J. Phys. B: Atom. Molec. Phys., Vol. 5, March 1972. Printed in Great Britain

A high energy approximation: III. Helium excitation by electrons

M B HIDALGO† and S GELTMAN

Joint Institute for Laboratory Astrophysics, National Bureau of Standards and University of Colorado, Boulder, Colorado, USA

MS received 1 October 1971

Abstract. The cross sections for the direct excitation of the 2^1S and 2^1P states of helium by electron impact are evaluated in a first order approximation in which the interaction between the incident electron and target nucleus is represented by a coulomb wave final state in the T matrix. The results obtained for the differential cross sections are much greater than those of the Born approximation for large scattering angles, and the present results are in much better agreement with experiment. The present calculation predicts inelastic differential cross sections which have the asymptotic energy dependence of E^{-3} for all nonzero angles of scattering. The total cross sections converge to the Born values at high energy.

1. Introduction

We have recently considered the application of a coulomb projected Born approximation to the processes of proton–hydrogen charge transfer (Geltman 1971, hereinafter referred to as I) and hydrogen atom excitation by electrons (Geltman and Hidalgo 1971, hereinafter referred to as II). The basic departure from the usual first Born approximation is to regard the coulomb interaction between the incident particle and the nucleus of the target atom as part of the unperturbed problem. The resulting T matrix element then contains a coulomb wave in the final state rather than the plane wave which is present in the Born approximation. For all electron–atom inelastic processes the interaction between the incident electron and the nucleus makes a vanishing contribution in the first Born approximation because of the orthogonality of the atomic eigenstates, while in the present (coulomb projected Born) approximation this effect is nonvanishing and yields important changes in the results.

In this note we apply the approximation to the 2^1S and 2^1P excitations of helium by electron impact. Of particular interest are the differential cross sections at large angles, where we have seen in II that there are very large differences with the predictions of the Born approximation. We will compare our results with the measurements of Opal and Beaty (1972).

2. Theory and atomic wavefunctions

Our present interest is in the direct amplitudes for the excitation of the 2^1S and 2^1P states of helium from the 1^1S ground state. Since we neglect exchange effects we cannot expect our results to be valid at low electron energies.

† Now at Physics Department, Texas A and M University, College Station, Texas, USA.

In complete analogy with the discussion in I and II, the direct excitation amplitude in the present approximation is

$$f_{\mathrm{P}}(p \to q) = -\frac{m}{2\pi\hbar^2}\left\langle \Psi_q^0, e^2\left(\frac{1}{r_{12}}+\frac{1}{r_{13}}\right)\exp(\mathrm{i}\boldsymbol{k}_p \cdot \boldsymbol{r}_1)\psi_p(\boldsymbol{r}_2, \boldsymbol{r}_3)\right\rangle \tag{1}$$

where the final state is

$$\Psi_q^0 = \exp\left(-\frac{\pi\alpha}{2}\right)\Gamma(1-\mathrm{i}\alpha)\exp(\mathrm{i}\boldsymbol{k}_q \cdot \boldsymbol{r}_1)\,{}_1F_1(\mathrm{i}\alpha; 1; -\mathrm{i}k_q r_1 - \mathrm{i}\boldsymbol{k}_q \cdot \boldsymbol{r}_1)$$
$$\times \psi_q(\boldsymbol{r}_2, \boldsymbol{r}_3) \tag{2}$$

and the coulomb parameter

$$\alpha = -\frac{2e^2 m}{\hbar^2 k_q}$$

represents the attractive field between the incident electron and the helium nucleus.

The spin parts of the helium atomic wavefunctions have been factored out and their overlaps contribute a trivial factor of unity since we are considering transitions only between singlet states. The ψ_p and ψ_q thus represent the atomic wavefunctions in coordinate space and they are symmetric functions in their two electron coordinates. Our choices of approximate atomic wavefunctions are the orthonormal set used by van den Bos (1969) for calculations on proton–helium scattering:

