1 Equations and graphs of Circles, Ellipses and Hyperbolas

1.1 Circles

1.1.1 Standard equation of a circle:

$\sqrt{(x-h)^2 + (y-k)^2} = r$

Where (h, k) is the center and r is the radius.

1.1.2 General equation of a circle:

$x^2 + y^2 + Dx + Ex + F = 0$

To go from general equation to standard equation, complete the square for both x and y.

1.1.3 Finding equation of circle from points:

Recall that the standard equation for a circle is

$\sqrt{(x-h)^2 + (y-k)^2} = r$

All points on the circle follow this equation. By substituting x and y for 3 points on the circle, we can solve for h and k. 3 points are required to uniquely define a circle.

1.2 Ellipses

1.2.1 Standard equation of an ellipse:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Where $-a$ and a are the x-intercepts and $-b$ and b are the y-intercepts for an ellipse centered at the origin.

1.2.2 Drawing an ellipse

1. Transform equation into standard form (divide such that RHS = 1)
2. Mark x and y intercepts
3. Connect the dots

1.3 Hyperbolas

1.3.1 Types of hyperbolas:

Type I (positive x^2): $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Type II (positive y^2): $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

Type I hyperbolas are "horizontal". They cut the x axis at $-a$ and a.

Type II hyperbolas are "vertical". They cut the y-axis at at $-b$ and b.

1.3.2 Finding oblique asymptotes of hyperbola:

$y = \pm\frac{bx}{a}$

2 Basic transformations

2.1 Translation and scaling

2.1.1 Parallel to y-axis

Scaling comes first.
Official phrasing:
Translation parallel to y-axis in positive/negative direction.
Scaling parallel to y-axis with scale factor a.

2.1.2 Parallel to x-axis

Translation comes first.
Note that directions are reversed.
Official phrasing:
Translation parallel to x-axis in positive/negative direction.
Scaling parallel to x-axis with scale factor $\frac{1}{a}$.