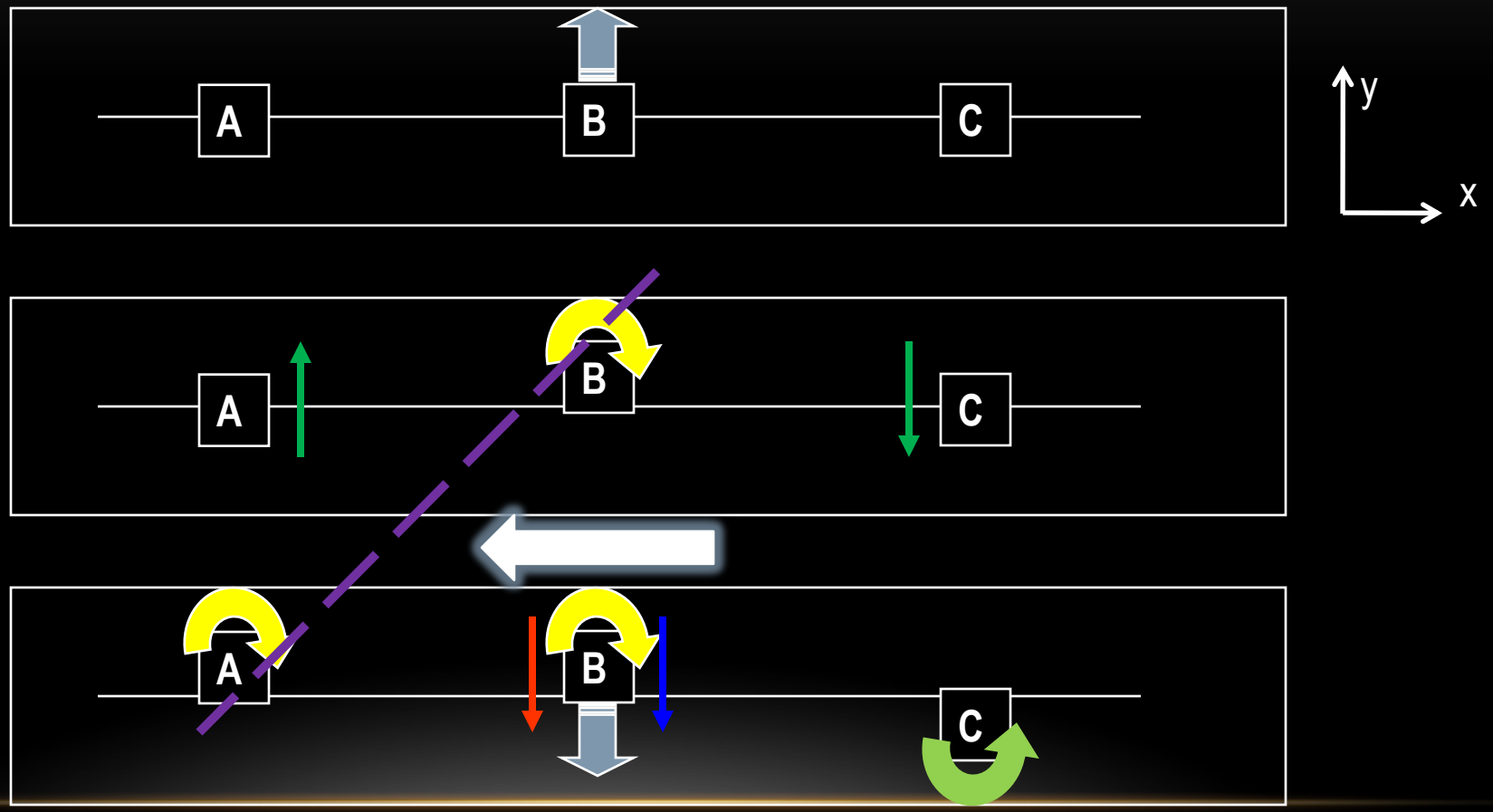
Linear Wave Basics

Physical interpretation of Rossby waves (vorticity waves)

Consider non-divergent, barotropic flow:
the absolute vorticity is conserved.
Note that f increases northward.

