrically by the sheet.

$\vec{D}$ is normal to the sheet,

$$\vec{D} = D_z \hat{a}_z.$$



Now, the total charge associated with sheet is,

$$Q = \int_S \rho_s \, ds = \rho_s \int_S ds = \rho_s A$$

$\rho_s A \rightarrow$ A is area of sheet ($\int_S ds$)

Total flux due to the sheet, is

$$\psi = \oint_S \vec{D} \cdot ds = D_z \int_{top} ds + D_z \int_{bottom} ds$$

$$\psi = D_z [A + A] = 2 A D_z$$

According to Gauss's law,

$$\psi = Q$$

$$D_z \, 2A = \rho_s A$$

$$D_z = \frac{\rho_s \cancel{A}}{2 \cancel{A}} = \frac{\rho_s}{2}$$

$$\boxed{\vec{D} = \frac{\rho_s}{2} \hat{a}_z}$$

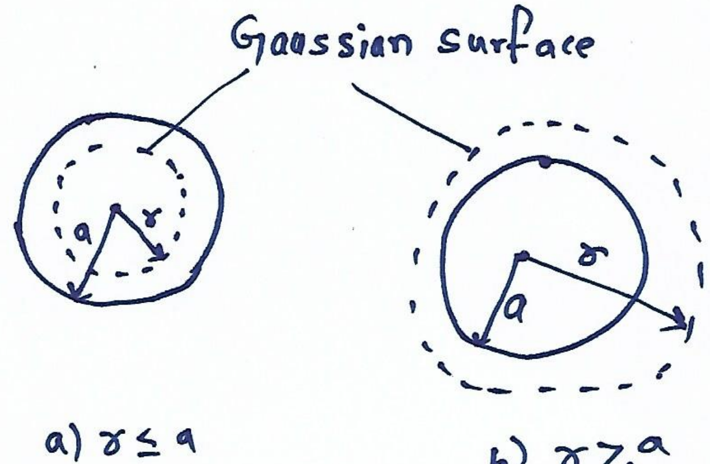
Electric flux density due to surface charge (or) sheet of charge.

$$\boxed{\vec{E} = \frac{\rho_s}{2 \epsilon_0} \hat{a}_z}$$

Electric field intensity due to sheet of charge.

D. Uniformly charged sphere

consider a sphere of radius 'a' with uniform charge ρ_0 C/m³. To determine $\vec{D}$ everywhere we construct surfaces for $r \le a$ and $r \geq a$ separately.



For $r \le a$, the total charge enclosed by spherical surface of radius 'r' is,

$$Q_{enclosed} = \int_v \rho_v \, dv = \rho_0 \int_v dv = \rho_0 \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{\infty} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \rho_0 \frac{4}{3}\pi r^3$$

and $\psi = \oint_s \vec{D}\cdot ds = D_r \oint_s ds = D_r \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi$

$= D_r \, 4\pi r^2$

apply Gauss law, $\psi = Q_{enclosed}$, gives,

$$\boxed{\vec{D} = \frac{r}{3}\rho_0 \, \hat{a}_r} \qquad 0 \le r \le a$$

For $r \geq a$,

the charge enclosed by the surface is,

$$Q_{enclosed} = \int_V \rho_v \, dv = \rho_0 \int_V dv = \rho_0 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \rho_0 \frac{4}{3} \pi a^3$$

and,

$$\psi = \oint_S \vec{D} \cdot ds = D_r \cdot 4\pi r^2$$

apply Gauss's law,

$$\psi = Q_{enclosed}$$

$$D_r \, 4\pi r^2 = \rho_0 \frac{4}{3} \pi a^3$$

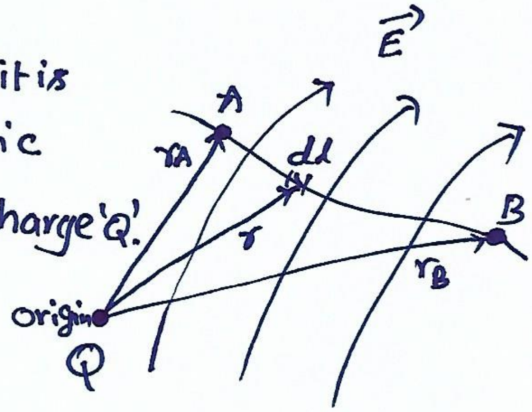
$$\boxed{\vec{D} = \frac{a^3}{3r^2} \rho_0 \hat{a}_r} \qquad r \geq a$$

$\therefore$ $\vec{D}$ everywhere is given by,

$$\vec{D} = \begin{cases} \frac{r}{3} \rho_0 \hat{a}_r & 0 < r \leq a \\ \frac{a^3}{3r^2} \rho_0 \hat{a}_r & r \geq a \end{cases}$$

Potential or potential difference

let a point charge 'Q', and it is placed at origin, $\vec{E}$ is electric field intensity due to point charge 'Q'.



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r.$$

Now, to move any charge from point 'A' to point 'B' in in the electric field $\vec{E}$, work is to be done.

The work done in displacing charge by 'dl' length is,

$$dW = -\vec{F}\cdot dl$$

$\vec{F}$ is force on charge in electric field $\vec{E}$

$$\Rightarrow dW = -Q_t\cdot\vec{E}\cdot dl$$

Now, total work done to move charge from point 'A' to point 'B', is

$$W = -Q_t\int_A^B \vec{E}\,dl \Rightarrow \frac{W}{Q_t} = -\int_A^B \vec{E}\,dl$$

This quantity is defined as potential difference between two points A & B (V_{AB})

$$\boxed{V_{AB} = \frac{W}{Q_t} = -\int_A^B \vec{E}\cdot dl}$$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r \cdot dr\,\hat{a}_r.$$

r_A, r_B are the distance vectors from origin

$$V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_{AB} = V_B - V_A.$$

V_A and V_B are the potential at point 'A' and point 'B'.

$\therefore$ Potential at any point 'r' due to point charge 'Q' is,

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}} \quad \text{~~Em~~ volts}$$

If there are 'N' Number of point Sources, $Q_1, Q_2, Q_3, \ldots Q_N$, located $r_1, r_2, r_3 \ldots r_N$ respectively, then the potential at point 'r' due to 'N' point charges, is,

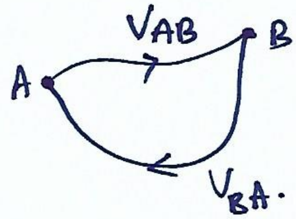
$$V(r) = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|} + \frac{Q_2}{4\pi\varepsilon_0 |r - r_2|} + \cdots + \frac{Q_N}{4\pi\varepsilon_0 |r - r_N|}$$

$$V(r) = \frac{1}{4\pi \varepsilon_0} \sum_{i=1}^{N} \frac{Q_i}{|r - r_i|}$$

Relationship between $\vec{E}$ and V – Maxwell's equation

The potential difference between two point 'A' and 'B' is, V_{AB} and $V_{AB} = -V_{BA}$.

that is $V_{AB} + V_{BA} = 0$



$$\oint_L \vec{E} \cdot dl = 0$$

Now apply stokes theorem,

$$\oint_L \vec{E} \cdot dl = \int_S (\nabla \times \vec{E})\, ds = 0$$

(or)

$$\boxed{\nabla \times \vec{E} = 0}$$ Maxwell's equation for static electric fields.

where, $\vec{E}$ is called conservative vector (or) irrotational vector.

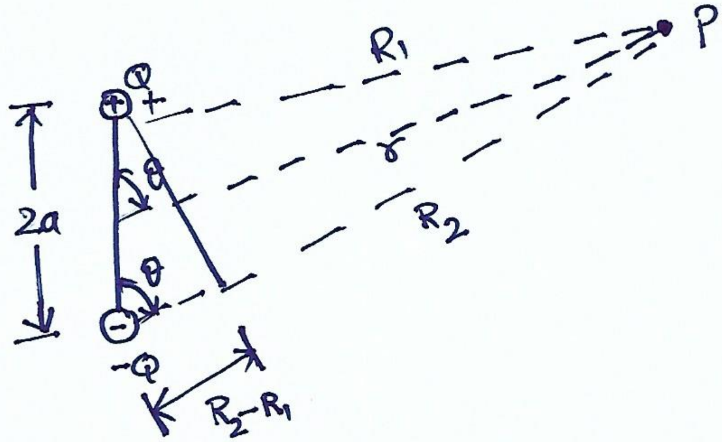
Thus electrostatic field is conservative field.

let $V = -\int \vec{E} \cdot dl$.

$dv = -\vec{E}$ ⇒ in vector form

$$\nabla V = -(E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z) \Rightarrow \boxed{\vec{E} = -\nabla V}$$

Potential - Electric Dipole



Dipole $\quad R_2 - R_1 = 2a\cdot\cos\theta.$

Potential at point 'P', $V = \dfrac{Q}{4\pi\varepsilon_0 R_1} - \dfrac{Q}{4\pi\varepsilon_0 R_2}$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$V = kQ\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

For $r >> a$ (i.e P is located at far away from the dipole), $R_1 \simeq R_2 = r$

Potential, $V = kQ\left[\dfrac{2a\cos\theta}{r^2}\right]$

$$\boxed{V = \frac{k}{r^2} p\cos\theta}$$

$P = Q\,(2a) =$ dipole moment

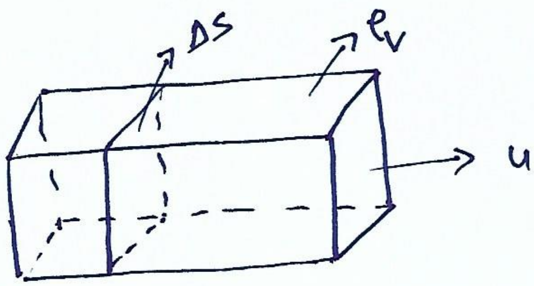
convection and Conduction current

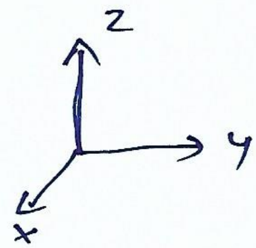
Convection current

Convection current does not involve conductors and doesnot satisfy ohm's law.

It occurs when current flows through insulating medium like vacuum.

Example ; Beam of electrons in a vacuum tube.





Current density, $J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y$ (A/m^2)

$\boxed{J = \rho_v u}$ convection current density.

u - velocity of charge.

Conduction current

conduction current requires conductor, which is characterized by large number of free electrons.

$\boxed{J = \sigma E}$ conduction current density.

E = applied electric field
σ = conductivity of conductor.

Dielectric constant, isotropic and homogeneous dielectrics

The materials or medium can be classified broadly into three types based on the parameter, conductivity (σ).

A material with good conductivity (or) high conductivity ($\sigma >> 1$) is refered to as metals.

A material with small conductivity is refered to as insulator (or) dielectric ($\sigma << 1$).

A material with conductivity lies between those of metals and insulators are called semiconductors.

Examples

Metals: copper and aluminium

Semiconductors: Silicon and Germanium

Insulators: Glass and rubber

another parameter which is used to characterize the materials is dielectric constant (ε_r).

Dielectric constant

The dielectric constant is a measure of the amount of electric potential energy, in the form of induced polarization that is stored in a given volume of material under the action of an electric field.

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

ε is permittivity of the dielectric

ε_0 is the permittivity of free space, $\varepsilon_0 = \frac{1}{36\pi \times 10^9}$ F/m

ε_r is the relative permittivity (or) dielectric constant

∴ The dielectricconstant (ε_r) is the ratio of the permittivity of diectric (ε) to that of free space (ε_0)

Materials classification based on 'ε'

A material is linear, if $\vec{D}$ varies linearly with $\vec{E}$

A material for which 'ε' doesnot vary in the region considered said to be homogeneous.

Material for which $\vec{D}$ and $\vec{E}$ are same direction said to be isotropic.

Dielectric materials classification

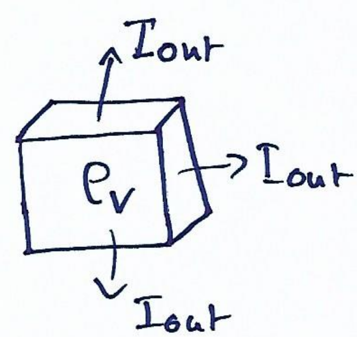
A dielectric material is linear, if $\vec{D} = \varepsilon \vec{E}$ and 'ε' does not change with the applied $\vec{E}$ field.

A dielectric material is homogeneous, if $\vec{D} = \varepsilon \vec{E}$ and 'ε' does not change from point to point.

A dielectric material is isotropic, if $\vec{D} = \varepsilon \vec{E}$ and 'ε' does not change with direction.

Continuity Equation and Relaxation time.

Consider a volume of charge, with density ρ_v, then net outward flow of current through the surface of volume charge is,



$$I_{out} = -\frac{dQ_{in}}{dt}$$

'Q_{in}' total charge enclosed by the surface.

$$\frac{-dQ_{in}}{dt} = \frac{-d}{dt}\int_V \rho_v \, dv = -\int_V \frac{\partial}{\partial t}\rho_v \, dv \quad \text{—— (1)}$$

then,

$$I_{out} = \oint_S \vec{J} \cdot ds$$

apply divergence theorem,

$$I_{out} = \oint_S \vec{J} \cdot ds = \int_V \nabla \cdot \vec{J} \, dv \quad \text{—— (2)}$$

(1) = (2)

$$-\int_V \frac{\partial}{\partial t} \rho_v \, dv = \int_V \nabla \cdot \vec{J} \, dv$$

$$\boxed{\therefore \quad \nabla \vec{J} = -\frac{\partial}{\partial t} \rho_v}$$

This equation is called continuity equation.

Relaxation time (T_r)

The time takes for a charge placed in the interior of amaterial to drop to $\frac{1}{e}$ or 36.8 % of initial value(T_r).

'T_r' is short for good conductors

'T_r' is long for dielectrics.

Poisson's and Laplace's Equations

we know Gauss law,

$$\nabla \cdot \vec{D} = \rho_v$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\therefore \nabla (\varepsilon \vec{E}) = \rho_v$$

we know, $\vec{E} = -\nabla V$

$$\nabla (\varepsilon (-\nabla V)) = \rho_v$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

This is known as poisson's equation.

The special case of this equation occurs when $\rho_v = 0$,

The above equation becomes, $\boxed{\nabla^2 V = 0}$

which is known as laplace's equation.

In cartesian co-ordinate system,

$$\boxed{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$

Summary of Electrostatics

1. Coulomb's law,

$$\vec{F} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 |r_2 - r_1|^3} (r_2 - r_1)$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2} \hat{a}_R$$

2. Electric field intensity, $\vec{E} = \frac{\vec{F}}{Q}$

$$\vec{E} = \frac{Q\,(r - r')}{4\pi \varepsilon_0 |r - r'|^3} \quad V/m$$

3. Electric field intensity due to line charge,

$$\vec{E} = \frac{\rho_L}{2\pi \varepsilon_0 \rho} \hat{a}_\rho \quad V/m$$

4. Electric field intensity due to Surface charge,

$$\vec{E} = \frac{\rho_s}{2 \varepsilon_0} \hat{a}_n \quad V/m$$

5. Electric field intensity due to volume charge,

$$\vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \hat{a}_r \quad V/m.$$

6. Electric flux density, $\vec{D} = \varepsilon_0 \vec{E}$

7. Electric flux density due to any charge can be derived. using $\vec{E}$.

8. Gauss law, $\psi = Q_{enclosed}$.

9. Maxwell's 1st equation, $\nabla \cdot \vec{D} = \rho_v$

10. Electric flux density due to point charge,

$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

11. Electric flux density due to line charge,

$$\vec{D} = \frac{\rho_L}{2\pi \rho} \hat{a}_\rho$$

12. Electric flux density due to infinite sheet of charge, $\vec{D} = \frac{\rho_s}{2} \hat{a}_z$.

13. Electric flux density due to uniformly charged volume,

$$\vec{D} = \begin{cases} \frac{r}{3} \rho_0 \hat{a}_r & 0 < r \leq a \\ \frac{a^3}{3r^2} \rho_0 \hat{a}_r & r \geq a \end{cases}$$

14. Electric potential at any point 'P' is,

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

15. Potential difference, between two points, A & B

$$V_{AB} = V_B - V_A = -\int_A^B \vec{E}\cdot dl$$

16. Maxwell's 2nd equation, $\nabla \times \vec{E} = 0$

17. Relation b/w $\vec{E}$ and V, $\vec{E} = -\nabla V$

18. Electric potential due to dipole is,

$$V = \frac{P}{4\pi\varepsilon_0 r^2} \cos\theta$$

19. Convection current density,

$$\vec{J} = \rho_v u$$

20 Conduction current density, $\vec{J} = \sigma \vec{E}$.

21. Dielectric constant, $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

22. Relaxation time, $T_r = \frac{\varepsilon}{\sigma}$

23. continuity equation (or) current continuity equation, $\nabla \cdot \vec{J} = -\frac{\partial}{\partial t} \rho_v$

24. Poisson's equation.

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

25. Laplace's equation, $\nabla^2 V = 0$

UNIT - II

MAGNETOSTATICS

Electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magentic (magnetostatic) field is produced.

There are two major law's governing magnetostatic fields: 1) Biot-Savart's law 2) Ampere's circuit law.

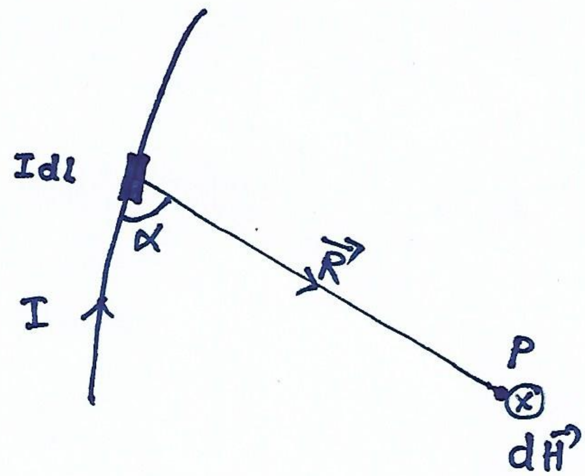
Biot-Savart's law is general law of magnetostatics like coulomb's law for electrostatics.

Just as Gauss law is speacial case of coulomb's law, Ampere's law is speacial case of Biot-savart law and is easily applied in problems involving symmetrical current distribution.

BIOT- SAVART'S LAW

Biot-Savart's law states that the differential magnetic field intensity $d\vec{H}$ produced at a point 'p' by the differential current element Idl is proportional to the product Idl and sine of angle 'α' between the element and line joining 'p' to element

and is inversely proportional to the square of the distance 'R' between 'p' and element.



⊗ $d\vec{H}$ is into the page.

fig. Magnetic field $d\vec{H}$ at point 'p' due to current element Idl.

That is, $d\vec{H} \propto Idl \sin\alpha$

$$d\vec{H} \propto \frac{1}{R^2}$$

(or)

$$d\vec{H} \propto \frac{Idl \sin\alpha}{R^2}$$

$$d\vec{H} = k \frac{Idl \sin\alpha}{R^2}$$

'k' is proportionality constant, $k = \frac{1}{4\pi}$

$$\therefore d\vec{H} = \frac{Idl \sin\alpha}{4\pi R^2}$$

$d\vec{H}$ is a vector, hence we can write as,

$$d\vec{H} = \frac{I dL \sin\alpha}{4\pi R^2} \hat{a}_R$$

By the definition of cross product, the above equation can be modified as,

$$d\vec{H} = \frac{I dL \times \hat{a}_R}{4\pi R^2} = \frac{I dL \times \vec{R}}{4\pi R^3}$$

$$R = |\vec{R}| \quad \text{and} \quad \hat{a}_R = \frac{\vec{R}}{R}$$

The magnetic field due to total wire of current I

is, $\vec{H} = \int d\vec{H}$

The direction of $d\vec{H}$ or $\vec{H}$ at point 'p' can be determined by right hand thumb rule, with right hand thumb pointing in the direction of current and right hand figers encircling the wire in the direction of $d\vec{H}$.

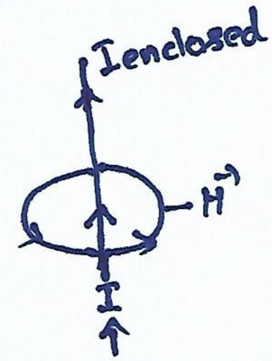
$\odot \rightarrow$ $\vec{H}$ is out of the page
$\otimes \rightarrow$ $\vec{H}$ is into the page.

AMPERE'S CIRCUIT LAW – MAXWELL'S EQUATION

Ampere's circuit law states that the line integral of $\vec{H}$ around a closed path is same as the net current I_{enc} enclosed by the path.

that is,

$$\oint_L \vec{H}\, dl = I_{enclosed}$$



This Ampere's circuit law is similar to the Gauss's law,
In Gauss's law we determine the total charge enclosed But here in Ampere's law we determine total current enclosed.

Ampere's law is easily applied to determine $\vec{H}$ when the current distribution is symmetrical.

Ampere's law is special case of Biot-Savart's Law.

Now take Ampere's circuit law equation,

$$\oint_L \vec{H} \cdot dl = I_{enclosed}$$

Apply stokes theorem to the left side term of the above equation,

$$\oint_L \vec{H} \cdot dl = \int_S (\nabla \times \vec{H})\, ds$$

But, the total current enclosed by the closed path can be expressed like this,

$$I_{enclosed} = \int_S \vec{J} \cdot ds$$

$\vec{J}$ is current density in the closed path

Now, the ampere's circuit law can be written as,

$$\int_S (\nabla \times \vec{H})\, ds = \int_S \vec{J} \cdot ds$$

Compare both the integrations, it clearly reveals that,

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

This is Maxwell's equation for Magnetostatic field.

