Net Present Value (NPV)

Discoint rate r: you're indifferent between receiving \$1 today and \$$\frac{1}{1+r}$ in one period.

Present Value (PV): $\boxed{PV(CF_t) = \frac{CF_t}{(1+r)^t}}$ how much a cash flow (CF) at time t is worth at time 0 (today). Computing a PV is often called "discounting".

NPV: $\boxed{NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}}$ summs over PVs of all cash flows in a project.

- **Scalability**: $NPV(\alpha CF_1, \ldots, \alpha CF_T) = \alpha NPV(CF_1, \ldots, CF_T)$
- **Additivity**: $NPV(X_1 + Y_1, \ldots, X_T + Y_T) = NPV(X_1, \ldots, X_T) + NPV(Y_1, \ldots, Y_T)$
- **Breaking up by time**: $NPV(CF_1, \ldots, CF_T) = NPV(CF_1, \ldots, CF_j) + NPV(CF_{j+1}, \ldots, CF_T)$

Future Value (FV): $\boxed{FV_T(CF_0) = CF_0(1+r)^T}$ how much a cash flow at time 0 (today) is worth in T periods.

Perpetuity:

- Constant recurring cash flow A forever starting **1 period from now**: $\boxed{PV = \frac{A}{r}}$
- Growing perpetuity starting **1 period from now** with cash flow A, growth rate g: $\boxed{PV = \frac{A}{r-g} (r > g)}$

Annuity:

- Constant recurring cash flow A for T periods starting **1 period from now**: (E.g. a loan)
  $\boxed{PV = \frac{A}{r}\left(1 - \frac{1}{(1+r)^T}\right)}$
  $\boxed{FV = PV \cdot (1+r)^T = A\frac{(1+r)^T - 1}{r}}$
- Growing annuity starting **1 period from now** with cash flow A, growth rate g for T periods.
  - If $r \neq g$ $\boxed{PV = \frac{A}{r-g}\left(1 - \frac{(1+g)^T}{(1+r)^T}\right)}$; $\boxed{FV = A\left(\frac{(1+r)^T - (1+g)^T}{r-g}\right)}$
  - If $r = g$ $\boxed{PV = T\left(\frac{A}{1+r}\right)}$; $\boxed{FV = T \cdot A \cdot (1+r)^{T-1}}$