$$\psi(1^1\mathrm{S}) = \chi_{1\mathrm{s}}(r_2)\chi_{1\mathrm{s}}(r_3)$$
$$\chi_{1\mathrm{s}}(r) = (4\pi)^{-1/2}N\{\exp(-\beta r)+\eta\exp(-\gamma r)\}$$
$$N = 2{\cdot}605 \qquad \beta = 1{\cdot}41$$
$$\gamma = 2{\cdot}61 \qquad \eta = 0{\cdot}799$$
$$\psi(2^1\mathrm{S}) = (2+2\Delta^2)^{-1/2}\{\phi_{1\mathrm{s}}(2, r_2)\chi_{2\mathrm{s}}(r_3)+\phi_{1\mathrm{s}}(2, r_3)\chi_{2\mathrm{s}}(r_2)\}$$
$$\Delta = \int \mathrm{d}\boldsymbol{r}\phi_{1\mathrm{s}}(2, r)\chi_{2\mathrm{s}}(r) = 0{\cdot}06996$$
$$\phi_{nlm}(Z, \boldsymbol{r}) = \text{hydrogenic orbital for nuclear charge Z}$$
$$\chi_{2\mathrm{s}}(r) = (4\pi)^{-1/2}N\{\exp(-\beta r)+\eta r\exp(-\gamma r)\}$$
$$N = 0{\cdot}6451 \qquad \beta = 1{\cdot}136$$
$$\gamma = 0{\cdot}464 \qquad \eta = -0{\cdot}2806$$
$$\psi(2^1\mathrm{P}) = 2^{-1/2}\{\phi_{1\mathrm{s}}(2, r_2)\phi_{2\mathrm{p}}(Z, \boldsymbol{r}_3)+\phi_{1\mathrm{s}}(2, r_3)\phi_{2\mathrm{p}}(Z, \boldsymbol{r}_2)\}$$
$$Z = 0{\cdot}97.$$

These are certainly not the most accurate helium wavefunctions available, but we cannot conveniently use correlated wavefunctions as they would lead to intractable integrals in (1) because of the presence of the coulomb function. The adequacy of these wavefunctions will be tested by using them in the Born approximation and comparing with Born results using much better wavefunctions.

The present calculation is much more laborious algebraically than is the corresponding calculation for hydrogen, but there are no differences in principle. The integrations

over the two atomic electrons are carried out first in (1), yielding an expression containing products of exponentials and polynomials in the incident electron radial coordinate and spherical harmonic angular dependence. The final integration over r_1 is then carried out using the methods described in II.

As a check on part of the algebraic and numerical work, the Born amplitude

$$f_B(p \to q) = -\frac{me^2}{2\pi\hbar^2}\left\langle \exp(i\boldsymbol{k}_q \cdot \boldsymbol{r}_1)\psi_q(\boldsymbol{r}_2, \boldsymbol{r}_3), \left(\frac{1}{r_{12}}+\frac{1}{r_{13}}\right) \times \exp(i\boldsymbol{k}_p \cdot \boldsymbol{r}_1)\psi_p(\boldsymbol{r}_2, \boldsymbol{r}_3)\right\rangle \tag{3}$$

is evaluated in the two ways

$$f_B(p \to q) = -\frac{me^2}{2\pi\hbar^2}\left(\frac{4\pi}{K^2}\right)\int\int d\boldsymbol{r}_2\, d\boldsymbol{r}_3 \psi_q^*\{\exp(i\boldsymbol{K} \cdot \boldsymbol{r}_2)+\exp(i\boldsymbol{K} \cdot \boldsymbol{r}_3)\}\psi_p$$

$$= -\frac{me^2}{2\pi\hbar^2}\int d\boldsymbol{r}_1 \exp(i\boldsymbol{K} \cdot \boldsymbol{r}_1)\mathscr{Y}_{pq}(\boldsymbol{r}_1) \tag{4}$$

where

$$\boldsymbol{K} = \boldsymbol{k}_p - \boldsymbol{k}_q \qquad \mathscr{Y}_{pq}(\boldsymbol{r}_1) = \int\int d\boldsymbol{r}_2\, d\boldsymbol{r}_3 \psi_q^*\left(\frac{1}{r_{12}}+\frac{1}{r_{13}}\right)\psi_p. \tag{5}$$

The equality of these two forms is a check on our evaluation of $\mathscr{Y}_{pq}(\boldsymbol{r}_1)$, which must be used in the present approximation.

