1 - FullyConnectedNets

March 11, 2019

1 Fully-Connected Neural Nets

In the previous homework you implemented a fully-connected two-layer neural network on CIFAR-10. The implementation was simple but not very modular since the loss and gradient were computed in a single monolithic function. This is manageable for a simple two-layer network, but would become impractical as we move to bigger models. Ideally we want to build networks using a more modular design so that we can implement different layer types in isolation and then snap them together into models with different architectures.

In this exercise we will implement fully-connected networks using a more modular approach. For each layer we will implement a forward and a backward function. The forward function will receive inputs, weights, and other parameters and will return both an output and a cache object storing data needed for the backward pass, like this:

```
def layer_forward(x, w):
    """ Receive inputs x and weights w """
    # Do some computations ...
    z = # ... some intermediate value
    # Do some more computations ...
    out = # the output

cache = (x, w, z, out) # Values we need to compute gradients
    return out, cache
```

The backward pass will receive upstream derivatives and the cache object, and will return gradients with respect to the inputs and weights, like this:

```
def layer_backward(dout, cache):
    """
    Receive derivative of loss with respect to outputs and cache,
    and compute derivative with respect to inputs.
    """
    # Unpack cache values
    x, w, z, out = cache

# Use values in cache to compute derivatives
    dx = # Derivative of loss with respect to x
```

```
dw = # Derivative of loss with respect to w
return dx, dw
```

After implementing a bunch of layers this way, we will be able to easily combine them to build classifiers with different architectures.

In addition to implementing fully-connected networks of arbitrary depth, we will also explore different update rules for optimization, and introduce Dropout as a regularizer and Batch Normalization as a tool to more efficiently optimize deep networks.

Acknowledgement: This exercise is adapted from Stanford CS231n.

```
In [18]: # As usual, a bit of setup
         import sys
         import time
         import numpy as np
         import matplotlib.pyplot as plt
         from libs.classifiers.fc_net import *
         from libs.data_utils import get_CIFAR10_data
         from libs.gradient_check import eval_numerical_gradient, eval_numerical_gradient_arra
         from libs.solver import Solver
         %matplotlib inline
         plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
         plt.rcParams['image.interpolation'] = 'nearest'
         plt.rcParams['image.cmap'] = 'gray'
         # for auto-reloading external modules
         \# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
         %load_ext autoreload
         %autoreload 2
         def rel_error(x, y):
             """ returns relative error """
             return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
         # python setup.py build_ext --inplace
The autoreload extension is already loaded. To reload it, use:
 %reload_ext autoreload
In [19]: # Load the (preprocessed) CIFAR10 data.
         data = get_CIFAR10_data()
         for k, v in data.items():
             print('%s: ' % k, v.shape)
X_train: (49000, 3, 32, 32)
y train: (49000,)
```

```
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

2 Affine layer: foward

Open the file libs/layers.py and implement the affine_forward function.

Once you are done you can test your implementation by running the following:

```
In [20]: # Test the affine_forward function
        num_inputs = 2
         input\_shape = (4, 5, 6)
        output_dim = 3
         input_size = num_inputs * np.prod(input_shape)
        weight_size = output_dim * np.prod(input_shape)
        x = np.linspace(-0.1, 0.5, num=input_size).reshape(num_inputs, *input_shape)
        w = np.linspace(-0.2, 0.3, num=weight_size).reshape(np.prod(input_shape), output_dim)
        b = np.linspace(-0.3, 0.1, num=output_dim)
        out, _ = affine_forward(x, w, b)
         correct_out = np.array([[ 1.49834967, 1.70660132, 1.91485297],
                                 [ 3.25553199, 3.5141327, 3.77273342]])
         # Compare your output with ours. The error should be around 1e-9.
        print('Testing affine_forward function:')
        print('difference: ', rel_error(out, correct_out))
Testing affine_forward function:
difference: 9.769849468192957e-10
```

3 Affine layer: backward

Now implement the affine_backward function and test your implementation using numeric gradient checking.