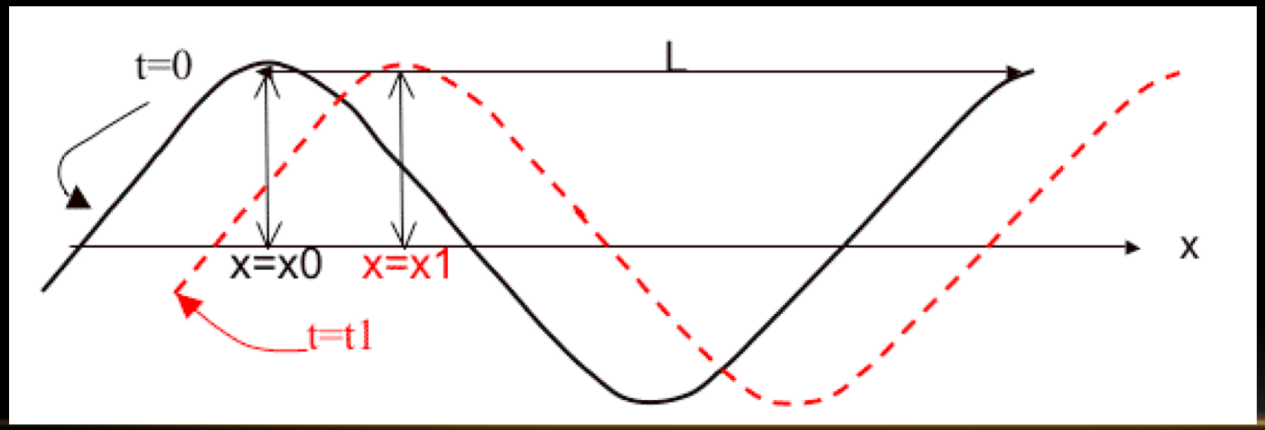


Phase Speed

- Phase speed: The speed of propagation of a mathematical surface of constant phase (or phase angle) of a time-harmonic wave (AMS Glossary).
- For one-dimensional sinusoidal waves:

$$\psi' = \mathrm{A}\ cos\left(kx - \nu t\right)$$

C = frequency / (wave number)



