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The ranges of validity of the Born and Bethe approximations for the single ionization of He^+ and Li^+ ions by electron impact

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Abstract. Absolute measurements have been made of cross sections for the single ionization of He^+ ions by electrons with energies which ranged from threshold (54·4 ev) to 10 kev. The results are in good agreement with those of an earlier experiment by Dolder *et al.* (1961) which was confined to energies below 1 kev. Cross sections for the single ionization of Li^+ have also been measured for electron energies between 3 and 25 kev. Comparison of these results with calculations (performed elsewhere) based on the Born and Bethe approximations shows that, for both ions, the former approximation gives results in excellent agreement with the experiments when the electron energies exceed twenty times threshold, whilst the Bethe approximation is only valid for energies greater than about 50 times threshold.

1. Introduction

The approximations most widely used in the theory of ionizing collisions were suggested by Born and Bethe (see, for example, Mott and Massey 1965, Rudge 1968). Both approximations neglect spin and coupling to states of the atom which lie between the initial state and the continuum, and the incident projectile is represented by a plane wave which is negligibly distorted by the collision. Bethe included the further assumption that the product of the momentum transfer and range of the interaction is small for all significant values of this quantity.

It is clear that these approximations will only be valid for fast projectiles and, to examine the energy range of their validity, measurements are reported of cross sections for the single ionization of He^+ and Li^+ by electrons with energies E up to a few hundred times threshold. In § 3 the results will be compared with previous experiments and theory. In both cases it transpires that Born's approximation agrees very closely with the measurements if E is greater than about twenty times threshold, whilst Bethe's approximation is only valid when E is almost three times larger.

Two features of these experiments should, perhaps, be emphasized. Firstly, the measurements were absolute, so that it was unnecessary to normalize the results by assuming any measured or calculated cross section. Secondly, the experiments deal with the two simplest singly-charged positive ions so that the theory is not complicated by inner-shell excitation or ionization.

2. Apparatus and method

Essential features of the crossed-beam method have been described by Harrison (1968) and Dolder (1969). The apparatus was the same as that used by Peart and Dolder (1969, to be referred to as I) with the exceptions of the gun used to produce electrons with energies greater than 3 kev, and the source of He^+ ions with its pumping system.

The electron gun used previously could not withstand potential differences greater than about 3 kv so the gun illustrated by figure 1 was developed to produce more energetic electron beams. The flat oxide-coated cathode Ca and control grid G were taken from a valve type 11E3, and electrons from the cathode were initially accelerated by the potential applied to the first anode A1. Electrons which emerged through the slit (1 mm × 8 mm) in this electrode were further accelerated to the second anode A2 where the beam height was limited by a narrow (1 mm × 10 mm) aperture. This electrode was held at the most positive potential in the gun to retain secondary electrons which might otherwise have traversed the ion beam. Opposing potential gradients at either side of A2 acted as a lens (e.g. Klemperer 1953) which focused electrons through the ion beam B and the wide apertures of the third anode A3. Anode A3 was an extended structure with two apertures to minimize the penetration of electric fields from the gun towards the ion beam. When electrons

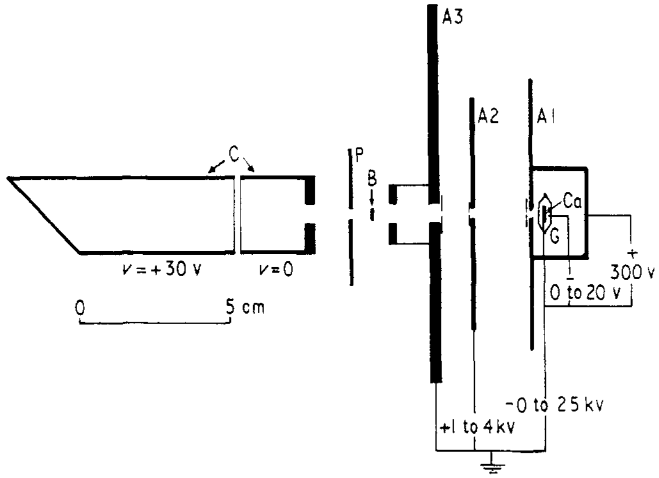

Figure 1. Elevation of the gun and collector used for fast electron beams. This shows the oxide-coated cathode Ca, control grid G, anodes A1, A2 and A3, ion beam B, defining plate P and Faraday cup C. Typical electrode potentials were: Ca, -1 to -25 kv; G, 0 to -25 v with respect to Ca; A1 $\simeq$ +200 v with respect to Ca; A2 $\simeq$ 3 kv; A3 = P = 0. The potentials of C are shown on the figure.

