Moment Generated Function	
Continuous R.V.X	$\phi_X(s)=\int_{-\infty}^{\infty} e^{sx} f_X(x)dx$
Discrete R.V.Y	$\phi_Y(s)=\sum_{y_i\in S_Y} e^{sy_i} P_Y(y_i)$
$\phi_X(s)$ has nth moment	$E[X^n]=\frac{d^n\phi_X(s)}{ds^n}\vert_{s=0}$.

Random Variable	PMF or PDF	MGF $\phi_X(s)$	
Bernoulli (p)	$P_X(x)=\begin{cases}1-p & x=0,\\ p & x=1,\\ 0 & otherwise.\end{cases}$	$1-p+pe^s$	$E[X]=p$ $Var[X]=p(1-p)$
Binomial (n,p)	$P_X(x)=\binom{n}{x}p^x(1-p)^{n-x}$	$(1-p+pe^s)^n$	$E[X]=np$ $Var[X]=np(1-p)$
Geometric (p)	$P_X(x)=\begin{cases}p(1-p)^{x-1} & x=1,2,...\\ 0 & otherwise,\end{cases}$	$\frac{pe^s}{1-(1-p)e^s}$	$E[X]=1/p$ $Var[X]=(1-p)/p^2$
Pascal (k,p)	$P_X(x)=\binom{x-1}{k-1}p^k(1-p)^{x-k}$	$(\frac{pe^s}{1-(1-p)e^s})^k$	$E[X]=k/p$ $Var[X]=k(1-p)/p^2$
Poisson (α)	$P_X(x)=\begin{cases}\alpha^x e^{-\alpha}/x! & x=0,1,2,...\\ 0 & otherwise,\end{cases}$	$e^{\alpha(e^s-1)}$	$E[X]=\alpha$ $Var[X]=\alpha$
Disc. Uniform (k,l)	$P_X(x)=\begin{cases}1/(l-k+1) & x=k,...,l\\ 0 & otherwise,\end{cases}$	$\frac{e^{sk}-e^{s(l+1)}}{(l-k+1)(1-e^s)}$	$E[X]=\frac{k+l}{2}$ $Var[X]=\frac{(l-k)(l-k+2)}{12}$
Constant (a)	$f_X(x)=\delta(x-a)$	e^{sa}	
Uniform (a,b)	$f_X(x)=\begin{cases}\frac{1}{b-a} & a<x<b\\ 0 & otherwise\end{cases}$	$\frac{e^{bs}-e^{as}}{s(b-a)}$	$E[X]=\frac{a+b}{2}$ $Var[X]=\frac{(b-a)^2}{12}$
Exponential (λ)	$f_X(x)=\begin{cases}\lambda e^{-\lambda x} & x\geq 0,\\ 0 & otherwise\end{cases}$	$\frac{\lambda}{\lambda-s}$	$E[X]=1/\lambda$ $Var[X]=1/\lambda^2$
Erlang (n,λ)	$f_X(x)=\begin{cases}\frac{\lambda^n x^{n-1}e^{-\lambda x}}{(n-1)!} & x\geq 0,\\ 0 & otherwise,\end{cases}$	$(\frac{\lambda}{\lambda-s})^n$	$E[X]=n/\lambda$ $Var[X]=n/\lambda^2$
Gaussian (μ,σ)	$f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$	$e^{s\mu+s^2\sigma^2/2}$	$E[X]=\mu$ $Var[X]=\sigma^2$

For two random variable X, Y. And W=g(x,y)	
Expected value	Discrete: $E[W]=\sum_{x\in S_X}\sum_{y\in S_Y} g(x,y)P_{X,Y}(x,y)$ Continuous: $E[W]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f_{X,Y}(x,y)dxdy$
Variance	$Var[X+Y]=Var[X]+Var[Y]+2E[(X-\mu_x)(Y-\mu_Y)]$
Covariance	$Cov[X,Y]=E[(X-\mu_x)(Y-\mu_Y)]$
Correlation	$r_{X,Y}=E[XY]$
Correlation coefficient	$\rho_{X,Y}=\frac{Cov[X,Y]}{\sqrt{Var[X]Var[Y]}}=\frac{Cov[X,Y]}{\sigma_X\sigma_Y}$

Iterated expectation	$E[E[X\vert Y]]=E[X]$
	$E[E[g(X)\vert Y]]=E[g(X)]$

For any set of R.V. $X_1,X_2,...,X_n$ and $W_n=X_1+X_2+...+X_n$	
Expected value	$E[W_n]=E[X_1]+E[X_2]+...+E[X_n]$
Variance	$Var[W_n]=\sum_{i=1}^{n}Var[X_i]+2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}Cov[X_i,X_j]$
	When $X_1,X_2,...,X_n$ are uncorrelated, $Var[W_n]=Var[X_1]+...+Var[X_n]$
MGF	When $X_1,X_2,...,X_n$ are independent, $\phi_W(s)=\phi_{X_1}(s)\phi_{X_2}(s)...\phi_{X_n}(s)$.