Rossby Wave Phase Speed

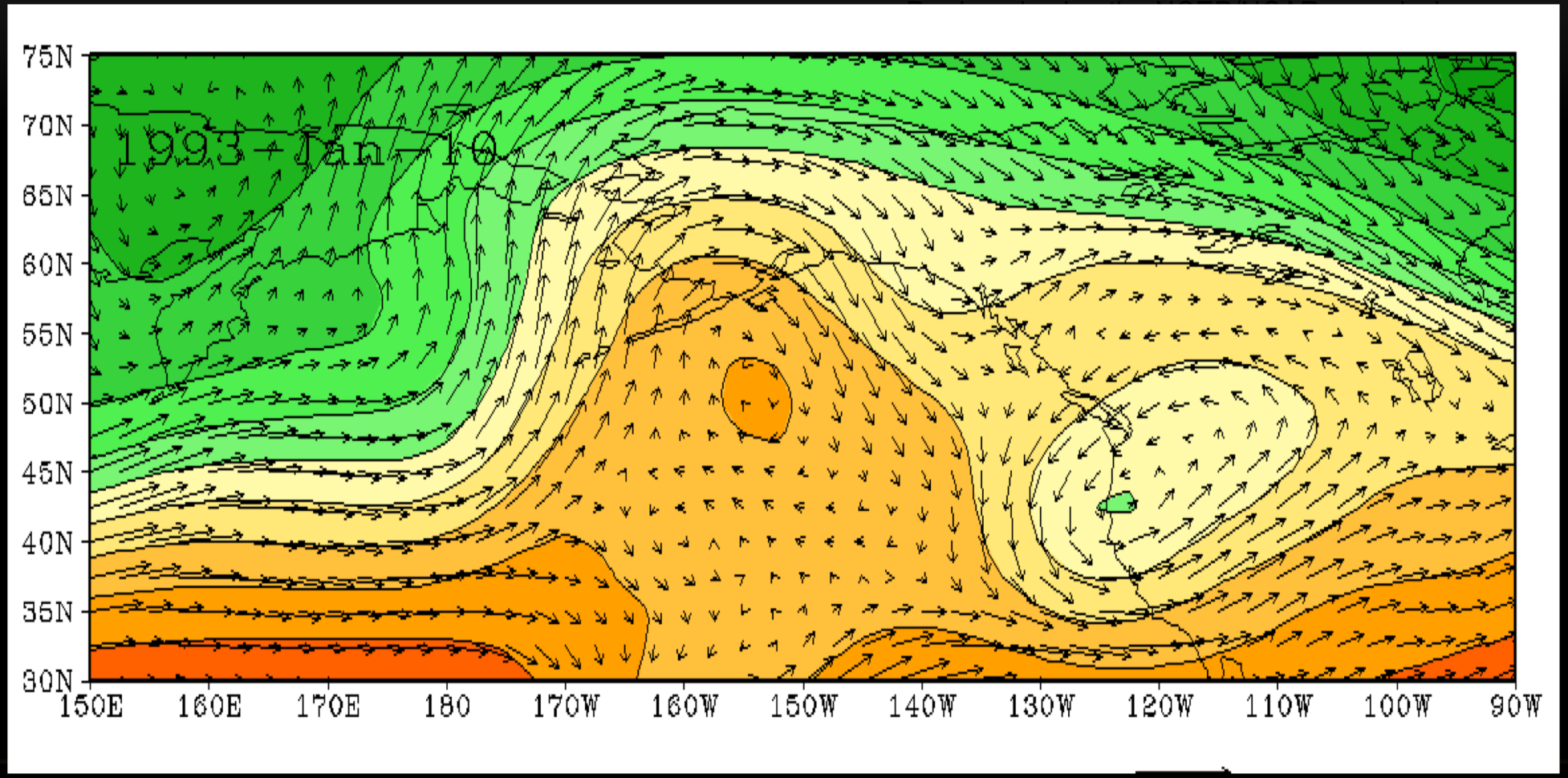
- For 1D Rossby waves,

$$c = \overline{u} - \frac{\beta}{K^2},$$

where the 1st RHS is the mean flow, β = df/dy, and K^2 is the wavenumber squared.

- The Rossby wave moves westward relative to the mean flow. However,
  - If the mean flow is easterly, c<0
  - If the mean flow is westerly, short waves propagate eastward (c>0), long waves propagate westward (c<0), and waves of a certain wavelength may be stationary (c=0)

Eastward Propagating Synoptic-Scale Waves: 500 hPa U, V and H (1993-01-10)



- Short waves propagate eastward in the mid-latitudes.

