ATOMIC DATA AND NUCLEAR DATA TABLES **58,** 67–75 (1994)

SIMPLIFIED MODEL OF ELECTRON SCATTERING ON ATOMIC HYDROGEN

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The simplified model of electron-hydrogen atom scattering which has all orbital angular momenta set to zero is solved to convergence at a range of energies from 1 to 400 eV using a general coupled-channel approach. The results are in complete agreement with the solution of this problem by Poet using a method that is applicable to this model only. The corresponding singlet and triplet T-matrix elements for the transitions from the ground state to each one of $1s$, $2s$, and $3s$ states are presented in tabular form. The $1s$, $2s$, $3s$, ionization, and total cross sections are also presented for each energy.

0092-640X/94 $6.00

CONTENTS

INTRODUCTION

The idea of solving a simplified model of electron-atom scattering exactly is that the solution may then be used to check the quality of the approximations in any scattering theory which are necessary in order to apply it to a realistic calculation. Most theories of this type are expanded in the partial-wave formalism, so they are readily directly tested against the exact solution of the simplified model that has all orbital angular momenta set to zero. This problem, first analyzed by Temkin[1] and Kyle and Temkin,[2] in electron-atomic hydrogen (e-H) scattering has been solved with a high degree of accuracy by Poet[3] using a method suited only to this restricted case. We will refer to this model of e-H scattering as the Poet–Temkin problem.

The Poet–Temkin problem is nontrivial and contains most of the difficulties associated with a full partial-wave calculation. It continues to be the model of choice for testing new methods of solving for the three-body Schrödinger equation for electron-atom scattering.[4,5] In our approach to electron-atom scattering calculations, we replace the sum and integral over the exact target discrete and continuum states by a Gaussian-type quadrature rule. This involves a finite sum over N square-integrable states, with the property that both approaches are equivalent in the limit as N goes to infinity. We call this method Convergent Close Coupling (CCC) as it is an expansion of standard close-coupling techniques to include square-integrable "continuum" states to convergence. We find[6] that we get excellent agreement with the exact results at the range of energies given by Poet.[3,7] Consequently, we believe that our method solves this model problem exactly.

The application of the CCC method to the full e-H scattering problem[8] has yielded remarkable agreement with the measurements of the total ionization cross section and spin asymmetry.[9] To date, CCC is the only theory of electron-atom scattering that is able to achieve this result. It is evident that in order to describe the ionization channel the target continuum needs to be treated. Standard close-coupling calculations that treat any number of discrete states yield identically zero for the ionization cross section. However, Bray[10] and Bray and McCarthy[11] showed that the effect of treating the target continuum was very large when calculating the elastic and first inelastic spin asymmetries in electron-sodium scattering at projectile energies above the ionization threshold and brought about excellent agreement with experiment. The method of calculation used was the Coupled-Channel Optical (CCO) method which treats the continuum indirectly via a complex nonlocal polarization potential. This established the importance of treating the continuum even when calculating scattering in just the elastic or first inelastic channels. It is therefore very useful for theorists to have a simple problem which can thoroughly test their method of treating the continuum. The Poet–Temkin model provides just such a test.

The aim of this paper is to present more detailed information for the Poet–Temkin model problem than that given by Poet. In addition to extending the energy range we also include ionization and total cross sections, as well as the T-matrix elements. This will provide a much greater range of information on which any scattering theory may be tested. For example, perturbative approaches may be tested to determine the energy range for which they will provide sufficiently accurate results, or we may check to see at above which energy exchange may be neglected. In general, the numerics of most scattering theories can be directly tested by comparing them with the results presented here.

The Hamiltonian for the nonrelativistic electron-atomic hydrogen scattering problem is given by

$$H = K_1 + v_1 + K_2 + v_2 + v_3, \quad (1)$$

where K_1 and K_2 are the projectile- and target-electron kinetic energy operators, and v_1, v_2, and v_3 are respectively the projectile-proton, target-proton, and projectile-target potentials. The corresponding partial-wave Lippmann–Schwinger equation for the reduced T matrix of the electron-hydrogen scattering problem in the momentum-space coupled-channel approach[8] is

$$\langle Lkln\|T^{SJ}\|n_0l_0k_0L_0\rangle = \langle Lkln\|V^{SJ}\|n_0l_0k_0L_0\rangle + \lim_{\varepsilon\to 0^+}\sum_{l',L'}\sum_{n'\in I}\int_0^\infty dk'k'^2\frac{\langle Lkln\|V^{SJ}\|n'l'k'L'\rangle}{E+i\varepsilon-\epsilon_{n'l'}-k'^2/2} \times\langle L'k'l'n'\|T^{SJ}\|n_0l_0k_0L_0\rangle, \quad (2)$$

where the projectile is denoted by linear momentum k and orbital momentum L, and the target state is denoted by principal quantum number n and orbital angular momentum l. The total orbital angular momentum is denoted by J, the total spin is denoted by S, and

$$E = \epsilon_{n_0l_0} + k_0^2/2 \quad (3)$$

is the on-shell energy. The matrix elements of V^{SJ} are given in Ref. 8.

