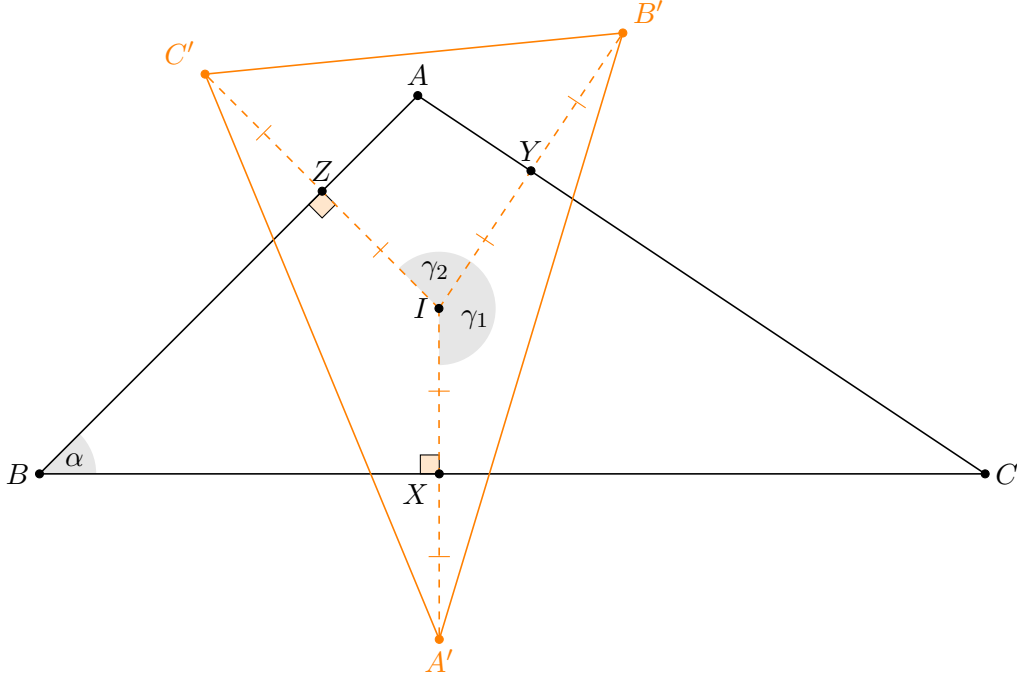


## Question 5

The in-center of a triangle is the point equidistant from the triangles sides.



Let  $\alpha = \angle ABC$ ,  $\gamma_1 = \angle A'IB'$  and  $\gamma_2 = \angle B'IC'$ , let  $X = BC \cap A'I$ ,  $Y = AC \cap B'I$ ,  $Z = AB \cap C'I$ .

Since  $C'$  is the reflection of the point  $I$  about the line  $AB$ , it follows that  $\angle IZB = 90^\circ$  and  $|IZ| = |ZC'|$ , similarly,  $\angle IXC = 90^\circ$ ,  $|IX| = |XA'|$ ,  $\angle IYA = 90^\circ$  and  $|IY| = |YB'|$ . Since  $IX$  is perpendicular to  $BC$ , it follows that the distance from  $I$  to the line  $BC$  is equal to the length of the segment  $IX$ , by similar reasoning, the distance from  $I$  to  $AC$  is  $|IY|$ , and the distance from  $I$  to  $AB$  is  $|IZ|$ , since  $I$  is the in-center of  $\triangle ABC$ , it follows that  $|IX| = |IY| = |IZ|$ . From this it follows that  $|IA'| = |IB'| = |IC'|$ .

Looking at quadrilateral  $BXIZ$ , we can deduce that  $\angle XIZ = 180^\circ - \alpha$ , hence  $\gamma_1 + \gamma_2 = 360^\circ - \angle XIZ = 180^\circ + \alpha$ . Now  $\triangle IC'B'$  is isosceles, so  $\angle IB'C' = \frac{180^\circ - \gamma_2}{2}$ , similarly,  $\angle IB'A' = \frac{180^\circ - \gamma_1}{2}$ . Hence

$$\begin{aligned}
 \angle A'B'C' &= \angle IB'C' + \angle IB'A' \\
 &= \frac{360^\circ - (\gamma_1 + \gamma_2)}{2} \\
 &= \frac{360^\circ - (180^\circ + \alpha)}{2} \\
 &= 90^\circ - \frac{\alpha}{2} \\
 &= 90^\circ - \frac{\angle ABC}{2}
 \end{aligned}$$

Hence  $\angle A'B'C'$  depends on  $\angle ABC$  only, and therefore does not depend on  $\angle BAC$