Westward Propagating Tropical Easterly Waves

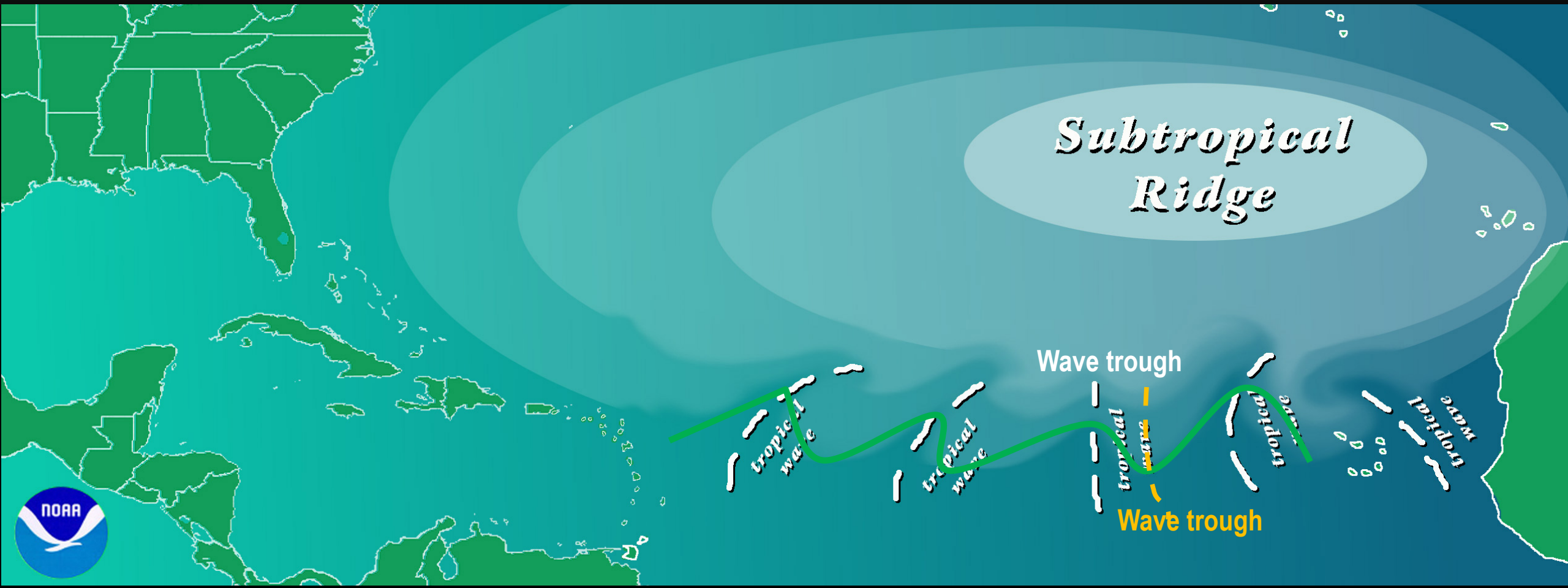
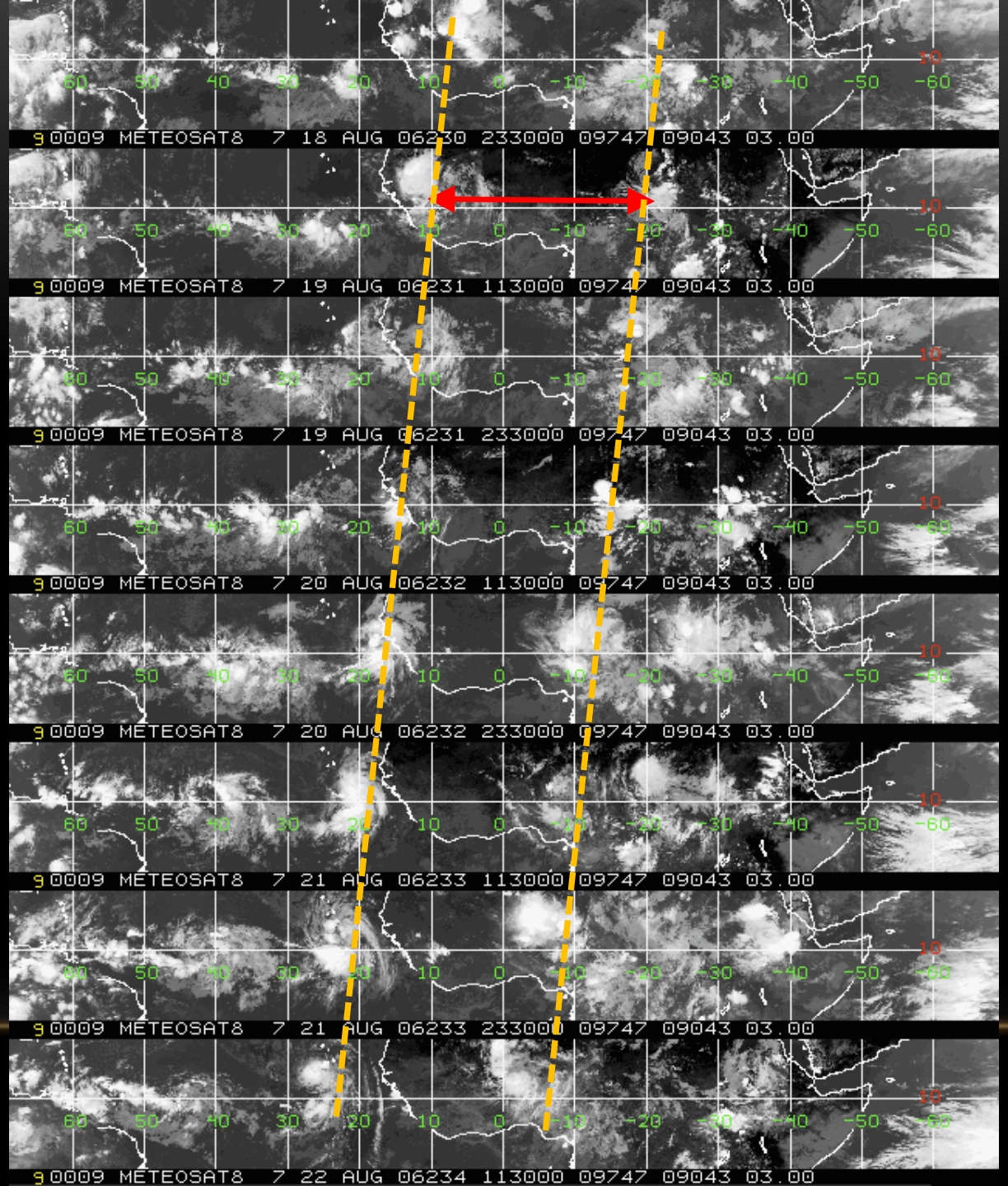