3. Results

In table 1 the results of the present approximation are compared with those of the Born approximation in which the same approximate atomic wavefunctions are used. We see the same general behaviour as was seen for hydrogen excitation in II. The two approximations approach one another for forward scattering only as the energy is increased. For every nonzero angle the two results have different energy dependences in the high energy limit. The Born $d\sigma/d\Omega$ goes as E^{-6} for s $\to$ s transitions and as E^{-7} for s $\to$ p transitions, while the present $d\sigma/d\Omega$ goes as E^{-3} for all inelastic transitions. The larger the scattering angle, the lower will be the energy at which this asymptotic energy dependence becomes valid.

The E^{-3} dependence of the inelastic differential cross section can be simply interpreted in a semiclassical model if we set

$$\frac{d\sigma(p \to q)}{d\Omega} = P(p \to q)\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}}. \tag{6}$$

The assumed Rutherford scattering of the incident electron on the nucleus goes as E^{-2} and the probability that the electron will cause an excitation while it traverses its coulomb trajectory varies as E^{-1} in the high energy limit. This simple model also explains the observations (Williams 1969a, Opal and Beaty 1972) that the angular dependences of the inelastic and elastic differential cross sections are very similar. That would follow from $P(p \to q)$ having a relatively weak dependence on angle for the larger scattering angles compared with the angular dependence of Rutherford scattering. The use of a Rutherford cross section in the above model should be valid at high energies for all angles

Table 1. Comparison of present (P) and Born (B) approximations for differential and total cross sections. ($d\sigma/d\Omega$ in units of a_0^2/sr^{-1})†