The difficulty with solving Eq. (2) is due to the infinite sum and integral over the complete set I of discrete and continuous target states n' for each particular l'. We solve this problem by replacing this sum and integral over exact target states $|n'l'\rangle$ by a finite sum over N square integrable states $|\bar{n}'l'\rangle$. These states are obtained by diagonalizing the target Hamiltonian in a Laguerre basis,[8] which has the advantage of being orthogonal and whose analytical properties are well known.[12] With this choice of basis it can be shown that in the limit $N \to \infty$ both approaches are equivalent. Furthermore, as N increases the lower states $|\bar{n}'l'\rangle$ converge rapidly to $|n'l'\rangle$ (see Table II in Ref. 13 for more details).

The quality of the basis is tested by repeating the model calculations with increasing N until convergence is obtained. We find that taking $N = 30$ is sufficient to get agreement with the exact answers of Poet[3,7] to an accuracy better than 1%. The complete description of the method of solution of Eq. (2) and the relation of the T-matrix elements to various physical observables, the K matrix, and other commonly used definitions may be found in Ref. 8 and references therein.

The simplified model of electron-hydrogen scattering which Poet solved corresponds to taking all orbital angular momenta to be zero in Eq. (2). The T-matrix elements resulting from such calculations are given in Table I. The corresponding cross section given by Poet[3,7] for the transition from the ground state $|10\rangle$ to state $|\bar{n}0\rangle$ in units of πa_0^2 is given by*

$$\sigma_{\bar{n}0}^S = \pi^2(2S+1)k_{\bar{n}}/k_1|\langle 0k_{\bar{n}}0\bar{n}\|T^{S0}\|10k_10\rangle|^2, \quad (4)$$

where $k_{\bar{n}}$ is given by the on-shell conservation of energy equation

$$E = k_{\bar{n}}^2/2 + \epsilon_{\bar{n}0} = k_1^2/2 + \epsilon_{10}, \quad (5)$$

and where $\epsilon_{\bar{n}0}$ is the energy of state $|\bar{n}0\rangle$. The $1s$, $2s$, $3s$, ionization, and total cross sections are given in Table II. The ionization cross section σ_i^S is approximated by summing the cross sections for states with positive energies, while the total cross section σ_t^S is obtained by summing the cross sections of all open channels. The latter may also be obtained via the optical theorem

$$\sigma_t^S = -\pi(2S+1)\mathrm{Im}(T_{1s}^{S0})/k_1. \quad (6)$$

We have done these calculations over the broad range of energies of 1 to 400 eV with very small steps in energy. We did this to demonstrate that by choosing N to be as large as 30 we have almost completely removed the effects of pseudo-resonances, which typically occur when a finite sum over a set of square-integrable states is taken to represent the sum and integral over the exact target states. This effect is only visible for the singlet ionization cross section around 30 eV. We believe that outside this and the resonance regions all quantities are accurate to approximately 1%.

The data presented in the tables are stored electronically and may be obtained via e-mail by sending a request to igor@csm.ph.flinders.edu.au. We expect it to be available for a period of at least five years after the publication of this paper.

Acknowledgments

We thank Professor Ian McCarthy for many useful discussions. We also acknowledge support from the Australian Research Council.

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* In the earlier work[6] the triplet cross sections were defined without the factor $2S + 1$

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EXPLANATION OF TABLES

TABLE I. Reduced T-Matrix Elements (in Atomic Units) of the Simplified Electron-Hydrogen Atom Scattering Problem Which Has All Orbital Angular Momenta Set to Zero

The two entries in each column are the real and imaginary parts of the matrix elements; superscripts denote powers of 10

E is the incident projectile energy in eV, and is given in atomic units by $k_1^2/2$

T_{1s}^{00} is the singlet elastic T-matrix element, i.e., $\langle 0k_1 01 \| T^{00} \| 10k_1 0 \rangle$

T_{2s}^{00} is the singlet T-matrix element for the transition from the ground state $1s$ to $2s$, i.e., $\langle 0k_2 02 \| T^{00} \| 10k_1 0 \rangle$

T_{3s}^{00} is the singlet T-matrix element for the transition from the ground state $1s$ to $3s$, i.e., $\langle 0k_3 03 \| T^{00} \| 10k_1 0 \rangle$

T_{1s}^{10} is the triplet elastic T-matrix element, i.e., $\langle 0k_1 01 \| T^{00} \| 10k_1 0 \rangle$