had traversed the ion beam, they passed through a defining plate P and were collected by a deep Faraday cup C which was positively biased to retain secondary electrons. The defining plate was pierced by a slit of the same height as the ion beam so that, when the electron current drawn by the plate was negligible, it could be assumed that all of the electrons had passed through the ion beam. Figure 1 shows typical potentials which were applied to the electrodes but it omits the insulators which supported the electrodes and the soft iron shield used to screen the gun from magnetic fields.

The source of Li^+ ions has already been described by Peart and Dolder (1968), but the He^+ source was developed from a design by Neff (1963). Essentially, it was a small hot cathode source with a magnetically confined discharge which could produce either metallic or gaseous ions and it had previously been used in this laboratory to produce Mg^+ and Mg^{2+} (Peart *et al.* 1969). When it was supplied with helium, a resolved collimated beam of He^+ with currents of order 10^{-7} A at 2 kev was obtained. If gaseous ion sources are used in a crossed-beam apparatus, it is essential that very little of the gas which issues from the source reaches the interaction region where the beams collide. Difficulties will otherwise arise from collisions between the beams and residual gas. Four additional stages of differential pumping were therefore installed between the source and the electron gun so that pressures of 5×10^{-8} torr could be maintained in the interaction region during the experiments. There were small differences in the techniques used for He^+ and Li^+ so the two experiments will be described separately.

2.1. *Experiments with He^+ ions*

For electron energies below 2 kev the technique was similar to that used in previous measurements of this cross section by Dolder *et al.* (1961, to be referred to as II). The only significant differences were the two-stage analyser used to separate the product ions, the much better vacua which were achieved and the single-particle detector. At electron energies greater than 2 kev the flux of He^{2+} produced by electron impacts was too small to be measured by an electrometer so that the device described in I was used to detect and count individual ions.

The present experiments gave signal-to-background ratios (for definition of SBR see Dolder 1969) between 10 and 100 whereas in II the SBR was only of order unity. This reduction in backgrounds was very advantageous because more accurate results could be obtained, and for electron energies where the cross section was largest it was sometimes possible to dispense with the beam modulation techniques (see II) used to separate signals from backgrounds. The greatest advantage, however, was that, in contrast to II, the measured cross section was zero below the ionization threshold. The small, but necessarily approximate, corrections which were applied to the results in II were therefore avoided. These corrections were largest near threshold so that the new results should be more accurate, particularly in this region.

The efficiency of the detector for He^{2+} ions was found by comparing cross sections measured with an accurate electrometer with those obtained with the particle multiplier. Several such comparisons were made for electron energies between 0·5 and 1·2 kev and a mean efficiency of 86 $\pm$ 4% was deduced. This calibration was repeated at intervals during the experiment and gave results which were consistently within these limits.

2.2. *Experiments with Li^+ ions*

Measurements of cross sections for the single ionization of Li^+ have already been published by Lineberger *et al.* (1966) and Peart and Dolder (1968) for electron energies up to 800 ev and 3 kev, respectively. The new measurements were confined to electron energies between 3 and 25 kev and they were obtained with the apparatus described in I. The only modifications necessary were the introduction of the high voltage electron gun and the movement of collector C_1 to an appropriate position.

The efficiency of the detector for Li^{2+} was found by using the apparatus to measure cross sections for the single ionization of Li^+ by electrons with energies of 1, 2 and 3 kev. Comparisons with previous measurements at these energies by Peart and Dolder gave 94 $\pm$ 6% for the mean detection efficiency.

3. Results and discussion

3.1. *Results for He^+ ions*

Table 1 summarizes the measured cross sections and 90% confidence limits of random error at each energy; the systematic errors are estimated to be less than $\pm$8%. Figure 2 compares the measurements at lower energies with the results of II. For electron energies above 400 ev the two curves virtually coincide, but at lower energies they diverge by an amount which is less than the errors assigned in either experiment.

