Linear Regression

Linear regression means the dependent variable is linear in the model coefficients.

- Linear: $y = b_0 + b_1x_1 + b_2x_2$
- Not linear: $y = b_0x_1^{b_1} + b_3x_2^{b_2}$

lm in R

Rentals as a function of temperature and humidity:

```
mod <- lm(data = df, rentals ~ temp
    + rel_humidity)
summary(mod)
```

Consider a "Best Fitting" Line

$$\boxed{\hat{y} = b_0 + b_1x}$$

Slope b_1 - sign is same as the sign of CORR(X,Y). CORR(X,Y) is between $[-1,1]$ and unit-less. But b_1 has units.

$$b_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \text{CORR}(X,Y)\cdot\frac{\text{SD}(Y)}{\text{SD}(X)}$$

Intercept - $b_0 = \bar{y} - b_1\bar{x}$

Mean-center data - if both y and x variables are mean-centered, rerun linear regression to get:

$$\hat{y}_i - \bar{y} = b_1(x_i - \bar{x})$$

- new intercept will be 0
- new slope remains b_1

Regression to the mean - If $x = \bar{x}$ (the average of all x values), the predicted $\hat{y} = \bar{y}$ (average of all y values), independent from x.

Independent Variables

Categorical variables ("factor variable" in R) - if encoded with one-hot, one category should be dropped to avoid perfect multicollinearity (a situation where one predictor variable can be perfectly predicted from the others). This omitted category serves as a reference category against which the other categories are compared. This approach is known as creating "dummy variables."

Evaluation

Residuals (error) - $e_i = y_i - \hat{y}_i$

Sum of Squared Residuals -

$$SSR = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Total Sum of Squares - a measure of the total variability in the observed data.

$$SST = \sum_{i=1}^{n}(y_i - \bar{y}_i)^2$$

R^2 - the proportion reduction in sum of squared residuals by the regression model compared to the baseline model (which always predicts the average value of all ys in the data: $\hat{y} = \bar{y}$).

$$\boxed{R^2 = 1 - \frac{SSR}{SST} = \frac{SST - SSR}{SST}}$$

- R^2 of a regression model lies between 0 and 1.
- Adding a new independent variable to a regression model can only increase R^2.
- R^2 on its own cannot judge how good a model is.

R^2 in multiple linear regression - equals the square of the correlation between the actual values y and the predicted values $\hat{y}$.

$$R^2 = (\text{CORR}(y,\hat{y}))^2$$

In the case of a perfect fit where every prediction of y_i is going to be correct, then $R^2 = 1$

R^2 in simple linear regression - equals the square of correlation between dependent variable y and independent variable x.

$$R^2 = (\text{CORR}(y,x))^2$$

Degrees of Freedom - the number of observations minus the number of parameters estimated (including the intercept): $df = n - p - 1$

Residual Variance - $\frac{SSR}{df}$

Residual Standard Error - provides a measure of the average distance that the observed values fall from the regression line.

$$RSE = \sqrt{\frac{SSR}{df}}$$

- Smaller RSE: the model's predictions are closer to the actual data points, suggesting a good fit.
- Larger RSE: the model's predictions are further from the actual data points, suggesting a poor fit.

Troubleshooting

The model's ability to predict the future and its interpretability can be impaired by

- The presence of irrelevant independent variables
- The presence of highly correlated independent variables
- The presence of "too many" variables relative to the size of the dataset

Irrelevant variables - have large p-values. Smaller p-value (R: `Pr(>|t|)`), the better. If lower than 0.05, consider it significant at the 5% significance level; otherwise, consider it non-significant at the 5% significance level. In R, count `*`

p-value of a variable x_i - the probability of observing a coefficient estimate as extreme, or more extreme, than the one actually obtained by the regression run in a similar sample assuming $\beta_i = 0$.

Highly correlated variables - Detected by inspecting the correlation matrix. (R: `cor(df)`).

Multicollinearity - When two variables are highly correlated (typically correlations higher than 0.75 in magnitude), resolve this by removing either of the independent variables and running the linear regression **again**.

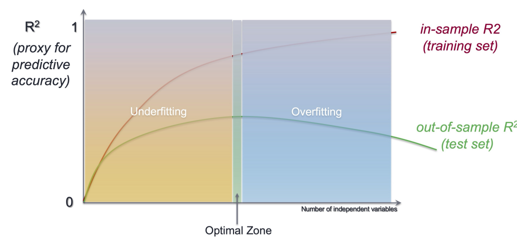
Overfitting ("too many" variables) - leads to poorly predicting future data. Remedy by (1) avoiding "non-significant" variables using the starts `*` as a guide and (2) out-of-sample testing.

Out-of-Sample Testing -

1. Partition data set into 70% trainning set and 30% test set before creating the regression model.
2. Generate a set of models (e.g. with different sets of variables) using only the training data.
3. Evaluate models on the test set using out-of-sample R^2.

Out-of-Sample R^2 -

$$\boxed{\text{OSR}^2 = 1 - \frac{\text{SSR of regression model applied to the test set}}{\text{SSR of baseline model applied to the test set}}}$$



Key takeaways -

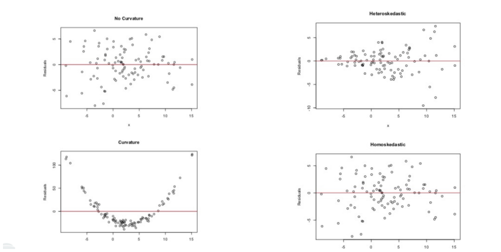
- Choose final model based on out-of-sample predictive quality metrics.
- Use metrics like R^2 and the standard error of regression in combination because they have different strength and weaknesses.
- Only include significant variables that are not highly correlated.
- Coefficients should "make sense".
- Use for interpolation rather than extrapolation.

Technical caution - for the p-value to be correct, the "unaccounted for" differences in the regression model ϵ (think of the residuals) needs to have zero mean, constant standard deviation, be independent and follow a normal distribution.

Linearity - there is no curvature

Homoskedasticity - the dispersion of $e_i = y_i - \hat{y}_i$ is not systematically smaller or larger for large x_i values compared to small x_i values.

Nonlinearity or Curvature in Residuals? Heteroskedasticity in error variances?



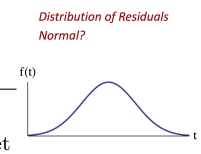
Top-right: hetero. Bottom-right:homo.

Normality - the residuals are approximately normal.

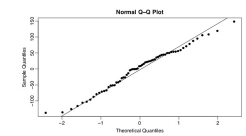
Checking Normality of Residuals.

QQ-Plot (quantile vs quantile plot) is commonly used here.

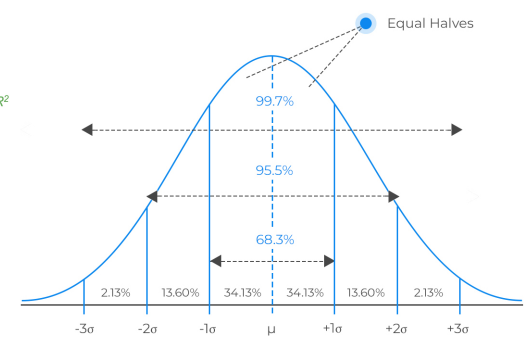
Make a normal quantile plot of the residuals interpretation:




```
qqnorm(resid.both)
qqline(resid.both)
```



Side bar



- about 68% of the total values lie within 1 standard deviation of the mean.
- about 95% lie within 2 standard deviations of the mean
- about 99.7% lie within 3 standard deviations of the mean.