Question 2

Note that $ab + bc + cd + da = (b + d)(a + c)$ so our equation becomes $(b + d)(a + c) = 2021$, since $a, b, c, d \in \mathbb{N}$, it follows that $b + d, a + c \in \mathbb{N}$. Now 2021 can be expressed as a product of two positive integers in only 2 ways, $1 \cdot 2021$ and $43 \cdot 47$. Now since $b, d \geq 1$, it follows that $b+d \geq 2$, so it cannot be that $b+d = 1$ and $a+c = 2021$. Similarly it cant be that $b+d = 2021$ and $a + c = 1$. Hence the solution to our equation are those (a, b, c, d) such that $b + d = 43$ and $a + c = 47$ or $b + d = 47$ and $a + c = 43$. There are 42 ways in which $b + d = 43$, namely $(b, d) = (1, 42), (2, 41), (3, 40), ..., (41, 2), (42, 1)$, similarly there are 46 ways in which $a + c = 47$, hence the total number of solutions is $42 \cdot 46 \cdot 2 = 3864$