```
In [21]: # Test the affine_backward function

x = np.random.randn(10, 2, 3)

w = np.random.randn(6, 5)

b = np.random.randn(5)

dout = np.random.randn(10, 5)
```

```
dx_num = eval_numerical_gradient_array(lambda x: affine_forward(x, w, b)[0], x, dout)
         dw num = eval numerical_gradient_array(lambda w: affine_forward(x, w, b)[0], w, dout)
         db_num = eval_numerical_gradient_array(lambda b: affine_forward(x, w, b)[0], b, dout)
         _, cache = affine_forward(x, w, b)
         dx, dw, db = affine_backward(dout, cache)
         print(db)
         # The error should be around 1e-10
         print('Testing affine_backward function:')
         print('dx error: ', rel_error(dx_num, dx))
         print('dw error: ', rel_error(dw_num, dw))
         print('db error: ', rel_error(db_num, db))
[ 2.6353553
              1.09234475 -2.13967024 -1.25132065 3.15974258]
Testing affine_backward function:
dx error: 1.1896014453808045e-10
dw error: 2.7872292672409854e-11
db error: 1.2277375245322469e-11
```

4 ReLU layer: forward

Implement the forward pass for the ReLU activation function in the relu_forward function and test your implementation using the following:

```
In [22]: # Test the relu_forward function
        x = np.linspace(-0.5, 0.5, num=12).reshape(3, 4)
        out, _ = relu_forward(x)
         correct_out = np.array([[ 0.,
                                              0.,
                                                          0.,
                                                                         0.,
                                                                                    ],
                                                            0.04545455, 0.13636364,],
                                               0.,
                                 [ 0.22727273, 0.31818182, 0.40909091, 0.5,
                                                                                    11)
         # Compare your output with ours. The error should be around 1e-8
        print('Testing relu_forward function:')
        print('difference: ', rel_error(out, correct_out))
Testing relu_forward function:
difference: 4.999999798022158e-08
```

5 ReLU layer: backward

Now implement the backward pass for the ReLU activation function in the relu_backward function and test your implementation using numeric gradient checking:

6 "Sandwich" layers

dw error: 2.634589600810808e-10 db error: 7.82667446064971e-12

There are some common patterns of layers that are frequently used in neural nets. For example, affine layers are frequently followed by a ReLU nonlinearity. To make these common patterns easy, we define several convenience layers in the file libs/layer_utils.py.

For now take a look at the affine_relu_forward and affine_relu_backward functions, and run the following to numerically gradient check the backward pass:

```
In [24]: from libs.layer_utils import affine_relu_forward, affine_relu_backward
        x = np.random.randn(2, 3, 4)
        w = np.random.randn(12, 10)
        b = np.random.randn(10)
         dout = np.random.randn(2, 10)
         out, cache = affine_relu_forward(x, w, b)
         dx, dw, db = affine_relu_backward(dout, cache)
         dx_num = eval_numerical_gradient_array(lambda x: affine_relu_forward(x, w, b)[0], x,
         dw_num = eval_numerical_gradient_array(lambda w: affine_relu_forward(x, w, b)[0], w,
         db_num = eval_numerical_gradient_array(lambda b: affine_relu_forward(x, w, b)[0], b,
        print('Testing affine_relu_forward:')
        print('dx error: ', rel_error(dx_num, dx))
        print('dw error: ', rel_error(dw_num, dw))
        print('db error: ', rel_error(db_num, db))
Testing affine_relu_forward:
dx error: 9.443149103609857e-10
```

7 Loss layers: Softmax

You implemented this loss function in the last assignment, so we'll give them to you for free here. You should still make sure you understand how it works by looking at the implementations in libs/layers.py.

You can make sure that the implementation is correct by running the following:

```
In [25]: num_classes, num_inputs = 10, 50
    x = 0.001 * np.random.randn(num_inputs, num_classes)
    y = np.random.randint(num_classes, size=num_inputs)

dx_num = eval_numerical_gradient(lambda x: softmax_loss(x, y)[0], x, verbose=False)
    loss, dx = softmax_loss(x, y)

# Test softmax_loss function. Loss should be 2.3 and dx error should be 1e-8
    print('\nTesting softmax_loss:')
    print('loss: ', loss)
    print('dx error: ', rel_error(dx_num, dx))
Testing softmax_loss:
loss: 2.3026065616859515
dx error: 7.200362009339885e-09
```

8 Two-layer network

In the previous assignment you implemented a two-layer neural network in a single monolithic class. Now that you have implemented modular versions of the necessary layers, you will reimplement the two layer network using these modular implementations.

Open the file libs/classifiers/fc_net.py and complete the implementation of the TwoLayerNet class. This class will serve as a model for the other networks you will implement in this assignment, so read through it to make sure you understand the API. You can run the cell below to test your implementation.