T_{2s}^{10} is the triplet T-matrix element for the transition from the ground state $1s$ to $2s$, i.e., $\langle 0k_2 02 \| T^{00} \| 10k_1 0 \rangle$

T_{3s}^{10} is the triplet T-matrix element for the transition from the ground state $1s$ to $3s$, i.e., $\langle 0k_3 03 \| T^{00} \| 10k_1 0 \rangle$

TABLE II. The $1s$, $2s$, $3s$, Ionization, and Total Cross Sections (in Units of πa_0^2) of the Simplified Electron-Hydrogen Scattering Problem

σ_{1s}^0 is the singlet elastic cross section

σ_{2s}^0 is the singlet $2s$ cross section

σ_{3s}^0 is the singlet $3s$ cross section

σ_i^0 is the singlet ionization cross section

σ_t^0 is the total singlet cross section

σ_{1s}^1 is the triplet elastic cross section

σ_{2s}^1 is the triplet $2s$ cross section

σ_{3s}^1 is the triplet $3s$ cross section

σ_i^1 is the triplet ionization cross section

σ_t^1 is the total triplet cross section

TABLE I. Reduced T-Matrix Elements (in Atomic Units) of the Simplified Electron-Hydrogen Atom Scattering Problem Which Has All Angular Momenta Set to Zero

