
Euler Problem #294

For a positive integer k , define $d(k)$ as the sum of the digits of k in its usual decimal representation. Thus $d(42) = 4 + 2 = 6$.

For a positive integer n , define $S(n)$ as the number of positive integers $k < 10^n$ with the following properties : k is divisible by 23 and $d(k) = 23$.

You are given that $S(9) = 263626$ and $S(42) = 6377168878570056$.

Find $S(1112)$ and give your answer mod 109.

Build the functions d and S , which are described in the problem

```
In[2]:= d[inputNum_] := Total[IntegerDigits[inputNum]]
```

```
In[3]:= d[94]
```

```
Out[3]= 13
```

For the function S , I should've included a function that computes how many numbers meet the condition, but I deleted that line so that I can look at the results myself (counting how many there are can be done after I run this function and select the numbers, anyways)

```
In[10]:= S[inputNum_] := Module[
  {integersBelow, selectedIntegers, selectedIntegers2},
  (*Positive integers below 10^n*)
  integersBelow = Range[1, 10inputNum - 1];
  (*Select the ones that are divisible by 23*)
  selectedIntegers = Select[integersBelow, Mod[#, 23] == 0 &];
  (*Select the numbers with d(k)=23*)
  selectedIntegers2 = Select[integersBelow, d[#] == 23 &]
]
```

I wanted to compute the test cases and the actual problem, but my program crashed every time I tried to use $S(9)$. Instead, I tested my code with smaller numbers, such as $S(3)$ and $S(4)$.

```
In[17]:= testCase1 = S[3]
Out[17]= {599, 689, 698, 779, 788, 797, 869, 878, 887, 896, 959, 968, 977, 986, 995}

In[18]:= Length@testCase1
Out[18]= 15
```

In[19]:= **testCase2 = S[4]**

Out[19]= {599, 689, 698, 779, 788, 797, 869, 878, 887, 896, 959, 968, 977, 986, 995, 1499, 1589, 1598, 1679, 1688, 1697, 1769, 1778, 1787, 1796, 1859, 1868, 1877, 1886, 1895, 1949, 1958, 1967, 1976, 1985, 1994, 2399, 2489, 2498, 2579, 2588, 2597, 2669, 2678, 2687, 2696, 2759, 2768, 2777, 2786, 2795, 2849, 2858, 2867, 2876, 2885, 2894, 2939, 2948, 2957, 2966, 2975, 2984, 2993, 3299, 3389, 3398, 3479, 3488, 3497, 3569, 3578, 3587, 3596, 3659, 3668, 3677, 3686, 3695, 3749, 3758, 3767, 3776, 3785, 3794, 3839, 3848, 3857, 3866, 3875, 3884, 3893, 3929, 3938, 3947, 3956, 3965, 3974, 3983, 3992, 4199, 4289, 4298, 4379, 4388, 4397, 4469, 4478, 4487, 4496, 4559, 4568, 4577, 4586, 4595, 4649, 4658, 4667, 4676, 4685, 4694, 4739, 4748, 4757, 4766, 4775, 4784, 4793, 4829, 4838, 4847, 4856, 4865, 4874, 4883, 4892, 4919, 4928, 4937, 4946, 4955, 4964, 4973, 4982, 4991, 5099, 5189, 5198, 5279, 5288, 5297, 5369, 5378, 5387, 5396, 5459, 5468, 5477, 5486, 5495, 5549, 5558, 5567, 5576, 5585, 5594, 5639, 5648, 5657, 5666, 5675, 5684, 5693, 5729, 5738, 5747, 5756, 5765, 5774, 5783, 5792, 5819, 5828, 5837, 5846, 5855, 5864, 5873, 5882, 5891, 5909, 5918, 5927, 5936, 5945, 5954, 5963, 5972, 5981, 5990, 6089, 6098, 6179, 6188, 6197, 6269, 6278, 6287, 6296, 6359, 6368, 6377, 6386, 6395, 6449, 6458, 6467, 6476, 6485, 6494, 6539, 6548, 6557, 6566, 6575, 6584, 6593, 6629, 6638, 6647, 6656, 6665, 6674, 6683, 6692, 6719, 6728, 6737, 6746, 6755, 6764, 6773, 6782, 6791, 6809, 6818, 6827, 6836, 6845, 6854, 6863, 6872, 6881, 6890, 6908, 6917, 6926, 6935, 6944, 6953, 6962, 6971, 6980, 7079, 7088, 7097, 7169, 7178, 7187, 7196, 7259, 7268, 7277, 7286, 7295, 7349, 7358, 7367, 7376, 7385, 7394, 7439, 7448, 7457, 7466, 7475, 7484, 7493, 7529, 7538, 7547, 7556, 7565, 7574, 7583, 7592, 7619, 7628, 7637, 7646, 7655, 7664, 7673, 7682, 7691, 7709, 7718, 7727, 7736, 7745, 7754, 7763, 7772, 7781, 7790, 7808, 7817, 7826, 7835, 7844, 7853, 7862, 7871, 7880, 7907, 7916, 7925, 7934, 7943, 7952, 7961, 7970, 8069, 8078, 8087, 8096, 8159, 8168, 8177, 8186, 8195, 8249, 8258, 8267, 8276, 8285, 8294, 8339, 8348, 8357, 8366, 8375, 8384, 8393, 8429, 8438, 8447, 8456, 8465, 8474, 8483, 8492, 8519, 8528, 8537, 8546, 8555, 8564, 8573, 8582, 8591, 8609, 8618, 8627, 8636, 8645, 8654, 8663, 8672, 8681, 8690, 8708, 8717, 8726, 8735, 8744, 8753, 8762, 8771, 8780, 8807, 8816, 8825, 8834, 8843, 8852, 8861, 8870, 8906, 8915, 8924, 8933, 8942, 8951, 8960, 9059, 9068, 9077, 9086, 9095, 9149, 9158, 9167, 9176, 9185, 9194, 9239, 9248, 9257, 9266, 9275, 9284, 9293, 9329, 9338, 9347, 9356, 9365, 9374, 9383, 9392, 9419, 9428, 9437, 9446, 9455, 9464, 9473, 9482, 9491, 9509, 9518, 9527, 9536, 9545, 9554, 9563, 9572, 9581, 9590, 9608, 9617, 9626, 9635, 9644, 9653, 9662, 9671, 9680, 9707, 9716, 9725, 9734, 9743, 9752, 9761, 9770, 9806, 9815, 9824, 9833, 9842, 9851, 9860, 9905, 9914, 9923, 9932, 9941, 9950}

In[20]:= **Length@testCase2**

Out[20]= 480