```
In [26]: N, D, H, C = 3, 5, 50, 7
    X = np.random.randn(N, D)
    y = np.random.randint(C, size=N)

std = 1e-2
    model = TwoLayerNet(input_dim=D, hidden_dim=H, num_classes=C, weight_scale=std)

print('Testing initialization ... ')
    W1_std = abs(model.params['W1'].std() - std)
    b1 = model.params['b1']
    W2_std = abs(model.params['W2'].std() - std)
    b2 = model.params['b2']
    assert W1_std < std / 10, 'First layer weights do not seem right'</pre>
```

```
assert W2_std < std / 10, 'Second layer weights do not seem right'
         assert np.all(b2 == 0), 'Second layer biases do not seem right'
        print('Testing test-time forward pass ... ')
        model.params['W1'] = np.linspace(-0.7, 0.3, num=D*H).reshape(D, H)
        model.params['b1'] = np.linspace(-0.1, 0.9, num=H)
        model.params['W2'] = np.linspace(-0.3, 0.4, num=H*C).reshape(H, C)
        model.params['b2'] = np.linspace(-0.9, 0.1, num=C)
        X = np.linspace(-5.5, 4.5, num=N*D).reshape(D, N).T
         scores = model.loss(X)
         correct_scores = np.asarray(
           [[11.53165108, 12.2917344, 13.05181771, 13.81190102, 14.57198434, 15.33206765,
            [12.05769098, 12.74614105, 13.43459113, 14.1230412, 14.81149128, 15.49994135,
            [12.58373087, 13.20054771, 13.81736455, 14.43418138, 15.05099822, 15.66781506,
         scores_diff = np.abs(scores - correct_scores).sum()
         assert scores_diff < 1e-6, 'Problem with test-time forward pass'
        print('Testing training loss (no regularization)')
        y = np.asarray([0, 5, 1])
        loss, grads = model.loss(X, y)
         correct loss = 3.4702243556
         assert abs(loss - correct_loss) < 1e-10, 'Problem with training-time loss'
        model.reg = 1.0
        loss, grads = model.loss(X, y)
         correct_loss = 26.5948426952
         assert abs(loss - correct_loss) < 1e-10, 'Problem with regularization loss'
        for reg in [0.0, 0.7]:
             print('Running numeric gradient check with reg = ', reg)
             model.reg = reg
             loss, grads = model.loss(X, y)
             for name in sorted(grads):
                 f = lambda _: model.loss(X, y)[0]
                 grad_num = eval_numerical_gradient(f, model.params[name], verbose=False)
                 print('%s relative error: %.2e' % (name, rel_error(grad_num, grads[name])))
Testing initialization ...
Testing test-time forward pass ...
Testing training loss (no regularization)
Running numeric gradient check with reg = 0.0
W1 relative error: 1.83e-08
W2 relative error: 3.20e-10
b1 relative error: 9.83e-09
b2 relative error: 4.33e-10
Running numeric gradient check with reg = 0.7
```

assert np.all(b1 == 0), 'First layer biases do not seem right'

```
W1 relative error: 2.53e-07
W2 relative error: 7.98e-08
b1 relative error: 1.35e-08
b2 relative error: 7.76e-10
```

9 Solver

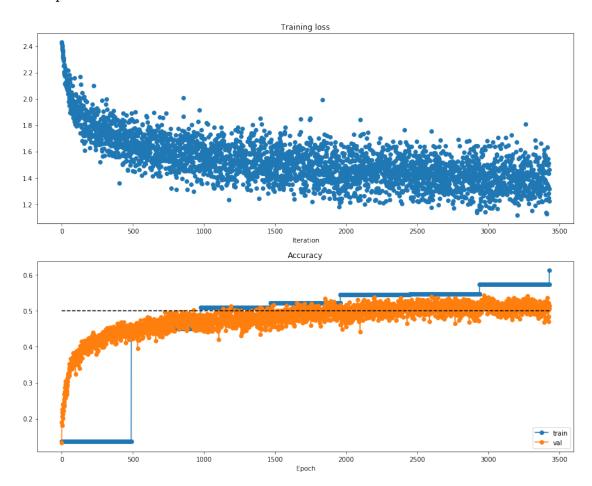
In the previous assignment, the logic for training models was coupled to the models themselves. Following a more modular design, for this assignment we have split the logic for training models into a separate class.

Open the file libs/solver.py and read through it to familiarize yourself with the API. After doing so, use a Solver instance to train a TwoLayerNet that achieves at least 50% accuracy on the validation set.

