

Diversification

Asset return characteristics

Buy an asset (e.g. a stock) at $t = 0$ at price P_0 . At time $t = 1$,

- its cash flow (dividend) is D_1 , and
- its price is P_1

(both are random variables). The risk-free rate is r_F .

Realized return: $r_1 = \frac{D_1 + P_1}{P_0} - 1$ Returns comes from both dividends and capital gains.

Expected return: $E[r_1] = \frac{E[D_1] + E[P_1]}{P_0} - 1$

Excess return: (realized) $r_1 - r_F$

Risk premium: (expected excess return)

$$E[r_1] - r_F$$

Mean (average) return: $\bar{r} = E[r] = \frac{1}{T} \sum_{t=1}^T r_t$

Would be same as the expected return $E[r_t]$ if expected returns are constant for all t .

Options

Options: Derivative contracts specifying a **right to buy (call option) or sell (put option)** an underlying asset at a specified price K (the **strike/exercise price**) on or before a specified date T (the **expiration/maturity date**).

- Call option::** right to buy the underlying asset at the strike price.
- Put option::** right to sell the underlying asset at the strike price.

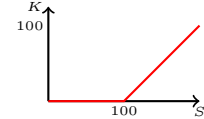
Exercise style:

- American option::** can be exercised at any time before expiration.
- European option::** can only be exercised at expiration.

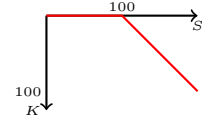
Option Payoff curves

- S : Price of the underlying asset at expiration
- K : Strike price of option
- Payoff \neq Profit.** To get profit (net payoff), need to subtract the option's cost.

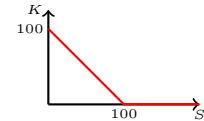
Payoff of buying a Call
 $\max(0, S - K)$



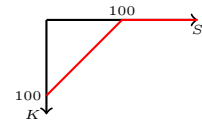
Payoff of selling a Call
 $-\max(0, S - K)$



Payoff of buying a Put
 $\max(0, K - S)$

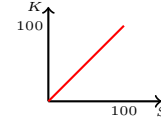


Payoff of selling a Put
 $-\max(0, K - S)$

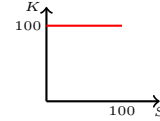


Payoff curves of other assets that can be used with options:

Payoff of underlying asset



Payoff of a \$100 FV ZC Bond



Option payoff and profit

- r : Risk-free interest rate (EAR)
- C : Call option price
- P : Put option price

Call option:

	$S < K$	$S = K$	$S > K$
Payoff	0	0	$S - K$
Profit	$-C(1+r)^T$	$-C(1+r)^T$	$S - K - C(1+r)^T$

Put option:

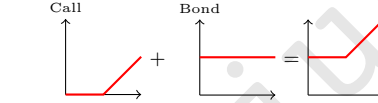
	$S < K$	$S = K$	$S > K$
Payoff	$K - S$	0	0
Profit	$K - S - P(1+r)^T$	$-P(1+r)^T$	$-P(1+r)^T$

An option is

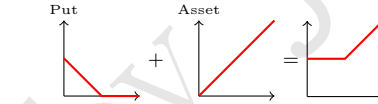
- in-the-money:** if it has positive payoff at expiration. A call option is in-the-money if $S > K$, and a put option is in-the-money if $S < K$.
- out-of-the-money:** if it has zero payoff at expiration. A call option is out-of-the-money if $S < K$, and a put option is out-of-the-money if $S > K$.
- at-the-money:** if it has zero payoff at expiration. A call option is at-the-money if $S = K$, and a put option is at-the-money if $S = K$.

Put-call parity: following portfolios have the same payoff at expiration:

- Long call with strike price K + Bond with face value K :**



- Long put with strike price K + Underlying asset:**



Given their identical payoffs, under no-arbitrage, they should have the same price:

- $P_{call} + P_{bond} = P_{put} + P_{asset}$
- $C + K(1+r)^{-T} = P + S$

Binomial option pricing model: Iterative approach to price options that makes the following simplifications:

- Discrete periods, in which stock price can either go up or down.
- We find the option price by a *no arbitrage* argument. Price is equal to the cost of purchasing a *replicating portfolio* whose payoffs

match the option payoff in each state. E.g., for a call, we solve: $\begin{cases} aS_u + bS_d = C_u \\ aS_d + bS_d = C_d \end{cases}$ where a is the number of shares of stock, b is the number of bonds, S_u and S_d are the stock prices if it goes up or down, and C_u and C_d are the call option prices if stock goes up or down.

- Under the binomial assumptions, **the probability of a stock moves up or down is irrelevant**, and the price of options can be determined solely using:
 - Current stock price S_0 , interest rate r , strike price K and time to maturity T ;
 - Magnitude of possible future changes of stock price (volatility), captured implicitly by the possible values the stock can take S_u and S_d .

Black-Scholes formula: Taking the limit of binomial model as the number of periods gets large, we obtain the B-S formula for the price of a European call option without dividends:

$$C(S, K, T, r, \sigma) = S \cdot N(x) - K(1+r)^{-T} \cdot N(x - \sigma\sqrt{T})$$

- S : current value of the underlying asset (in \$)
- K : strike price of the option (in \$)
- T : option maturity (in years)
- r : annual risk-free interest rate
- σ : annualized standard deviation of the underlying asset's return (volatility)
- $N(\cdot)$: cumulative normal distribution function (NORM.S.DIST(x, TRUE)). These $N(\cdot)$ terms **capture the replicating portfolio weights**.

Real options

Option-like payoffs appear in many contexts outside of financial markets. Management can be thought of as the act of creating and optimally exercising real options. **E.g.:** follow-on products, R&D investments, delaying product launches, abandoning projects, etc.