E (eV)		0°	15°	30°	45°	67·5°	90°	135°	180°	$\sigma(\pi a_0^2)$
					$1^1S \rightarrow 2^1S$					
100	P	1.37^{-1}†	6.88^{-2}	1.35^{-2}	2.64^{-3}	7.82^{-4}	4.26^{-4}	1.93^{-4}	1.48^{-4}	1.70^{-2}
	B	1.48^{-1}	8.03^{-2}	1.87^{-2}	3.40^{-3}	3.23^{-4}	4.91^{-5}	4.56^{-6}	2.14^{-6}	1.93^{-2}
200	P	1.59^{-1}	4.31^{-2}	3.39^{-3}	5.41^{-4}	1.52^{-4}	6.75^{-5}	2.51^{-5}	1.85^{-5}	9.44^{-3}
	B	1.68^{-1}	4.89^{-2}	4.03^{-3}	3.40^{-4}	1.63^{-5}	1.71^{-6}	1.13^{-7}	4.88^{-8}	1.02^{-2}
300	P	1.68^{-1}	2.62^{-2}	1.19^{-3}	1.93^{-4}	5.13^{-5}	2.11^{-5}	7.45^{-6}	5.43^{-6}	6.54^{-3}
	B	1.75^{-1}	2.92^{-2}	1.19^{-3}	6.66^{-5}	2.31^{-6}	2.05^{-7}	1.18^{-8}	4.93^{-9}	6.91^{-3}
400	P	1.73^{-1}	1.65^{-2}	5.26^{-4}	8.97^{-5}	2.28^{-5}	9.06^{-6}	3.13^{-6}	2.28^{-6}	5.01^{-3}
	B	1.78^{-1}	1.80^{-2}	4.40^{-4}	1.88^{-5}	5.37^{-7}	4.33^{-8}	2.30^{-9}	9.46^{-10}	5.22^{-3}
500	P	1.76^{-1}	1.07^{-2}	2.72^{-4}	4.86^{-5}	1.20^{-5}	4.66^{-6}	1.60^{-6}	1.16^{-6}	4.06^{-3}
	B	1.81^{-1}	1.16^{-2}	1.89^{-4}	6.70^{-6}	1.67^{-7}	1.26^{-8}	6.39^{-10}	2.60^{-10}	4.20^{-3}
700	P	1.79^{-1}	4.94^{-3}	9.85^{-5}	1.88^{-5}	4.45^{-6}	1.70^{-6}	5.78^{-7}	4.20^{-7}	2.94^{-3}
	B	1.83^{-1}	5.23^{-3}	4.77^{-5}	1.30^{-6}	2.72^{-8}	1.90^{-9}	9.08^{-11}	3.64^{-11}	3.02^{-3}
900	P	1.81^{-1}	2.53^{-3}	4.60^{-5}	9.12^{-6}	2.10^{-6}	7.98^{-7}	2.71^{-7}	1.97^{-7}	2.31^{-3}
	B	1.84^{-1}	2.62^{-3}	1.58^{-5}	3.63^{-7}	6.78^{-9}	4.53^{-10}	2.09^{-11}	8.30^{-12}	2.35^{-3}
1100	P	1.83^{-1}	1.41^{-3}	2.51^{-5}	5.07^{-6}	1.15^{-6}	4.36^{-7}	1.48^{-7}	1.07^{-7}	1.90^{-3}
	B	1.85^{-1}	1.42^{-3}	6.25^{-6}	1.27^{-7}	2.20^{-9}	1.42^{-10}	6.42^{-12}	2.54^{-12}	1.93^{-3}
1300	P	1.84^{-1}	8.32^{-4}	1.52^{-5}	3.10^{-6}	6.97^{-7}	2.63^{-7}	8.93^{-8}	6.49^{-8}	1.61^{-3}
	B	1.85^{-1}	8.21^{-4}	2.81^{-6}	5.21^{-8}	8.53^{-10}	5.40^{-11}	2.40^{-12}	9.45^{-13}	1.63^{-3}
1500	P	1.84^{-1}	5.19^{-4}	9.85^{-6}	2.02^{-6}	4.53^{-7}	1.71^{-7}	5.80^{-8}	4.21^{-8}	1.40^{-3}
	B	1.86^{-1}	4.99^{-4}	1.39^{-6}	2.40^{-8}	3.77^{-10}	2.35^{-11}	1.03^{-12}	4.04^{-13}	1.42^{-3}
					$1^1S \rightarrow 2^1P$					
100	P	5.29^{0}	8.22^{-1}	6.41^{-2}	1.03^{-2}	2.10^{-3}	7.78^{-4}	2.59^{-4}	1.87^{-4}	1.99^{-1}
	B	5.70^{0}	5.12^{-1}	3.16^{-2}	2.46^{-3}	1.03^{-4}	9.23^{-6}	4.86^{-7}	1.94^{-7}	1.47^{-1}
200	P	1.29^{1}	2.24^{-1}	7.77^{-3}	1.23^{-3}	2.60^{-4}	9.71^{-5}	3.27^{-5}	2.37^{-5}	1.26^{-1}
	B	1.39^{1}	1.64^{-1}	3.04^{-3}	1.07^{-4}	2.29^{-6}	1.44^{-7}	5.44^{-9}	1.99^{-9}	1.05^{-1}
300	P	2.08^{1}	8.28^{-2}	2.11^{-3}	3.60^{-4}	7.78^{-5}	2.91^{-5}	9.82^{-6}	7.12^{-6}	9.58^{-2}
	B	2.22^{1}	6.32^{-2}	5.60^{-4}	1.32^{-5}	2.06^{-7}	1.11^{-8}	3.66^{-10}	1.30^{-10}	8.28^{-2}
400	P	2.88^{1}	3.70^{-2}	8.38^{-4}	1.52^{-4}	3.30^{-5}	1.24^{-5}	4.19^{-6}	3.04^{-6}	7.48^{-2}
	B	3.06^{1}	2.84^{-2}	1.48^{-4}	2.70^{-6}	3.50^{-8}	1.71^{-9}	5.26^{-11}	1.84^{-11}	6.88^{-2}
500	P	3.70^{1}	1.88^{-2}	4.14^{-4}	7.79^{-5}	1.70^{-5}	6.38^{-6}	2.16^{-6}	1.57^{-6}	6.25^{-2}
	B	3.89^{1}	1.41^{-2}	4.95^{-5}	7.52^{-7}	8.57^{-9}	3.94^{-10}	1.16^{-11}	3.99^{-12}	5.92^{-2}
700	P	5.34^{1}	6.17^{-3}	1.46^{-4}	2.85^{-5}	6.26^{-6}	2.35^{-6}	7.98^{-7}	5.80^{-7}	4.98^{-2}
	B	5.56^{1}	4.35^{-3}	8.58^{-6}	1.02^{-7}	9.79^{-10}	4.18^{-11}	1.16^{-12}	3.95^{-13}	4.68^{-2}
900	P	6.99^{1}	2.55^{-3}	6.76^{-5}	1.35^{-5}	2.96^{-6}	1.12^{-6}	3.80^{-7}	2.76^{-7}	4.07^{-2}
	B	7.23^{1}	1.63^{-3}	2.16^{-6}	2.17^{-8}	1.88^{-10}	7.67^{-12}	2.05^{-13}	6.96^{-14}	3.90^{-2}
1100	P	8.65^{1}	1.23^{-3}	3.68^{-5}	7.41^{-6}	1.63^{-6}	6.15^{-7}	2.10^{-7}	1.53^{-7}	3.48^{-2}
	B	8.89^{1}	7.03^{-4}	6.87^{-7}	6.16^{-9}	4.95^{-11}	1.96^{-12}	5.14^{-14}	1.73^{-14}	3.36^{-2}
1300	P	1.03^{2}	6.64^{-4}	2.23^{-5}	4.50^{-6}	9.93^{-7}	3.75^{-7}	1.28^{-7}	9.31^{-8}	3.06^{-2}
	B	1.06^{2}	3.36^{-4}	2.59^{-7}	2.12^{-9}	1.62^{-11}	6.27^{-13}	1.62^{-14}	5.44^{-15}	2.97^{-2}
1500	P	1.20^{2}	3.92^{-4}	1.45^{-5}	2.94^{-6}	6.49^{-7}	2.45^{-7}	8.38^{-8}	6.10^{-8}	2.75^{-2}
	B	1.22^{2}	1.73^{-4}	1.10^{-7}	8.42^{-10}	6.16^{-12}	2.35^{-13}	6.01^{-15}	2.02^{-15}	2.66^{-2}