```
In [51]: # X_val: (1000, 3, 32, 32)
      # X_train: (49000, 3, 32, 32)
       # X_test: (1000, 3, 32, 32)
       # y_val: (1000,)
       # y_train: (49000,)
      # y_test: (1000,)
       # model = TwoLayerNet(input_dim=D, hidden_dim=H, num_classes=C, weight_scale=std)
       # TODO: Use a Solver instance to train a TwoLayerNet that achieves at least
       # 50% accuracy on the validation set.
      model = TwoLayerNet(hidden_dim=200, reg=0.4)
      solver = Solver(model, data,
                   update_rule='sgd',
                   optim_config={
                    'learning_rate': 1e-3,
                   },
                   lr_decay=0.95,
                   num_epochs=7, batch_size=100,
                   print_every=100)
      solver.train()
      solver.check_accuracy(data['X_val'], data['y_val'])
       END OF YOUR CODE
       (Epoch 0 / 7) (Iteration 1 / 3430) loss: 2.431635 train acc: 0.137000 val_acc: 0.133000
(Epoch 0 / 7) (Iteration 101 / 3430) loss: 2.029064 train acc: 0.137000 val_acc: 0.354000
(Epoch 0 / 7) (Iteration 201 / 3430) loss: 1.730774 train acc: 0.137000 val_acc: 0.405000
(Epoch 0 / 7) (Iteration 301 / 3430) loss: 1.772304 train acc: 0.137000 val_acc: 0.407000
```

(Epoch 0 / 7) (Iteration 401 / 3430) loss: 1.825738 train acc: 0.137000 val_acc: 0.435000