$\therefore \nabla \times \vec{H} \neq 0$
$= \vec{J}$

That means the magnetostatic field is not conservative. (or) the magnetostatic field is rotational where as the electrostatic field is conservative.

$\therefore$ i.e $\nabla \times \vec{E} = 0$

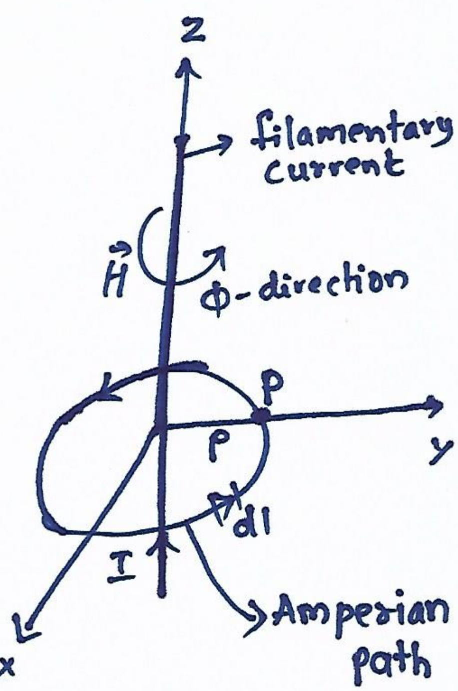
APPLICATIONS OF AMPERE'S LAW

Ampere's circuit law is used to determine $\vec{H}$ for some symmetrical current distributions.

A. Infinite line of current
B. Infinite sheet of current
C. Infinitely long co-axial transmission line.

Infinite line current

Consider an infinitely long filamentary current I along the z-axis.
To determine $\vec{H}$ at point 'P', draw a closed path to pass through 'P'.
This closed path is called Amperian path.



The magnetic field intensity, $\vec{H}$ is constant along the closed path provided 'ρ' is constant.

According to ampere's law,

$$I = \oint_L \vec{H} \cdot dl$$

$$= \int H_\phi \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi$$

$$= H_\phi \rho \int_0^{2\pi} d\phi$$

$$I = H_\phi \rho \, 2\pi$$

(or) $$\boxed{\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

cylindrical co-ordinate system.

$$\vec{H} = H_\rho \hat{a}_\rho + H_\phi \hat{a}_\phi + H_z \hat{a}_z$$

$$dl = \partial\rho \, \hat{a}_\rho + \rho \, d\phi \, \hat{a}_\phi + dz \, \hat{a}_z$$

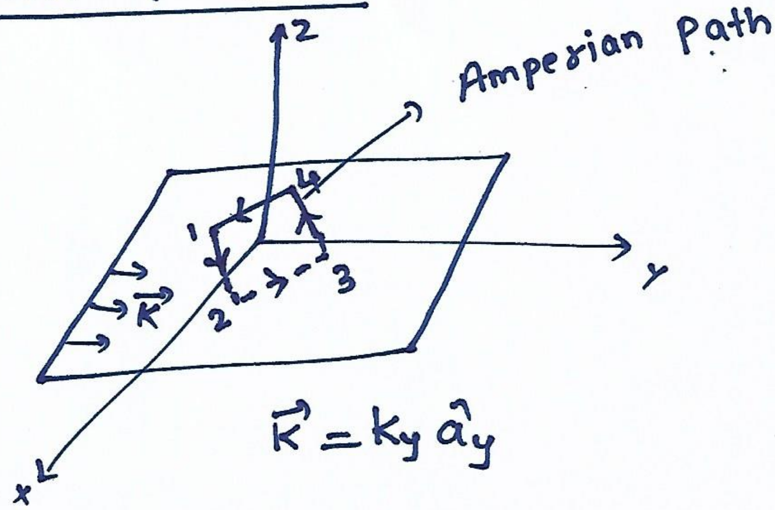
$z = 0$

$\rho =$ constant

$\partial\rho = 0$ & $\partial_z = 0$

$$dl = \rho \, d\phi \, \hat{a}_\phi$$

Infinite sheet of current



$$\vec{K} = K_y \hat{a}_y$$

Consider an infinite sheet of current in $z=0$ plane(xy plane). The sheet has uniform current density $\vec{K}$, $\vec{K} = K_y \hat{a}_y$ A/m.

1-2-3-4-1 is the the closed path around the sheet of current. This is called amperian path.

According to ampere's law,

$$I = \oint_L \vec{H} \cdot dl = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) \vec{H} \cdot dl$$

$$K_y b = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) \vec{H} \cdot dl$$

where, $\vec{H} = \begin{cases} H_o \hat{a}_x & z > 0 \\ -H_o \hat{a}_x & z < 0 \end{cases}$

where H_o yet to be determined, evaluating the integral of $\vec{H}$ along the closed path.

$$\oint \vec{H} dl = \int_1^2 \vec{H} \cdot dl + \int_2^3 \vec{H} dl + \int_3^4 \vec{H} dl + \int_4^1 \vec{H} dl.$$

$$= 0(-a) + (-H_o)(-b) + 0(a) + H_o(b)$$

$$\oint \vec{H} dl = 2 H_o b$$

$K_y b = 2 H_0 b$

$$H_0 = \frac{K_y}{2}$$

$$\therefore \quad H = \begin{cases} \frac{1}{2} K_y \hat{a}_x & z > 0 \\ -\frac{1}{2} K_y \hat{a}_x & z < 0 \end{cases}$$

In general, for infinite sheet of current density $\vec{K}$ A/m,

$$\boxed{\vec{H} = \frac{1}{2} \vec{K} \times \hat{a}_n}$$

where $\hat{a}_n$ is unit normal vector directed from the current sheet to point of interest.

Magnetic FLUX DENSITY

Magnetic flux density, $\vec{B}$ is defined as,

$$\vec{B} = \mu \vec{H}$$

$\mu = \mu_0 \mu_r$

μ_r = relative permeability ($\mu_r = 1$ for Space).

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$\therefore \vec{B}$ for free space, $\vec{B} = \mu_0 \vec{H}$

The magnetic flux through any closed surface 'S' is,

$$\psi = \oint_S \vec{B} \cdot ds$$

The magnetic field lines (or) flux line are closed one for small a small magnet, also we can't separate N and S pole.

The total flux through the closed surface in a magnetic field must be zero.

$$\boxed{\oint_S \vec{B} \cdot ds = 0}$$

This equation is called law of conservation of magnetic flux or Gaussian law for magnetostatic fields.

Now, apply the divergence theorem to the above equation,

$$\oint_S \vec{B} \cdot ds = \int_V \nabla \cdot \vec{B} \cdot dv = 0$$

(or)

$$\boxed{\nabla \cdot \vec{B} = 0}$$

This is maxwell's 4^{th} equation.

This shows magnetic fields have no source or sink. The magnetic field lines are always continuous.

Maxwell's equations

Differential form	Integral form	Remarks
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D}\, ds = \int_V \rho_v\, dv$	Gauss law.
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot ds = 0$	Non existance of mag. pole
$\nabla \times \vec{E} = 0$	$\oint_L \vec{E} \cdot dl = 0$	conservative nature of electric field
$\nabla \times \vec{H} = \vec{J}$	$\oint_L \vec{H} \cdot dl = \int_S \vec{J} \cdot ds$	Ampere's law

MAGNETIC POTENTIAL

we know,

$$\nabla \cdot \vec{B} = 0$$

The vector identity, $\nabla \cdot \nabla \times (\text{vector}) = 0$

To satisfy this vector identity, $\nabla \cdot \vec{B} = 0$ can be written as,

$$\nabla \cdot \nabla \times (\text{Vector}) = 0$$

$$\vec{B} = \nabla \times \text{Vector}$$

Vector is defined as, magnetic vector potential. and is denoted with 'A'.

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

for line current, $\vec{A} = \int_L \frac{\mu_0 I \, dL}{4\pi R}$

Derivation of magnetic flux from magnectic ~~scala~~ vector potential,

$$\boxed{\psi = \oint_S \vec{B} \cdot ds = \int_S (\nabla \times \vec{A}) \cdot ds = \int_L \vec{A} \cdot dL.}$$

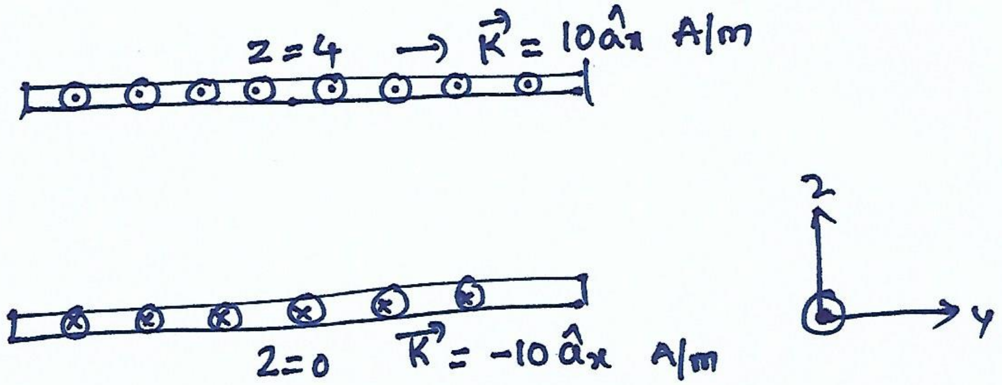
Problems

Planes, $z=0$ and $z=4$ carry current $\vec{K} = -10\hat{a}_x$ A/m and $\vec{K} = 10\hat{a}_x$ A/m, respectively. Determine $\vec{H}$ at

a) $(1, 1, 1)$ b) $(0, -3, 10)$

Sol

The parallel current sheets are placed at $z=0$ and $z=4$



The magnetic field intensity due to current sheets at $z=0$ and $z=4$ are $\vec{H_0}$ and $\vec{H_4}$ respectively.

a) At $(1, 1, 1)$ which is between the plates $(0 < z = 1 < 4)$

$$\vec{H_0} = \frac{\vec{K}}{2} \times \hat{a}_n = \frac{1}{2}(-10\hat{a}_x) \times \hat{a}_z = 5\hat{a}_y \text{ A/m}$$

$$\vec{H_4} = \frac{\vec{K}}{2} \times \hat{a}_n = \frac{1}{2}(10\hat{a}_x) \times (-\hat{a}_z) = 5\hat{a}_y \text{ A/m.}$$

Hence, $\vec{H} = 10\hat{a}_y$ A/m

b) At $(0,-3,10)$ which is above two sheets or plates $(z=10>4>0)$,

$$\vec{H_0} = \frac{1}{2}(-10\hat{a}_x)\times\hat{a}_z = 5\,\hat{a}_y \quad A/m$$

$$\vec{H_4} = \frac{1}{2}(10\,\hat{a}_x)\times\hat{a}_z = -5\,\hat{a}_y \quad A/m$$

Hence, $\vec{H} = \vec{H_0} + \vec{H_4} = 0.$

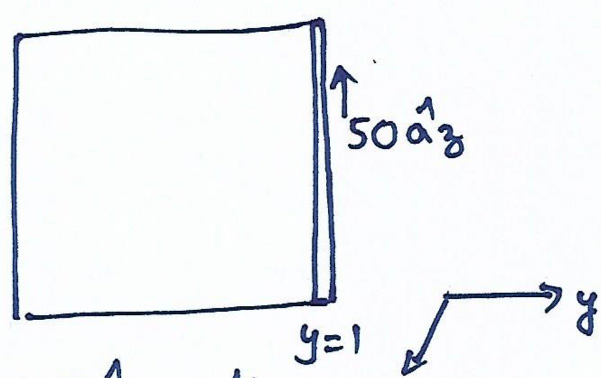
Problem 2

plane $y=1$ carries current $\vec{K} = 50\,\hat{a}_z$ mA/m. Find $\vec{H}$ at

a) $(0,0,0)$

b) $(1,5,-3)$

Sol



a) $\vec{H}$ at $(0,0,0)$

$$\vec{H} = \frac{1}{2}\vec{K}\times\hat{a}_n$$

$$\vec{H} = -\frac{1}{2}(50\hat{a}_z)\times\hat{a}_y = 25\,\hat{a}_x \text{ mA/m.}$$ $\longrightarrow$ below sheet

b) $\vec{H}$ at $(1,5,-3)$ $\Rightarrow$ $\vec{H} = \frac{1}{2}\,50\,\hat{a}_z\times\hat{a}_y$ } above sheet

$$= -25\,\hat{a}_x \text{ mA/m}$$

MAGNETIC FORCE

The forces due to magnetic fields:

a) Force on a moving charged particle

b) Force on a current element

c) Force between two current elements.

The electric force $\vec{F_e}$ on a stationary or moving electric change Q in an electric field is:

$$\vec{F_e} = Q\vec{E}$$

This shows that if Q is positive, $\vec{F_e}$ and $\vec{E}$ have the same direction.

$\vec{F_e}$ is independent of the velocity charge.

The magnetic field can exert force only on a moving charge. if a charge 'Q' is moving with a velocity 'u' in a magnetic field $\vec{B}$, the magnetic force $\vec{F_m}$ experienced by the charge is:

$$\vec{F_m} = Qu \times \vec{B}$$

$\vec{F_m}$ is perpendicular to both 'u' and $\vec{B}$

$\vec{F_m}$ is depends on the charge velocity

For a moving charge 'Q' in the presence of both electric and magnetic fields, the total force on the charge is given by,

$$\vec{F} = Q\vec{E} + Q u \times \vec{B}$$
$$\vec{F} = Q(\vec{E} + u \times \vec{B})$$

The force on a current element Idl in a magnetic field $\vec{B}$ is, $\quad d\vec{F} = Idl \times \vec{B}$

If the current 'I' is through a closed path 'L' or circuit, the force on the circuit is given by:

$$\vec{F} = \oint_L IdL \times \vec{B}$$

The Force on Surface current element $\vec{K}ds$:

$$\vec{F} = \oint_S \vec{K}ds \times \vec{B}$$

The Force on volume current element $\vec{J}dv$:

$$\vec{F} = \oint_V \vec{J}\,dv \times \vec{B}$$

A charged Particle moves with a uniform velocity $4\hat{a}_x$ m/s in a region where $\vec{E} = 20\,\hat{a}_y$ V/m and $\vec{B} = B_0\hat{a}_z$ wb/m². Determine B_0 Such that the velocity of particle remains constant.

Sol

Given, velocity of the charged particle, $u = 4\hat{a}_x$ m/s

The electric field $\vec{E} = 20\,\hat{a}_y$ V/m

The magnetic field, $\vec{B} = B_0\,\hat{a}_z$ wb/m²

We know, Force on Charged particle in electric and magnetic fields is,

$$\vec{F} = Q(\vec{E} + u \times \vec{B})$$

According to Newtons law $\vec{F} = ma = m\dfrac{du}{dt}$

$$\therefore \vec{F} = m\frac{du}{dt} = Q(\vec{E} + u \times \vec{B})$$

If the Velocity of charged particle is constant, acceleration is zero.

$$0 = Q(\vec{E} + \vec{u} \times \vec{B})$$

$$-\vec{E} = \bar{u} \times \vec{B}$$

$$-20\hat{a}_y = 4\,\hat{a}_x \times B_0\,\hat{a}_z = 4B_0(\hat{a}_x \times \hat{a}_z)$$

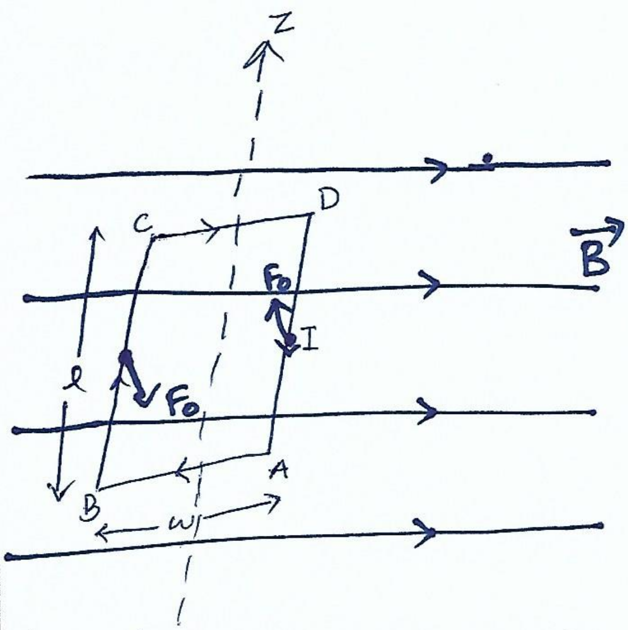
$$-20\hat{a}_y = -4B_0\,\hat{a}_y$$

Thus $B_0 = 5$

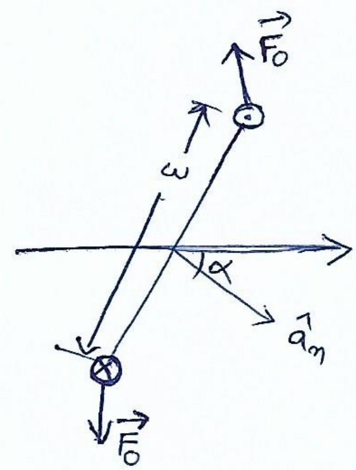
MAGNETIC TORQUE AND MOMENT

The torque (or mechanical moment of force) on the loop is the vector product of the force $\vec{F}$ and the moment arm $\vec{r}$.

$$\vec{T} = \vec{r} \times \vec{F}$$ ~~Netto~~ Newtons. Meters



a) Rectangular planar loop in a uniform magnetic field.



b) cross-sectional view of part (a)

Rectangular loop of length 'l' and width 'w' placed in a uniform magnetic field $\vec{B}$ as shown in the figure.

we know that the magnetic force due to $\vec{B}$ is in normal direction of $\vec{B}$.

Thus the force exerted on BC side and DA side, no force exerted on AB and CD sides.

The Force directions are shown in the figure.

Thus

$$\vec{F} = \int_B^C I\, dl \times \vec{B} + \int_A^D -I\, dL \times \vec{B}$$

$$= I \int_B^C dl \times \vec{B} + I \int_A^B dl \times \vec{B}$$

$$= I \int_B^C dz\, \hat{a}_z \times \vec{B} - I \int_A^B dz\, \hat{a}_z \times \vec{B}$$

Or

$$\vec{F} = \vec{F}_0 - \vec{F}_0 = 0$$

$|F_0| = IB\ell$, no force exerted on the loop as a whole. However, $\vec{F}_0$ and $-\vec{F}_0$ act at different points on the loop, there by creating a couple.

If the normal to the plane (loop) makes an angle 'α' with $\vec{B}$ as shown in figure b,

the torque on the loop is,

$$\vec{T} = \vec{F_0} \times w$$

$$|\vec{T}| = |\vec{F_0}|\, w \sin\alpha$$

or

$$T = BIlw \sin\alpha$$

$$T = BIS \sin\alpha \qquad \boxed{lw = S}$$

we define the quantity,

$$m = IS\,\hat{a}_n$$

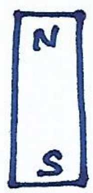
m is magnetic dipole moment

The magnetic dipole moment is product of current and area of the loop. Its direction is normal to the loop.

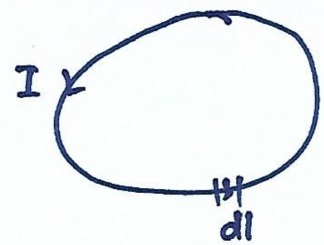
$$\boxed{\vec{T} = m \times \vec{B}}$$

MAGNETIC DIPOLE

A bar magnet or a small current loop is usually referred to as a magnetic dipole.



Bar magnet



Small current loop

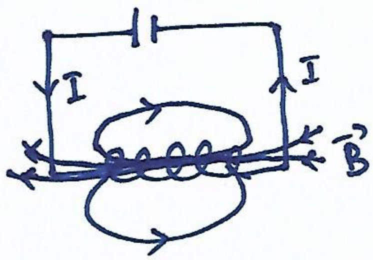
The magnetic vector potential $\vec{A}$ at point $p(r,\theta,\phi)$ in the space is,

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{dl}{r}$$

once, we know $\vec{A}$, we can determine magnetic flux density, $\vec{B}$ from,

$$\vec{B} = \nabla \times \vec{A}$$

INDUCTORS AND INDUCTANCES



circuit carrying current I produce magnetic field $\vec{B}$ that causes a flux $\psi = \int B \cdot ds$

to pass through each turn of the ckt as shown in figure. If ckt has 'N' number of turns, we define flux linkage 'λ' as,

$$\lambda = N\psi$$

If the medium around the ckt is linear,

$$\lambda \propto I$$

$$\lambda = LI$$

'L' is called inductance of the circuit.