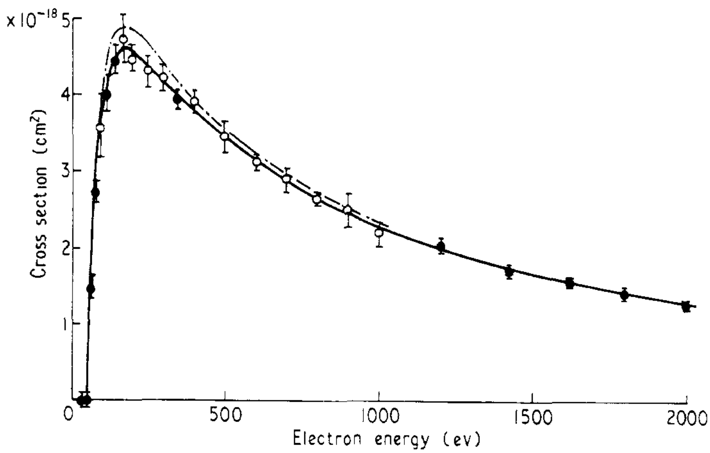

Figure 2. Comparison of present results for the single ionization of He^+ by electrons with energies up to 2 kev with previous measurements by Dolder *et al.* (1961). Present measurements: ○ obtained with the electrometer; ● obtained with the single-particle detector. The vertical bars show 90% confidence limits of random error at each energy and the chain curve denotes results of the earlier experiment.

Table 1. Cross sections for the single ionization of He^+ ions by electron impact

Electron energy (ev)	Cross section (10^{-18} cm^2)	Random error ($\pm$ %)
54·5	0·10	
68	1·50	10
83	2·74	5
98	3·58	12
118	4·01	6
145	4·46	4
170	4·75	7
200	4·47	3
250	4·30	5
300	4·23	4
350	3·96	3
400	3·90	4
500	3·44	6
600	3·12	3
700	2·90	5
800	2·64	3
900	2·51	9
1000	2·20	8
1200	2·08	4
1400	1·74	3
1600	1·59	2
1800	1·44	5
2000	1·28	4
2250	1·25	3
3000	0·970	8
4000	0·755	5
5000	0·604	6
6000	0·504	7
7000	0·431	9
8000	0·393	11
9000	0·346	10
10000	0·316	15

The random errors are 90% confidence limits at each electron energy and the systematic errors are assessed to be less than $\pm$ 8% at all energies.

If the cross sections measured by Fite and Brackmann (1958) for atomic hydrogen are scaled classically and superimposed on figure 2, they will be seen to lie about 4% above the present results for electron energies approximately ten times threshold. This is contrary to expectations, but Fite and Brackmann's results at these energies are respectively about 4% and 20% greater than those of Boksenberg (1961) and Rothe *et al.* (1962).

The behaviour at high energies is best illustrated by plotting the cross sections σ as σE against ln E, and comparing them with calculations for the hydrogenic series recently made by Omidvar (1969) in the Born and Bethe approximations. Figure 3 indicates that these approximations are respectively valid for electron energies greater than about 1 and 2·5 kev. No curve has been drawn through the present points, but the broken line represents the results of II.

3.2. *Results for Li^+ ions*

The present results are plotted in figure 4 in the form of $\sigma\beta^2$ against $\ln\{\beta^2/(1-\beta^2)\}-\beta^2$, where $\beta = v/c$, to give a linear relation in the relativistic form of Bethe's approximation (e.g. Mott and Massey 1965). The ionization threshold for Li^+ is 75·6 ev so that electron energies up to 25 kev were necessary to obtain a sufficient range of results in the Bethe region. At these energies relativistic effects could not be ignored.