For the random sum of iid random variables $R=X_1+...+X_N$,
$E[R]=E[N]E[X]$, $Var[R]=E[N]Var[X]+Var[N](E[X])^2$

Let $\{X_1,X_2...\}$ be a collection of iid random variables, each with MGF $\phi_X(s)$, and let N be a nonnegative integer-valued random variable that is independent of $\{X_1,X_2...\}$. The random sum $R=X_1+...+X_N$ has moment generating function $\phi_R(s)=\phi_N(ln\phi_X(s))$.

Central Limit Theorem Approximation
Let $W_n = X_1 + ... + X_n$ be the sum of n iid random variables, each with $E[X] = \mu_X$ and $Var[X] = \sigma_X^2$. The central limit theorem approximation to the CDF of W_n is

$$F_{W_n}(w) \approx \Phi(\frac{w - n\mu_X}{\sqrt{n\sigma_X^2}})$$

Chernoff Bound
For an arbitrary random variable X and a constant c,

$$P[X \geq c] \leq \min_{s \geq 0} e^{-sc}\phi_X(s)$$

Sample Mean
For iid random variable $X_1, ..., X_n$ with PDF $f_X(x)$, the sample mean of X is the random variable

$$M_n(X) = \frac{X_1 + ... + X_n}{n}$$

The sample mean $M_n(X)$ has expected value and variance

$$E[M_n(X)] = E[X], \quad Var[M_n(X)] = \frac{Var[X]}{n}$$

Markov Inequality
For a random variable X such that $P[X < 0] = 0$ and a constant c,

$$P[X \geq c^2] \leq \frac{E[X]}{c^2}.$$

Chebyshev Inequality
For an arbitrary random variable Y and constant $c > 0$,

$$P[|Y - \mu_Y| \geq c] \leq \frac{Var[Y]}{c^2}.$$

Confidential interval
For any constant $c > 0$,
(a) $P[|M_n(X) - \mu_X| \geq c] \leq \frac{Var[X]}{nc^2} = \alpha$
(b) $P[|M_n(X) - \mu_X| < c] \geq 1 - \frac{Var[X]}{nc^2} = 1 - \alpha$
Confidential interval = 2c **confidence coefficient=**$1 - \alpha$
Theorem 10.4
Let X be a Gaussian (μ, σ) random variable. A confidence interval estimate of μ of the form $M_n(X) - c \leq \mu \leq M_n(X) + c$
has confidence coefficient $1 - \alpha$ where

$$\alpha/2 = Q(\frac{c\sqrt{n}}{\sigma}) = 1 - \Phi(\frac{c\sqrt{n}}{\sigma}).$$

De Moivre-Laplace Formula
For a binomial (n,p) random variable K,

$$P[k_1 \leq K \leq k_2] \approx \Phi(\frac{k_2 + 0.5 - np}{\sqrt{np(1-p)}}) - \Phi(\frac{k_1 - 0.5 - np}{\sqrt{np(1-p)}})$$

Maximum A posteriori Probability (MAP) Binary Hypothesis Test
Given a binary hypothesis testing experiment with outcome s, the following rule leads to the lowest possible value of P_{ERR}:

$$s \in A_0 \text{ if } P[H_0|s] \geq P[H_1|s]; \qquad s \in A_1 \quad \text{otherwise.}$$

For an experiment that produces a random vector X, the MAP hypothesis test is

$$\text{Discrete: } x \in A_0 \text{ if } \frac{P_{X|H_0}(x)}{P_{X|H_1}(x)} \geq \frac{P[H_1]}{P[H_0]}; \, x \in A_1 \text{ otherwise.}$$
$$\text{Continuous: } x \in A_0 \text{ if } \frac{f_{X|H_0}(x)}{f_{X|H_1}(x)} \geq \frac{P[H_1]}{P[H_0]}; \, x \in A_1 \text{ otherwise.}$$

For an experiment that produces a random vector X, the minimum cost hypothesis test is

$$\text{Discrete: } x \in A_0 \text{ if } \frac{P_{X|H_0}(x)}{P_{X|H_1}(x)} \geq \frac{P[H_1]C_{01}}{P[H_0]C_{10}}; \, x \in A_1 \text{ otherwise.}$$
$$\text{Continuous: } x \in A_0 \text{ if } \frac{f_{X|H_0}(x)}{f_{X|H_1}(x)} \geq \frac{P[H_1]C_{01}}{P[H_0]C_{10}}; \, x \in A_1 \text{ otherwise.}$$

$$\boxed{\begin{array}{c} P_{ERR} = P[A_1|H_0]P[H_0] + P[A_0|H_1]P[H_1] \\ P_{MISS} = P[A_0|H_1] = P[X \leq x_0|H_1] = \Phi(x_0 - v) \\ P_{FA} = P[A_1|H_0] = P[X > x_0|H_0] = 1 - \Phi(x_0) \end{array}}$$