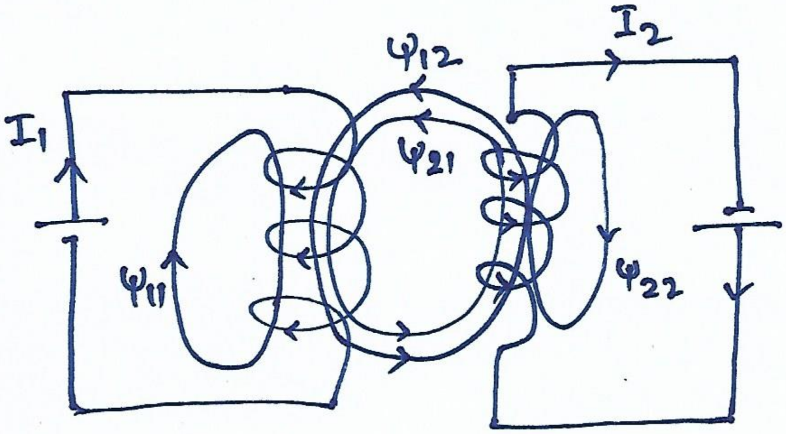
$$\boxed{L = \frac{\lambda}{I} = \frac{N\psi}{I}}$$ Henry (H)

This inductance is commonly referred to as 'self-inductance' since the linkages are produced by the inductor it'self.

The magnetic energy stored in the inductor is,

$$\boxed{W_m = \frac{1}{2} LI^2}$$

If there are two circuits carrying current 'I_1' and 'I_2' magnetic interaction exist between the circuits.



Magnetic interaction between the circuits.

$\psi_{11} \rightarrow$ flux passing through ckt1 due to current I_1 in ckt1
$\psi_{12} \rightarrow$ flux passing through ckt1 due to current I_2 in ckt2
$\psi_{21} \rightarrow$ flux passing through ckt2 due to current I_1 in ckt1
$\psi_{22} \rightarrow$ flux passing through ckt2 due to current I_2 in ckt2

$$\psi_{12} = \int_{S_1} B_2 \cdot ds$$

we define the mutual inductance M_{12} as,

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \psi_{12}}{I_2}$$

Similarly, $$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \psi_{21}}{I_1}$$

self inductances of ckt1 and ckt2 are,

$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1 \psi_1}{I_1}$$

$$L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2 \psi_2}{I_2}$$

Then, the total energy in the magnetic field is,

$$W_m = W_1 + W_2 + W_{12}$$

$$W_m = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M_{12} I_1 I_2$$

'$\pm$' depends on current direction.

MAGNETIC ENERGY

The potential energy in electrostatic field was derived as,

$$W_E = \frac{1}{2} \int \vec{D} \cdot \vec{E} \, dv = \frac{1}{2} \int \varepsilon E^2 \, dv$$

Thus the energy in a magnetostatic field in a linear medium is

$$W_m = \frac{1}{2} \int \vec{B} \cdot \vec{H} \, dv = \frac{1}{2} \int \mu H^2 \, dv$$

Summary of Magnetostatics

1. Biot- Savart's law ; $d\vec{H} = \dfrac{I dl \times \hat{a}_R}{4\pi R^2} = \dfrac{I dl \times \vec{R}}{4\pi R^3}$

(or)

$$d\vec{B} = \frac{\mu_0 \, I dl \times \vec{R}}{4\pi R^3}$$

2. Ampere's circuit Law: $\oint_l \vec{H} \cdot dl = I_{enclosed}$

3. Maxwell's 3^{rd} equation , $\nabla \times \vec{H} = \vec{J}$

4. Magnetic field intensity due to infinite line current

$$\vec{H} = \frac{I}{2\pi\rho}\,\hat{a}_\phi$$

5. Magnetic field intensity due to infinite sheet of Current

$$\vec{H} = \frac{1}{2}\vec{K} \times \hat{a}_n$$

6. Magnetic flux density, $\vec{B} = \mu_0 \vec{H}$ (for free space)

7. Magenetic flux through a surface 'S' is,

$$\Psi = \int_S \vec{B} \cdot ds$$

8. Maxwell's 4^{th} equation,

$$\nabla \cdot \vec{B} = 0$$

9. Magnetic vector potential, $\vec{A} = \int_L \frac{\mu_0 I \, dl}{4\pi R}$

for line current

10. $\vec{B} = \nabla \times \vec{A}$

11. Magnetic flux, ψ and $\vec{A}$ relation.

$$\psi = \oint_L \vec{A} \cdot dl.$$

12. Magnetic force on point charge 'Q'

$$\vec{F}_m = Qu \times \vec{B}$$

Magnetic force on small current element,

$$d\vec{F}_{dl} = I dl \times \vec{B}$$

Magnetic force on closed path (or) circuit which is carrying current 'I' is,

$$\vec{F} = \oint_L I \, dl \times \vec{B}$$

Similary magnetic force on surface and volume currents is,

$$\vec{F}_s = \int_S \vec{K} ds \times \vec{B} \quad \text{and} \quad \vec{F}_v = \int_V \vec{J} dv \times \vec{B}$$

13. Magnetic Torque, $\vec{T} = \vec{r} \times \vec{F}$

14. Magnetic dipole moment, $\vec{m} = I S \hat{a}_m$.

15. self inductance, $L = \frac{N\psi}{I}$

16. Energy (or) magnetic energy stored in the inductor, $W_m = \frac{1}{2} L I^2$

17. Mutual inductance

$$M_{12} = \frac{N_1 \psi_{12}}{I_2}$$

$$M_{21} = \frac{N_2 \psi_{21}}{I_1}$$

18. Potential energy in electrostatic field is,

$$W_E = \frac{1}{2} \int \vec{D} \cdot \vec{E} \, dv = \frac{1}{2} \int \epsilon E^2 \, dv$$

19. The energy in the magnetostatic field in the linear medium,

$$W_E = \frac{1}{2} \int \vec{B} \cdot \vec{H} \, dv = \frac{1}{2} \int \mu H^2 \, dv$$

UNIT- III

Maxwell's Equations

The electrostatic fields are usually produced by static electric charges, whereas magnetostatic fields are due to motion of electric charges with uniform velocity (direct current) or static magnetic charges (magnetic poles).

Time varying fields or waves are usually due to accelerated charges or time varying currents as shown in below figure.

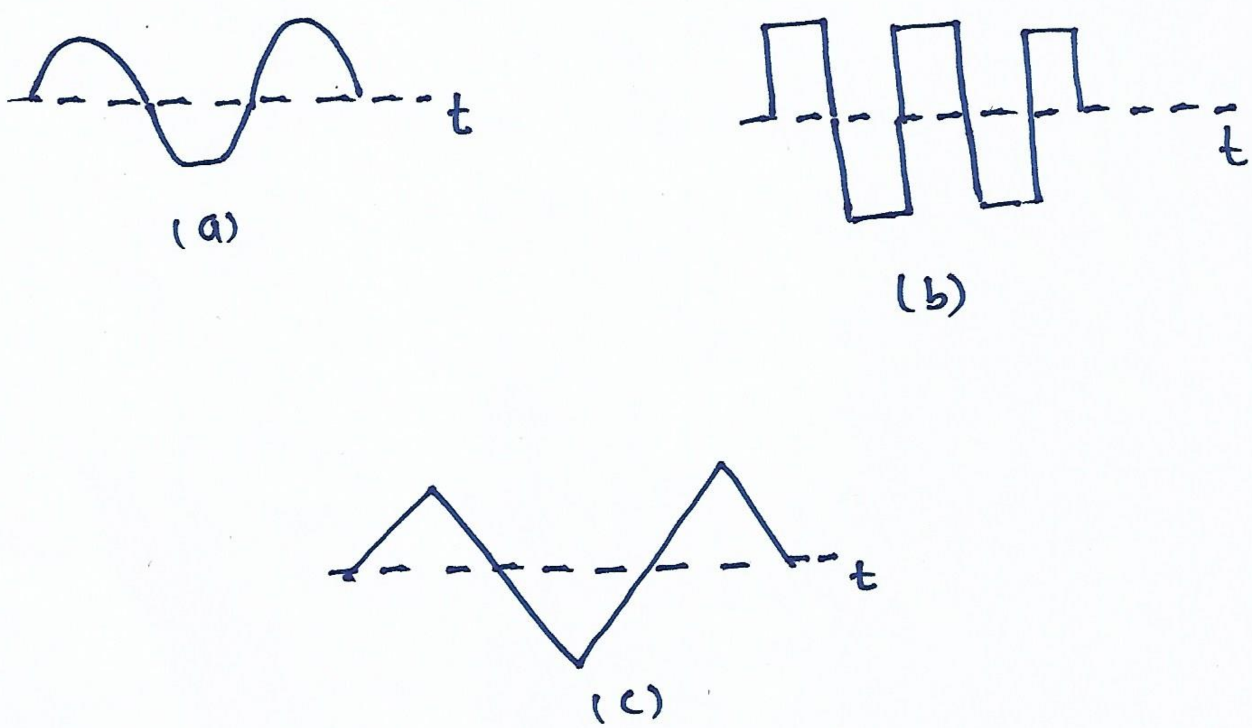


Fig. Examples of time varying current

In this unit we will study Faraday's law and maxwell's equations for time-varying fields.

FARADAY's LAW

According to Faraday's ~~law~~ Experiment, static magnetic field produces no current flow, but time varying field produces an induced voltage (electromotive force) in a closed circuit, which causes a flow of current.

"The induced emf, Vemf in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit"

This is called Faraday's law.

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\psi}{dt}$$

where, $\lambda = N\psi$ is the flux linkage,

N = number of turns in the circuit

ψ = flux through each turn

The negative sign indicates that the induced voltage acts in such a way as to oppose the flux producing it. This is called "Lenz's Law"

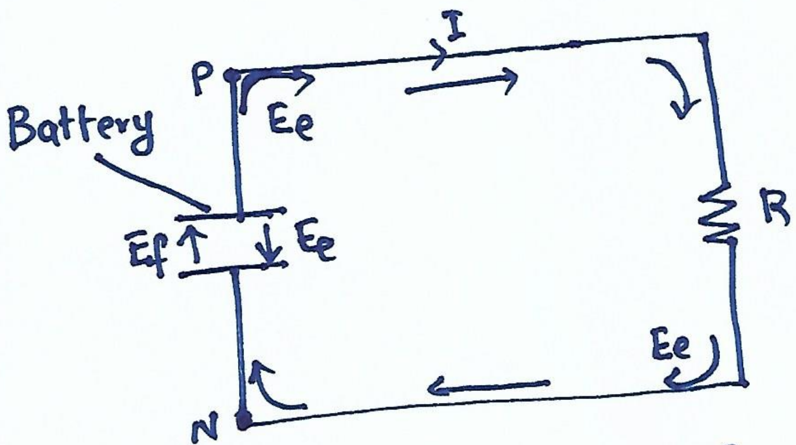


Fig. A circuit showing emf-producing field E_f and electrostatic field E_e.

Here, battery is a source of emf. $\vec{E_f}$ is emf produced field. Due to accumulation of charges at battery terminals, an electrostatic field exist (E_e).

The total Electric field at any point is,

$$\vec{E} = \vec{E_f} + \vec{E_e}$$

$\vec{E_f}$ is zero outside the battery

$\vec{E_e}$ inside the battery is opposite to that outside it.

Integration over the closed ckt is

$$\oint_L \vec{E} \cdot dl = \oint_L \vec{E_f} \cdot dl + 0 = \int_N^P E_f \cdot dl.$$

$\oint_L \vec{E_e} \cdot dl = 0$ because $\vec{E_e}$ is conservative.

The emf of the battery is equa to line integration of emf produced field,

$$V_{emf} = \int_N^P \vec{E_f} \cdot dl = -\int_N^P \vec{E_e} \cdot dl$$

$\vec{E_f}$ and $\vec{E_e}$ are equal but opposite within the battery

This is also called potential difference b/w battery open ckt terminals $(V_P - V_N)$.

TRANSFORMER & MOTIONAL ELECTROMOTIVE FORCES

Faraday's law for a ckt having Single turn $(N=1)$,

$$V_{emf} = -\frac{d\varphi}{dt}$$

The above equation can be written as,

$$V_{emf} = \oint_L \vec{E} \cdot dl = \frac{-d}{dt} \int_S \vec{B} \cdot ds$$

here, $\varphi = \int_S \vec{B} \cdot ds$, S is surface area of the circuit bounded by the closed path L.

The variation of flux with time cacused ~~by~~ in three ways.

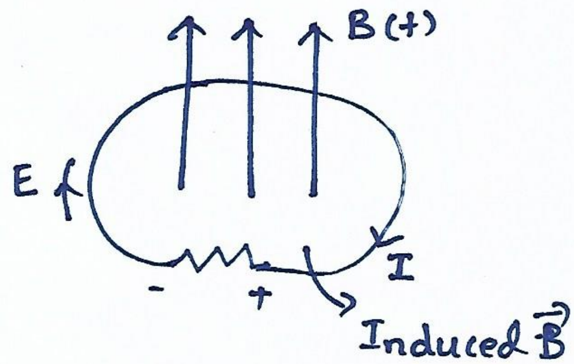
1. By having a stationary loop in a time-varying $\vec{B}$ field.

2. By having a time-varying loop area in a static $\vec{B}$ field

3. By having a time varying loop area in a time-varying $\vec{B}$ field.

1. Stationary loop in time-varying $\vec{B}$ field

A stationary conducting loop is in a time-varying magnetic field $\vec{B}$.



$$V_{emf} = \oint_L \vec{E} \cdot dl = - \int_S \frac{\partial}{\partial t} \vec{B} \cdot ds$$

This emf induced by the time varying $\vec{B}$ field in the stationary loop is often referred to as transformer emf, since it is due to transformer action.

By applying stokes theorem to the middle term in the above equation,

$$\int_S (\nabla \times \vec{E}) \, ds = - \int \frac{\partial \vec{B}}{\partial t} \cdot ds$$

for the two integrals to be equal, their integrads must be equal;

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

This is one of the maxwell's equations for the time varying fields. It shows that the time varying $\vec{E}$ field is not conservative ($\nabla \times \vec{E} \neq 0$).

Ampere's law inconsistency and Displacement current

For static EM fields,

$$\nabla \times \vec{H} = \vec{J}$$

Apply divergence on both the sides,

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$$

From vectors identify,

$\nabla \cdot \nabla \times (\text{vector}) = 0$ that means,

$$\nabla \cdot (\nabla \times \vec{H}) = 0 = \nabla \cdot \vec{J} \quad \text{——— (1)}$$

But from the current continuity equation,

$$\nabla \cdot \vec{J} = -\frac{\partial}{\partial t} \rho_v \neq 0 \quad \text{——— (2)}$$

(1) and (2) are obviously in compatable for time varying conditions.

This is called inconsistency of ampere's law.

To satisfy current continuity equation,
$\nabla \times \vec{H} = \vec{J}$ must be modified, for that Maxwell's added a term, So that it becomes,

$$\nabla \times \vec{H} = \vec{J} + \vec{J_d}$$

$\vec{J_d}$ is displacement current density.

Again consider, $\nabla \cdot (\nabla \times \vec{H}) = \nabla(\vec{J} + \vec{J_d})$

$$\nabla \cdot \vec{J} = -\nabla \cdot \vec{J_d}$$

$$-\frac{\partial}{\partial t} \rho_v = -\nabla \cdot \vec{J_d}$$

we know, $\nabla \cdot \vec{D} = \rho_v$

$$\frac{\partial}{\partial t}(\nabla \cdot \vec{D}) = \nabla \cdot \vec{J_d}$$

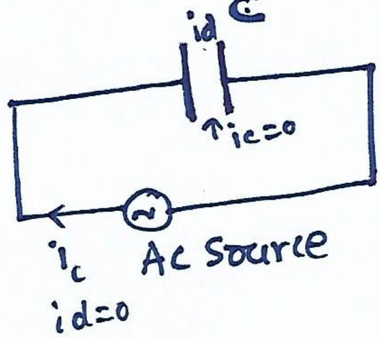
$$\nabla \cdot \frac{\partial}{\partial t} \vec{D} = \nabla \cdot \vec{J_d}$$

$$\boxed{\vec{J_d} = \frac{\partial \vec{D}}{\partial t}}$$

$$\nabla \times \vec{H} = \vec{J_c} + \vec{J_d}$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Displacement current



Maxwell observed the magnetic field the wire in the left side circuit. That magnetic field is produced by the conduction current.

Surprisingly, he also observed magnetic field around the capacitor gap. without any coduction current.

Here he introduced new parameter i.e displacement current, because of this displacement current only a magnetic field is produced.

The displacement current density $\vec{J_d}$

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t}$$

Because of the electric flux change with time this displacement is produced.

At any point of the ckt, we could notice only one current either conduction (or) displacement

MAXWELL'S EQUATIONS IN FINAL FORMS

Maxwell put laws of electromagnetism in the form of four equations.

The final form of maxwell's equations in time varying conditions is as shown below.

Differential form	Integral form	Remarks
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot ds = \int_V \rho_v \, dv$	Gauss law
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot ds = 0$	Non-existence of isolated magnetic charge. (or) Gauss law
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_L \vec{E} \cdot dl = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot ds$	Faraday's Law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint_L \vec{H} \cdot dl = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) ds$	Ampere's Law.

word statement of maxwell's equations

I. $\nabla \cdot \vec{D} = \rho_v$ (or) $\oint_S \vec{D} \cdot ds = \int_V \rho_v dv$

"The total electric displacement (or) flux through the surface enclosing a volume is equal to the total charge within the volume"

II $\nabla \cdot \vec{B} = 0$ (or) $\oint_S \vec{B} \cdot ds = 0$

"The net magnetic flux emerging through any closed surface is zero."

III $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$ (or) $\oint_L \vec{E} \cdot dl = \frac{-\partial}{\partial t} \int_S \vec{B} \cdot ds$

"The electromotive force around a closed path is equal to the time derivative of magnetic flux through any surface bounded by the path".

IV $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ (or) $\oint_L \vec{H} \cdot dl = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) ds$

"The magnetomotive force around a closed path is equal to the conduction current plus time derivative of electric flux through any surface bounded by the path".

UNIT- IV

Wave characteristics

Electromagnetic wave (EM wave) is nothing but a wave which consists both electric ($\vec{E}$) and magnetic ($\vec{H}$) fields.

The existence of EM waves was predicted by James clerk Maxwell, after that Hertz succeded in generating and detecting EM waves.

"In general, waves are means of transporting energy or information".

The typical examples of EM waves are,

Radio waves
TV Signals
Radar beams
Cellphone signals
Satellite signals.

In this chapter, we will study about EM wave propagation in different mediums with help of Maxwell's equations.

wave equation for free-Space Condition

The final form of Maxwell's equations,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \cdot \vec{B} = 0$$

we already know, the relations

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{J} = \sigma \vec{E}$$

Now for free space medium, $\sigma = 0$, $\rho_v = 0$, $\mu = \mu_0$

$\varepsilon = \varepsilon_0$

The maxwell's equations in free space become

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} = 0\,\vec{E} + \frac{\partial \vec{D}}{\partial t} = \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = 0$$

$$\nabla \cdot \vec{B} = 0$$

let the 1st equation$(\nabla \times \vec{E})$,

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

apply curl on both sides

$$\nabla \times \nabla \times \vec{E} = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

from vector identity, $\nabla \times \nabla \times \vec{E} = \nabla \cdot \nabla \cdot \vec{E} - \nabla^2 \vec{E}$

$$\nabla \cdot \nabla \cdot \vec{E} - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$[\because \nabla \cdot \vec{D} = 0]$

$$\nabla \cdot \vec{E} = \nabla\left(\frac{\vec{D}}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}(\nabla \cdot \vec{D}) = 0$$

$$-\nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \mu_0 \vec{H}) = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \vec{H})$$

$$-\nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$\nabla^2 \vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}(\varepsilon_0 \vec{E}) = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \longrightarrow ①$$

Similarly from $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$, we get,

$$\boxed{\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \longrightarrow ②$$

eqn ① & ② are called wave equations.

The uniform plane wave

A uniform plane wave, in which both fields $\vec{E}$ and $\vec{H}$ lie in transverse plane .i.e The plane whose normal is the direction of propagation.

Further both fields are of constant magnitude in the transverse plane. Because of this reason such a wave is sometimes called a transverse Electromagnetic wave (TEM wave).

Assume that wave travelling in x-direction, then it allows spatial variations of $\vec{E}$ only in x-direction.

that means,

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

This consist only one term i.e.

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

This equation we may be written in terms of the components, of $\vec{E}$ as,

$$\frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial t^2}$$

In a region in which there is no charge

$$\nabla \cdot \vec{E} = \frac{1}{\varepsilon} \nabla \cdot \vec{D} = 0$$

That is,

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

For a uniform plane wave, $\vec{E}$ is independent of y and z. (no spatial variations in y and z direction).

$$\frac{\partial E_x}{\partial x} + 0 + 0 = 0$$

$$\frac{\partial E_x}{\partial x} = 0$$

That means $E_x = 0$.

Therefore a uniform plane wave progressing (travelling) in x-direction has no x-component of $\vec{E}$ and $\vec{H}$. And $\vec{E}$ and $\vec{H}$ have components only in the directions normal to the direction of wave propagation.

$$E_x = H_x = 0$$

$$E_y \neq 0, \quad E_z \neq 0, \quad H_y \neq 0, \quad H_z \neq 0$$

Wave propagation in Lossy Dielectrics

Lossy dielectric is a medium in which an EM wave, as it propagates, loses power owing to imperfect dielectric.