See page 71 for Explanation of Tables

E (eV)	T^{00}_{1s}	T^{00}_{2s}	T^{00}_{3s}	T^{10}_{1s}	T^{10}_{2s}	T^{10}_{3s}
1	$6.77^{-2} - 1.17^{+0}$			$5.54^{-1} - 3.94^{-1}$		
2	$-2.07^{-1} - 7.74^{-1}$			$4.11^{-1} - 4.69^{-1}$		
3	$-2.64^{-1} - 5.50^{-1}$			$3.03^{-1} - 4.89^{-1}$		
4	$-2.68^{-1} - 4.12^{-1}$			$2.21^{-1} - 4.86^{-1}$		
5	$-2.55^{-1} - 3.23^{-1}$			$1.57^{-1} - 4.72^{-1}$		
6	$-2.38^{-1} - 2.62^{-1}$			$1.07^{-1} - 4.54^{-1}$		
7	$-2.21^{-1} - 2.20^{-1}$			$6.79^{-2} - 4.33^{-1}$		
8	$-2.06^{-1} - 1.90^{-1}$			$3.65^{-2} - 4.11^{-1}$		
9	$-1.94^{-1} - 1.70^{-1}$			$1.14^{-2} - 3.91^{-1}$		
9.9	$-1.85^{-1} - 1.64^{-1}$			$-6.97^{-3} - 3.73^{-1}$		
10.0	$-1.85^{-1} - 1.68^{-1}$			$-8.73^{-3} - 3.71^{-1}$		
10.05	$-1.85^{-1} - 1.72^{-1}$			$-9.63^{-3} - 3.70^{-1}$		
10.1	$-1.85^{-1} - 1.83^{-1}$			$-1.05^{-2} - 3.69^{-1}$		
10.15	$-1.65^{-1} - 2.66^{-1}$			$-1.13^{-2} - 3.68^{-1}$		
10.18	$-1.49^{-1} - 7.57^{-2}$			$-1.18^{-2} - 3.68^{-1}$		
10.2	$-1.74^{-1} - 1.24^{-1}$			$-1.23^{-2} - 3.67^{-1}$		
10.3	$-1.69^{-1} - 1.47^{-1}$	$1.14^{-1} - 1.55^{-1}$		$-1.40^{-2} - 3.65^{-1}$	$3.52^{-3} - 3.97^{-3}$	
10.4	$-1.69^{-1} - 1.48^{-1}$	$3.62^{-2} - 1.51^{-1}$		$-1.57^{-2} - 3.63^{-1}$	4.82^{-3} 3.00^{-3}	
10.5	$-1.68^{-1} - 1.47^{-1}$	$-2.81^{-3} - 1.37^{-1}$		$-1.73^{-2} - 3.61^{-1}$	5.36^{-3} 1.91^{-3}	
10.6	$-1.68^{-1} - 1.47^{-1}$	$-2.66^{-2} - 1.25^{-1}$		$-1.89^{-2} - 3.59^{-1}$	5.68^{-3} 1.07^{-3}	
10.7	$-1.67^{-1} - 1.46^{-1}$	$-4.35^{-2} - 1.13^{-1}$		$-2.05^{-2} - 3.57^{-1}$	6.16^{-3} 1.08^{-4}	
10.8	$-1.66^{-1} - 1.45^{-1}$	$-5.58^{-2} - 1.03^{-1}$		$-2.20^{-2} - 3.55^{-1}$	$6.29^{-3} - 7.19^{-4}$	
10.885	$-1.65^{-1} - 1.44^{-1}$	$-6.40^{-2} - 9.54^{-2}$		$-2.33^{-2} - 3.54^{-1}$	$6.29^{-3} - 1.37^{-3}$	
11.837	$-1.53^{-1} - 1.35^{-1}$	$-9.97^{-2} - 4.41^{-2}$		$-3.63^{-2} - 3.37^{-1}$	$4.80^{-3} - 7.06^{-3}$	
11.973	$-1.51^{-1} - 1.33^{-1}$	$-9.99^{-2} - 4.28^{-2}$		$-3.79^{-2} - 3.34^{-1}$	$4.37^{-3} - 7.61^{-3}$	