```
(Epoch 1 / 7) (Iteration 501 / 3430) loss: 1.451621 train acc: 0.451000 val_acc: 0.451000
(Epoch 1 / 7) (Iteration 601 / 3430) loss: 1.672821 train acc: 0.451000 val_acc: 0.454000
(Epoch 1 / 7) (Iteration 701 / 3430) loss: 1.701232 train acc: 0.451000 val_acc: 0.464000
(Epoch 1 / 7) (Iteration 801 / 3430) loss: 1.421941 train acc: 0.451000 val_acc: 0.470000
(Epoch 1 / 7) (Iteration 901 / 3430) loss: 1.620550 train acc: 0.451000 val acc: 0.466000
(Epoch 2 / 7) (Iteration 1001 / 3430) loss: 1.566371 train acc: 0.510000 val_acc: 0.465000
(Epoch 2 / 7) (Iteration 1101 / 3430) loss: 1.407347 train acc: 0.510000 val acc: 0.477000
(Epoch 2 / 7) (Iteration 1201 / 3430) loss: 1.536224 train acc: 0.510000 val_acc: 0.456000
(Epoch 2 / 7) (Iteration 1301 / 3430) loss: 1.445338 train acc: 0.510000 val acc: 0.473000
(Epoch 2 / 7) (Iteration 1401 / 3430) loss: 1.644816 train acc: 0.510000 val_acc: 0.479000
(Epoch 3 / 7) (Iteration 1501 / 3430) loss: 1.463599 train acc: 0.522000 val_acc: 0.456000
(Epoch 3 / 7) (Iteration 1601 / 3430) loss: 1.461250 train acc: 0.522000 val_acc: 0.489000
(Epoch 3 / 7) (Iteration 1701 / 3430) loss: 1.509319 train acc: 0.522000 val_acc: 0.482000
(Epoch 3 / 7) (Iteration 1801 / 3430) loss: 1.458530 train acc: 0.522000 val_acc: 0.492000
(Epoch 3 / 7) (Iteration 1901 / 3430) loss: 1.562339 train acc: 0.522000 val_acc: 0.525000
(Epoch 4 / 7) (Iteration 2001 / 3430) loss: 1.468374 train acc: 0.545000 val_acc: 0.478000
(Epoch 4 / 7) (Iteration 2101 / 3430) loss: 1.235128 train acc: 0.545000 val_acc: 0.498000
(Epoch 4 / 7) (Iteration 2201 / 3430) loss: 1.481097 train acc: 0.545000 val acc: 0.503000
(Epoch 4 / 7) (Iteration 2301 / 3430) loss: 1.273442 train acc: 0.545000 val_acc: 0.473000
(Epoch 4 / 7) (Iteration 2401 / 3430) loss: 1.549268 train acc: 0.545000 val acc: 0.483000
(Epoch 5 / 7) (Iteration 2501 / 3430) loss: 1.471234 train acc: 0.546000 val acc: 0.509000
(Epoch 5 / 7) (Iteration 2601 / 3430) loss: 1.756919 train acc: 0.546000 val acc: 0.540000
(Epoch 5 / 7) (Iteration 2701 / 3430) loss: 1.294480 train acc: 0.546000 val_acc: 0.505000
(Epoch 5 / 7) (Iteration 2801 / 3430) loss: 1.407933 train acc: 0.546000 val_acc: 0.520000
(Epoch 5 / 7) (Iteration 2901 / 3430) loss: 1.470201 train acc: 0.546000 val_acc: 0.482000
(Epoch 6 / 7) (Iteration 3001 / 3430) loss: 1.436696 train acc: 0.573000 val_acc: 0.532000
(Epoch 6 / 7) (Iteration 3101 / 3430) loss: 1.305904 train acc: 0.573000 val_acc: 0.509000
(Epoch 6 / 7) (Iteration 3201 / 3430) loss: 1.262684 train acc: 0.573000 val_acc: 0.522000
(Epoch 6 / 7) (Iteration 3301 / 3430) loss: 1.372571 train acc: 0.573000 val_acc: 0.512000
(Epoch 6 / 7) (Iteration 3401 / 3430) loss: 1.437906 train acc: 0.573000 val_acc: 0.496000
Out[51]: 0.544
In [52]: # Run this cell to visualize training loss and train / val accuracy
        plt.subplot(2, 1, 1)
        plt.title('Training loss')
        plt.plot(solver.loss history, 'o')
        plt.xlabel('Iteration')
        plt.subplot(2, 1, 2)
        plt.title('Accuracy')
        plt.plot(solver.train_acc_history, '-o', label='train')
        plt.plot(solver.val acc history, '-o', label='val')
        plt.plot([0.5] * len(solver.val_acc_history), 'k--')
        plt.xlabel('Epoch')
        plt.legend(loc='lower right')
```



10 Multilayer network

Next you will implement a fully-connected network with an arbitrary number of hidden layers. Read through the FullyConnectedNet class in the file libs/classifiers/fc_net.py. Implement the initialization, the forward pass, and the backward pass.

10.1 Initial loss and gradient check

As a sanity check, run the following to check the initial loss and to gradient check the network both with and without regularization. Do the initial losses seem reasonable?

For gradient checking, you should expect to see errors around 1e-6 or less.

```
In [53]: N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))
```

```
for reg in [0, 3.14]:
             print('Running check with reg = ', reg)
             model = FullyConnectedNet([H1, H2], input_dim=D, num_classes=C,
                                     reg=reg, weight_scale=5e-2, dtype=np.float64)
             loss, grads = model.loss(X, y)
             print('Initial loss: ', loss)
             for name in sorted(grads):
                 f = lambda _: model.loss(X, y)[0]
                 grad_num = eval_numerical_gradient(f, model.params[name], verbose=False, h=1e-
                 print('%s relative error: %.2e' % (name, rel_error(grad_num, grads[name])))
Running check with reg = 0
Initial loss: 2.3033827265087945
W1 relative error: 2.54e-07
W2 relative error: 9.35e-07
W3 relative error: 2.74e-07
b1 relative error: 3.16e-08
b2 relative error: 2.36e-09
b3 relative error: 1.85e-10
Running check with reg = 3.14
Initial loss: 6.595748222690213
W1 relative error: 6.80e-09
W2 relative error: 8.77e-07
W3 relative error: 2.33e-07
b1 relative error: 2.09e-08
b2 relative error: 1.50e-07
b3 relative error: 2.18e-10
```

As another sanity check, make sure you can overfit a small dataset of 50 images. First we will try a three-layer network with 100 units in each hidden layer. You will need to tweak the learning rate and initialization scale, but you should be able to overfit and achieve 100% training accuracy within 20 epochs.