Lossy dielectric is a partially conducting medium with $\sigma \neq 0$,

consider a linear, isotropic, homogeneous, lossy dielectric medium that is charge free ($\rho_v = 0$), the maxwell equations are,

$$\nabla \cdot \vec{D} = \nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{B} = \nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{H} = (\sigma + j\omega\epsilon)\vec{E}$$

$$\nabla \times \vec{E} = -j\omega\mu \vec{H}$$

Take $\nabla \times \vec{E} = -j\omega\mu \vec{H}$

$$\nabla^2 \vec{E} = j\omega\mu (\sigma + j\omega\epsilon)\vec{E}$$

(or) $$\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0 \quad \text{——} \quad (1)$$

where, $$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

γ is called propagation constant of the medium.

By similar procedure,

$$\nabla^2 \vec{H} - \gamma^2 \vec{H} = 0 \quad (2)$$

(1) & (2) called homogeneous vector helmholtz's equations.

γ is a complex quantity, we may let

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = (\alpha + j\beta)^2 = \alpha^2 - \beta^2 + j2\alpha\beta$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

$$Re\{\gamma^2\} = \alpha^2 - \beta^2 = -\omega^2\mu\epsilon$$

$$|\gamma| = \sqrt{\alpha^2+\beta^2} \qquad |\gamma^2| = \sqrt{(\omega\mu)^2}\cdot\sqrt{\sigma^2+\omega^2\epsilon^2}$$

$$|\gamma|^2 = \alpha^2+\beta^2 \qquad |\gamma^2| = \omega\mu\sqrt{\sigma^2+\omega^2\epsilon^2}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

The sollution for the differential equation.

$$(\nabla^2 - \gamma^2) E(x) = 0.$$

$$E(x) = E_0 e^{-\gamma x} + E_0' e^{\gamma x}$$

E_0, E_0' are constants, wave propagation we assume is x-direction, $E_0' = 0$.

$$E(x,t) = Re\left[E(x) e^{j\omega t} \hat{a}_y \right]$$

$$E(x,t) = E_0 e^{-\alpha x} \cos(\omega t - \beta x) \hat{a}_y$$

and

$$H(x,t) = Re\left\{ H_0 e^{-\alpha x} e^{j(\omega t - \beta x)} \hat{a}_z \right\}$$

where

$$H_0 = \frac{E_0}{\eta}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}.$$

α is called attenuation constant (or) coefficient

β is called phase constant

velocity (or) phase velocity of the wave is

$$u = \frac{\omega}{\beta} \quad , \quad \lambda = \frac{2\pi}{\beta}.$$

and

loss tangent, $= \dfrac{|\vec{J}|}{|\vec{J}_D|} = \dfrac{|\sigma \vec{E}|}{|j\omega\varepsilon\vec{E}|} = \dfrac{\sigma}{\omega\varepsilon}$

$$\tan\theta = \frac{\sigma}{\omega\varepsilon}.$$

PLANE WAVES IN LOSSLESS DIELECTRICS

In lossless dielectric, $\sigma << \omega\epsilon$,

$$\sigma \simeq 0, \quad \varepsilon = \varepsilon_o \varepsilon_r, \quad \mu = \mu_o \mu_r$$

After substiting these values,

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu\epsilon}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}, \qquad \lambda = \frac{2\pi}{\beta}$$

Also

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \angle 0^\circ$$

PLANE WAVES IN FREE SPACE

In free space, $\sigma = 0$, $\varepsilon = \varepsilon_o$, $\mu = \mu_o$

$$\alpha = 0, \quad \beta = \omega\sqrt{\mu_o \varepsilon_o} = \frac{\omega}{c}$$

$$u = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = c, \qquad \lambda = \frac{2\pi}{\beta}$$

where, $c = 3\times10^8$ m/s.

$$\eta = \eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi = 377\Omega.$$

PLANE WAVES IN GOOD CONDUCTORS

In good conductors, $\sigma >> \omega \epsilon$ $\quad \frac{\sigma}{\omega \epsilon} >> 1$

$$\sigma \cong \infty \quad , \quad \epsilon = \epsilon_0 \quad , \quad \mu = \mu_0 \mu_r .$$

$$\alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}} = \sqrt{\pi f \mu \sigma}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu \sigma}} \quad , \quad \lambda = \frac{2\pi}{\beta}$$

Also,

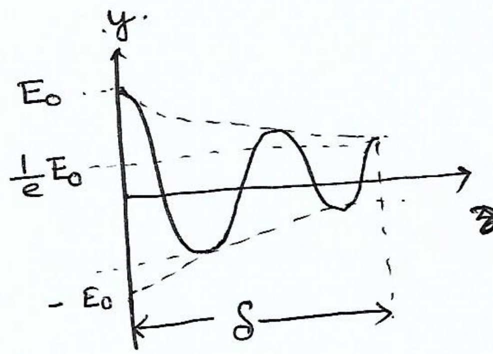
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^0$$

SKIN DEPTH (OR) PENETRATION DEPTH

When wave travels in conducting medium, its amplitude is attenuated by the factor $e^{-\alpha z}$. The distance δ, through which the wave amplitude decreases to a factor e^{-1} (37% of the original value) is called skin depth (or) penetration depth of the medium.

$$E_0 e^{-\alpha \delta} = E_0 e^{-1}$$

or $\boxed{\delta = \frac{1}{\alpha}}$



$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha}}$$

SURFACE RESISTANCE

The surface (or) skin resistance R_s (in Ω) is defined as the real part of 'η' for a good conductor.

$$R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\pi f \mu}{\sigma}}$$

$$\boxed{R_s = \sqrt{\frac{\pi f \mu}{\sigma}} \quad \Omega}$$

POLARIZATION

The orientation of Electricfield is defined as 'polarization'

For example, a uniform plane wave travelling in the z-direction with the $\vec{E}$ and $\vec{H}$ vectors lying in the xy plane.

If $\vec{E}_y = 0$ and E_x is present, the wave is said tobe polarized in the x-direction.

If E_y present, $E_x = 0$, the wave is said tobe polarized in the y-direction.

If both E_x and E_y present and are inphase, the direction of resultant vector is ~~Catt~~ constant with time, the wave is said tobe linearly polarized.

If both E_x and E_y are not inphase, that means they reach their maximum at different instants of time, then the direction of resultant vector vary with time. The wave is said to be elliptically polarized.

E_y and E_x present have the equal magnitudes and 90° phase difference, the wave is said tobe circularly polarized.

POYNTING VECTOR AND POYNTING THEOREM

The instantaneous power density at any point is obtained by poynting vector, '$\wp$'.

$$\boxed{\wp = \vec{E} \times \vec{H}}$$

Poynting theorem states that net power flowing out of ~~the~~ a given volume 'v' is equal to the time rate of decrease in the energy stored within 'v' minum the ohmic losses.

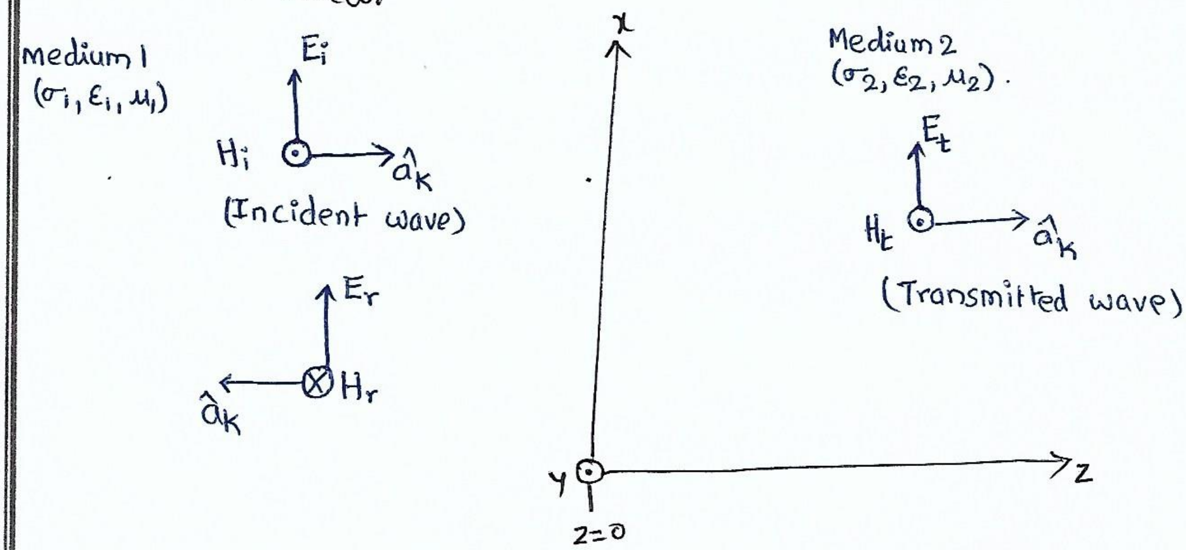
$$\wp_{avg} = \frac{1}{2} \, Re \, (\vec{E}_s \times \vec{H}^*)$$

$$P = \int_S \wp_{avg} \cdot ds$$

total avg power crossing a given surface.

REFLECTION OF A PLANE WAVE AT NORMAL INCIDENCE

when a plane wave from one medium meets a different medium, it's partly reflected and partly transmitted. The portion of the incident wave that is reflected or transmitted depends on the constitutive parameters $(\varepsilon, \mu, \sigma)$ of the two media involved.



A plane wave propagating along the +z direction in medium 1 $(\sigma_1, \varepsilon_1, \mu_1)$ meets the medium 2 $(\sigma_2, \varepsilon_2, \mu_2)$ at the boundary z=0, as shown in figure.

The incident wave $\vec{E}$ and $\vec{H}$ fields are,

$$E_i(z) = E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x$$

$$H_i(z) = H_{io}\, e^{-\gamma_1 z}\, \hat{a}_y = \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, \hat{a}_y$$

The portion of the incident wave reflected along $-z$ direction, in medium 1, then the reflected wave field components are,

$$E_r(z) = E_{ro}\, e^{\gamma_1 z}\, \hat{a}_x.$$

$$H_r(z) = H_{ro}\, e^{\gamma_1 z} (-\hat{a}_y) = -\frac{E_{ro}}{\eta_1} e^{\gamma_1 z}\, \hat{a}_y$$

The portion of the incident wave transmitted along $+z$-direction in medium 2, the field components of transmitted wave are,

$$E_t(z) = E_{to}\, e^{-\gamma_2 z}\, \hat{a}_x$$

$$H_t(z) = H_{to}\, e^{-\gamma_2 z}\, \hat{a}_y = \frac{E_{to}}{\eta_2} e^{-\gamma_2 z}\, \hat{a}_y.$$

Here, we assume that E_i, E_r, E_t have the same polarization. E_{io}, E_{ro}, E_{to} are the magnitudes of incident, reflected and transmitted electric field magnitudes at $z=0$.

In medium 1, incident and reflected waves are there, In medium 2 only transmitted wave is there,

$$E_1 = E_i + E_r \qquad H_1 = H_i + H_r$$

$$E_2 = E_t \qquad H_2 = H_t.$$

Since the waves are transverse, $\vec{E}$ and $\vec{H}$ are entirely tangential to the interface. Hence at $z=0$ $E_{1tan} = E_{2tan}$ $H_{1tan} = H_{2tan}$

$E_i(0) + E_r(0) = E_t(0) \longrightarrow E_{io} + E_{ro} = E_{to}$

$H_i(0) + H_r(0) = H_t(0) \longrightarrow \frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2}$

from the above two eqn's

$$E_{ro} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{io}$$

and

$$E_{to} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{io}$$

Now we can define, the reflection and transmission co-efficients are,

reflection co-efficient, $\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

Transmission coefficient, $\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$

NOTE

1. $E_{to} = \tau E_{io}$
2. $1 + \Gamma = \tau$
3. Γ and τ are dimensionless and maybe complex
4. $0 \leq |\Gamma| \leq 1$.

General Case

consider, medium1 is a perfect dielectric ($\sigma = 0$) and medium2 is perfect conductor ($\sigma \simeq \infty$)

For this case, $\eta_2 = 0$, hence, $\Gamma = -1$ and $\tau = 0$, and wave is totally reflected. The totally reflected wave combines with incident wave to form a standing wave.

Standing wave consists incident wave and reflected wave of equal amplitude and opposite direction.

$$E_1 = E_i + E_r = \left(E_{io} e^{-\gamma_1 z} + E_{ro} e^{\gamma_1 z}\right) \hat{a}_x$$

But, $\Gamma = \dfrac{E_{ro}}{E_{io}} = -1$ $\quad \sigma_1 = 0, \ \alpha_1 = 0, \ \gamma_1 = j\beta_1$

$$E_1 = -E_{io}\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right) \hat{a}_x$$

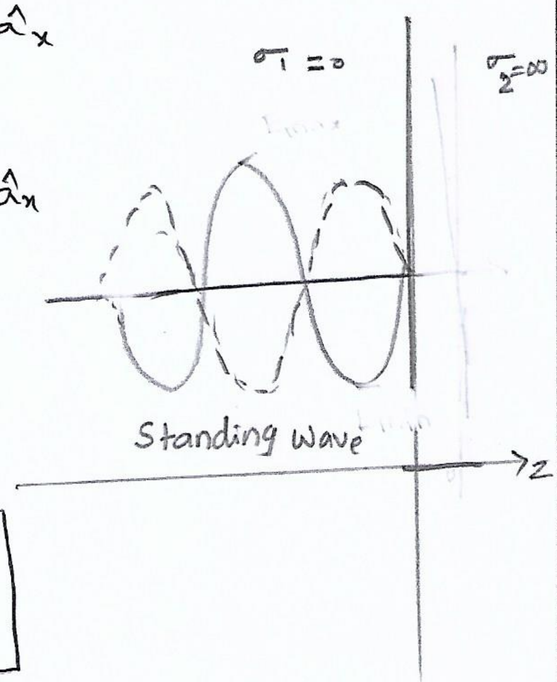
(or)

$$E_1 = -2j E_{io} \sin\beta_1 z \ \hat{a}_x$$

$$E_1 = Re\{E_1 . e^{j\omega t}\}$$

$$\boxed{E_1 = 2E_{io} \sin\beta_1 z \cdot \sin \omega t \ \hat{a}_x}$$

$$\boxed{H_1 = \frac{2E_{io}}{\eta_1} \cos\beta_1 z \ \cos\omega t \ \hat{a}_y}$$



Standing wave

when medium1 and medium2 are lossless,

$\sigma_1 = \sigma_2 = 0$, η_1 and η_2 are real numbers.

Case 1

If $\eta_2 > \eta_1$, $\Gamma > 0$, there is standing wave in medium1, and there is also a transmitted wave in medium2.

However, the incident and reflected waves have the amplitudes that are not equal in magnitude.

The relative maximum of $|E_1|$ occurs at

$$-\beta_1 Z_{max} = n\pi$$

$$Z_{max} = \frac{-n\pi}{\beta_1} = \frac{-n\pi}{\left(\frac{2\pi}{\lambda_1}\right)} = \frac{-n\lambda_1}{2}, \quad n = 0, 1, 2, ---$$

and minimum value of $|E_1|$ occurs at

$$-\beta_1 Z_{min} = (2n+1)\frac{\pi}{2}$$

$$Z_{min} = \frac{-(2n+1)\pi}{2\beta_1} = \frac{-2(n+\frac{1}{2})\pi}{2\times\frac{2\pi}{\lambda_1}} = \frac{-(n+\frac{1}{2})}{2}\lambda_1$$

$n = 0, 1, 2, - - -$

Case 2

If $\eta_2 < \eta_1$, $\Gamma < 0$, standing wave is there in medium1 and transmitted wave in medium 2.

$|E_1|$ maximum occurs at

$$Z_{max} = \frac{-(2n+1)\pi}{2\beta_1} = \frac{(2n+1)}{4}\lambda_1, \quad n = 0, 1, 2, \ldots$$

$|E_1|$ minimum occurs at

$$Z_{min} = -\frac{n\pi}{\beta_1} = -\frac{n\lambda_1}{2}, \quad n = 0, 1, 2, \ldots$$

Note

$|H_1|$ minimum occurs whenever there is $|E_1|$ maximum, and vice-versa

Transmitted wave in medium2 is purely ~~transm~~ travelling wave

The ratio of $\frac{|E_1|_{max}}{|E_1|_{min}}$ (or) $\frac{|H_1|_{max}}{|H_1|_{min}}$ is called standing wave ratio.

$$S = \frac{|E_1|_{max}}{|E_1|_{min}} = \frac{|H_1|_{max}}{|H_1|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

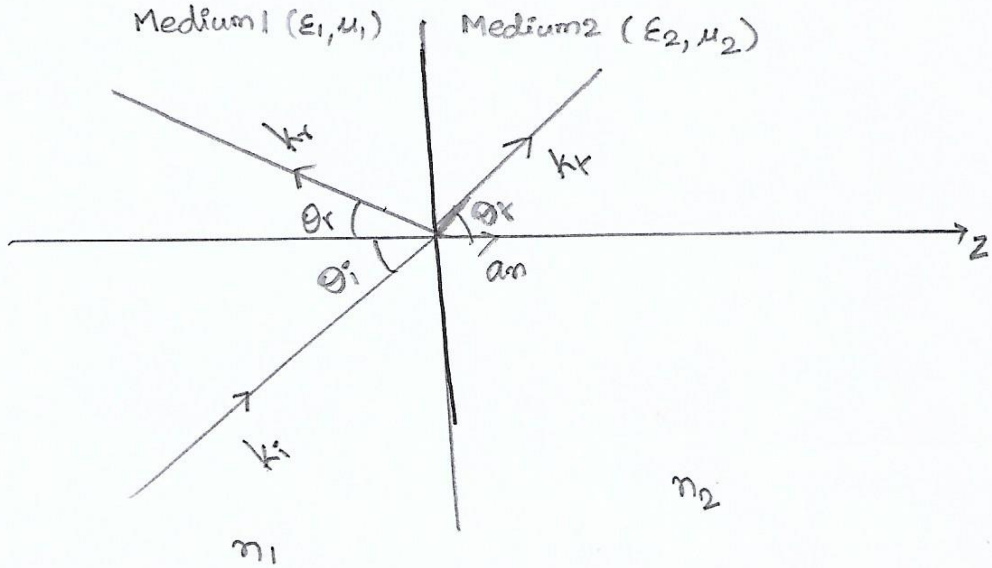
(or)

$$|\Gamma| = \frac{S-1}{S+1}$$

$|\Gamma| \leq 1$, $1 \leq S \leq \infty$. The standing wave ratio is dimensionless and is expressed in decibels (dB) as,

$$S\,(dB) = 20 \log_{10}(S)$$

REFLECTION OF A PLANE WAVE AT OBLIQUE INCIDENCE



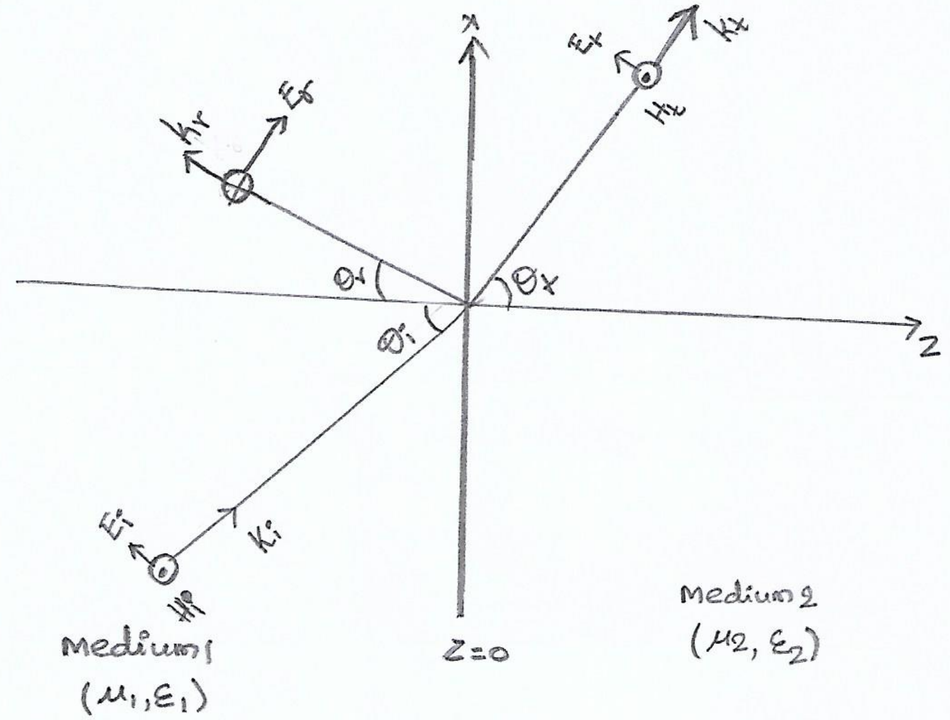
According to the Snell's law

$$\eta_1 \sin\theta_i = \eta_2 \sin\theta_t$$

$$\eta_1 = \frac{c}{\left(\frac{1}{\sqrt{\mu_1 \epsilon_1}}\right)} = \frac{c}{u_1}, \quad \eta_2 = c\sqrt{\mu_2 \epsilon_2} = c/u_2$$

η_1 and η_2 are refractive index of medium1 and medium2.