† The superscript indicates the power of 10 to which the entry must be raised.

except a small forward cone, in which the screening of the nuclear field by the atomic electrons becomes important. The large angle elastic scattering results of Williams (1969b) and Opal and Beaty are seen to be well represented by the Rutherford cross section for electron scattering on a bare nucleus.

Since the predominant contribution to the total cross section comes from the forward direction the Born and present approximations give the same result for the total cross section at high energies. Specifically, we find that the two approximations for the total cross section are within 1% of one another at 1600 eV for the 2^1S case and at 4200 eV for the 2^1P case. We regard the confluence of the Born and present total cross sections as an indication of the energy at which both total cross sections become correct. At somewhat lower energies we would expect the correct total cross section to lie between these two results, assuming the atomic wavefunctions were exact. Note that the energies for 1% agreement in the total cross sections are much higher for helium than for corresponding excitations in hydrogen (see II). This would indicate that the energy at which one would expect the Born approximation to be valid varies from atom to atom. All of the above remarks are within the context of nonrelativistic theory. It should be kept in mind that relativistic effects of the order of a few percent enter for electron energies of 10^4 eV.

A well known property of the Born approximation is that it gives generalized oscillator strengths, defined as

$$F(p \to q) = \frac{1}{4}K^2(k_p^2 - k_q^2)a_0^2 \frac{1}{2l_p+1} \sum_{m_p m_q} |f(p \to q)|^2 \tag{7}$$

which for a given transition depend only on the magnitude of the momentum transfer K. In the present approximation the scattering amplitude and hence also the generalized oscillator strength will depend on the incident momentum k_p as well as on K. In figures 1 and 2 the departures of the present from the Born generalized oscillator strengths are shown. Also indicated on these figures are the differences between the Born results using our present atomic wavefunctions and the 'exact' Born results, which are based on extremely elaborate correlated atomic wavefunctions (Kim and Inokuti 1968, Bell *et al* 1969). In the 2^1S case there is no difference in the Born results at high K for either choice of atomic wavefunctions, while at low K our approximate atomic wavefunctions lead to a Born generalized oscillator strength lower than the 'exact' one. In the 2^1P case the approximate atomic wavefunctions lead to a Born generalized oscillator strength which is slightly below the 'exact' one at low K and about 50% above the 'exact' one for high K. The discrepancies of about 15% in the 2^1S case and 4% in the 2^1P case between the present Born and 'exact' Born values for the low K oscillator strengths indicates that our total cross sections in table 1 are too low by about 15% and 4%, respectively, because of inaccuracies in our atomic wavefunctions. Unfortunately, we cannot estimate the error in the coulomb projected Born differential cross sections arising from inexact atomic wavefunctions since different parts of the atomic wavefunctions are most heavily weighted in the two scattering approximations.