Figure from NOAA: https://www.aoml.noaa.gov/hrd/tcfaq/Twaves.jpg



https://radarmet.atmos.colostate.edu/namma/logs/NAMMA-TOGA_Radar_Scientist_Log_start-2Sep.html

Stationary Rossby Waves

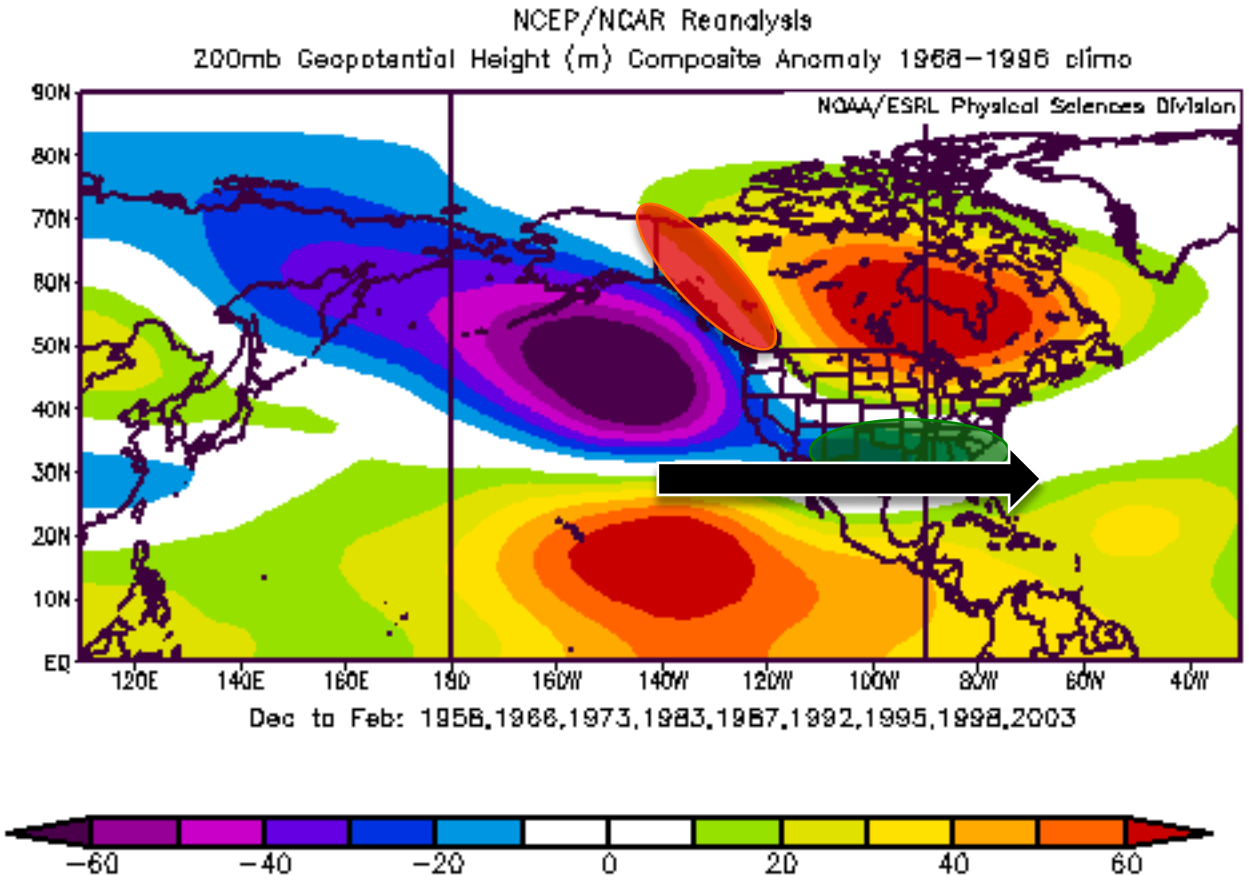


Figure produced at https://psl.noaa.gov/cgi-bin/data/composites/printpage.pl

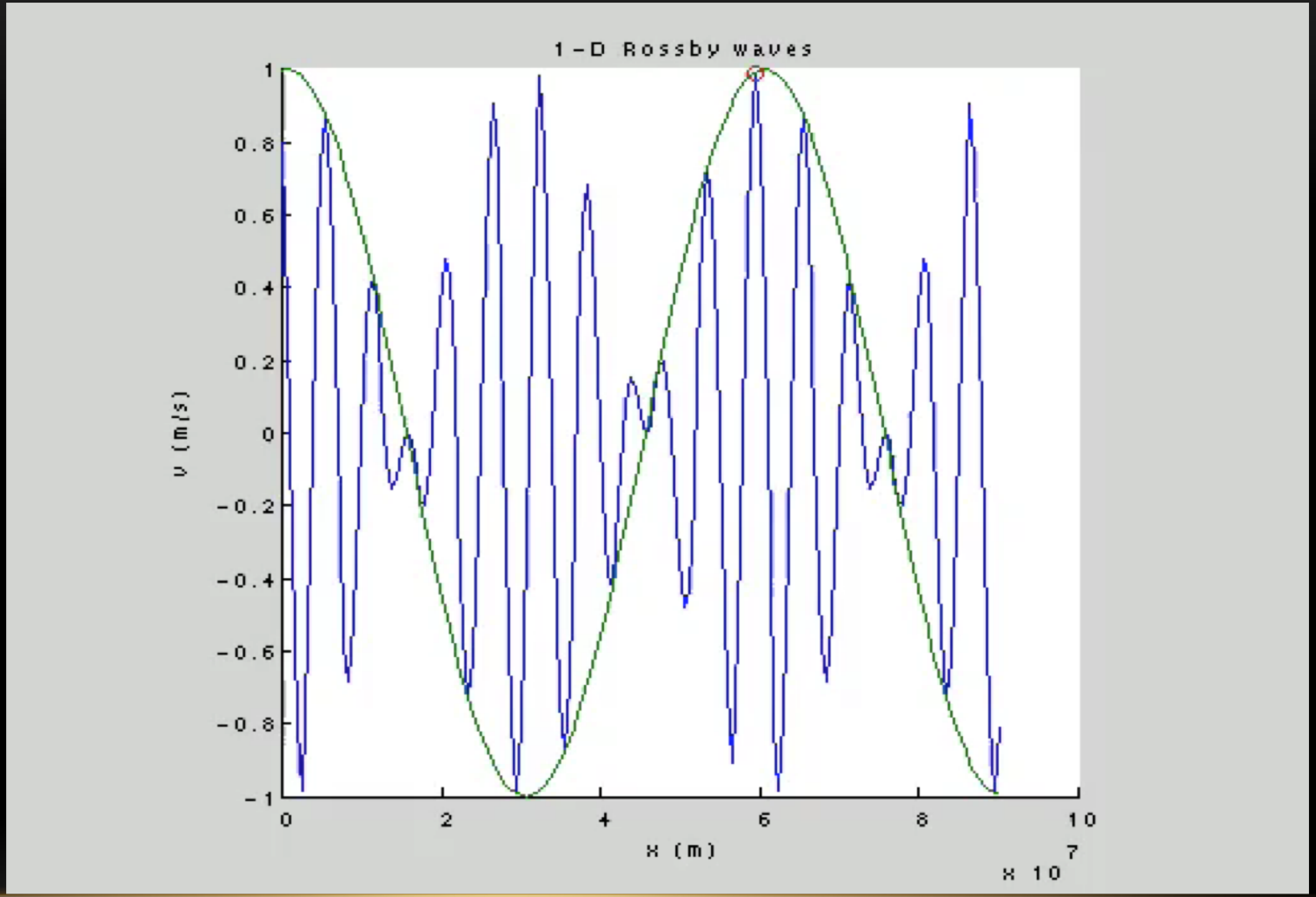
Group Velocity

- Group velocity: The velocity at which a group of waves, and the wave energy, travels. Often denoted as Cg.
- Non-dispersive waves: the phase speed c is not a function of wavenumber, and the waves of different wavenumbers all move at the same speed. c=cg, and the shape of the total pattern remains the same.
- Dispersive waves: the phase speed c is a function of wavenumber, and the waves of different wavenumbers propagate at different speeds. The wave “envelope” generally broadens in time and the wave energy is dispersed in space.
- Rossby waves are dispersive waves.