A. Parallel polarization



The reflection co-efficient, $\Gamma_{\parallel} = \dfrac{E_{ro}}{E_{io}} = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

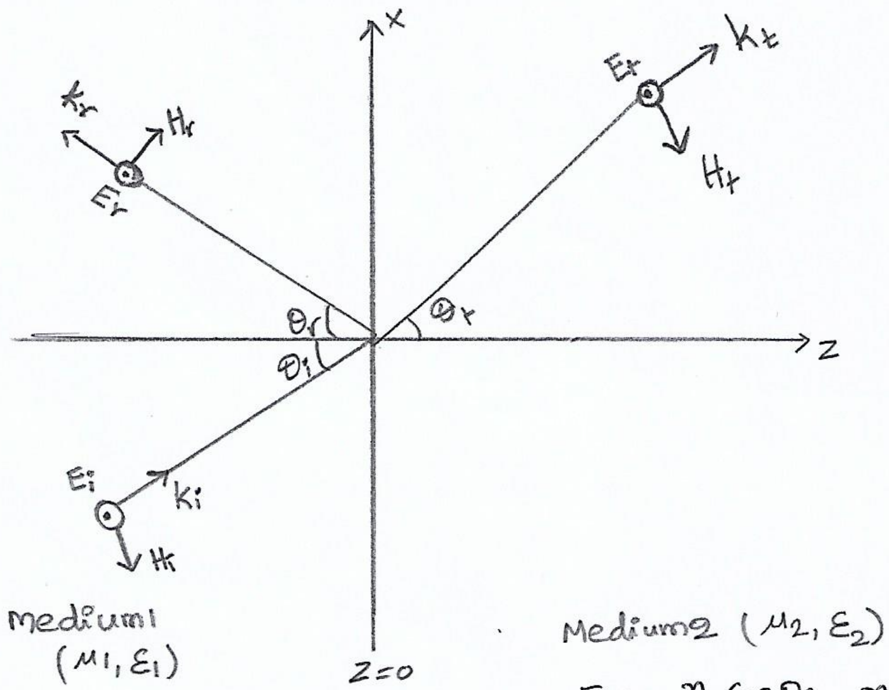
The transmission coefficient, $\tau = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$

Brewster angle ($\theta_{B\parallel}$) : The incident angle at which, no reflection takes place is called 'Brewster angle' ($\theta_{B\parallel}$).

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_{B\parallel}$$

$$\tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}$$

B. PERPENDICULAR POLARIZATION.



The reflection co-efficient, $\Gamma_\perp = \dfrac{E_{ro}}{E_{io}} = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_2 \cos\theta_t}$

The transmission co-efficient, $\tau_\perp = \dfrac{E_{to}}{E_{io}} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

Brewster angle $\theta_{B\perp}$

$$\eta_2 \cos\theta_{B\perp} = \eta_1 \cos\theta_t$$

$$\sin^2\theta_{B\perp} = \frac{1 - \mu_1\varepsilon_2 / \mu_2\varepsilon_1}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}$$

For non-magnetic media $(\mu_1 = \mu_2 = \mu_0)$, $\theta_{B\perp}$ does not exist.

if $\mu_1 \neq \mu_2$ and $\varepsilon_1 = \varepsilon_2$

$$\tan\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$$

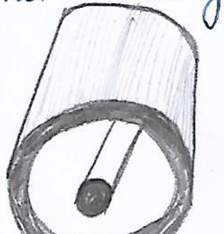
UNIT – V

TRANSMISSION LINES

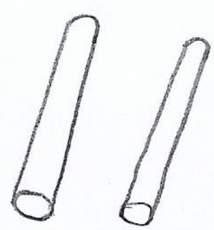
In unit-IV, we have been studied wave propagation in unbounded media, media of infinite extent. Another means of transmitting power or information is by guided structure. The typical examples of such structures are transmission lines and waveguides.

Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies). A transmission lines basically consists of two or more parallel conductors used to connect a source to a load.

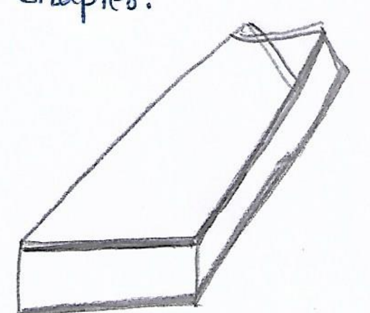
Transmission line problems are usually solved by means of EM field theory and electric circuit theory. The concepts of wave propagation such as propagation constant, reflection co-efficient and standing wave ratio are used in this chapter.



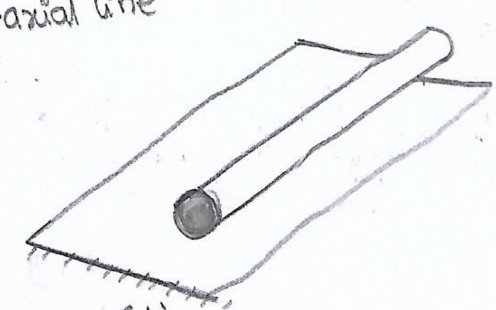
(a) Co-axial line



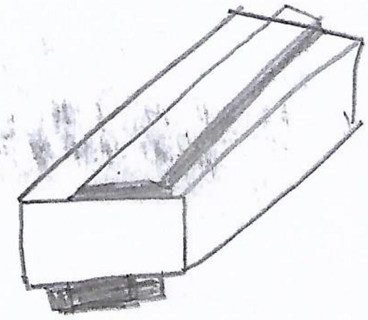
(b) Two-wire line



(c) planar line



(d) wire above conducting plane



(e) microstrip line

TRANSMISSION LINE PARAMETERS

Any transmission line can be described interms of its line parameters, resistance per unit length 'R', inductance per unit length 'L', conductance per unit length 'G' and capacitance per unit length 'C'.

These parameters R, L, G and C are not discrete(or) lumped, they are distributed, uniformly along the length of the line.

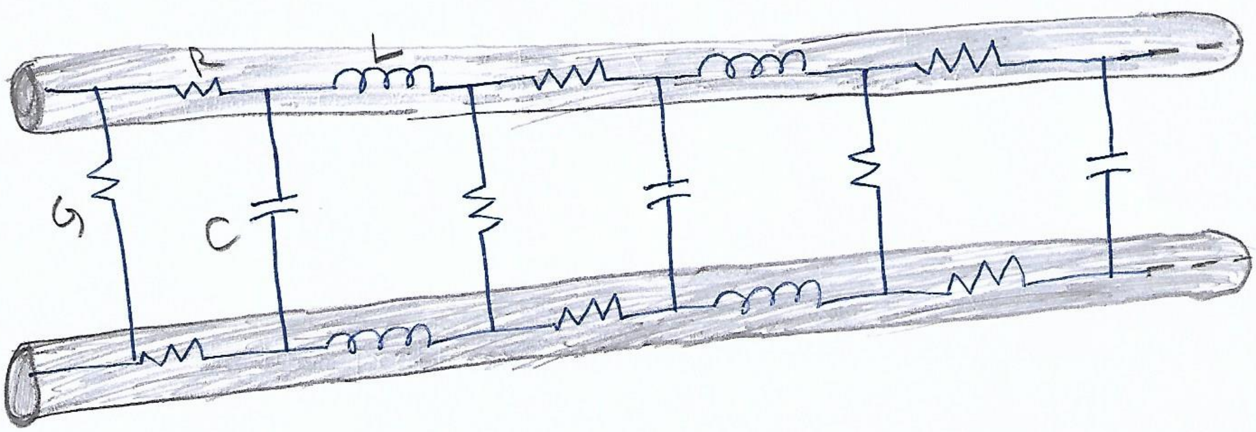
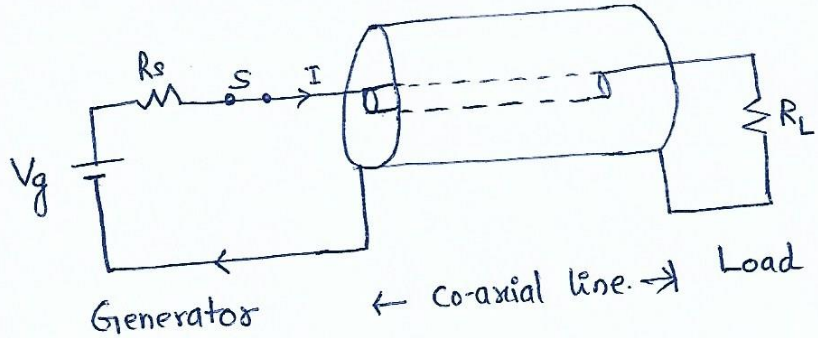
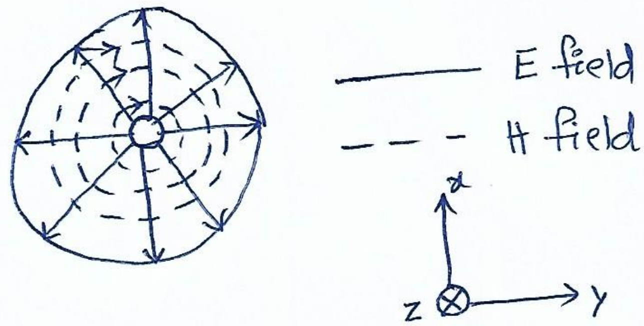


Table. Distributed line parameters.

Parameters	co-axial line	Two-wire Line	planar Line.
R (Ω/m)	$\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$	$\frac{1}{\pi a\delta\sigma_c}$	$\frac{2}{w\delta\sigma_c}$
L (H/m)	$\mu/2\pi \ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{d}{2a}\right)$	$\frac{\mu d}{w}$
G (S/m)	$2\pi\sigma/\ln(b/a)$	$\pi\sigma/\cosh^{-1}\left(\frac{d}{2a}\right)$	$\frac{\sigma w}{d}$
C (F/m)	$2\pi\varepsilon/\ln\frac{b}{a}$	$\frac{\pi\varepsilon}{\cosh^{-1}\left(\frac{d}{2a}\right)}$	$\frac{\varepsilon w}{d}$ $w \gg d$.



(a)



(b)

Fig. (a) Co-axial line connecting the generator to load
(b) E and H fields on co-axial line.

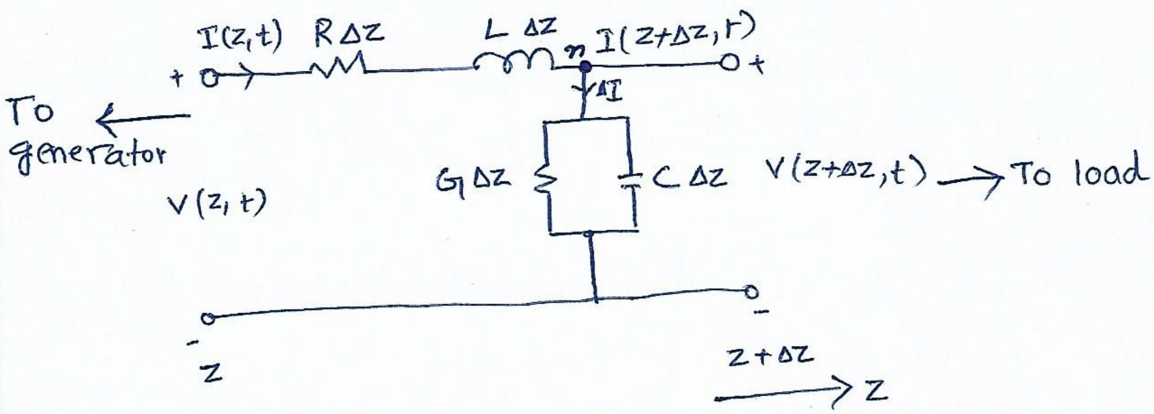
In the above figure, we can see a co-axial line connected b/w source (generator) and load. which supports transverse electromagnetic (TEM) wave propagation. The electric and magnetic fields are shown in Figure (b).

TRANSMISSION LINE EQUATIONS

Two-conductor transmission line support a TEM wave, the electric and magnetic fields on the line are perpendicular to each other and transverse to the direction of wave propagation. The TEM wave Electric field $\vec{E}$ and magneti field $\vec{H}$ are uniquely related to voltage 'V' and current 'I'.

$$V = -\int \vec{E} \cdot dl \qquad I = \oint \vec{H} \cdot dl$$

Let us consider a small portion of transmission line, Δz, in this we assume wave propagates along the +z direction,



Apply KVL to outer loop of the circuit,

$$V(z,t) = R\Delta z\, I(z,t) + L\,\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t)$$

or

$$-\frac{V(z+\Delta z, t) - V(z,t)}{\Delta z} = R I(z,t) + L \frac{\partial I(z,t)}{\partial t}$$

$\Delta z \to 0$

$$\boxed{-\frac{\partial}{\partial z} V(z,t) = R I(z,t) + L \frac{\partial}{\partial t} I(z,t)}$$

Similarly, apply KCL at node 'n'

$\cancel{\Delta z \to 0}$

$$I(z,t) = I(z+\Delta z, t) + \Delta I$$

$$= I(z+\Delta z, t) + G\Delta z\, V(z+\Delta z, t) + C\Delta z \frac{\partial V(z+\Delta z, t)}{\partial t}$$

(or)

$$-\frac{I(z+\Delta z, t) - I(z,t)}{\Delta z} = G\, V(z+\Delta z, t) + C\cancel{\Delta z}\frac{\partial V(z+\Delta z, t)}{\partial t}$$

$\Delta z \to 0$

$$\boxed{-\frac{\partial I(z,t)}{\partial z} = G\, V(z,t) + C\frac{\partial V(z,t)}{\partial t}} \quad \text{—— } (2)$$

If we assume hormonic time dependence, so that

$$V(z,t) = Re\left[V_s(z)\, e^{j\omega t}\right]$$

$$I(z,t) = Re\left[I_s(z)\, e^{j\omega t}\right]$$

where $V_s(z)$ and $I_s(z)$ are the phasor forms of $V(z,t)$ and $I(z,t)$, eqn (1) and (2) become

$$-\frac{d}{dz}V_s = (R + j\omega L)\, I_s$$

$$-\frac{d}{dz}I_s = (G + j\omega C)\, V_s$$

$$-\frac{d^2}{dz^2}V_s = (R + j\omega L)\frac{d}{dz}I_s$$

$$-\frac{d^2}{dz^2}V_s = -(R + j\omega L)(G + j\omega C)\, V_s$$

$$\frac{d^2}{dz^2}V_s - \gamma^2 V_s = 0 \quad \text{—— } (3)$$

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

Similarly,

$$\frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0 \quad \text{—— (4)}$$

γ – propagation constant,

sollution for eqn (3) and (4)

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

V_0^+, V_0^-, I_0^+ and I_0^- are the wave amplitudes, + and – denotes wave travelling directions. in +z and –z directions.

$$V(z,t) = Re\left[V_s(z)\, e^{j\omega t}\right]$$

$$= V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z)$$

The characteristic impedence of the transmission line (Z_0) is defined as,

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C}$$

(or)

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = R_0 + jX_0$$

A. LOSS LESS LINE ($R=0=G$)

A transmission line is said to be lossless if the conductors of the line are perfect ($\sigma_c=\infty$) and dielectric medium separating them is lossless ($\sigma=0$).

$$R=0=G=0$$

$$\alpha=0, \quad \gamma=j\beta=j\omega\sqrt{LC}$$

$$u=\frac{\omega}{\beta}=\frac{1}{\sqrt{LC}}=f\lambda$$

$$X_0=0, \quad Z_0=R_0=\sqrt{\frac{L}{C}}.$$

B. DISTORTIONLESS LINE ($R/L=G/C$)

For distortionless line, attenuation constant 'α' is frequency independent and β is linearly dependent on frequency.

$$\frac{R}{L}=\frac{G}{C}$$

$$\gamma=\sqrt{RG\left(1+\frac{j\omega L}{R}\right)\left(1+\frac{j\omega C}{G}\right)}$$

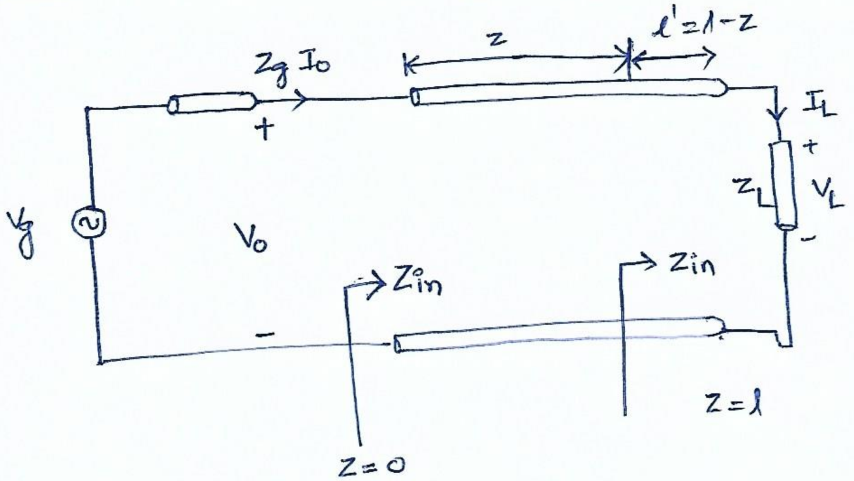
$$=\sqrt{RG}\left(1+\frac{j\omega C}{G}\right)=\alpha+j\beta$$

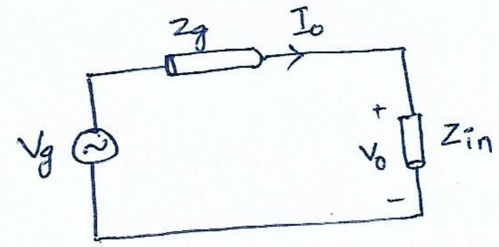
(or) $$\alpha=\sqrt{RG} \qquad \beta=\omega\sqrt{LC}$$

$$Z_0=\sqrt{\frac{R(1+j\omega L/R)}{G(1+j\omega C/G)}}=\sqrt{\frac{R}{G}}=\sqrt{\frac{L}{C}}=R_0+jX_0$$

$$u=\frac{1}{\sqrt{LC}}=f\lambda$$

INPUT IMPEDANCE, STANDING WAVE RATIO AND POWER





equivalent circuit

consider a transmission line of length 'l', characterized by 'γ' and Z_o, connected to a load Z_L.

The generator 'V_g' see the line with load as an input impedance Z_{in}. let the transmission line extend from $z=0$ at the generator to $z=l$ at the load. The voltage and current waves,

$$V_s(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$$

at input terminals of transmission line,

$$V_o = V(z=0), \qquad I_o = I(z=0).$$

Substituting these in the above equations,

$$V_0 = V_0^+ e^{-\gamma(0)} + V_0^- e^{\gamma(0)}$$

$$I_0 = \frac{V_0^+}{Z_0} e^{-\gamma(0)} + \frac{V_0^-}{Z_0} e^{\gamma(0)}$$

$$V_0^+ = \frac{1}{2}(V_0 + Z_0 I_0)$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_0)$$

From the equivalent circuit,

$$V_0 = \frac{Z_{in}}{Z_{in}+Z_g} V_g \quad , \quad I_0 = \frac{V_g}{Z_{in}+Z_g}$$

on the other hand, at load

$$V_L = V(z=l) \quad , \quad I_L = I(z=l)$$

$$V_L = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}$$

$$I_L = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l}$$

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{\gamma l} \quad , \quad V_0^- = \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma l}$$

The input impedance at any point on the line, $Z_{in} = V_s(z)/I_s(z)$

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_0(V_0^+ + V_0^-)}{(V_0^+ - V_0^-)}$$

$$= \frac{Z_0\left[\left(\frac{1}{2} V_L + Z_0 I_L\right) e^{\gamma l} + \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma l}\right]}{\left[\frac{1}{2}(V_L + Z_0 I_L) e^{\gamma l} + \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma l}\right]}$$

$$\tanh \gamma l = \frac{\sinh \gamma l}{\cosh \gamma l} = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]$$

For lossless line $\alpha = 0$, $\gamma = j\beta$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

Reflection co-efficient (Γ_L)

$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Standing wave ratio (SWR)

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = S Z_0$$

$$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{S}$$

The average input power at distance 'l' from the load is given by,

$$P_{avg} = \frac{|V_0^+|^2}{2Z_0}\left(1-|\Gamma|^2\right)$$

A. Shorted line ($Z_L = 0$)

$$Z_{sc} = Z_{in}\Big|_{Z_L=0} = jZ_0 \tan\beta l$$

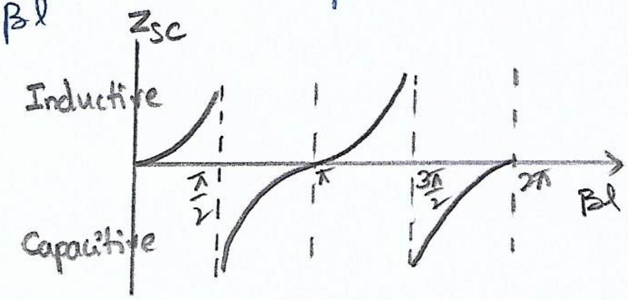
$$\Gamma_L = -1, \quad S = \infty$$

B. open-circuited line ($Z_L = \infty$)

$$Z_{oc} = \lim_{Z_L \to \infty} Z_{in} = \frac{Z_0}{j\tan\beta l} = -jZ_0 \cot\beta l.$$

$$\Gamma_L = 1, \quad S = \infty$$

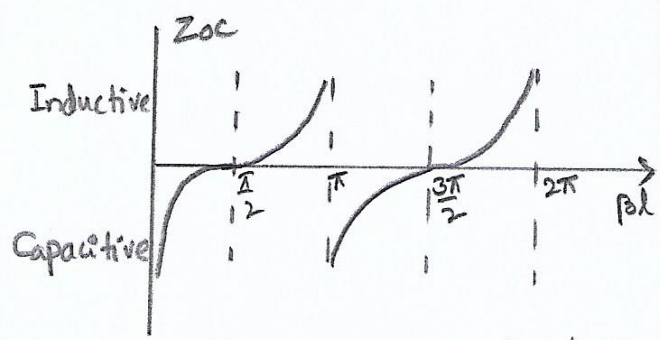
$$Z_{sc} \cdot Z_{oc} = Z_0^2$$



C. Matched line ($Z_L = Z_0$)

$$Z_{in} = Z_0$$

$$\Gamma_L = 0, \quad S = 1$$



I/P Impedence of lossless line a) shorted b) opened

THE SMITH CHART

The smith chart is most commonly used of the graphical techniques. The smith chart employ in calculations of transmission line characteristics such as Γ_L, S, Z_{in}.

