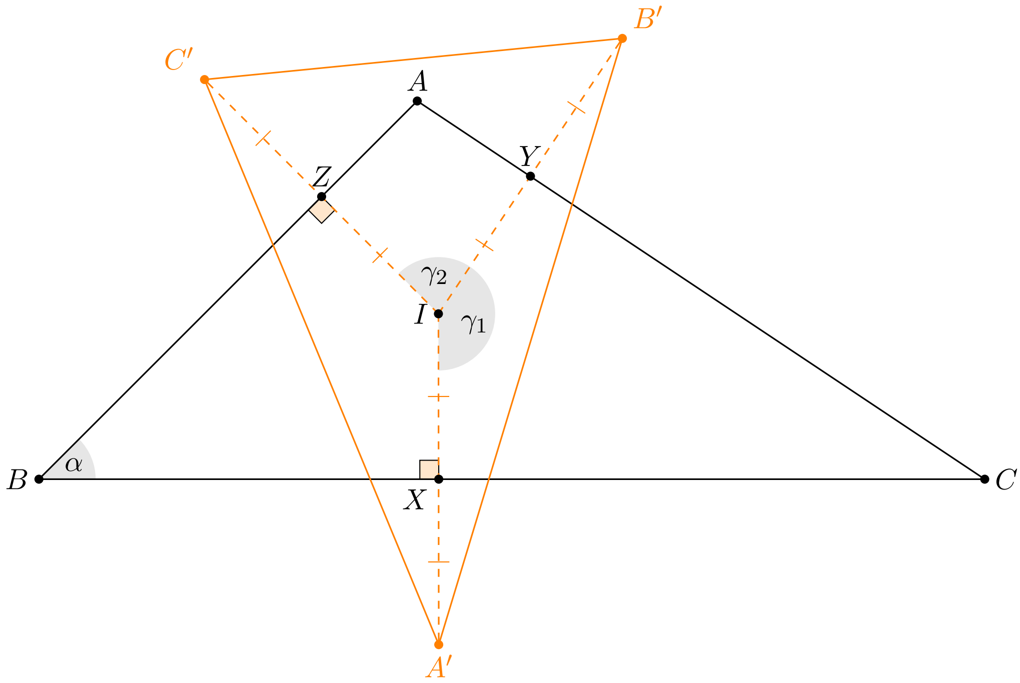
Question 5

The in-center of a triangle is the point equidistant from the triangles sides.



Let $\alpha = \angle ABC$, $\gamma_1 = \angle A'IB'$ and $\gamma_2 = \angle B'IC'$, let $X = BC \cap A'I$, $Y = AC \cap B'I$, $Z = AB \cap C'I$.

Since C' is the reflection of the point I about the line AB, it follows that $\angle IZB = 90°$ and $|IZ| = |ZC'|$, similarly, $\angle IXC = 90°$, $|IX| = |XA'|$, $\angle IYA = 90°$ and $|IY| = |YB'|$. Since IX is perpendicular to BC, it follows that the distance from I to the line BC is equal to the length of the segment IX, by similar reasoning, the distance from I to AC is $|IY|$, and the distance from I to AB is $|IZ|$, since I is the in-center of $\triangle ABC$, it follows that $|IX| = |IY| = |IZ|$. From this it follows that $|IA'| = |IB'| = |IC'|$.

Looking at quadrilateral $BXIZ$, we can deduce that $\angle XIZ = 180° - \alpha$, hence $\gamma_1 + \gamma_2 = 360° - \angle XIZ = 180° + \alpha$. Now $\triangle IC'B'$ is isosceles, so $\angle IB'C' = \dfrac{180° - \gamma_2}{2}$, similarly, $\angle IB'A' = \dfrac{180° - \gamma_1}{2}$. Hence

$$\begin{aligned}
\angle A'B'C' &= \angle IB'C' + \angle IB'A' \\
&= \frac{360° - (\gamma_1 + \gamma_2)}{2} \\
&= \frac{360° - (180° + \alpha)}{2} \\
&= 90° - \frac{\alpha}{2} \\
&= 90° - \frac{\angle ABC}{2}
\end{aligned}$$

Hence $\angle A'B'C'$ depends on $\angle ABC$ only, and therefore does not depend on $\angle BAC$