Another example: 1-D Rossby waves

$$c_p = U - \frac{\beta_o}{k^2}$$

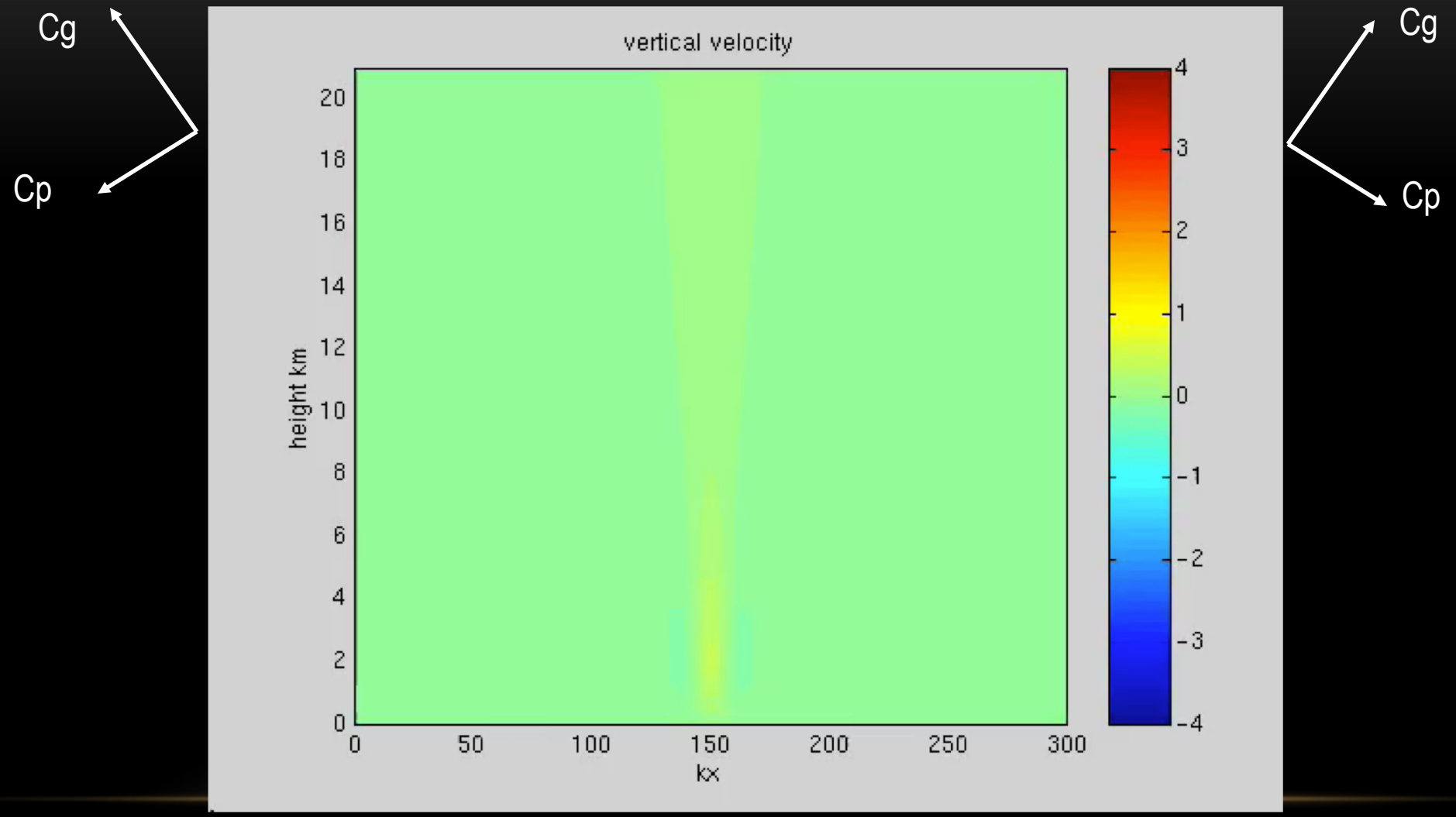
$$c_g = U + \frac{\beta_o}{k^2}$$



Assume *U=0*, **C_g>0: eastward (following the wave envelop)**

C_p<0: westward (following the red circle)