The smith chart is constructed using
constant resistance ~~res~~ circles (r- circle)
constant reactance circles (x- circle)
S- circles or constant standing wave ratio circles (s-circle)

Resistance circles,

$$\text{center at } (\Gamma_r, \Gamma_i) = \left(\frac{r}{1+r}, 0\right)$$

$$\text{radius} = \frac{1}{1+r}$$

$$\frac{Z_L}{Z_0} = r + jx$$

Reactance circles,

$$\text{center at } (\Gamma_r, \Gamma_i) = \left(1, \frac{1}{x}\right)$$

$$\text{radius} = \frac{1}{x}$$

S- circles centered at origin with S varying from 1 to ∞.

Some important points about smith chart.

1. At P_{sc}, $Z_L = 0 + j0$, $r = 0, x = 0$, at P_{oc}, $Z_L = \infty + j\infty$

$r = \infty$ and $x = \infty$

2. A complete revolution (360°) around the smith chart represents a distance of $\lambda/2$ on the line.

clockwise movement → moving towards generator

counter clockwise movement → moving toward load

3. The voltage V_{max} occurs where $Z_{in,max}$ is located on chart.

The voltag V_{min} occurs where $Z_{in,min}$ is located on chart.

4. The smith chart is used as impedance chart and admittance chart.

5. There are three scales on peripheory of smith chart to determine the distance from the load or generator.

Problem

A lossless transmission line with $Z_0 = 50\,\Omega$ is 30 m long and operates at 2 MHz. The line is terminated with load $Z_L = 60 + j40\,\Omega$. If $u = 0.6c$ on the line, find

a) The reflection coefficient Γ

b) The standing wave ratio

c) The input impedance.

Sol (without the smith chart)

a) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 + j40 - 50}{60 + j40 + 50} = \frac{10 + j40}{110 + j40}$$

$$\Gamma = 0.3523 \angle 56^\circ$$

b) $$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.3523}{1 - 0.3523} = 2.088.$$

c) $$u = \frac{\omega}{\beta} \quad (\text{or}) \quad \beta l = \frac{\omega l}{u} = \frac{2\pi \times 2\times10^6 \times 30}{0.6 \times 3\times10^8} = \frac{2\pi}{3} = 120^\circ$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

$$Z_{in} = 23.97 + j1.35 \ \Omega$$

Method 2 (using the Smith chart)

(a) Calculate the normalized load impedance

$$z_L = \frac{Z_L}{Z_0} = \frac{60 + j40}{50} = 1.2 + j0.8$$

Locate z_L on smith chart, at point 'p', where $r = 1.2$ circle and the $x = 0.8$ circle meet.

To get Γ at z_L, extend OP to meet the $r = 0$ circle at Q and measure OP and OQ.

$$|\Gamma| = \frac{OP}{OQ} = \frac{3.2\ \text{cm}}{9.1\ \text{cm}} = 0.3516$$

The angle between OS and OP,

$$\theta_\Gamma = \text{angle } POS = 56^\circ$$

Thus $\Gamma = 0.3516 \angle 56^\circ$

(b) To obtain the standing wave ratio 's', draw the circle with radius OP and center at O. The value of 'r' at this point is 's'.

$$S = r \quad (\text{for } r > 1)$$

$$S = 2.1$$

(c) To obtain Z_{in}, first express 'l' in terms of λ (or) degrees.

$$\lambda = \frac{u}{f} = \frac{0.6(3 \times 10^8)}{2 \times 10^6} = 90 \text{ m}$$

$$l = 30 \text{ m} = \frac{30}{90}\lambda = \frac{\lambda}{3} \rightarrow \frac{720^\circ}{3} = 240^\circ$$

That means we move towards the generator 240° on S-circle from point P to point G. At 'G' we obtain

$$z_{in} = 0.47 + j0.03$$

Hence, $Z_{in} = Z_0\, z_{in} = 50(0.47 + j0.03) = 23.5 + j1.5\ \Omega$.

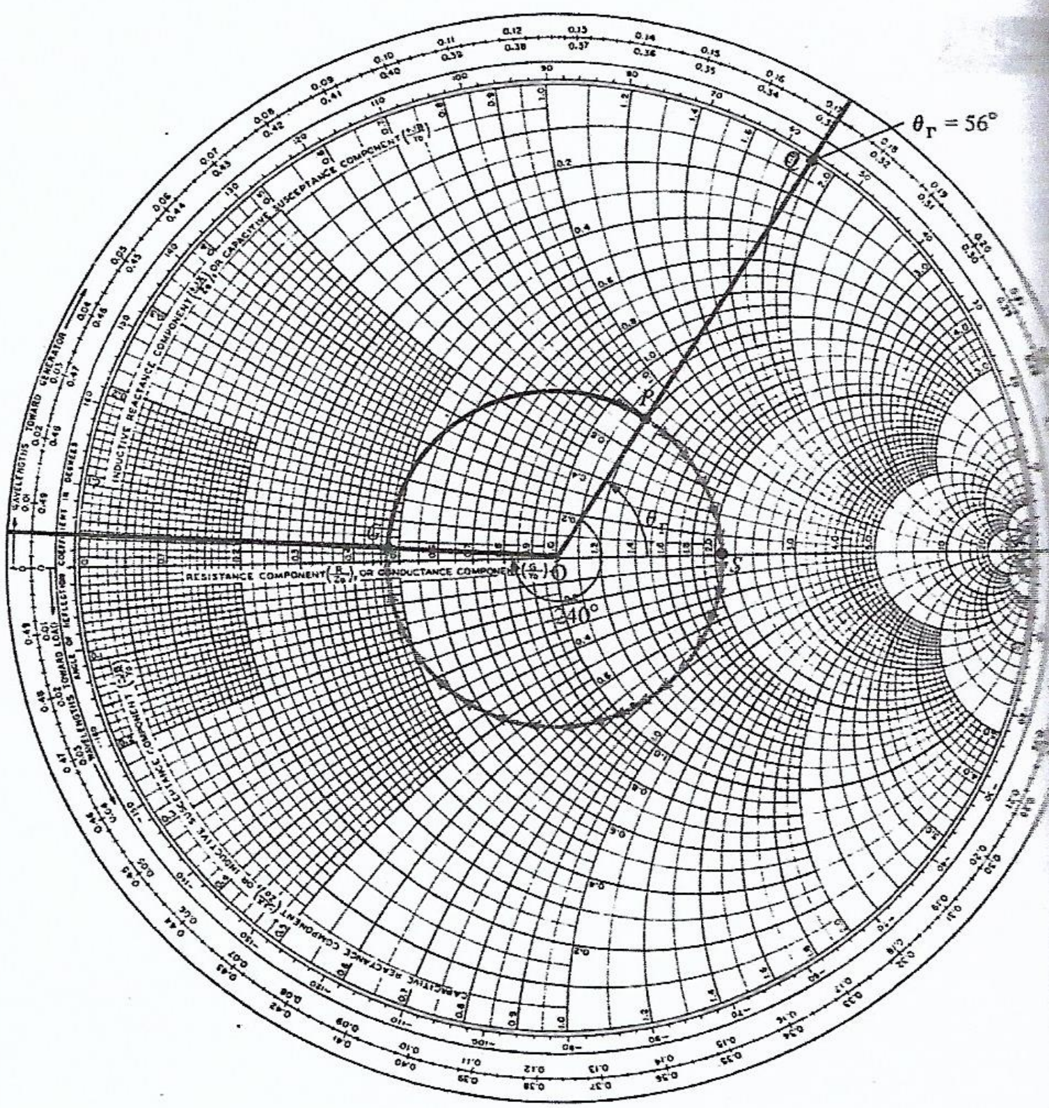


Figure 10.15 Smith chart for Example 10.4.

Note that OP = 3.2 cm and OQ = 9.1 cm were taken from the Smith chart used by the thor; the Smith chart in Figure 10.15 is reduced, but the ratio of OP/OQ remains the

Angle θ_Γ is read directly on the chart as the angle between OS and OP, that is,

$$\theta_\Gamma = \text{angle } POS = 56°$$

Thus

$$\Gamma = 0.3516 \underline{/56°}$$

(b) To obtain the standing wave ratio s, draw a circle with radius OP and center This is the constant s or $|\Gamma|$ circle. Locate point S where the s-circle meets the Γ_r- [This is easily shown by setting $\Gamma_i = 0$ in eq. (10.49a).] The value of r at this point that is

$$s = r \text{ (for } r \geq 1)$$
$$= 2.1$$

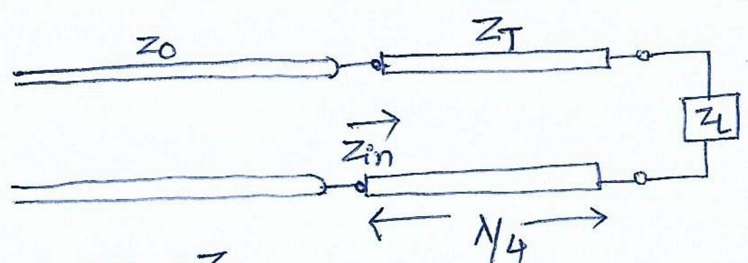
SOME APPLICATIONS OF TRANSMISSION LINES

Transmission lines are used for load matching and impedance measurements.

A. Quarter-wave Transformer (Matching)

when $Z_0 \neq Z_L$, we say that the load is mismatched and reflected wave exists on the line. The matching is achieved by using shorted sections of transmission lines(Z_T).

A quarter wave transformer changes the impedence of the load to another value so that the matching is possible.



we want $Z_{in} = Z_0$

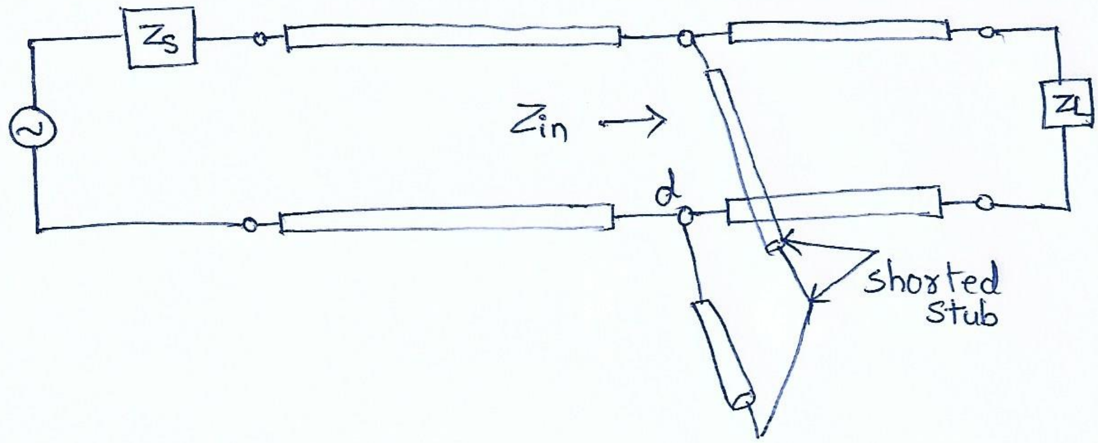
$$Z_{in} = Z_T \frac{Z_L + jZ_T \tan \pi/2}{Z_T + jZ_L \tan \pi/2} = \frac{Z_T^2}{Z_L}$$

for $Z_{in} = Z_0$, $Z_T^2 = Z_0 \cdot Z_L$

$$\boxed{Z_T = \sqrt{Z_0 \cdot Z_L}}$$

B. Single-Stub Tuning

Another device for performing matching is a single stub which is shunted across the transmission line at $z=-d$ from the load.



The location 'd' is choosen so that the admittance looking towards the load is $Y_0 + jB$ ($Y_0 = \frac{1}{Z_0}$). The length of short stub is choosen so that its admittance is $-jB$. Hence, when the stub is connected in parallel to the transmission line at $z = -d$, the impedance $Z_{in} = Z_0$, so that matching condition is achieved.

shorted stub has impedance and admittance given by

$$Z_s = jZ_0 \tan \beta l.$$

$$Y_s = -jY_0 \cot \beta l$$

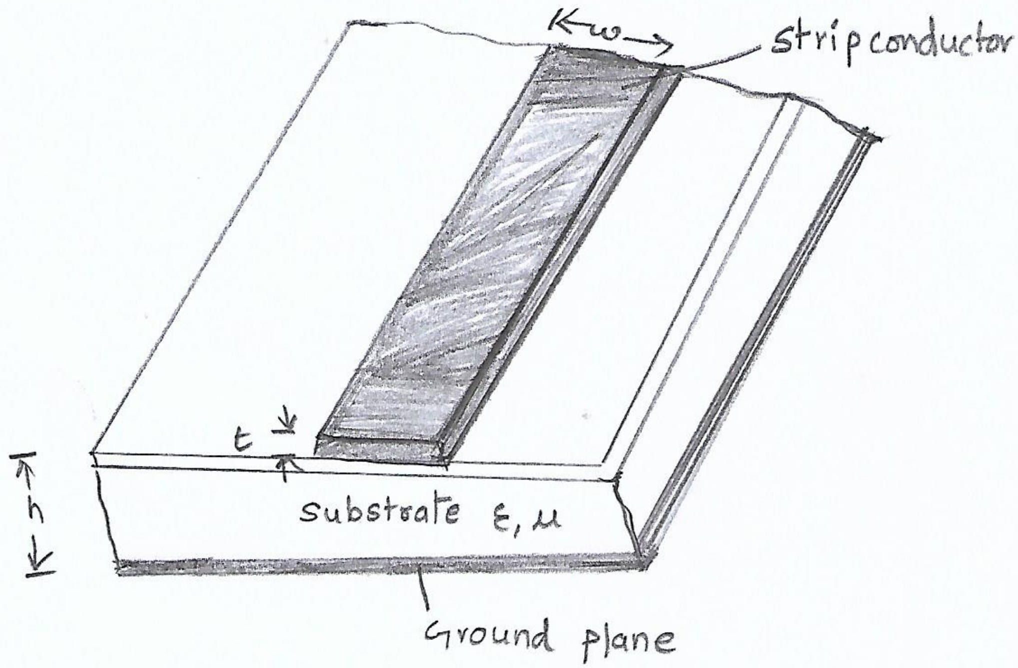
An open-circuited stub can also be used, and the impedance and admittance are given by

$$Z_{op} = -jZ_0 \cot \beta l$$

$$Y_{op} = jY_0 \tan \beta l$$

MICROSTRIP TRANSMISSION LINES

Microstrip line consists of a single ground plane and an open strip conductor separated by dielectric substrate. provided the frequency is not too high, the microstrip line will propagate a wave that, for all practical purposes, is a TEM wave.



The effective relative permittivity, ε_{eff}

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\sqrt{1 + 12\, h/w}}$$

The characteristic impedance, Z_0

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(\dfrac{8h}{w} + \dfrac{w}{4h}\right) & \dfrac{w}{h} \leq 1 \\[2ex] \dfrac{1}{\sqrt{\varepsilon_{eff}}} \dfrac{120\pi}{\left[w/h + 1.393 + 0.667 \ln\left(\dfrac{w}{h} + 1.444\right)\right]} & \dfrac{w}{h} \geq 1 \end{cases}$$

UNIT-I

2 Marks Questions

1. Define coulombs law
2. Define Gauss's law.
3. Classify charge distributions.
4. Define electric field and electric flux density.
5. What is Electric potential? Write relation between **V** and **E**
6. What is dipole? Write the expression for electric potential due to dipole.
7. Define Relaxation time? How it would be for conductor and dielectric materials.
8. Differentiate homogeneous and isotropic dielectric materials.
9. Write the Poisson's and Laplace's equations and their significance.
10. Differentiate convection current and conduction current

10 Marks Questions

1. Define Coulomb's law. Point charges of 1mC and -2mC are located at (3, 2, -1) and (-1, -1, 4) respectively. Calculate electric force (**F**) on a 10 nC charge located at (0, 3, 1).
2. Define electric field intensity. Point charges of 5nC and -2nC are located at (2, 0, 4) and (-3, 0, 5) respectively. Find the electric field **E** at (1, -3, 7).
3. Define electric flux density? Determine **D** at (4, 0, 3) if there is a point charge -5π mC at (4, 0, 0) and a line charge 3π mC/m along the y-axis.
4. State and prove Gauss's law. Express Gauss's law in both integral and differential forms. Also discuss its applications.
5. (a) Derive Possion's and Laplace's equations from fundamentals

   (b) Point charges 1mC and -2mC are located at (3,2,-1) and (-1,-1,4) respectively. Calculate the electric force on a 10nc charge located at(0, 3, 1) and the electric field intensity at that point.
6. Derive the expression of electric field intensity (**E**) for a) Line charge distribution b) Surface charge distribution.
7. Discuss about current continuity equation, Poisson's equation and Laplace's equation.
8. Define electric potential. Derive relation between **E** and **V**

9. (a) Determine electric flux density, **D** at (4, 0, 3) , if there is point charge of -5π mC at (4, 0, 0) and line charge 3π mC/m along the y-axis.

   (b) What are the isotropic and homogeneous dielectric materials.

UNIT-II

2 Marks Questions

1. What is Biot-Savarts's law.
2. Define Amperes circuit law or Amperes law.
3. Define magnetic flux density (B).
4. Explain the law of conservation of magnetic field (or) Gauss law for magnetostatic fields.
5. Write Maxwell's equations derived from the magnetostatics.
6. Write about magnetic force on a moving point charge, line current, surface current and volume current.
7. Define magnetic Torque and (T) and magnetic dipole moment.
8. Compare or differentiate magnetic dipole with electric dipole.
9. Write about self-inductance and mutual inductance.
10. Write the expression for the energy in electrostatic field and magneto static field.

10 Marks Questions

1. Define Ampere's circuit law and Determine the magnetic field intensity (H) due to infinite length of line current and infinite sheet current.
2. (a) A charged particle of mass 2kg and charge 3C starts at point (1, -2, 0) with velocity 4ax + 3az m/s in an electric field 12 ax + 10 ay V/m. At time t = 1sec, find

   (i) The acceleration of the particle, (ii) Its velocity

   (b) Justify that the magnetostatic fields are non-conservative fields.
3. Derive Maxwell's equations for magnetostatic fields.
4. Define Ampere's law. Planes, z=0 and z=4 carry current k= $-10a_x$ A/m and k= $10a_x$ A/m respectively. Determine **H** at (i) (1, 1, 1) (ii) (0, -3, 10).

5. Define Ampere's law. Plane y=1 carries current k= 50 a_x mA/m. Find **H** at (i) (0, 0, 0) (ii) (1, 5, -3)
6. (a) Explain any one of Gauss's law application.

   (b) If an electric field in free space is given by E = $a_x + 2a_y + 5a_z$ V/m, find the Electric flux density when €r = 4.
7. (a) Describe Magnetic torque and moment

   (b) Do Isolated magnetic charge available? Justify your answer
8. Discuss about the magnetic force on different types of charge distribution
9. What is the magnetic force on moving charged particle?. A charged particle moves with a uniform velocity $4a_x$ m/s in a region where **E=** $20a_y$ v/m and **B**=B_0a_z wb/m^2. Determine B_0 such that the velocity of the particle remains constant.
10. (a) Derive any one of the Magnetostatic Maxwell's equation.

    (b) A charge of 12 C has velocity of $5a_x + 2a_y - 3a_z$ m/s. Determine 'F' on the charge in the field of B = $4a_x + 4a_y + 3a_z$ Wb/m^2.

UNIT-III

2 Marks Questions

1. Define faraday's law
2. What is displacement current
3. Write the Maxwell's equations for free space condition.
4. List any two Maxwell's equations for time varying fields.
5. Write the word statements for Maxwell's final equations.
6. How the boundary conditions helps to understand electrostatics
7. Write Maxwell's equations in integral form
8. Calculate J_d if €r = 1 and Ey = 20cosωt
9. What are the different ways in which an emf is induced around a loop?
10. What is inconsistency in Ampere's law?

10 Marks Questions

1. (a) State and explain Faraday's law

   (b) Explain Maxwell's equations integral and differential form with their word statements.
2. (a) Illustrate the need of Displacement current in Electromagnetic fields.

   (b) Explain the boundary conditions of Electromagnetic fields in dielectric to dielectric interface.
3. (a) Prove that Etan1 = Etan2 & Dn1 = Dn2.

   (b) Explain inconsistency of Ampere's law.
4. Represent Maxwell's equations in all possible forms and write word statements.
5. (a) Discuss electromagnetic boundary conditions between conductor and dielectric media.

   (b)What is inconsistency in Ampere's law? How it is rectified by the Maxwell.

UNIT-IV

2 Marks Questions

1. What is uniform plane wave
2. Define intrinsic impedance
3. Define skin depth or depth of penetration
4. Write Helmholtz's wave equation
5. Define poynting theorem
6. Define standing wave ratio. What is its typical range?
7. Define Normal & Oblique incidences.
8. Define reflection coefficient? What is its typical range?
9. Define Brewster angle.
10. Write the expression for attenuation constant and phase constant for free space media.