12.081	$-1.56^{-1} - 1.25^{-1}$	$-3.56^{-2} - 6.47^{-2}$		$-3.92^{-2} - 3.33^{-1}$	$4.03^{-3} - 8.03^{-3}$	
12.083	$-1.61^{-1} - 1.26^{-1}$	$-8.19^{-3} - 4.39^{-2}$		$-3.92^{-2} - 3.33^{-1}$	$4.02^{-3} - 8.04^{-3}$	
12.085	$-1.64^{-1} - 1.32^{-1}$	-3.59^{-3} 1.02^{-2}		$-3.93^{-2} - 3.33^{-1}$	$4.01^{-3} - .05^{-3}$	
12.089	$-1.55^{-1} - 1.39^{-1}$	-8.98^{-2} 2.80^{-2}		$-3.93^{-2} - 3.32^{-1}$	$4.00^{-3} - 8.07^{-3}$	
12.109	$-1.51^{-1} - 1.34^{-1}$	$-9.68^{-2} - 2.43^{-2}$	7.36^{-2} 7.68^{-2}	$-3.96^{-2} - 3.33^{-1}$	$3.94^{-3} - 8.15^{-3}$	-5.15^{-4} 7.22^{-5}
12.245	$-1.49^{-1} - 1.31^{-1}$	$-9.79^{-2} - 2.65^{-2}$	$4.33^{-2} - 3.23^{-2}$	$-4.12^{-2} - 3.30^{-1}$	$3.47^{-3} - 8.58^{-3}$	-1.70^{-4} 6.23^{-4}
12.5	$-1.47^{-1} - 1.29^{-1}$	$-9.71^{-2} - 2.07^{-2}$	$3.91^{-3} - 4.95^{-2}$	$-4.40^{-2} - 3.26^{-1}$	$2.57^{-3} - 9.35^{-3}$	5.77^{-4} 6.31^{-4}
12.6	$-1.46^{-1} - 1.29^{-1}$	$-9.62^{-2} - 1.74^{-2}$	$-4.80^{-3} - 5.05^{-2}$	$-4.50^{-2} - 3.24^{-1}$	$2.22^{-3} - 9.60^{-3}$	7.71^{-4} 2.17^{-4}
12.7	$-1.45^{-1} - 1.28^{-1}$	$-9.54^{-2} - 1.37^{-2}$	$-9.31^{-3} - 5.14^{-2}$	$-4.61^{-2} - 3.23^{-1}$	$1.85^{-3} - 9.80^{-3}$	9.04^{-4} 1.04^{-4}
12.75	$-1.44^{-1} - 1.28^{-1}$	$-1.02^{-1} - 7.63^{-3}$	$1.80^{-2} - 3.80^{-2}$	$-4.66^{-2} - 3.22^{-1}$	$1.67^{-3} - 9.97^{-3}$	1.06^{-3} 3.38^{-4}
12.8	$-1.44^{-1} - 1.27^{-1}$	$-9.57^{-2} - 1.21^{-2}$	$-1.69^{-2} - 4.47^{-2}$	$-4.71^{-2} - 3.21^{-1}$	$1.49^{-3} - 1.01^{-2}$	1.13^{-3} 2.46^{-4}
12.9	$-1.43^{-1} - 1.26^{-1}$	$-9.51^{-2} - 9.09^{-3}$	$-2.05^{-2} - 4.38^{-2}$	$-4.80^{-2} - 3.20^{-1}$	$1.08^{-3} - 1.04^{-2}$	1.93^{-3} 1.05^{-3}
13	$-1.42^{-1} - 1.25^{-1}$	$-9.48^{-2} - 6.70^{-3}$	$-2.37^{-2} - 4.09^{-2}$	$-4.91^{-2} - 3.18^{-1}$	$7.60^{-4} - 1.05^{-2}$	$1.37^{-3} - 6.06^{-5}$
14	$-1.33^{-1} - 1.18^{-1}$	-8.78^{-2} 1.15^{-2}	$-3.99^{-2} - 1.81^{-2}$	$-5.78^{-2} - 3.02^{-1}$	$-2.78^{-3} - 1.14^{-2}$	$1.26^{-3} - 1.85^{-3}$
15	$-1.26^{-1} - 1.12^{-1}$	-7.92^{-2} 2.43^{-2}	$-4.33^{-2} - 3.62^{-3}$	$-6.50^{-2} - 2.88^{-1}$	$-5.92^{-3} - 1.13^{-2}$	$2.98^{-4} - 2.99^{-3}$

