CALCULUS FORMULES

DIFFERENTIATIE REGELS

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x)+g(x)) = f'(x)+g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(cf(x)) = cf'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(f(x)g(x)) = f'(x)g(x)+f(x)g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{f(x)}\right) = -\frac{f'(x)}{(f(x))^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = f'(g(x))g'(x)$$

ELEMENTAIRE AFGELEIDEN

$$\frac{\mathrm{d}}{\mathrm{d}x}x^r = rx^{r-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x \ln a \quad (a>0)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x} \quad (x>0)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sec x = \sec x \tan x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\csc x = -\csc x \cot x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cot x = -\csc^2 x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}|x| = \operatorname{sgn} x = \frac{x}{|x|}$$

TRIGONOMETRISCHE IDENTITEITEN

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

$\csc^2 x = 1 + \cot^2 x$

$\sin(-x) = -\sin x$

$\sin(\pi - x) = -\sin x$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\cos(-x) = \cos x$

$\cos(\pi - x)\cos x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\sin(x \pm y) = \sin x \cos y = \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\dfrac{1+\cos 2x}{2}$

KWADRATISCHE VERGELIJKING

$$\text{Als } Ax^2 + Bx + C = 0, \text{dan } x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

VECTORIDENTITEITEN

Als $u_1\boldsymbol{i} + u_2\boldsymbol{j} + u_3\boldsymbol{k}$

$v_1\boldsymbol{i} + v_2\boldsymbol{v} + v_3\boldsymbol{k}$

$w_1\boldsymbol{i} + w_2\boldsymbol{w} + w_3\boldsymbol{k}$

(dot product) $\boldsymbol{u}\cdot\boldsymbol{v} = \boldsymbol{u}_1\boldsymbol{v}_1 + \boldsymbol{u}_2\boldsymbol{v}_2 + \boldsymbol{u}_3\boldsymbol{v}_3$

(cross product) $\boldsymbol{u}\times\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$= (u_2v_3 - u_3v_2)\boldsymbol{i} + (u_3v_1 - u_1v_3)\boldsymbol{j} + (u_1v_2 - u_2v_1)\boldsymbol{k}$

lengte van $\boldsymbol{u} = |u| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

hoek tussen $\boldsymbol{u}$ en $\boldsymbol{v} = \cos^{-1}\left(\dfrac{\boldsymbol{u}\cdot\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}\right)$

drievoudig product

$\boldsymbol{u}\cdot(\boldsymbol{v}\times\boldsymbol{w}) = \boldsymbol{v}\cdot(\boldsymbol{w}\times\boldsymbol{u}) = \boldsymbol{w}\cdot(\boldsymbol{u}\times\boldsymbol{v})$

$\boldsymbol{u}\times(\boldsymbol{v}\times\boldsymbol{w}) = \boldsymbol{u}\times(\boldsymbol{u}\cdot\boldsymbol{w})\boldsymbol{v} - (\boldsymbol{u}\cdot\boldsymbol{v})\boldsymbol{w}$

IDENTITEITEN M.B.T. GRADIENT, DIVERGENTIE, CURL EN LAPLACIAAN

$\nabla = \boldsymbol{i}\dfrac{\partial}{\partial x} + \boldsymbol{j}\dfrac{\partial}{\partial y} + \boldsymbol{k}\dfrac{\partial}{\partial z}$ "del" of "nabla" operator

$\boldsymbol{F}(x,y,z) = F_1(x,y,z)\boldsymbol{i} + F_2(x,y,z)\boldsymbol{j} + F_3(x,y,z)\boldsymbol{k}$

$\nabla\phi(x,y,z) = \dfrac{\partial\phi}{\partial x}\boldsymbol{i} + \dfrac{\partial\phi}{\partial y}\boldsymbol{j} + \dfrac{\partial\phi}{\partial z}\boldsymbol{k}$

$\nabla\cdot\boldsymbol{F}(x,y,z) = \dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_2}{\partial y} + \dfrac{\partial F_3}{\partial z}$

$$\nabla\times\boldsymbol{F}(x,y,z) = \operatorname{curl}\boldsymbol{F}(x,y,z) = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\boldsymbol{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\boldsymbol{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\boldsymbol{k}$$

$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$\nabla\cdot(\phi\boldsymbol{F}) = (\nabla\phi)\times\boldsymbol{F} + \phi(\nabla\cdot\boldsymbol{F})$

$\nabla\times(\phi\boldsymbol{F}) = (\nabla\phi)\times\boldsymbol{F} + \phi(\nabla\times\boldsymbol{F})$

$\nabla\times(\nabla\phi) = \boldsymbol{0}$

$$\nabla^2\phi(x,y,z) = \nabla\cdot\nabla\phi(x,y,z) = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$\nabla\cdot(\boldsymbol{F}\times\boldsymbol{G}) = (\nabla\times\boldsymbol{F})\cdot\boldsymbol{G} - \boldsymbol{F}\cdot(\nabla\times\boldsymbol{G})$