10 marks Questions

1. Explain the wave propagation in good conductors medium and derive attenuation constant (α), phase constant (β) and intrinsic impedance (η).
2. (a) Derive the wave equations (E&H).

   (b) Write and explain Poynting theorem.
3. Derive wave equations (E&H) for free space and conducting media. Discuss all relations between **E** and **H** of a uniform plane wave.
4. Discuss plane wave propagation in lossless dielectric, free space and good conductor media.
5. (a) Find the depth of penetration, δ of an EM wave in copper at f = 50 kHz for copper, σ = 5 x 107 mho/m, μr = 1, €r = 1.

   (b) Explain the properties of EM wave incident with normal incidence.
6. In a medium E= $16e^{-0.05x}$ sin ($2x10^8$ t – 2x) V/m find (i) Propagation constant. (ii) Wavelength. (iii) Speed of the wave.
7. (a) Describe the wave polarization phenomenon

   (b) Discuss about wave propagation in lossy dielectric media?
8. In a lossless medium η=40π, μ_r=1 and H=2cos(ωt-z)a_x+5sin(ωt-z)a_y A/m. Find ε_r, ω and E.
9. Explain reflection and refraction of plane wave at normal incidence.
10. Explain reflection and refraction of plane wave at oblique incidence.

UNIT-V

2 Marks Questions

1. List different types of transmission lines.
2. What are the primary and secondary parameters of transmission line?
3. Define Distortion less transmission line.
4. Define lossless transmission line.
5. Draw the input impedance variation of a two-wire transmission line.
6. Calculate SWR of a transmission line if reflection Coefficient is 0.6.

7. What is input impedance value for shorted line, open circuited line and Matched line.
8. List out the applications of a smith chart.
9. Differentiate single stub matching and double stub matching.
10. Sketch the schematic of micro strip transmission line.

10 Marks Questions

1. Elucidate the need of a transmission line? Derive the Transmission line equations.
2. (a) Derive the equation for Input Impedance of a two-wire Transmission line?

   (b) Describe the impact of Reflection coefficient on transmission line
3. (a) Draw the equivalent circuit of a 2-wire transmission line. Explain primary & secondary constants.

   (b) Discuss the variation of Z_{in} for a lossless transmission line when the load is SC,OC and matched.
4. (a) Write a short note on lossless and distortion less transmission line.

   (b) A lossless transmission line used in a TV receiver has a capacitance of 50 pF/m and inductance of 200 nH/m. Find the characteristic for sections of a line 10 meters long.
5. Explain about single stub matching and quarter wave transformer.
6. Explain the procedure to find reflection coefficient, standing wave ratio and input impedance of a lossless transmission line using smith chart.
7. A distortionless line has Z_o =60Ω, α=20 mNp/m, u= 0.6c, where c is speed of light in vacuum. Find R, L, G, C and λ at 100 MHz.
8. (a) Define lossless line and distortion less line.

   (b) An airline has a characteristic impedance of 70Ω and phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and capacitance per meter of the line.
9. (a) Discuss quarter wave transformer.

   (b) A lossless transmission line used in a TV receiver has a capacitance of 50 pF/m and inductance of 200 nH/m. Find the characteristic for sections of a line 10 meters long.

10. A lossless transmission line with Z_o=50Ω is 30 m long and operates at 2 MHz. The line is terminated with a load Z_L=60+j40 Ω. If u=0.6c on the line, find

(i) The reflection coefficient

(ii) The standing wave ratio

(iii) The input impedance

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(Autonomous)

II B.Tech II SEM MID-II Examinations June-2024

(ECE)

Subject: Electromagnetics and Transmissions lines(A30414) **Date: 06-06-2024**

Time: 1 hour 30 minutes **SET NO:1** **Max. Marks: 30**

Answer all the Questions

1. (a) State and explain Faraday's law

   (b) Explain Maxwell's equations integral and differential form with their word statements

MARKS:5,5	UNIT- III	CO: 1	Cognitive Level : Understand

(OR)

2. a) Illustrate the need of Displacement current in Electromagnetic fields.

   (b) Explain the boundary conditions of Electromagnetic fields in dielectric to dielectric interface.

MARKS:5,5	UNIT- III	CO: 2	Cognitive Level: Analyze, Understand

3. (a) Describe the wave polarization phenomenon

   (b) Discuss about wave propagation in lossy dielectric media?

MARKS:5,5	UNIT- IV	CO: 3	Cognitive Level: Understand

(OR)

4. In a medium E= $16e^{-0.05x}$ sin ($2x10^8$ t – 2x) V/m find (i) Propagation constant. (ii) Wavelength. (iii) Speed of the wave.

Marks: 10	Unit: IV	CO: 3	Cognitive Level: Apply

5. (a) Derive the equation for Input Impedance of a two-wire Transmission line?

   (b) Describe the impact of Reflection coefficient on transmission line

Marks: 5,5	Unit: V	CO: 4	Cognitive Level: Remember, Understand

(**OR**)

6. (a) Discuss quarter wave transformer.

   (b) A lossless transmission line used in a TV receiver has a capacitance of 50 pF/m and inductance of 200 nH/m. Find the characteristic for sections of a line 10 meters long.

MARKS: 4,6	UNIT- V	CO: 4	COGNITIVE LEVEL: Remember, Apply

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(Autonomous)
II B.Tech II SEM MID-II Examinations June 2024
(ECE)

Subject: Electromagnetics and Transmissions lines(A30414) **Date:06-06-2024**
Time: 1 hour 30 minutes **SET NO: 2** **Max. Marks: 30**

Answer all the Questions

1. (a) Prove that Etan1 = Etan2 & Dn1 = Dn2.
   (b) Explain inconsistency of Ampere's law

MARKS:5,5	UNIT- III	CO: 2	Cognitive Level : Understand

(OR)

2. Represent Maxwell's equations in all possible forms and write word statements.

MARKS: 10	UNIT- III	CO: 1	Cognitive Level : Understand

3. Explain the wave propagation in good conductors medium and derive attenuation constant (α), phase constant (β) and intrinsic impedance (η).

MARKS:10	UNIT- IV	CO: 3	Cognitive Level: Understand

(OR)

4. (a) Describe the wave polarization phenomenon
   (b) Discuss about wave propagation in lossy dielectric media?

Marks: 5,5	Unit: IV	CO: 3	Cognitive Level: Remember

5. A distortionless line has Z_o =60Ω, α=20 mNp/m, u= 0.6c, where c is speed of light in vacuum. Find R, L, G, C and λ at 100 MHz.

Marks: 10	Unit: V	CO: 4	Cognitive Level: Apply

(**OR**)

6. A lossless transmission line with Z_o=50Ω is 30 m long and operates at 2 MHz. The line is terminated with a load Z_L=60+j40 Ω. If u=0.6c on the line, find
   (i) The reflection coefficient
   (ii) The standing wave ratio
   (iii) The input impedance

MARKS: 10	UNIT- V	CO: 4	Cognitive Level: Apply

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II B.Tech II SEM MID-II Examinations June-2024
(ECE)

Subject: Electromagnetics and Transmissions lines(A30414) **Date: 06-06-2024**
Time: 1 hour 30 minutes **SET NO: 3** **Max. Marks: 30**

Answer all the Questions

1. (a) Discuss electromagnetic boundary conditions between conductor and dielectric media.
(b)What is inconsistency in Ampere's law? How it is rectified by the Maxwell.

MARKS:5, 5	UNIT- III	CO: 2	Cognitive Level: Understand, Analyze

(OR)

2. Represent Maxwell's equations in all possible forms and write word statements.

MARKS:10	UNIT- III	CO: 1	Cognitive Level : Understand

3. Analyse the plane wave propagation in lossless dielectric, free space and good conductor media.

MARKS:10	UNIT- IV	CO: 3	Cognitive Level: Analyze

(OR)

4. In a medium E= $16e^{-0.05x}\sin(2x10^8\ t - 2x)$ V/m find (i) Propagation constant. (ii) Wavelength. (iii) Speed of the wave.

Marks: 10	Unit: IV	CO: 3	Cognitive Level: Apply

5. (a) Draw the equivalent circuit of a 2-wire transmission line. Explain primary & secondary constants.
(b) Discuss the variation of Z_{in} for a lossless transmission line when the load is SC,OC and matched.

Marks: 5,5	Unit: V	CO: 4	Cognitive Level: Remember, Analyze

(OR)

6. What is smith chart? Explain the procedure to find reflection coefficient, standing wave ratio and input impedance of a lossless transmission line using smith chart.

MARKS: 10	UNIT- V	CO: 4	Cognitive Level: Remember

Signature of the Staff **Signature of HOD**

G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL
(Autonomous)
II B.Tech II SEM MID-II Examinations June-2024
(ECE)

Subject: Electromagnetics and Transmissions lines(A30414) **Date: 06-06-2024**
Time: 1 hour 30 minutes **SET NO: 4** **Max. Marks: 30**

Answer all the Questions

1. (a) State and explain Faraday's law

   (b) Explain Maxwell's equations integral and differential form with their word statements

MARKS:5,5	UNIT- III	CO: 1	Cognitive Level: Understand

(OR)

2. a) Illustrate the need of Displacement current in Electromagnetic fields.

   (b) Explain the boundary conditions of Electromagnetic fields in dielectric to dielectric interface.

MARKS:5,5	UNIT- III	CO: 2	Cognitive Level: Analyze, Understand

3. (a) Describe the wave polarization phenomenon
   (b) Discuss about wave propagation in lossy dielectric media?

MARKS:5,5	UNIT- IV	CO: 3	Cognitive Level: Understand

(OR)

4. (a) Describe the wave polarization phenomenon
   (b) Discuss about wave propagation in lossy dielectric media?

Marks: 5,5	Unit: IV	CO: 3	Cognitive Level: Remember

5. A distortionless line has Z_o =60Ω, α=20 mNp/m, u= 0.6c, where c is speed of light in vacuum. Find R, L, G, C and λ at 100 MHz.

Marks: 10	Unit: V	CO: 4	Cognitive Level: Apply

(**OR**)

6. A lossless transmission line with Zo=50 Ohms is 30 meters long and operated at 2 MHz. The line is terminated with load Z_L=60+j40 ohms. If u=0.6c on line, find a) reflection coefficient b) standing wave ratio and c) input impedance using smith chart.

Marks: 10	Unit: V	CO: 4	Cognitive Level: Apply

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G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY::KURNOOL

II B.Tech II SEM (R20) MID-I Examinations April-2024

(ECE)

Subject: Electromagnetics & Transmission Lines (A30414) **Date: 04-04-2024**

Time: 20 minutes **Max. Marks: 10**

Roll No.:______________ **SET NO. 1** **Invigilator Signature_________**

1 $\nabla.(\nabla X\boldsymbol{A}) =$ _______________ [ ]
A) 1 B) 0 C) $\nabla.(\nabla.\boldsymbol{A})$ D) Infinity

2 Point Charges Q1=1nC and Q2=2 nC are at a distance apart. Which of the following is correct. [ ]
A) The force on Q1 is repulsive B) Force on Q2 is same magnitude as that on Q1
C) As the distance between Q1 and Q2 increases, force decreases D) All

3 The divergence theorem is ______________ [ ]
A) $\oint A\, dL = \int(\nabla XA)\ dS$ B) $\oint A\, dS = \int(\nabla.A)\ dV$
C) $\oint A\, dL = \int(\nabla.A)\ dS$ D) $\oint A\, dS = \int(\nabla XA)\ dV$

4 The relation between Magnetic Field and vector potential [ ]
A) B=$\nabla X\boldsymbol{E}$ B) **A=** $\nabla X\boldsymbol{F}$ C) $\boldsymbol{B} = \nabla X\boldsymbol{A}$ D) **E=** -∇V

5 Magnetic Force on a charge **Q** moving with velocity **u** is________ [ ]
A)F_m= Qu x B B) F_m =EQ C) F_m= Q x u D) F_m =0

6 The inductance developed in one circuit due to magnetic flux linkage of another circuit is called _____ [ ]
A) Self-inductance B) Mutual-inductance C) Zero inductance D) None

7 Stokes theorem _______ [ ]
A) $\oint A\, dL = \int(\nabla XA)\ dS$ B) $\oint A\, dS = \int(\nabla.A)\ dV$ C) $\oint A\, dL = \int(\nabla.A)\ dS$ D) $\oint A\, dS = \int(\nabla XA)\ dV$

8 The relation between electric field and electric potential is _______________ [ ]
A) B=$\nabla X\boldsymbol{E}$ B) **A=** $\nabla X\boldsymbol{F}$ C) $\boldsymbol{B} = \nabla X\boldsymbol{A}$ D) **E=** -∇V

9 The inductance developed in one circuit or coil due to magnetic flux linkage of another circuit is called _____ [ ]
A) Self-inductance B) Mutual-inductance C) Zero inductance D) None

10 The electric flux density **D** =_________________ [ ]
A) $\in \boldsymbol{E}$ B) $\mu\boldsymbol{H}$ C) ρv D) 0

11 The electric field due to a line charge of density $\rho l\left(\frac{C}{m}\right)$ is **E** =______________________

12 The conduction current density, J=__________________

13 The magnetic field due to a surface current of density (K) is **H=**_____________________

14 The electric force on a point charge (**F**)=__________________

15 The magnetic torque (T)=__________________________

16	Gauss's Law	[ ]	A) $\nabla^2 V = 0$
17	Ampere's Law	[ ]	B) $\nabla.J = -\frac{\partial}{\partial t}\rho\vartheta$
18	Faraday's Law	[ ]	C) $\nabla.\boldsymbol{D} = \rho v$
19	Laplacian equation	[ ]	D) . $\nabla X\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}$
20	Continuity equation	[ ]	E) $\nabla X\boldsymbol{H} = \boldsymbol{J}$

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G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY::KURNOOL
II B.Tech II SEM (R20) MID-I Examinations April-2024
(ECE)

Subject: Electromagnetics & Transmission Lines (A30414) **Date: 04-04-2024**
Time: 20 minutes **Max. Marks: 10**
Roll No.:______________ **SET NO. 2** **Invigilator Signature_________**

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1 $\nabla.(\nabla X\boldsymbol{A}) =$ _______________ [ ]
A) 0 B) 1 C) $\nabla.(\nabla.\boldsymbol{A})$ D) Infinity

2 Point Charges Q1=1nC and Q2=2 nC are at a distance apart. Which of the following is correct. [ ]
A) The force on Q1 is repulsive B) Force on Q2 is same magnitude as that on Q1
C) As the distance between Q1 and Q2 increases, force decreases D) All

3 The divergence theorem is ______________ [ ]
A) $\oint A\, dL = \int(\nabla XA)\ dS$ B) $\oint A\, dS = \int(\nabla.A)\ dV$
C) $\oint A\, dL = \int(\nabla.A)\ dS$ D) $\oint A\, dS = \int(\nabla XA)\ dV$

4 The inductance developed in one circuit due to magnetic flux linkage of same circuit is called _____ [ ]
A) Self-inductance B) Mutual-inductance C) Zero inductance D) None

5 Magnetic Force on a charge **Q** moving with velocity **u** is________ [ ]
A)F_m= Qu x B B) F_m =EQ C) F_m= Q x u D) F_m =0

6 The relation between Magnetic Field and vector potential
A) B=$\nabla X\boldsymbol{E}$ B) **A**= $\nabla X\boldsymbol{F}$ C) $\boldsymbol{B} = \nabla X\boldsymbol{A}$ D) **E**= -∇V

7 Stokes theorem _______ [ ]
A) $\oint A\, dL = \int(\nabla XA)\ dS$ B) $\oint A\, dS = \int(\nabla.A)\ dV$ C) $\oint A\, dL = \int(\nabla.A)\ dS$ D) $\oint A\, dS = \int(\nabla XA)\ dV$

8 The relation between electric field and electric potential is _______________ [ ]
A) B=$\nabla X\boldsymbol{E}$ B) **A**= $\nabla X\boldsymbol{F}$ C) $\boldsymbol{B} = \nabla X\boldsymbol{A}$ D) **E**= -∇V

9 The inductance developed in one circuit or coil due to magnetic flux linkage of another circuit is called _____ [ ]
A) Self-inductance B) Mutual-inductance C) Zero inductance D) None

10 The electric flux density **D** =_________________ [ ]
A) $\in \boldsymbol{E}$ B) $\mu\boldsymbol{H}$ C) ρv D) 0

11 The electric field due to a line charge of density $\rho l\left(\frac{C}{m}\right)$ is **E** =______________________

12 The displacement current density, Jd=_________________

13 The magnetic field due to a line current is **H=**_____________________

14 The electric force on a point charge (**F**)=__________________

15 The magnetic torque (T)=__________________________

16 Gauss's Law [ ] A) . $\nabla X\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}$

17 Ampere's Law [ ] B) $\nabla.J = -\frac{\partial}{\partial t}\rho\vartheta$

18 Faraday Law [ ] C) $\nabla.\boldsymbol{D} = \rho v$

19 Laplacian equation [ ] D) $\nabla^2 V = 0$

20 Continuity equation [ ] E) $\nabla X\boldsymbol{H} = \boldsymbol{J}$

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G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY::KURNOOL
II B.Tech II SEM (R20) MID-I Examinations April-2024
(ECE)

Subject: Electromagnetics & Transmission Lines (A30414) **Date: 04-04-2024**
Time: 20 minutes **Max. Marks: 10**
Roll No.:______________ **SET NO. 3** **Invigilator Signature_________**

1 The electric flux density **D** =_________________ [ ]
A) $\in \boldsymbol{E}$ B) $\mu \boldsymbol{H}$ C) ρv D) 0

2 The divergence theorem is _______________ [ ]
$A)$ $\oint A\, dL = \int (\nabla X A)\ dS$ B) $\oint A\, dS = \int (\nabla . A)\ dV$
C) $\oint A\, dL = \int (\nabla . A)\ dS$ D) $\oint A\, dS = \int (\nabla X A)\ dV$

3 The relation between Magnetic Field and vector potential [ ]
A) B=$\nabla X\boldsymbol{E}$ B) **A**= $\nabla X\boldsymbol{F}$ C) $\boldsymbol{B} = \nabla X\boldsymbol{A}$ D) **E**= -∇V

4 Magnetic Force on a charge **Q** moving with velocity **u** is________ [ ]
A)F_m= Qu x B B) F_m =EQ C) F_m= Q x u D) F_m =0

5 The inductance developed in one circuit due to magnetic flux linkage of another circuit is called _____ [ ]
A) Self-inductance B) Mutual-inductance C) Zero inductance D) None

6 Stokes theorem ________ [ ]
A) $\oint A\, dL = \int (\nabla X A)\ dS$ B) $\oint A\, dS = \int (\nabla . A)\ dV$ C) $\oint A\, dL = \int (\nabla . A)\ dS$ D) $\oint A\, dS = \int (\nabla X A)\ dV$

7 Point Charges Q1=1nC and Q2=2 nC are at a distance apart. Which of the following is correct. [ ]
A) The force on Q1 is repulsive B) Force on Q2 is same magnitude as that on Q1
C) As the distance between Q1 and Q2 increases, force decreases D) All

8 $\nabla.(\nabla X\boldsymbol{A}) =$ _______________
A) 1 B) 0 C) $\nabla.(\nabla.\boldsymbol{A})$ D) Infinity

9 The relation between electric field and electric potential is _______________ [ ]
A) B=$\nabla X\boldsymbol{E}$ B) **A**= $\nabla X\boldsymbol{F}$ C) $\boldsymbol{B} = \nabla X\boldsymbol{A}$ D) **E**= -∇V

10 The inductance developed in one circuit or coil due to magnetic flux linkage of another circuit is called _____ [ ]
A) Self-inductance B) Mutual-inductance C) Zero inductance D) None

11 The electric field due to a surface charge of density $\rho s \left(\frac{C}{m2}\right)$ is **E** =______________________

12 The convection current density, J=__________________

13 The magnetic field due to a surface current of density (K) is **H**=_____________________

14 The electric force on a point charge (**F**)=__________________

15 The magnetic torque (T)=__________________________

16 Gauss's Law [ ] A) $\nabla X\boldsymbol{H} = \boldsymbol{J}$

17 Ampere's Law [ ] B) $\nabla . J = -\frac{\partial}{\partial t}\rho\vartheta$

18 Faraday's Law [ ] C) $\nabla.\boldsymbol{D} = \rho v$

19 Laplacian equation [ ] D) . $\nabla X\boldsymbol{E}$ = - $\frac{\partial}{\partial t}\boldsymbol{B}$

20 Continuity equation [ ] E) $\nabla^2 V = 0$

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G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY::KURNOOL
II B.Tech II SEM (R20) MID-I Examinations April-2024
(ECE)

Subject: Electromagnetics & Transmission Lines (A30414) **Date: 04-04-2024**
Time: 20 minutes **Max. Marks: 10**
Roll No.:______________ **SET NO. 4** **Invigilator Signature_________**

1 $\nabla.(\nabla X\boldsymbol{A}) =$ _______________ [ ]
A) 1 B) 0 C) $\nabla.(\nabla.\boldsymbol{A})$ D) Infinity

2 Point Charges Q1=1nC and Q2=2 nC are at a distance apart. Which of the following is correct. [ ]
A) The force on Q1 is repulsive B) Force on Q2 is same magnitude as that on Q1
C) As the distance between Q1 and Q2 increases, force decreases D) All