TABLE I. Reduced T-Matrix Elements (in Atomic Units) of the Simplified Electron-Hydrogen Atom Scattering Problem Which Has All Angular Momenta Set to Zero

See page 71 for Explanation of Tables

E (eV)	T^{00}_{1s}	T^{00}_{2s}	T^{00}_{3s}	T^{10}_{1s}	T^{10}_{2s}	T^{10}_{3s}
16	$-1.20^{-1} - 1.08^{-1}$	-7.04^{-2} 3.17^{-2}	-3.98^{-2} 6.98^{-3}	$-7.08^{-2} - 2.74^{-1}$	$-8.52^{-3} - 1.05^{-2}$	$-9.24^{-4} - 3.57^{-3}$
17	$-1.14^{-1} - 1.03^{-1}$	-6.40^{-2} 3.77^{-2}	-3.77^{-2} 1.19^{-2}	$-7.56^{-2} - 2.62^{-1}$	$-1.06^{-2} - 9.44^{-3}$	$-2.09^{-3} - 3.70^{-3}$
18	$-1.09^{-1} - 1.00^{-1}$	-5.65^{-2} 4.02^{-2}	-3.26^{-2} 1.62^{-2}	$-7.94^{-2} - 2.50^{-1}$	$-1.22^{-2} - 8.10^{-3}$	$-3.13^{-3} - 3.53^{-3}$
19	$-1.05^{-1} - 9.74^{-2}$	-4.99^{-2} 4.27^{-2}	-2.90^{-2} 1.95^{-2}	$-8.26^{-2} - 2.40^{-1}$	$-1.35^{-2} - 6.67^{-3}$	$-3.99^{-3} - 3.17^{-3}$
20	$-1.01^{-1} - 9.47^{-2}$	-4.45^{-2} 4.39^{-2}	-2.57^{-2} 2.13^{-2}	$-8.51^{-2} - 2.30^{-1}$	$-1.44^{-2} - 5.23^{-3}$	$-4.69^{-3} - 2.70^{-3}$
21	$-9.79^{-2} - 9.21^{-2}$	-3.98^{-2} 4.44^{-2}	-2.29^{-2} 2.22^{-2}	$-8.70^{-2} - 2.20^{-1}$	$-1.50^{-2} - 3.82^{-3}$	$-5.24^{-3} - 2.18^{-3}$
22	$-9.50^{-2} - 8.97^{-2}$	-3.58^{-2} 4.42^{-2}	-2.03^{-2} 2.23^{-2}	$-8.86^{-2} - 2.12^{-1}$	$-1.54^{-2} - 2.48^{-3}$	$-5.66^{-3} - 1.63^{-3}$
23	$-9.21^{-2} - 8.74^{-2}$	-3.25^{-2} 4.42^{-2}	-1.84^{-2} 2.26^{-2}	$-8.98^{-2} - 2.04^{-1}$	$-1.56^{-2} - 1.22^{-3}$	$-5.96^{-3} - 1.08^{-3}$
24	$-8.91^{-2} - 8.53^{-2}$	-2.92^{-2} 4.44^{-2}	-1.66^{-2} 2.31^{-2}	$-9.07^{-2} - 1.96^{-1}$	$-1.57^{-2} - 4.33^{-5}$	$-6.17^{-3} - 5.53^{-4}$
25	$-8.68^{-2} - 8.40^{-2}$	-2.57^{-2} 4.36^{-2}	-1.43^{-2} 2.30^{-2}	$-9.13^{-2} - 1.89^{-1}$	-1.57^{-2} 1.04^{-3}	$-6.30^{-3} - 4.37^{-5}$
26	$-8.53^{-2} - 8.22^{-2}$	-2.35^{-2} 4.22^{-2}	-1.29^{-2} 2.22^{-2}	$-9.17^{-2} - 1.82^{-1}$	-1.56^{-2} 2.04^{-3}	-6.37^{-3} 4.38^{-4}
27	$-8.28^{-2} - 8.04^{-2}$	-2.12^{-2} 4.21^{-2}	-1.16^{-2} 2.24^{-2}	$-9.19^{-2} - 1.76^{-1}$	-1.54^{-2} 2.95^{-3}	-6.39^{-3} 8.83^{-4}
28	$-8.12^{-2} - 7.94^{-2}$	-1.86^{-2} 4.08^{-2}	-9.93^{-3} 2.18^{-2}	$-9.20^{-2} - 1.70^{-1}$	-1.51^{-2} 3.78^{-3}	-6.37^{-3} 1.29^{-3}
29	$-7.99^{-2} - 7.76^{-2}$	-1.73^{-2} 4.00^{-2}	-9.25^{-3} 2.12^{-2}	$-9.19^{-2} - 1.64^{-1}$	-1.48^{-2} 4.55^{-3}	-6.31^{-3} 1.68^{-3}
30	$-7.79^{-2} - 7.69^{-2}$	-1.49^{-2} 3.91^{-2}	-7.72^{-3} 2.09^{-2}	$-9.18^{-2} - 1.59^{-1}$	-1.45^{-2} 5.23^{-3}	-6.22^{-3} 2.03^{-3}
31	$-7.73^{-2} - 7.51^{-2}$	-1.42^{-2} 3.80^{-2}	-7.33^{-3} 2.01^{-2}	$-9.15^{-2} - 1.54^{-1}$	-1.42^{-2} 5.85^{-3}	-6.11^{-3} 2.35^{-3}
32	$-7.54^{-2} - 7.42^{-2}$	-1.23^{-2} 3.73^{-2}	-6.17^{-3} 1.99^{-2}	$-9.12^{-2} - 1.50^{-1}$	-1.38^{-2} 6.41^{-3}	-5.99^{-3} 2.64^{-3}
33	$-7.44^{-2} - 7.26^{-2}$	-1.15^{-2} 3.65^{-2}	-5.73^{-3} 1.94^{-2}	$-9.08^{-2} - 1.45^{-1}$	-1.34^{-2} 6.92^{-3}	-5.86^{-3} 2.91^{-3}