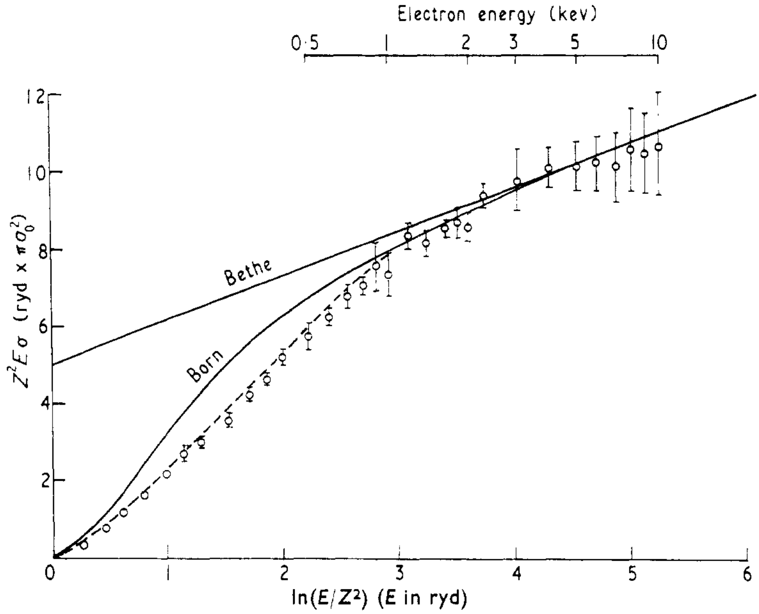

Figure 3. Comparison of measured cross sections σ for He^+ ($Z = 2$) with calculations by Omidvar (1969) in the Born and Bethe approximations. ○ present measurements; – – – – Dolder *et al.* (1961).

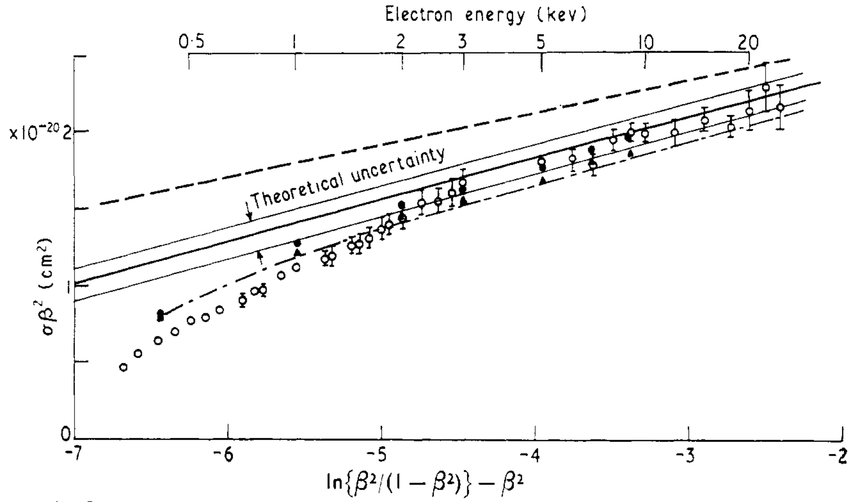

Figure 4. Comparison of measured cross sections σ for Li^+ with theory. ○ measured cross sections with 90% confidence limits of random error. Theoretical results: —·—·- Moores and Nussbaumer 1969 (Coulomb–Born); ● length, ▲ velocity, Economides and McDowell 1969 (Born); – – – – Bell and Kingston 1967 (gradient only) (Bethe); ——— Kim and Inokuti 1969 (Bethe). At energies below 3 kev the measurements by Peart and Dolder (1968) are plotted. β denotes the electron velocity relative to that of light.

Results of four quantum calculations of this cross section are included in figure 4. In the limit of Bethe's approximation Kim and Inokuti (1969) have obtained cross sections with an estimated theoretical uncertainty of 3%. They used the method of oscillator strength sum rules described by Inokuti *et al.* (1967) and Kim and Inokuti (1969). Similar methods have also been used by Bell and Kingston (1967) to obtain the slope (but not the magnitude of the intercept at the ordinate) of the broken line in figure 4. The Coulomb–Born and Born approximations have been employed by Moores and Nussbaumer (1969) and Economides and McDowell (1969) respectively and, in the latter case, results in both the 'length' and 'velocity' formulations are shown. The figure also includes the previous experimental values by Peart and Dolder for energies below 3 kev.

The agreement between all the results in figure 4 is very satisfactory and the ranges of validity of the Born and Bethe approximations are clearly seen. The present results for Li^+ are listed in table 2 with the 90% confidence limits of random error at each energy. Systematic errors were assessed to be less than ±8%.

Table 2. Cross sections for the single ionization of Li^+ ions by fast electron impact

Electron energy (ev)	Cross section (10^{-18} cm^2)	Random error (± %)
4000	1·078	3
5000	0·936	2
6000	0·791	3
7000	0·666	4
8000	0·636	4
9000	0·586	3
10000	0·529	4
12000	0·443	5
15000	0·371	3
17500	0·313	3
20000	0·291	6
22500	0·280	8
25000	0·238	7

The random errors are 90% confidence limits at each electron energy and the systematic errors are assessed to be less than ±8% at all energies.

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