$\nabla\times(\boldsymbol{F}\times\boldsymbol{G}) = \boldsymbol{F}(\nabla\cdot\boldsymbol{G}) - \boldsymbol{G}(\nabla\cdot\boldsymbol{F}) - (\boldsymbol{F}\cdot\nabla)\boldsymbol{G} + (\boldsymbol{G}\cdot\nabla)\boldsymbol{F}$

$\nabla(\boldsymbol{F}\cdot\boldsymbol{G}) = \boldsymbol{F}\times(\nabla\times\boldsymbol{G}) + \boldsymbol{G}\times(\nabla\times\boldsymbol{F}) + (\boldsymbol{F}\cdot\nabla)\boldsymbol{G} + (\boldsymbol{G}\cdot\nabla)\boldsymbol{F}$

$\nabla\cdot(\nabla\times\boldsymbol{F}) = 0$

$$\nabla\times(\nabla\times\boldsymbol{F}) = \nabla(\nabla\cdot\boldsymbol{F}) - \nabla^2\boldsymbol{F}$$

VARIANTEN FUNDAMENTELE STELLING VAN CALCULUS

1 dimensie

$$\int_a^b f'(t)dt = f(b) - f(a)$$

$$\int_C \nabla\phi\cdot d\boldsymbol{r} = \phi(\boldsymbol{r}(a))$$ als C de kromme $\boldsymbol{r} = \boldsymbol{r}(t)$ is, met $a \le t \le b$

Theorema van Green

$$\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dA = \oint_C \boldsymbol{F}\cdot d\boldsymbol{r} = \oint_C F_1(x,y) + F_2(x,y)dy$$

waarbij C de positieve grens is van R

Theorema van Stokes

$$\iint_S \nabla\times\boldsymbol{F}\cdot\hat{\boldsymbol{N}}dS = \oint_C \boldsymbol{F}\cdot d\boldsymbol{r} = \oint_C F_1(x,y,z)dx + F_2(x,y,z)dy + F_3(x,y,z)dz$$

waarbij C de georienteerde grens is van S

3D varianten

Divergentie theorema

$$\iiint_D \nabla\cdot\boldsymbol{F}dV = \oiint_S \boldsymbol{F}\cdot\hat{\boldsymbol{N}}dS$$

$$\iiint_D \nabla\phi dV = \oiint_S \phi\hat{\boldsymbol{N}}dS$$

$$\iiint_D \nabla\times\boldsymbol{F}dV = -\oiint_S \boldsymbol{F}\times\hat{\boldsymbol{N}}dS$$

FORMULES OVER KROMMEN IN DRIEDIMENSIONALE RUIMTE

Kromme: $\boldsymbol{r} = \boldsymbol{r}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j} + z(t)\boldsymbol{k}$

Velociteit: $\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = v\hat{\boldsymbol{T}}$

Snelheid: $v = |\boldsymbol{v}| = \frac{\mathrm{d}s}{\mathrm{d}t}$

Booglengte: $s = \int_{t_0}^t v dt$

Versnelling $\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}$

Tangentiele componenten: $\boldsymbol{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{\boldsymbol{T}}$

Unit tangent: $\hat{\boldsymbol{T}} = \frac{\boldsymbol{v}}{v}$

Binormaal: $\hat{\boldsymbol{B}} = \frac{\boldsymbol{v}\times\boldsymbol{a}}{|\boldsymbol{v}\times\boldsymbol{a}|}$

Normaal: $\hat{\boldsymbol{N}} = \hat{\boldsymbol{B}}\times\hat{\boldsymbol{T}} = \frac{d\hat{\boldsymbol{T}}/dt}{|d\hat{\boldsymbol{T}}/dt|}$

Kromming: $\kappa = \frac{|\boldsymbol{v}\times\boldsymbol{a}|}{v^3}$

Kromtegraad: $\rho = \frac{1}{\kappa}$

Torsie $\tau = \frac{(\boldsymbol{v}\times\boldsymbol{a})\cdot(d\boldsymbol{a}/dt)}{|\boldsymbol{v}\times\boldsymbol{a}|^2}$

Frenet-Serret formules

$\frac{\mathrm{d}\hat{\boldsymbol{T}}}{\mathrm{d}s} = \kappa\hat{\boldsymbol{N}}$

$\frac{\mathrm{d}\hat{\boldsymbol{N}}}{\mathrm{d}s} = -\kappa\hat{\boldsymbol{T}} + \tau\hat{\boldsymbol{B}}$

$\frac{\mathrm{d}\hat{\boldsymbol{B}}}{\mathrm{d}s} = -\tau\hat{\boldsymbol{N}}$