

$$\sum_{i=1}^n \|x^{(i)} - z\|^2$$

$$\nabla \frac{d}{dz} \sum_{i=1}^n \|x^{(i)} - z\|^2 = 2 \sum_{i=1}^n (x^{(i)} - z)$$

$$\text{By equating } 2 \sum_{i=1}^n (x^{(i)} - z) = 0,$$

$$\text{we get } 0 = \sum_{i=1}^n (x^{(i)} - z)$$

$$0 = -nz + \sum_{i=1}^n x^{(i)}$$

$$nz = \sum_{i=1}^n x^{(i)}$$

$$z = \frac{1}{n} \sum_{i=1}^n x^{(i)} \quad (\text{proven})$$