15.761 Introduction to Operations Management

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1 Process Flow Analysis

1.1 Process flow diagram

Attention!

- Always a good idea to sketch out a diagram even if not asked for
- Label the resources. e.g. cashier / barista / Christian / Marisa with dashed boxes
- Label all the final outcomes
- Be consistent with the time unit: hours vs minutes, stick to one
- Differentiate "average rate" vs "average time duration" provided for each step: one is the inverse of another
- You can depart more than the arrival:
  - Arrival rate of a later step is $min\left[\lambda_{\text{previous step}}, (N_{\text{previous step}} \cdot \mu_{\text{previous step}})\right]$
  - Depart rate of the current step is $min\left[\lambda, N \cdot \mu\right]$
- When interpreting a drawn diagram be **careful with** $\boxed{\textbf{change of the “perspective”.}}$ e.g. diagram may reflect the flow of a patient but we may need to change to a doctor's perspective

1.2 Throughput time

The time that elapses from when the job starts the process to the time it ends the process.

1.3 Little's Law

The only law in operations management.

$$L = \lambda W \tag{1}$$

- L: average number of jobs in system
- λ: average job arrival rate
- W: average throughput time / average time each job spend in the process

Caution: watch out for unit conversion, especially time-related units.

2 Capacity

$$\text{capacity utilization} = \frac{\text{capacity required}}{\text{capacity available}} \tag{2}$$

Three ways to change capacity utilization:

1. Increase resources: maintain speed but add time available or add resources

2. Work faster: in same amount of time

3. Shift demand

If we need to distinct peak and non-peak times instead of just looking at the average behavior, simply break the time interval into two time intervals: peak and non-peak.

2.1 Capacity measured in terms of units

$$\text{capacity required} = \#\text{ of jobs} \tag{3}$$

$$\text{capacity available} = \frac{\text{time available}}{\text{cycle time}} \tag{4}$$

2.2 Capacity measured in terms of time

$$\text{capacity required} = (\#\text{ of jobs})(\text{cycle time}) \tag{5}$$

$$\text{capacity available} = \text{time available} \tag{6}$$

2.3 Adjusted for start-up

$$\text{capacity available} = \frac{\text{time available} - \text{throughput time}}{\text{cycle time}} + 1 \tag{7}$$

If looking for the number of "whole" units can be made, round **down** to the nearest whole number.

3 Congestion Analysis

3.1 Deterministic variability - inventory buildup diagrams

The balance equation:

$$\begin{aligned}(\#\text{ of jobs in system at end of period}) = &(\#\text{ of jobs } \textbf{in} \text{ system at start of period})\\ &+ (\#\text{ of jobs } \textbf{arriving} \text{ to system during period})\\ &- (\#\text{ of jobs serviced (departed system) during period})\end{aligned} \tag{8}$$

Can replace "system" with "queue" or "service".

$$\text{buildup rate} = \text{arrival rate} - \text{departure rate} \tag{9}$$

Buildup rate can be negative if there is inventory being worked off. Otherwise, the departure rate cannot exceed the arrival rate.

$$\text{average inventory} = \frac{\textbf{area} \text{ under the inventory buildup curve}}{\text{total time interval}} \tag{10}$$

Also applicable to average queue size if the "inventory" is considered as jobs waiting in a queue.

4 Queuing Analysis

4.1 Setup

- A: time between successive job arrivals (a.k.a "interarrival time")
- TODO

4.2 Capacity utilization ρ

$$\rho = \frac{\lambda}{N\mu} \tag{11}$$

- λ: job arrival rate
- N: number of servers
- μ: a server's expected service rate, which is the inverse of the average service time $\bar{S}$

$$\mu = \frac{1}{\bar{S}} \tag{12}$$

4.3 Coefficient of variation of the interarrival time CV_S

$$CV_S = TODO \tag{13}$$

4.4 Coefficient of variation of the service time CV_A

$$CV_A = TODO \tag{14}$$

4.5 Expected number of jobs in the queue L_q

$$L_q = \frac{\rho^{\sqrt{2(N+1)}}}{1-\rho} \frac{C_A^2 + C_S^2}{2} \tag{15}$$

Midterm exam until here.

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5 Exam Cautions

- Draw a supply chain graph clearly showing:
  - the producer
  - the distributor
  - the retailer
  - the customer
- Fathom, the "seller" and "buyer" roles are relative to which stage in the supply chain?