34	$-7.31^{-2} - 7.22^{-2}$	-9.84^{-3} 3.53^{-2}	-4.66^{-3} 1.88^{-2}	$-9.03^{-2} - 1.41^{-1}$	-1.30^{-2} 7.37^{-3}	-5.71^{-3} 3.15^{-3}
35	$-7.20^{-2} - 7.04^{-2}$	-9.18^{-3} 3.50^{-2}	-4.41^{-3} 1.86^{-2}	$-8.98^{-2} - 1.37^{-1}$	-1.26^{-2} 7.78^{-3}	-5.56^{-3} 3.37^{-3}
36	$-7.14^{-2} - 6.99^{-2}$	-8.23^{-3} 3.36^{-2}	-3.79^{-3} 1.77^{-2}	$-8.93^{-2} - 1.33^{-1}$	-1.22^{-2} 8.15^{-3}	-5.41^{-3} 3.57^{-3}
40	$-6.75^{-2} - 6.59^{-2}$	-5.10^{-3} 3.09^{-2}	-2.07^{-3} 1.63^{-2}	$-8.68^{-2} - 1.20^{-1}$	-1.07^{-2} 9.26^{-3}	-4.77^{-3} 4.17^{-3}
45	$-6.45^{-2} - 6.18^{-2}$	-2.79^{-3} 2.74^{-2}	-8.38^{-4} 1.43^{-2}	$-8.35^{-2} - 1.06^{-1}$	-9.01^{-3} 1.00^{-2}	-4.00^{-3} 4.62^{-3}
50	$-6.16^{-2} - 5.78^{-2}$	-1.19^{-3} 2.50^{-2}	-8.67^{-5} 1.30^{-2}	$-8.01^{-2} - 9.56^{-2}$	-7.50^{-3} 1.04^{-2}	-3.32^{-3} 4.84^{-3}
55	$-5.89^{-2} - 5.46^{-2}$	2.55^{-4} 2.25^{-2}	7.28^{-4} 1.16^{-2}	$-7.69^{-2} - 8.65^{-2}$	-6.21^{-3} 1.05^{-2}	-2.71^{-3} 4.92^{-3}
60	$-5.69^{-2} - 5.13^{-2}$	9.42^{-4} 2.07^{-2}	1.03^{-3} 1.05^{-2}	$-7.38^{-2} - 7.90^{-2}$	-5.10^{-3} 1.04^{-2}	-2.23^{-3} 4.92^{-3}
70	$-5.35^{-2} - 4.67^{-2}$	2.03^{-3} 1.73^{-2}	1.52^{-3} 8.79^{-3}	$-6.82^{-2} - 6.72^{-2}$	-3.38^{-3} 1.00^{-2}	-1.43^{-3} 4.76^{-3}
80	$-5.05^{-2} - 4.22^{-2}$	2.58^{-3} 1.51^{-2}	1.72^{-3} 7.60^{-3}	$-6.34^{-2} - 5.83^{-2}$	-2.14^{-3} 9.52^{-3}	-8.53^{-4} 4.51^{-3}
90	$-4.79^{-2} - 3.86^{-2}$	2.89^{-3} 1.33^{-2}	1.81^{-3} 6.65^{-3}	$-5.92^{-2} - 5.14^{-2}$	-1.23^{-3} 8.93^{-3}	-4.37^{-4} 4.24^{-3}
100	$-4.57^{-2} - 3.55^{-2}$	3.02^{-3} 1.19^{-2}	1.82^{-3} 5.90^{-3}	$-5.56^{-2} - 4.60^{-2}$	-5.58^{-4} 8.36^{-3}	-1.21^{-4} 3.97^{-3}
120	$-4.18^{-2} - 3.08^{-2}$	3.15^{-3} 9.63^{-3}	1.82^{-3} 4.73^{-3}	$-4.96^{-2} - 3.79^{-2}$	3.48^{-4} 7.32^{-3}	2.91^{-4} 3.48^{-3}
140	$-3.87^{-2} - 2.69^{-2}$	3.12^{-3} 8.14^{-3}	1.75^{-3} 3.97^{-3}	$-4.50^{-2} - 3.21^{-2}$	8.84^{-4} 6.46^{-3}	5.30^{-4} 3.06^{-3}
160	$-3.60^{-2} - 2.39^{-2}$	3.03^{-3} 6.99^{-3}	1.66^{-3} 3.39^{-3}	$-4.12^{-2} - 2.78^{-2}$	1.20^{-3} 5.74^{-3}	6.73^{-4} 2.72^{-3}
180	$-3.38^{-2} - 2.14^{-2}$	2.89^{-3} 6.12^{-3}	1.56^{-3} 2.92^{-3}	$-3.80^{-2} - 2.45^{-2}$	1.40^{-3} 5.15^{-3}	7.56^{-4} 2.44^{-3}
200	$-3.18^{-2} - 1.94^{-2}$	2.77^{-3} 5.40^{-3}	1.48^{-3} 2.61^{-3}	$-3.54^{-2} - 2.19^{-2}$	1.52^{-3} 4.65^{-3}	8.03^{-4} 2.20^{-3}
250	$-2.78^{-2} - 1.57^{-2}$	2.50^{-3} 4.16^{-3}	1.31^{-3} 2.00^{-3}	$-3.03^{-2} - 1.72^{-2}$	1.64^{-3} 3.71^{-3}	8.42^{-4} 1.75^{-3}
300	$-2.48^{-2} - 1.31^{-2}$	2.25^{-3} 3.36^{-3}	1.17^{-3} 1.60^{-3}	$-2.67^{-2} - 1.41^{-2}$	1.63^{-3} 3.05^{-3}	8.27^{-4} 1.44^{-3}
350	$-2.25^{-2} - 1.12^{-2}$	2.05^{-3} 2.79^{-3}	1.05^{-3} 1.33^{-3}	$-2.39^{-2} - 1.19^{-2}$	1.58^{-3} 2.58^{-3}	7.95^{-4} 1.22^{-3}
400	$-2.05^{-2} - 9.77^{-3}$	1.87^{-3} 2.38^{-3}	9.53^{-4} 1.14^{-3}	$-2.17^{-2} - 1.03^{-2}$	1.50^{-3} 2.22^{-3}	7.54^{-4} 1.05^{-3}

