

1. Compute the derivative $f'(x)$ of the logistic sigmoid

$$f(x) = \frac{1}{1 + \exp(-x)}$$

$$\text{sol)} \quad f(x) = \frac{1}{1 + e^{-x}}$$

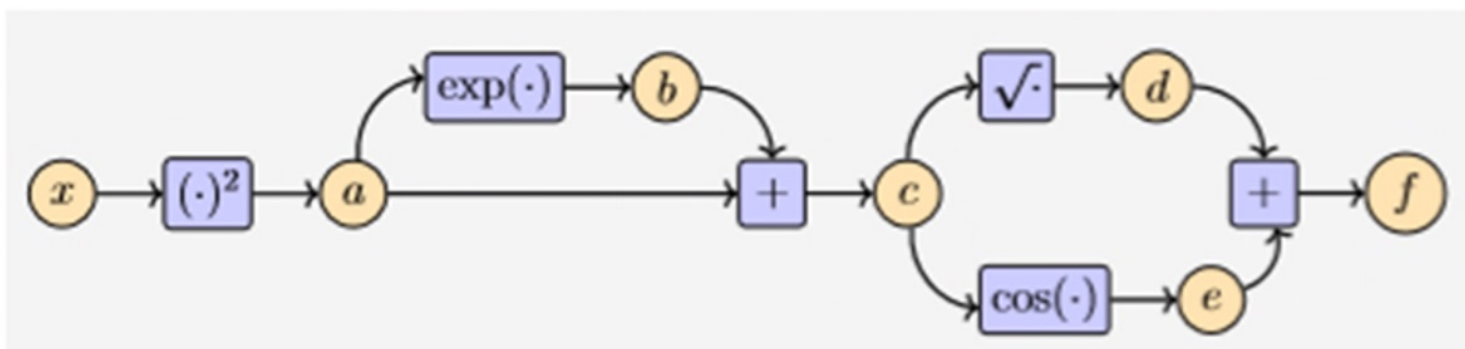
$$f'(x) = \frac{+e^{-x}}{(1+e^{-x})^2} = \frac{+e^{-x}}{1+2e^{-x}+e^{-2x}} = \frac{+1}{e^x+2+e^{-x}} = + \frac{1}{\underline{e^x+e^{-x}+2}}$$

2. Compute the $\frac{df}{dt}$ of the $f(x_1, x_2) = x_1^2 + 2x_2$, where $x_1 = \sin t$ and $x_2 = \cos t$

$$f(x_1, x_2) = x_1^2 + 2x_2$$

$$\begin{aligned} \frac{f(x_1, x_2)}{dt} &= 2x_1 \left(\frac{dx_1}{dt} \right) + 2 \left(\frac{dx_2}{dt} \right) = 2(\sin t)(\cos t) + 2(-\sin t) \\ &= \underline{2\sin t \{ (\cos t) - 1 \}} \end{aligned}$$

3. Fill the blank



• $a = x^2$

• $b = e^{x^2}$

• $c = 1 + e^{x^2}$

• $d = \sqrt{1 + e^{x^2}}$

• $e = \cos(1 + e^{x^2})$

• $f = \sqrt{1 + e^{x^2}} + \cos(1 + e^{x^2})$

▶ $\frac{\partial a}{\partial x} = 2x$

▶ $\frac{\partial b}{\partial a} =$

▶ $\frac{\partial c}{\partial a} =$

▶ $\frac{\partial d}{\partial c} =$

▶ $\frac{\partial e}{\partial c} =$

▶ $\frac{\partial f}{\partial d} =$

↓ 아래에서 따로 계산

▶ $\frac{\partial f}{\partial c} =$

▶ $\frac{\partial f}{\partial b} =$

▶ $\frac{\partial f}{\partial a} =$

▶ $\frac{\partial f}{\partial x} =$

$$\frac{\partial b}{\partial a} = \frac{\partial b}{\partial x} \cdot \frac{\partial x}{\partial a} = (2xe^{x^2}) \cdot \left(\frac{1}{2x}\right) = e^{x^2}$$

$$\frac{\partial c}{\partial a} = \frac{\partial c}{\partial x} \cdot \frac{\partial x}{\partial a} = (2x + 2xe^{x^2}) \cdot \left(\frac{1}{2x}\right) = 1 + e^{x^2}$$

$$\frac{\partial d}{\partial c} = \frac{\partial d}{\partial x} \cdot \frac{\partial x}{\partial c} = \left(\frac{2x + 2xe^{x^2}}{2\sqrt{1 + e^{x^2}}}\right) \cdot \left(\frac{1}{2x + 2xe^{x^2}}\right) = \frac{1}{2\sqrt{1 + e^{x^2}}}$$

$$\frac{\partial e}{\partial c} = \frac{\partial e}{\partial x} \cdot \frac{\partial x}{\partial c} = (-\sin(1 + e^{x^2})) \cdot (2x + 2xe^{x^2}) \cdot \left(\frac{1}{2x + 2xe^{x^2}}\right) = -\sin(1 + e^{x^2})$$

$$\frac{\partial f}{\partial d} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial d} = \left[\frac{2x + 2xe^{x^2}}{2\sqrt{1 + e^{x^2}}} + (-\sin(1 + e^{x^2}))(2x + 2xe^{x^2}) \right] \cdot \left(\frac{2\sqrt{1 + e^{x^2}}}{2x + 2xe^{x^2}}\right)$$

$$= 1 - 2\sqrt{1 + e^{x^2}} \sin(1 + e^{x^2})$$

$$\frac{\partial f}{\partial c} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial c} = \left[(2x + 2xe^{x^2}) \left(\frac{1}{2\sqrt{1 + e^{x^2}}} - \sin(1 + e^{x^2}) \right) \right] \cdot \left(\frac{1}{2x + 2xe^{x^2}}\right)$$

$$= \frac{1}{2\sqrt{1 + e^{x^2}}} - \sin(1 + e^{x^2})$$

$$\begin{aligned}\frac{\partial f}{\partial b} &= \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial b} = \left[(2x + 2xe^{x^2}) \left(\frac{1}{2\sqrt{x^2 + e^{x^2}}} - \sin(x^2 + e^{x^2}) \right) \right] \left(\frac{1}{2xe^{x^2}} \right) \\ &= \left(\frac{1}{e^{x^2}} + 1 \right) \left(\frac{1}{2\sqrt{x^2 + e^{x^2}}} - \sin(x^2 + e^{x^2}) \right)\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial a} &= \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial a} = \left[(2x + 2xe^{x^2}) \left(\frac{1}{2\sqrt{x^2 + e^{x^2}}} - \sin(x^2 + e^{x^2}) \right) \right] \left(\frac{1}{2x} \right) \\ &= (1 + e^{x^2}) \left(\frac{1}{2\sqrt{x^2 + e^{x^2}}} - \sin(x^2 + e^{x^2}) \right)\end{aligned}$$

$$\frac{\partial f}{\partial x} = (2x + 2xe^{x^2}) \left(\frac{1}{2\sqrt{x^2 + e^{x^2}}} - \sin(x^2 + e^{x^2}) \right)$$