6 Newsvendor Inventory Model

Goal: Find the optimal order quantity Q^*, that only be placed once, based on:

- **purchase cost** c: each unit's cost
- **sales price** p: each unit's sold price
- **salvage value** s: each unit's salvage price; if each unsold unit incurs a disposal cost, the value is $\boxed{\text{negative}}$.
- Assume: $p > c > s$

6.1 Two "costs" needed to know

$$\text{overage cost} = c - s \tag{16}$$

Again: remember that s can be negative.

$$\text{underage cost} = p - c \tag{17}$$

6.2 Critical Fractile

Expresses the underage cost as a percentage of the combined underage and overage costs.

$$\text{critical fractile (cf)} = \frac{\text{underage cost}}{\text{underage cost} + \text{overage cost}} = \frac{p - c}{p - s} \tag{18}$$

6.3 Optimal Order Quantity Q^*

$$Q^* = F^{-1}(\text{cf}) \tag{19}$$

where $F^{-1}(\alpha)$ denotes inverse cumulative distribution function (CDF).

6.3.1 Example

If demand is $\mathcal{N}(\mu, \sigma)$, then $Q^* = \mu + \text{Z-score}(\text{cf}) \cdot \sigma$

6.4 Multi-product problem with capacity constraint

Find a single z-value that equalizes the fill rate across all products:

$$z = \frac{K - \sum_{i=1}^{N} \mu_i}{\sum_{i=1}^{N} \sigma_i} \tag{20}$$

- N products that must be ordered
- K number units limit across all N products

Assumption: all products all have the exact same overage and underage costs. If they differ, the optimal order quantities have to take into account the other SKU's stocking level to accurately calculate the combined overage and underage costs.

7 Bullwhip Effect

Exists when order volatility (i.e. variance) increases as you move upstream (i.e. farther from the customer / closer to the factory).

7.1 Causes

- Promotion and volume discounts
- Sales incentives
- No information sharing
- Long lead-times (easily forget about the in-transit inventory)
- Anxiety, emotions, distrust, ...

7.2 Negativities

- High production costs (switch-over, overtime, etc.)
- Forcasting challenge
- High transportation costs
- High inventory but low service level
- Hurts trust among business partners

7.3 Solution

Communicate more.

8 Base Stock Policy

$$\begin{aligned}\text{average inventory on-hand} &= \text{expected cycle stock} + \text{expected safety stock} \\ &= \frac{r\mu}{2} + z\sigma\sqrt{r+L}\end{aligned} \tag{21}$$

- r **review period**: the interval of time that elapses before an inventory evaluation occurs
- L **replenishment time**: the elapsed time between placing an order and having the ordered unit available to satisfy demand
- α **cycle service level (CSL)**: the probability that the SKUL will have sufficient inventory on-hand to meet all demand in an order cycle
- Assume demand follows $\mathcal{N}(\mu, \sigma)$

8.1 If given an average inventory on-hand target of X (time-period-units of supply)

$$\text{current average on-hand inventory level} = X \cdot \mu \tag{22}$$

where μ is the mean of the normally distributed demand on the same time-period-unit.

8.1.1 Impute service level

$$\begin{aligned}X\mu &= \frac{r\mu}{2} + z\sigma\sqrt{r+L} \\ \Rightarrow z &= \frac{X\mu - \frac{r\mu}{2}}{\sigma\sqrt{r+L}} \\ \Rightarrow \text{service level} &= \Phi(z)\end{aligned} \tag{23}$$

where $\Phi(z)$ is the CDF for the standard normal random variable.
In Excel: `=NORM.S.DIST(z, TRUE)`

9 Economic Order Quantity (EOQ) model

$$Q^* = \sqrt{\frac{2K\lambda}{h}} \tag{24}$$

- K: fixed order cost incurred every time an order is placed
- λ: demand in units per <u>year</u>
- $h = Ic$ inventory holding cost: per unit <u>per year</u> cost to hold inventory
  - c: per unit purchasing cost
  - I: <u>annual</u> interest rate

If the deterministic demand always arrives in increments of x where EOQ cannot be evenly divided by x, update EOQ to be the nearest multiply of the x.

9.1 Reorder point

$$R = L\mu \tag{25}$$

10 Convert the underlying time unit of a normal distribution

Given weekly demand $W \sim \mathcal{N}(\mu_W, \sigma_W)$,

- Monthly demand $M \sim \mathcal{N}(4 \cdot \mu_W, \sqrt{4} \cdot \sigma_W)$ if $M = (W + W + W + W)$ and W are i.i.d.
- Daily demand $D \sim \mathcal{N}(\frac{\mu_W}{5}, \frac{\sigma_W}{\sqrt{5}})$, if $W = (D + D + D + D + D)$ and D are i.i.d.