In figures 3, 4, 5, and 6 we compare our results with the measurements of Truhlar *et al* (1970) and Opal and Beaty (1972) at the energies of 82 and 200 eV. Although 82 eV is too low an energy to expect validity of the present approximation, we note that it is still in much better accord with the measured differential cross sections than is the Born approximation. At 200 eV and at the larger angles our present approximation falls below experiment by factors of about 4 and 2 for the 2^1S and 2^1P cases, respectively.

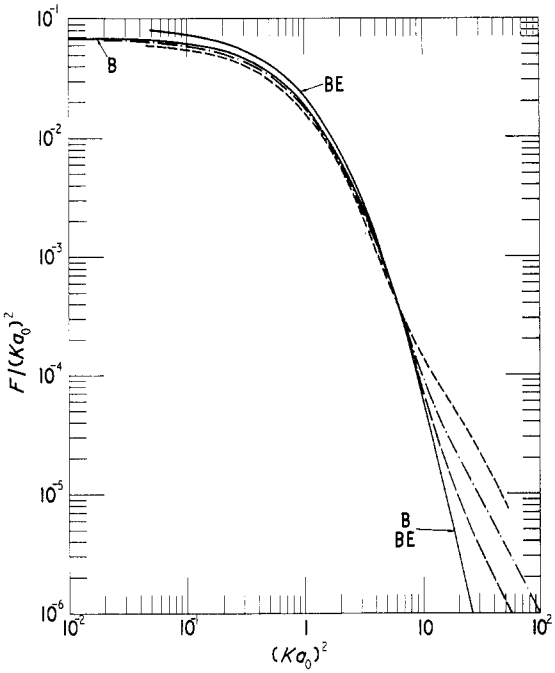

Figure 1. Generalized oscillator strength divided by $(Ka_0)^2$ as a function of $(Ka_0)^2$ for $1^1S \rightarrow 2^1S$ transition. BE, 'exact' Born approximation; B, Born approximation with present atomic wavefunctions; results in the present approximation: – – – 200eV; – · – 500 eV; ——— 1500 eV.

The same order of discrepancy between theory and experiment was present at 200 eV for hydrogen in II.

We also may note in figures 3, 4, 5, and 6 as well as in figures 1 and 2 that the Born and present differential cross sections do not always approach one another monotonically in going from large to small scattering angles. They appear to cross at least once.

The amount of computer time required in the present approximation is rather small. All of the differential cross sections in table 1 in the present approximation took about ten minutes on a CDC 6400, while all the corresponding Born results took about six seconds.

4. Conclusions

The new experimental results on the 2^1S and 2^1P excitations of helium (Opal and Beaty 1972) give further confirmation that the Born approximation is grossly inadequate at 200 eV in describing the differential cross sections at all but the smallest angles. The present approximation significantly reduces the disagreement with experiment at 82 and 200 eV.

The reason for the qualitative success of the present approximation is its inclusion of the interaction between the incident electron and the nucleus, an effect which is neglected entirely in the first Born approximation but is included in higher Born approximations. The present approximation is thus analogous to a complete summation of the part of the Born series containing the interaction between the incident electron and the nucleus, but it is only first order in the incident electron–atomic electron interactions.