An example of Dispersive waves: internal gravity waves

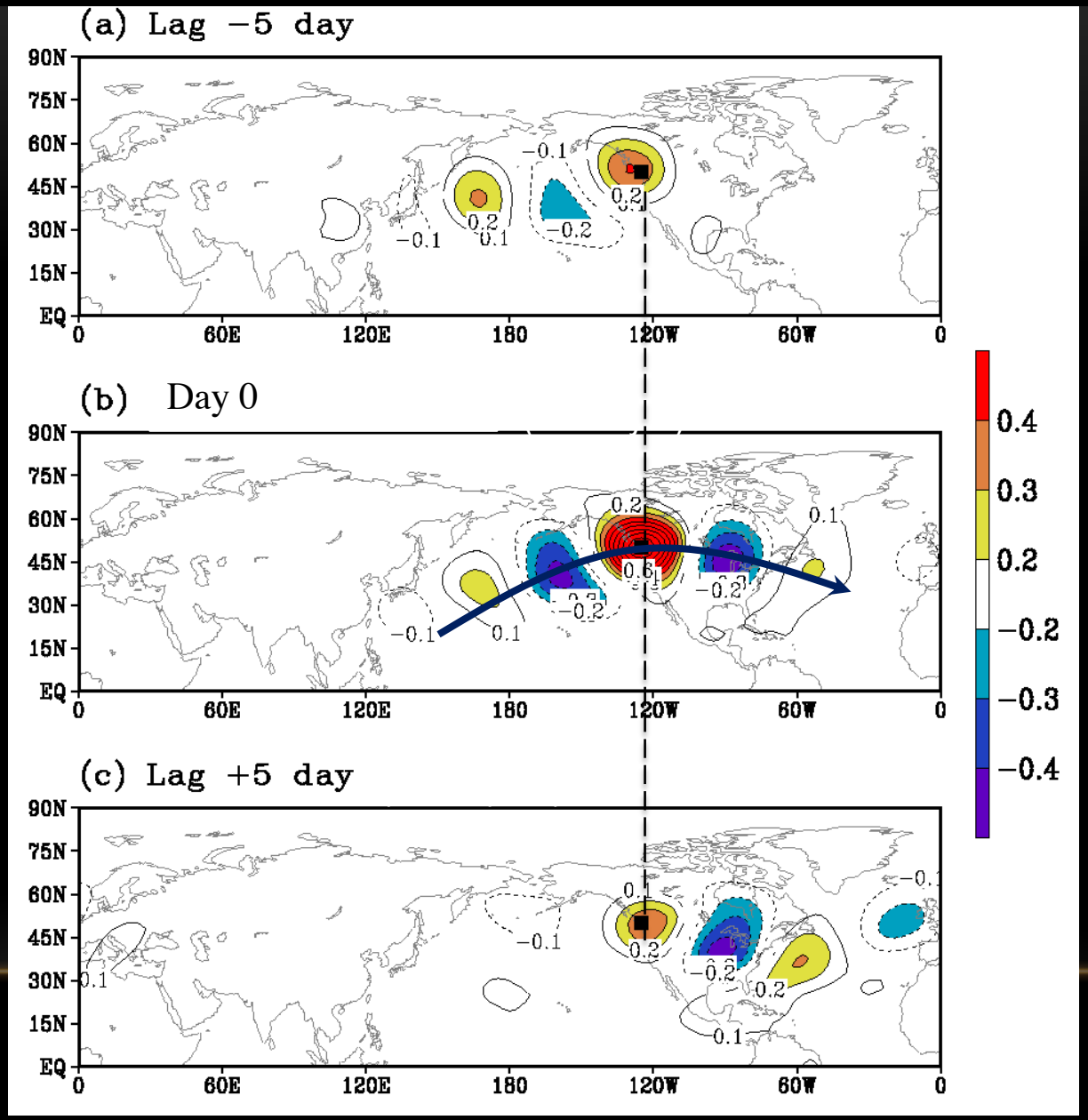


The vertical phase velocity and the vertical group velocity have opposite directions.

From Chapter 5 in Holton and Hakim 2013

2-D Rossby Waves

Due to the beta-effect and the mean flow variations, stationary Rossby waves often take a great circle route: they extend poleward and eastward from the tropics and then recurve back equatorward at a certain latitude.



Wang 2004

References

- Holton, J. and G. Hakim, 2013: An Introduction to Dynamic Meteorology, Fifth Edition. Chapter 5: Atmospheric Oscillations.