Net Present Value (NPV)

Discount rate r: you're indifferent between receiving \$1 today and $\$\frac{1}{1+r}$ in one period.

Present Value (PV): $\boxed{PV(CF_t) = \frac{CF_t}{(1+r)^t}}$ how much a cash flow (CF) at time t is worth at time 0 (today). Computing a PV is often called "discounting".

NPV: $\boxed{NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}}$ summs over PVs of all cash flows in a project.

- **Scalability**: $NPV(\alpha CF_1, \ldots, \alpha CF_T) = \alpha NPV(CF_1, \ldots, CF_T)$
- **Additivity**: $NPV(X_1 + Y_1, \ldots, X_T + Y_T) = NPV(X_1, \ldots, X_T) + NPV(Y_1, \ldots, Y_T)$
- **Breaking up by time**: $NPV(CF_1, \ldots, CF_T) = NPV(CF_1, \ldots, CF_j) + NPV(CF_{j+1}, \ldots, CF_T)$

Future Value (FV): $\boxed{FV_T(CF_0) = CF_0(1+r)^T}$ how much a cash flow at time 0 (today) is worth in T periods.

Perpetuity:

- Constant recurring cash flow A forever starting **1 period from now**: $\boxed{PV = \frac{A}{r}}$
- Growing perpetuity starting **1 period from now** with cash flow A, growth rate g: $\boxed{PV = \frac{A}{r-g} (r > g)}$

Annuity:

- Constant recurring cash flow A for T periods starting **1 period from now**: (E.g. a loan)
  $\boxed{PV = \frac{A}{r}\left(1 - \frac{1}{(1+r)^T}\right)}$
  $\boxed{FV = PV \cdot (1+r)^T = A\frac{(1+r)^T - 1}{r}}$
- Growing annuity starting **1 period from now** with cash flow A, growth rate g for T periods.
  - If $r \neq g$ $\boxed{PV = \frac{A}{r-g}\left(1 - \frac{(1+g)^T}{(1+r)^T}\right)}$; $\boxed{FV = A\left(\frac{(1+r)^T - (1+g)^T}{r-g}\right)}$
  - If $r = g$ $\boxed{PV = T\left(\frac{A}{1+r}\right)}$; $\boxed{FV = T \cdot A \cdot (1+r)^{T-1}}$

Annual Percentage Rate (APR) & Effective Annual Rate (EAR):

$\boxed{(1 + EAR) = \left(1 + \frac{APR}{k}\right)^k = (1+r)^k}$ where k is the number of compounding periods per year and r is the per-period (e.g. monthly) interest rate.

- APR $= r \cdot k$
- EAR i.e. Annual Percentage Yield (APY)

Mortgage-related terms

Principal: the amount of \$ borrowed in a lending agreement. E.g. Buy a \$1,000,000 house with a 20% down payment, the principal is \$800,000.

Interest:

- **Fixed rate**: No matter what happens to interest rates around the world, you would still be charged interest at this same rate.
- **Adjustable rate (ARM)**: E.g. an adjustable rate of 3% above the federal funds rate (the Fed's benchmark rate). If this rate is around 4.5%, you would be charged a 7.5% interest rate. If in the next month the Fed raises to 5%, you would be charged an 8% interest rate.

Amortization schedule: sequence of payments made through the loan's lifetime. A part of the payments goes to reduce (i.e. amortize) the principal owed, and the rest goes to pay the interest on the loan.

Collateral: An asset offered by the borrower as a guarantee in a loan. If you fail to make payments, the bank can take the collateral.

Refinancing: Paying off an existing loan with a new loan that has better terms. E.g. lower interest rate, lower monthly payment, shorter loan term.

E.g.: a 30-year fixed-rate mortgage, APR 9% compounded monthly. Fixed monthly payment = \$3000. First payment will start next month and last until the contract expires in 30 years.

- **How much borrowed when took out the mortgage?**: use the constant annuity formula, where $A = \$3000$, $r = \frac{0.09}{12} = 0.0075$, $T = 30 \times 12 = 360$. $PV = \frac{A}{r}\left(1 - \frac{1}{(1+r)^T}\right) = \frac{3000}{0.0075}\left(1 - \frac{1}{(1+0.0075)^{360}}\right) = \$372,845.60$
- **10 years later, how much must pay back to the bank if sell the house?**: (i.e. NPV of the remaining principal amount as of this future date.) $T = 20 \times 12 = 240$
  $PV = \frac{3000}{0.0075}\left(1 - \frac{1}{(1+0.0075)^{240}}\right) = \$333,434.86$
- **Why the remaining principal is still so high after 10 years' worth of repayments?**: Because the amortization schedule is front-loaded. I.e. Early on in the life of the mortgage, the vast majority of each monthly payment is interest: interest is applied to the remaining principal amount, and this remaining principal is highest in month 1 and decreases over time. Because the majority of each monthly payment in the early years is interest, the principal repayment amounts are small, and the remaining principal de-creases very slowly. (It's only toward the end of the mortgage that interest payments decline enough to repay the principal more quickly.)

Inflation i: the change in CPI $\boxed{1 + i_{t+1} = \frac{CPI_{t+1}}{CPI_t}}$

- **"Nominal"**: not adjusted for inflation
- **"Real"**: adjusted for inflation

Real rate of return: $r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1+i} - 1$

$\boxed{r_{\text{real}} \approx r_{\text{nominal}} - i}$

Treat inflation consistently for NPV:

$\boxed{PV(CF_T) = \frac{CF_{\text{nominal,T}}}{(1+r_{\text{nominal,T}})^T} = \frac{CF_{\text{real,T}}}{(1+r_{\text{real,T}})^T}}$ ("$_T$" denotes the cash flow at time T)