3 The divergence theorem is ______________ [ ]
$A)$ $\oint A\, dL = \int(\nabla XA)\ dS$ B) $\oint A\, dS = \int(\nabla.A)\ dV$
C) $\oint A\, dL = \int(\nabla.A)\ dS$ D) $\oint A\, dS = \int(\nabla XA)\ dV$

4 The relation between Magnetic Field and vector potential [ ]
A) B=$\nabla X\boldsymbol{E}$ B) **A=** $\nabla X\boldsymbol{F}$ C) $\boldsymbol{B} = \nabla X\boldsymbol{A}$ D) **E=** -∇V

5 Magnetic Force on a charge **Q** moving with velocity **u** is________ [ ]
A)F_m= Qu x B B) F_m =EQ C) F_m= Q x u D) F_m =0

6 The inductance developed in one circuit due to magnetic flux linkage of another circuit is called _____ [ ]
A) Self-inductance B) Mutual-inductance C) Zero inductance D) None

7 Stokes theorem _______ [ ]
A) $\oint A\, dL = \int(\nabla XA)\ dS$ B) $\oint A\, dS = \int(\nabla.A)\ dV$ C) $\oint A\, dL = \int(\nabla.A)\ dS$ D) $\oint A\, dS = \int(\nabla XA)\ dV$

8 The relation between electric field and electric potential is _______________ [ ]
A) B=$\nabla X\boldsymbol{E}$ B) **A=** $\nabla X\boldsymbol{F}$ C) $\boldsymbol{B} = \nabla X\boldsymbol{A}$ D) **E=** -∇V

9 The inductance developed in one circuit or coil due to magnetic flux linkage of another circuit is called _____ [ ]
A) Self-inductance B) Mutual-inductance C) Zero inductance D) None

10 The electric flux density **D =**_________________ [ ]
A) $\in \boldsymbol{E}$ B) $\mu\boldsymbol{H}$ C) ρv D) 0

11 The magnetic field due to a surface current of density (K) is **H=**_____________________
12 The conduction current density, J=__________________
13 The magnetic torque (T)=__________________________
14 The electric force on a point charge (**F**)=__________________
15 The electric field due to a line charge of density $\rho l\ \left(\frac{C}{m}\right)$ is **E** =______________________

16	Gauss's Law	[ ]	A) $\nabla^2 V = 0$
17	Ampere's Law	[ ]	B) $\nabla.J = -\frac{\partial}{\partial t}\rho\vartheta$
18	Faraday's Law	[ ]	C) $\nabla X\boldsymbol{H} = \boldsymbol{J}$
19	Laplacian equation	[ ]	D) . $\nabla X\boldsymbol{E}$ = - $\frac{\partial}{\partial t}\boldsymbol{B}$
20	Continuity equation	[ ]	E) $\nabla.\boldsymbol{D} = \rho v$

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G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL

(Autonomous)

II B.Tech II SEM MID-I Examinations April-2024

(ECE)

Subject: Electromagnetics and Transmissions lines(A30414) **Date: 04-04-2024**

Time: 1 hour 30 minutes **SET NO:1** **Max. Marks: 30**

Answer all the Questions

1. Explain about different types of charge distributions. Two point charges 5 nC and -2nC are located at (2, 0, 4) and (-3, 0, 5) respectively. Determine the force on 1 nC point charge located at (1,-3, 7) and electric field (**E**) at (1,-3, 7).

MARKS:10	UNIT- I	CO: 1	Cognitive Level : Understand, Apply

(OR)

2. (a) Derive Maxwell's equation from the Gauss's law for electrostatic fields?
(b) Two planes x=2 and y= -3, respectively carry charges 10 nC/m^2 and 15 nC/m^2. If the line x=0, z=2 carries charge 10π nC/m, calculate **E** at (1, 1, -1) due to the three charge distributions.

MARKS:5,5	UNIT- I	CO: 1	Cognitive Level : Understand, Apply

3. Explain about electric flux and flux density? A point charge of 30nC is located at origin, while plane y=3 carries charge 10 nC/m^2. Find electric flux density at (0,4,3).

MARKS:10	UNIT- I	CO: 1	Cognitive Level: Apply

(OR)

4. Explain Ampere's law? Determine magnetic field intensity (**H**) due to infinite line current and sheet current.

Marks: 10	Unit: II	CO: 1	Cognitive Level: Apply

5. a) Describe magnetic torque and moment.
b) Do isolated magnetic charge exist ? Justify your answer.

Marks: 5,5	Unit: II	CO: 1	Cognitive Level: Remember, Understand

(**OR**)

6. a) Discuss about magnetic force on different current distributions.

b) A charged particle moves with a uniform velocity $4a_x$ m/s in a region where **E=** $20a_y$ v/m and **B**=B_0a_z wb/m^2. Determine B_0 such that the velocity of the particle remains constant.

MARKS: 4,6	UNIT- II	CO: 1	COGNITIVE LEVEL: Remember, Apply

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G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL
(Autonomous)
II B.Tech II SEM MID-I Examinations April 2024
(ECE)

Subject: Electromagnetics and Transmissions lines(A30414) **Date:04-04-2024**
Time: 1 hour 30 minutes **SET NO: 2** **Max. Marks: 30**

Answer all the Questions

1. (a) Explain Gauss's law applications in brief.
(b) Point charges of 5nC and -2nC are located at (2, 0, 4) and (-3, 0, 5) respectively. Calculate electric force (**F**) on a 1 nC charge located at (1, -3, 7).

MARKS:5,5	UNIT- I	CO: 1	Cognitive Level : Understand, Apply

(OR)

2. (a) Derive current continuity equation.
(b) A circular ring of radius 'a' carries a uniform charges ρ_LC /m and is placed on the xy-plane with axis the same as z-axis. Determine E(0,0,h).

MARKS:5,5	UNIT- I	CO: 1	Cognitive Level : Understand, Apply

3. Determine electric flux density due point charge, line charge and surface charge? Determine electric flux density (**D**) at (4, 0, 3), if there is a point charge -5π mC at (4, 0, 0) and a line charge 3π mC/m along the y-axis.

MARKS:10	UNIT- I	CO: 1	Cognitive Level: Apply

(OR)

4. Explain Ampere's law? Determine magnetic field intensity (**H**) due to infinite line current and sheet current.

Marks: 10	Unit: II	CO: 2	Cognitive Level: Apply

5. a) State and Explain Biot-Savart's Law
b) Magnetic flux through any closed surface is zero. Justify your answer

Marks: 5,5	Unit: II	CO: 2	Cognitive Level: Remember, Analyze

(**OR**)

6. a) Discuss about Self-inductance and Mutual inductance.
b) A charged particle moves with a uniform velocity 4a_x m/s in a region where **E=** 20a_y v/m and **B**=B_0a_z wb/m^2. Determine B_0 such that the velocity of the particle remains constant.

MARKS: 4,6	UNIT- II	CO: 1	COGNITIVE LEVEL: Remember, Apply

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G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL
(Autonomous)
II B.Tech II SEM MID-I Examinations April-2024
(ECE)

Subject: Electromagnetics and Transmissions lines(A30414) **Date: 04-04-2024**
Time: 1 hour 30 minutes **SET NO: 3** **Max. Marks: 30**

Answer all the Questions

1. a) Explain Coulombs law? Find force on a point charge 'Q' due to 'n' number of point charges.
   b) Point charges of 1mC and -2mC are located at (3, 2, -1) and (-1, -1, 4) respectively. Calculate electric force (**F**) on a 10 nC charge located at (0, 3, 1).

MARKS:4, 6	UNIT- I	CO: 1	Cognitive Level : Understand, Apply

(OR)

2. Define Electric field intensity? Find the force on a point charge 'q' located at (0,0, h) due to a surface charge of density ρ_s (C/m^2) uniformly distributed over the circular disc $\rho \leq a$, Z=0 m.

MARKS:10	UNIT- I	CO: 1	Cognitive Level : Apply

3. (a) By applying Gauss's law determine electric flux density due to a line charge.
   (b) Determine electric flux density (**D**) at (0, 4, 3), if a point charge of 30 nC is located at the origin, while y=3 carries charge 10 nC/m^2.

MARKS:5,5	UNIT- I	CO: 1	Cognitive Level: Apply, Apply

(OR)

4. a) Distinguish between self-inductance and mutual inductance.
   b) What is Ampere's law? Determine magnetic field intensity (**H**) due to infinite line current.

Marks: 5,5	Unit: II	CO: 1	Cognitive Level: Analyze, Understand

5. a) Discuss about different types current and magnetic field due to them with help of Biot-Savart's Law.
   b) Do isolated magnetic charge available? Justify your answer.

Marks: 5,5	Unit: II	CO: 1	Cognitive Level: Remember, Understand

(**OR**)

6. a) Write short note magnetic force ($\mathbf{F_m}$)
   b) A current sheet Z=1 carries current **K**= 50 a_x mA/m. Find **H** at (i) (0, 0, 10) (ii) (1, 5, -3)

MARKS: 4,6	UNIT- II	CO: 1	COGNITIVE LEVEL: Remember, Apply

Signature of the Staff **Signature of HOD**

G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL
(Autonomous)
II B.Tech II SEM MID-I Examinations April-2024
(ECE)

Subject: Electromagnetics and Transmissions lines(A30414) **Date: 04-04-2024**
Time: 1 hour 30 minutes **SET NO: 4** **Max. Marks: 30**

Answer all the Questions

1. a) What are isotropic and homogeneous dielectric materials.
   b) Point charges of 5nC and -2nC are located at (2, 0, 4) and (-3, 0, 5) respectively. Calculate electric force (**F**) on a 1 nC charge located at (1, -3, 7).

MARKS:4, 6	UNIT- I	CO: 1	Cognitive Level : Remember, Apply

(OR)

2. (a) Derive relation between **E** and **V**.
   (b) Point charges 1mC and -2mC are located at (3, 2, -1) and (-1, -1, 4) respectively. Determine electric field intensity (**E**) at (0, 3, 1).

MARKS:5,5	UNIT- I	CO: 1	Cognitive Level : Understand, Apply

3. a) Derive Possion's and Laplace's equations from fundamentals
   b) The point charges, -1nC, 4nC, and 3nC are located at (0,0,0), (0,0,1) and (1,0,0) respectively. Find energy in the system.

MARKS:5,5	UNIT- I	CO: 1,5	Cognitive Level: Understand, Apply

(OR)

4. Explain Ampere's law? Apply Ampere's law and determine magnetic field intensity (**H**) due to line current and sheet current.

Marks: 10	Unit: II	CO: 2	Cognitive Level: Apply

5. a) State and Explain Biot-Savart's Law
   b) Magnetic flux through any closed surface is zero. Justify your answer

Marks: 5,5	Unit: II	CO: 1	Cognitive Level: Remember, Analyze

(**OR**)

6. a) Explain Torque and magnetic dipole.

   b) A current sheet Z=4 carries current **K**= 25 a_y mA/m. Find H at (i) (0, 0, 10) (ii) (1, 5, -3)

MARKS: 4,6	UNIT- II	CO: 1	COGNITIVE LEVEL: Remember, Apply

Signature of the Staff **Signature of HOD**

G. PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL

II B.Tech II SEM (R20) MID-II Examinations, JUNE -2024

(ECE)

Subject: Electromagnetics & Transmission Lines (A30414) **Date:** 06-06-2024

Time: 20 minutes **Max. Marks:**10

Roll No.:_______________ **SET NO. 3** **Invigilator Signature**_________

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1. The range of standing wave ratio is________________ [ ]
   A) 1 to 10 B) 1 to α C) 0 to 1 D) -1 to 0
2. For loss less transmission line [ ]
   A) R=0=G B) R/L = G/C C) R=G D) R=G=1
3. The input impedance of the short circuited line(Z_{in}) =______________ [ ]
   A) α B) 0 C) $jZ_0\tan\beta l$ D) $-jZ_0\cot\beta l$
4. The displacement current density (J_D)=__________________ [ ]
   A) $\frac{\partial D}{\partial t}$ B) $\frac{\partial E}{\partial t}$ C) $\frac{\partial A}{\partial t}$ D) None
5. The induced emf, V_{emf} = _______________ [ ]
   A) $-\frac{\partial B}{\partial t}$ B) $-\frac{\partial \psi}{\partial t}$ C) $\frac{\partial D}{\partial t}$ D) None
6. The boundary condition for conductor and free space media is ______ [ ]
   A) $D_t=0$ B) $D_n=\rho_s$ C) Both A and B D) None
7. The skin depth is (δ)= ___________ [ ]
   A) 0 B) 1/α C) $(\alpha)^2$ D) None
8. The propagation constant , ϒ =______ [ ]
   A) α-jβ B) α+jβ C) $(1/\alpha)^2$ D) 0
9. The wave velocity (u)_________________ [ ]
   A) $\frac{\alpha}{\beta}$ B) $\frac{2\pi}{\beta}$ C) $\frac{\omega}{\beta}$ D) $\frac{c}{f}$
10. For free space, the conductivity σ = __________ [ ]
    A) 0 B) 1 C) 0.1 D) Infinite
11. The Poynting vector (P) =_____________
12. The conduction current density, J_c =_______________
13. The loss tangent, tanθ=______________
14. The length of quarter wave transformer is ________________
15. A lossless TX line has the characteristic impedance of 50Ω connected a load of 100+j50 Ω, the normalized load impedance______________________
16. The primary constants of a Tx line are R, L ,G and C (T/F)
17. The induced V_{emf} opposes the flux linkage with the circuit (T/F)
18. The time varying electric flux produces magnetic field (T/F)
19. The circularly polarized plane wave travelling in z- direction have Ex=Ey (T/F)
20. The reflection coefficient range varies from -1 to 1 (T/F)

G.PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL

II B.Tech II SEM (R20) MID-II Examinations, JUNE-2024

(ECE)

Subject: Electromagnetics & Transmission Lines (A30414) **Date:**06-06-2024

Time: 20 minutes **Max. Marks:**10

Roll No.:_______________ **SET NO. 4** **Invigilator Signature**_________

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1. The boundary condition for conductor and free space media is ______ [ ]
A) $D_t = 0$ B) $D_n = \rho_s$ C) Both A and B D) None

2 The intrinsic impedance (η) =_________________ [ ]
A) E_o B) H_o C) E_o/H_o D) None

3 The propagation constant , Υ =______ [ ]
A) α-jβ B) α+jβ C) β+jα D) jβ

4 The wave velocity_________________ [ ]
A) $\frac{\alpha}{\beta}$ B) $\frac{2\pi}{\beta}$ C) $\frac{\omega}{\beta}$ D) $\frac{c}{f}$

5 In free space medium the conductivity, σ = __________ [ ]
A) 0 B) 1 C) 0.1 D) Infinite

6 The range of reflection coefficient ($|\Gamma|$) is________________ [ ]
A) 1 to 10 B) 0 to α C) 0 to 1 D) -1 to 0

7 For lossless transmission line [ ]
A) R=0=G B) R/L = G/C C) R=G D) R=G=1

8 The input impedance of the short circuited line(Z_{in}) =______________ [ ]
A) α B) 0 C) $jZ_0\tan\beta l$ D) - $jZ_0\cot\beta l$

9 The displacement current density (J_D)=__________________ [ ]
A) $\frac{\partial D}{\partial t}$ B) $\frac{\partial E}{\partial t}$ C) $\frac{\partial A}{\partial t}$ D) None

10 The induced emf, V_{emf} = _______________ [ ]
A) - $\frac{\partial B}{\partial t}$ B) - $\frac{\partial \psi}{\partial t}$ C) $\frac{\partial D}{\partial t}$ D) None

11 The conduction current density=_______________

12 The loss tangent, $\tan\theta$=______________

13 The characteristic impedance (Z_o) of a 2-wire transmission line =________________

14 A lossless TX line has the characteristic impedance of 50Ω connected a load of 100+j50 Ω, the normalized load impedance______________________

15 Poynting vector, **P**=_____________

16 The primary constants of a Tx line are only L and C (T/F)

17 The induced V_{emf} opposes the flux linkage with the circuit (T/F)

18 The time varying electric flux produces electric field (T/F)

19 The circularly polarized plane wave travelling in z- direction have Ex=Ey (T/F)

20 The standing wave ratio 's' range varies from 0 to α (T/F)

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G.PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL

II B.Tech II SEM (R20) MID-II Examinations, JUNE-2024

(ECE)

Subject: Electromagnetics & Transmission Lines (A30414) **Date:** 06-06-2024

Time: 20 minutes **Max. Marks:**10

Roll No.:_______________ **SET NO. 1** **Invigilator Signature**_________

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1. The boundary condition for Dielectric-Dielectric media is ______ [ ]
A) $D_t=0$ B) $D_n=\rho_s$ C) Both A and B D) None

2 The intrinsic impedance (η) =_________________ [ ]
A) $E_o.H_o$ B) H_o/E_o C) E_o/H_o D) None

3 The propagation constant , ϒ =______ [ ]
A) α-jβ B) α+jβ C) β+jα D) jβ

4 The wave velocity_________________ [ ]
A) $\frac{\alpha}{\beta}$ B) $\frac{2\pi}{\beta}$ C) $\frac{\omega}{\beta}$ D) $\frac{c}{f}$

5 In good conducting medium the conductivity, σ = __________ [ ]
A) 0 B) 1 C) 0.1 D) Infinite

6 The range of reflection coefficient (| Γ|) is________________ [ ]
A) 1 to 10 B) 0 to α C) 0 to 1 D) -1 to 0

7 For distortion less transmission line [ ]
A) R=0=G B) R/L = G/C C) R=G D) R=G=1

8 The input impedance of the short circuited line(Z_{in}) =______________ [ ]
A) α B) 0 C) $jZ_0\tan\beta l$ D) - $jZ_0\cot\beta l$

9 The displacement current density (J_D)=__________________ [ ]
A) $\frac{\partial D}{\partial t}$ B) $\frac{\partial E}{\partial t}$ C) $\frac{\partial A}{\partial t}$ D) None

10 The induced emf, V_{emf} = _______________ [ ]
A) $-\frac{\partial B}{\partial t}$ B) $-\frac{\partial \psi}{\partial t}$ C) $\frac{\partial D}{\partial t}$ D) None

11 The modified Amperes law, $\nabla X H =$_____________

12 The skin depth (δ)=______________

13 Z_o for short circuited transmission line =_______________

14 A lossless TX line has the characteristic impedance of 50Ω connected a load of 100+j50 Ω, the normalized load impedance______________________

15 Poynting vector **P**=_____________

16 The primary constants of a Tx line are only L and C (T/F)

17 The induced V_{emf} opposes the flux linkage with the circuit (T/F)

18 The time varying electric flux produces electric field (T/F)

19 The elliptically polarized plane wave travelling in z- direction have Ex=Ey (T/F)

20 The standing wave ratio 's' range varies from 0 to α (T/F)

G.PULLAIAH COLLEGE OF ENGINEERING AND TECHNOLOGY:: KURNOOL

II B.Tech II SEM (R20) MID-II Examinations, JUNE-2024

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Time: 20 minutes **Max. Marks:**10

Roll No.:_______________ **SET NO. 2** **Invigilator Signature**_________

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1. The boundary condition for conductor and free space media is ______ [ ]
   A) $D_t = 0$ B) $D_n = \rho_s$ C) Both A and B D) None
2. The time varying current produces_________________ [ ]
   A) E field B) H field C) E and H field D) None
3. The propagation constant , ϒ =______ [ ]
   A) α-jβ B) α+jβ C) β+jα D) jβ
4. The wave velocity_________________ [ ]
   A) $\frac{\alpha}{\beta}$ B) $\frac{2\pi}{\beta}$ C) $\frac{\omega}{\beta}$ D) $\frac{c}{f}$
5. In free space medium the conductivity, σ = __________ [ ]
   A) 0 B) 1 C) 0.1 D) Infinite
6. The range of reflection coefficient (|Γ|) is________________ [ ]
   A) 1 to 10 B) 0 to α C) 0 to 1 D) -1 to 0
7. For ldistortionless transmission line [ ]
   A) R=0=G B) R/L = G/C C) R=G D) R=G=1
8. The input impedance of the short circuited line(Z_{in}) =______________ [ ]
   A) α B) 0 C) $jZ_0\tan\beta l$ D) $-jZ_0\cot\beta l$
9. The displacement current density (J_D)=__________________ [ ]
   A) $\frac{\partial D}{\partial t}$ B) $\frac{\partial E}{\partial t}$ C) $\frac{\partial A}{\partial t}$ D) None
10. The induced emf, V_{emf} = ________________ [ ]
    A) $-\frac{\partial B}{\partial t}$ B) $-\frac{\partial \psi}{\partial t}$ C) $\frac{\partial D}{\partial t}$ D) None
11. The conduction current density (Jc)=________________
12. The loss tangent, tanθ=_______________
13. The intrinsic impedance (η) =__________________
14. A lossless TX line has the characteristic impedance of 50Ω connected a load of 100+j50 Ω, the normalized load impedance_______________________
15. The average power of plane wave P_{avg} =______________
16. The primary constants of a Tx line are R,L,G and C (T/F)
17. The induced V_{emf} opposes the flux linkage with the circuit (T/F)
18. The reflection coefficient (Γ) is -1 when medium-I is dielectric and medium-II is conducting medium (T/F)
19. The circularly polarized plane wave travelling in z- direction have Ex=Ey (T/F)
20. The standing wave ratio ‘s’ range varies from 0 to α (T/F)