15.761 Introduction to Operations Management

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1 Process Flow Analysis

1.1 Process flow diagram

Attention!

- Always a good idea to sketch out a diagram even if not asked for
- Label the resources. e.g. cashier / barista / Christian / Marisa with dashed boxes
- Label all the final outcomes
- Be consistent with the time unit: hours vs minutes, stick to one
- Differentiate "average rate" vs "average time duration" provided for each step: one is the inverse of another
- You can depart more than the arrival:
  - Arrival rate of a later step is $min\,[\lambda_{\text{previous step}}, (N_{\text{previous step}} \cdot \mu_{\text{previous step}})]$
  - Depart rate of the current step is $min\,[\lambda, N \cdot \mu]$
- When interpreting a drawn diagram be **careful with** $\boxed{\textbf{change of the “perspective”.}}$ e.g. diagram may reflect the flow of a patient but we may need to change to a doctor's perspective

1.2 Throughput time

The time that elapses from when the job starts the process to the time it ends the process.

1.3 Little's Law

The only law in operations management.

$$L = \lambda W \tag{1}$$

- L: average number of jobs in system
- λ: average job arrival rate
- W: average throughput time / average time each job spend in the process

Caution: watch out for unit conversion, especially time-related units.

2 Capacity

$$\text{capacity utilization} = \frac{\text{capacity required}}{\text{capacity available}} \tag{2}$$

Three ways to change capacity utilization:

1. Increase resources: maintain speed but add time available or add resources
2. Work faster: in same amount of time
3. Shift demand

If we need to distinct peak and non-peak times instead of just looking at the average behavior, simply break the time interval into two time intervals: peak and non-peak.

2.1 Capacity measured in terms of units

$$\text{capacity required} = \text{\# of jobs} \tag{3}$$

$$\text{capacity available} = \frac{\text{time available}}{\text{cycle time}} \tag{4}$$

2.2 Capacity measured in terms of time

$$\text{capacity required} = (\text{\# of jobs})(\text{cycle time}) \tag{5}$$

$$\text{capacity available} = \text{time available} \tag{6}$$

2.3 Adjusted for start-up

$$\text{capacity available} = \frac{\text{time available} - \text{throughput time}}{\text{cycle time}} + 1 \tag{7}$$

If looking for the number of "whole" units can be made, round **down** to the nearest whole number.

3 Congestion Analysis

3.1 Deterministic variability - inventory buildup diagrams

The balance equation:

$$\begin{aligned}(\text{\# of jobs in system at end of period}) = &(\text{\# of jobs } \textbf{in} \text{ system at start of period}) \\ &+ (\text{\# of jobs } \textbf{arriving} \text{ to system during period}) \\ &- (\text{\# of jobs serviced (departed system) during period})\end{aligned} \tag{8}$$

Can replace "system" with "queue" or "service".

$$\text{buildup rate} = \text{arrival rate} - \text{departure rate} \tag{9}$$

Buildup rate can be negative if there is inventory being worked off. Otherwise, the departure rate cannot exceed the arrival rate.

$$\text{average inventory} = \frac{\textbf{area} \text{ under the inventory buildup curve}}{\text{total time interval}} \tag{10}$$

Also applicable to average queue size if the "inventory" is considered as jobs waiting in a queue.

4 Queuing Analysis

4.1 Setup

- A: time between successive job arrivals (a.k.a "interarrival time")
- TODO

4.2 Capacity utilization ρ

$$\rho = \frac{\lambda}{N\mu} \tag{11}$$

- λ: job arrival rate
- N: number of servers
- μ: a server's expected service rate, which is the inverse of the average service time $\bar{S}$

$$\mu = \frac{1}{\bar{S}} \tag{12}$$

4.3 Coefficient of variation of the interarrival time CV_S

$$CV_S = TODO \tag{13}$$

4.4 Coefficient of variation of the service time CV_A

$$CV_A = TODO \tag{14}$$

4.5 Expected number of jobs in the queue L_q

$$L_q = \frac{\rho^{\sqrt{2(N+1)}}}{1-\rho} \frac{C_A^2 + C_S^2}{2} \tag{15}$$