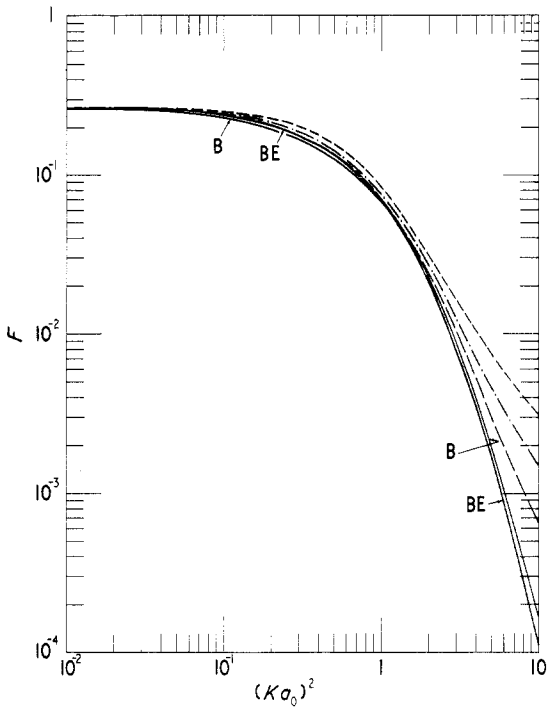

Figure 2. Generalized oscillator strength as a function of $(Ka_0)^2$ for $1^1S \rightarrow 2^1P$ transition. BE, 'exact' Born approximation; B, Born approximation with present atomic wavefunctions; results in the present approximation: – – – 200 eV; – · – · 500 eV; — — — 1500 eV.

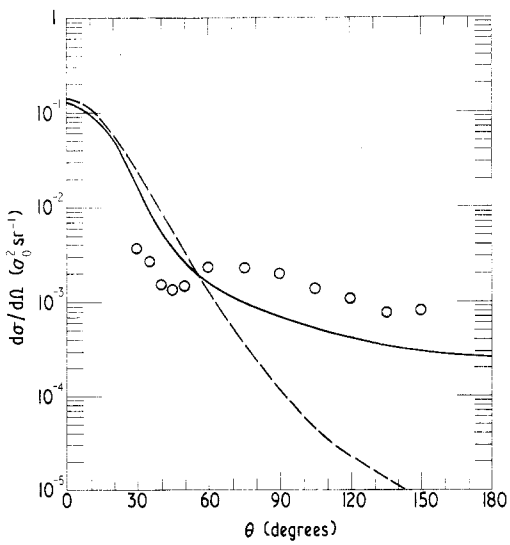

Figure 3. Differential cross section for $1^1S \rightarrow 2^1S$ transition at 82 eV. ——— present approximation; — — — Born approximation; ○ experiment (Opal and Beaty).

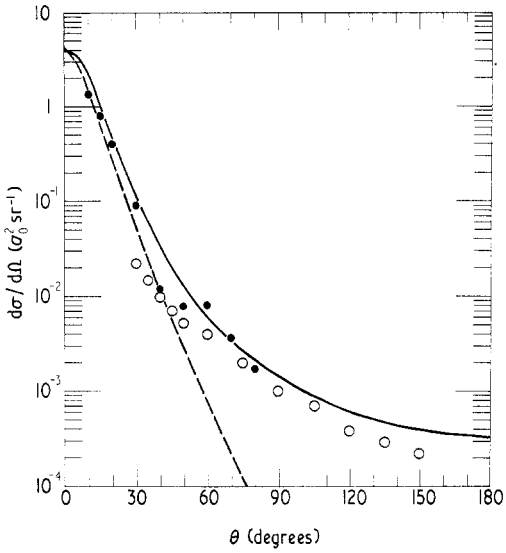

Figure 4. Differential cross section for $1^1S \rightarrow 2^1P$ transition at 82 eV. ——— present approximation; — — — Born approximation; ○ experiment (Opal and Beaty); ● experiment (Truhlar *et al*).

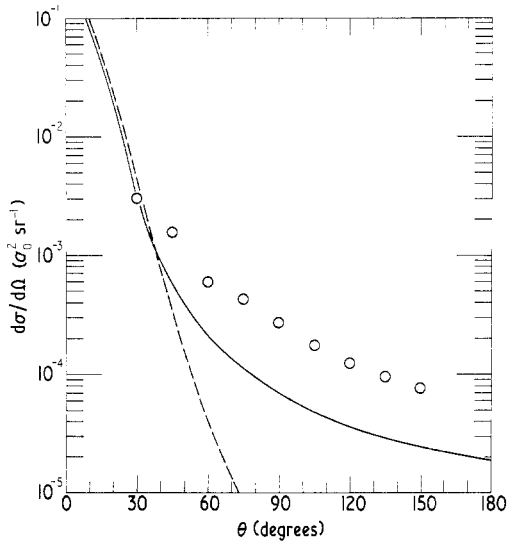

Figure 5. Differential cross section for $1^1S \rightarrow 2^1S$ transition at 200 eV. ——— present approximation; — — — Born approximation; ○ experiment (Opal and Beaty).