```
# TODO: Use a three-layer Net to overfit 50 training examples.
        layers = [100, 100, 100]
       overfitmodel = FullyConnectedNet(layers,reg=0, weight scale=weight scale)
       overfitsolver = Solver(overfitmodel, small data,
                      update rule='sgd',
                      optim_config={
                        'learning_rate': learning_rate,
                      },
                      lr_decay=0.95,
                      num_epochs=20, batch_size=4,
                      print_every=10)
       overfitsolver.train()
       overfitsolver.check_accuracy(small_data['X_val'], small_data['y_val'])
        END OF YOUR CODE
        plt.plot(solver.loss history, 'o')
       plt.title('Training loss history')
       plt.xlabel('Iteration')
       plt.ylabel('Training loss')
       plt.show()
(Epoch 0 / 20) (Iteration 1 / 250) loss: inf train acc: 0.060000 val acc: 0.140000
(Epoch 0 / 20) (Iteration 11 / 250) loss: 6.329886 train acc: 0.060000 val_acc: 0.112000
(Epoch 0 / 20) (Iteration 21 / 250) loss: 24.873686 train acc: 0.060000 val acc: 0.128000
(Epoch 1 / 20) (Iteration 31 / 250) loss: 8.315066 train acc: 0.520000 val_acc: 0.130000
(Epoch 1 / 20) (Iteration 41 / 250) loss: 0.000000 train acc: 0.520000 val_acc: 0.127000
(Epoch 2 / 20) (Iteration 51 / 250) loss: inf train acc: 0.720000 val acc: 0.140000
(Epoch 2 / 20) (Iteration 61 / 250) loss: 0.000007 train acc: 0.720000 val_acc: 0.142000
(Epoch 2 / 20) (Iteration 71 / 250) loss: 1.846825 train acc: 0.720000 val_acc: 0.143000
(Epoch 3 / 20) (Iteration 81 / 250) loss: 0.000122 train acc: 0.840000 val_acc: 0.141000
(Epoch 3 / 20) (Iteration 91 / 250) loss: 0.000060 train acc: 0.840000 val acc: 0.133000
(Epoch 4 / 20) (Iteration 101 / 250) loss: 0.000000 train acc: 0.980000 val_acc: 0.130000
(Epoch 4 / 20) (Iteration 111 / 250) loss: 0.000000 train acc: 0.980000 val acc: 0.131000
(Epoch 4 / 20) (Iteration 121 / 250) loss: 0.000001 train acc: 0.980000 val_acc: 0.131000
(Epoch 5 / 20) (Iteration 131 / 250) loss: 0.000000 train acc: 1.000000 val acc: 0.130000
(Epoch 5 / 20) (Iteration 141 / 250) loss: 0.000001 train acc: 1.000000 val_acc: 0.130000
(Epoch 6 / 20) (Iteration 151 / 250) loss: 0.000000 train acc: 1.000000 val acc: 0.131000
(Epoch 6 / 20) (Iteration 161 / 250) loss: 0.000000 train acc: 1.000000 val_acc: 0.131000
(Epoch 6 / 20) (Iteration 171 / 250) loss: 0.000000 train acc: 1.000000 val_acc: 0.131000
(Epoch 7 / 20) (Iteration 181 / 250) loss: 0.000016 train acc: 1.000000 val acc: 0.131000
(Epoch 7 / 20) (Iteration 191 / 250) loss: 0.000001 train acc: 1.000000 val_acc: 0.131000
(Epoch 8 / 20) (Iteration 201 / 250) loss: 0.000289 train acc: 1.000000 val acc: 0.131000
(Epoch 8 / 20) (Iteration 211 / 250) loss: 0.000022 train acc: 1.000000 val_acc: 0.131000
```

```
(Epoch 8 / 20) (Iteration 221 / 250) loss: 0.000000 train acc: 1.000000 val_acc: 0.131000 (Epoch 9 / 20) (Iteration 231 / 250) loss: 0.000000 train acc: 1.000000 val_acc: 0.131000 (Epoch 9 / 20) (Iteration 241 / 250) loss: 0.000000 train acc: 1.000000 val_acc: 0.131000
```