Another calculation which explicitly takes account of the interaction with the nucleus is the distorted wave calculation of Madison and Shelton (1971), and it also gives qualitative agreement with experiment for the 2^1P excitation at 82 eV. In that calculation both the incident and final scattered waves in the T matrix are assumed to be distorted by the static potential produced by the helium atom in its ground state. The part of this potential which is most influential in the large angle scattering is the short

range part, which is essentially coulombic. A practical difficulty with distorted wave methods of this type is that they must be done numerically in partial wave expansions since one does not have analytic three dimensional solutions for scattering in the static potentials of atoms.

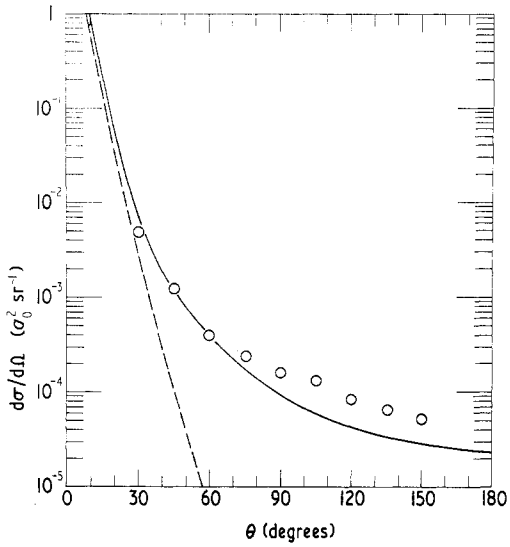

Figure 6. Differential cross section for $1^1S \rightarrow 2^1P$ transition at 200 eV. ——— present approximation; — — — Born approximation; ○ experiment (Opal and Beaty).

The maximum energy (200 eV) of the large angle experiments to date is not high enough to provide a test of whether the present approximation gives the correct high energy behaviour for differential inelastic cross sections. It would be very desirable to have additional experiments at much higher energies to test the angular dependences predicted in table 1 and the prediction that for fixed angle the inelastic differential cross sections fall as E^{-3}.

Acknowledgments

We are indebted to C B Opal and E C Beaty for keeping us informed on their results while their experiments were in progress. This research was partially supported by the Advanced Research Projects Agency, The Department of Defence and was monitored by U S Army Research Office–Durham, Box CM, Duke Station, Durham, North Carolina, 27706, under Contract No. DA-31-124-ARO-D-139.

References

Bell K L, Kennedy D J and Kingston A E 1969 *J. Phys.* B: *Atom. molec. Phys.* **2** 26–43
van den Bos J 1969 *Phys. Rev.* **181** 191–7
Geltman S 1971 *J. Phys.* B: *Atom. molec. Phys.* **4** 1288–98
Geltman S and Hidalgo M B 1971 *J. Phys.* B: *Atom. molec. Phys.* **4** 1299–307
Kim Y K and Inokuti M 1968 *Phys. Rev.* **175** 176–88

Madison D H and Shelton W N 1971 *Abstr. 7th Int. Conf. Physics Electronic and Atomic Collisions* (Amsterdam: North-Holland) pp 768–70

Opal C B and Beaty E C 1972 *J. Phys.* B: *Atom. molec. Phys.* **5** 627–35

Truhlar D G *et al* 1970 *Phys. Rev.* A **1** 778–802

Williams K G 1969a *Abstr. 6th Int. Conf. Physics Electronic and Atomic Collisions* (Cambridge, Mass.: MIT) pp 731–4

—— 1969b *Abstr. 6th Int. Conf. Physics Electronic and Atomic Collisions* (Cambridge, Mass.: MIT) pp 735–7