TABLE II. The $1s$, $2s$, $3s$, Ionization, and Total Cross Sections (in Units of πa_0^2) of the Simplified Electron-Hydrogen Scattering Problem

See page 71 for Explanation of Tables

E (eV)	σ^0_{1s}	σ^0_{2s}	σ^0_{3s}	σ^0_{i}	σ^0_{t}	σ^1_{1s}	σ^1_{2s}	σ^1_{3s}	σ^1_{i}	σ^1_{t}
1	1.36^{+1}				1.36^{+1}	1.37^{+1}				1.37^{+1}
2	6.35^{+0}				6.35^{+0}	1.15^{+1}				1.15^{+1}
3	3.68^{+0}				3.68^{+0}	9.83^{+0}				9.83^{+0}
4	2.39^{+0}				2.39^{+0}	8.46^{+0}				8.46^{+0}
5	1.68^{+0}				1.68^{+0}	7.35^{+0}				7.35^{+0}
6	1.24^{+0}				1.24^{+0}	6.44^{+0}				6.44^{+0}
7	9.65^{-1}				9.65^{-1}	5.69^{+0}				5.69^{+0}
8	7.81^{-1}				7.81^{-1}	5.06^{+0}				5.06^{+0}
9	6.59^{-1}				6.59^{-1}	4.53^{+0}				4.53^{+0}
9.9	6.05^{-1}				6.05^{-1}	4.12^{+0}				4.12^{+0}
10.0	6.14^{-1}				6.14^{-1}	4.08^{+0}				4.08^{+0}
10.05	6.28^{-1}				6.28^{-1}	4.06^{+0}				4.06^{+0}
10.1	6.67^{-1}				6.67^{-1}	4.04^{+0}				4.04^{+0}
10.15	9.67^{-1}				9.67^{-1}	4.02^{+0}				4.02^{+0}
10.18	2.75^{-1}				2.75^{-1}	4.01^{+0}				4.01^{+0}
10.2	4.50^{-1}				4.50^{-1}	4.00^{+0}				4.00^{+0}
10.3	4.96^{-1}	3.52^{-2}			5.31^{-1}	3.96^{+0}	8.03^{-5}			3.96^{+0}
10.4	4.99^{-1}	3.26^{-2}			5.32^{-1}	3.92^{+0}	1.31^{-4}			3.92^{+0}
10.5	4.97^{-1}	3.15^{-2}			5.29^{-1}	3.88^{+0}	1.61^{-4}			3.88^{+0}
10.6	4.92^{-1}	3.16^{-2}			5.24^{-1}	3.84^{+0}	1.92^{-4}			3.84^{+0}
10.7	4.87^{-1}	3.14^{-2}			5.19^{-1}	3.80^{+0}	2.42^{-4}			3.80^{+0}
10.8	4.81^{-1}	3.20^{-2}			5.13^{-1}	3.76^{+0}	2.79^{-4}			3.77^{+0}
10.885	4.76^{-1}	3.26^{-2}			5.09^{-1}	3.73^{+0}	3.08^{-4}			3.73^{+0}
11.837	4.13^{-1}	4.36^{-2}			4.57^{-1}	3.41^{+0}	8.02^{-4}			3.41^{+0}
11.973	4.03^{-1}	4.48^{-2}			4.48^{-1}	3.36^{+0}	8.78^{-4}			3.36^{+0}
12.081	3.98^{-1}	2.13^{-2}			4.19^{-1}	3.33^{+0}	9.44^{-4}			3.33^{+0}
12.083	4.14^{-1}	7.78^{-3}			4.22^{-1}	3.33^{+0}	9.44^{-4}			3.33^{+0}
12.085	4.42^{-1}	4.63^{-4}			4.42^{-1}	3.33^{+0}	9.46^{-4}			3.33^{+0}
12.089	4.32^{-1}	3.45^{-2}			4.66^{-1}	3.33^{+0}	9.50^{-4}			3.33^{+0}
12.109	4.03^{-1}	3.90^{-2}	3.92^{-3}		4.45^{-1}	3.32^{+0}	9.61^{-4}	2.81^{-7}		3.32^{+0}
12.245	3.91^{-1}	4.15^{-2}	3.20^{-3}		4.36^{-1}	3.28^{+0}	1.04^{-3}	1.37^{-6}		3.28^{+0}
12.5	3.78^{-1}	4.17^{-2}	4.39^{-3}		4.24^{-1}	3.20^{+0}	1.19^{-3}	3.90^{-6}		3.21^{+0}
12.6	3.73^{-1}	4.11^{-2}	5.09^{-3}		4.19^{-1}	3.18^{+0}	1.25^{-3}	3.81^{-6}		3.18^{+0}
12.7	3.69^{-1}	4.06^{-2}	5.89^{-3}		4.16^{-1}	3.15^{+0}	1.31^{-3}	5.35^{-6}		3.15^{+0}
12.75	3.66^{-1}	4.63^{-2}	3.96^{-3}		4.16^{-1}	3.13^{+0}	1.35^{-3}	8.30^{-6}		3.13^{+0}
12.8	3.64^{-1}	4.14^{-2}	5.30^{-3}		4.11^{-1}	3.12^{+0}	1.39^{-3}	9.31^{-6}		3.12^{+0}
12.9	3.59^{-1}	4.12^{-2}	5.78^{-3}		4.07^{-1}	3.09^{+0}	1.47^{-3}	3.58^{-5}		3.09^{+0}
13	3.55^{-1}	4.13^{-2}	5.82^{-3}		4.03^{-1}	3.06^{+0}	1.51^{-3}	1.47^{-5}		3.07^{+0}
14	3.15^{-1}	4.03^{-2}	7.01^{-3}	1.76^{-4}	3.67^{-1}	2.81^{+0}	2.14^{-3}	5.51^{-5}	1.63^{-7}	2.81^{+0}
15	2.84^{-1}	3.83^{-2}	8.23^{-3}	1.72^{-3}	3.38^{-1}	2.58^{+0}	2.74^{-3}	1.18^{-4}	4.69^{-5}	$.59^{+0}$

TABLE II. The $1s$, $2s$, $3s$, Ionization, and Total Cross Sections (in Units of πa_0^2) of the Simplified Electron-Hydrogen Scattering Problem

See page 71 for Explanation of Tables

E (eV)	σ^0_{1s}	σ^0_{2s}	σ^0_{3s}	σ^0_i	σ^0_t	σ^1_{1s}	σ^1_{2s}	σ^1_{3s}	σ^1_i	σ^1_t
16	2.58^{-1}	3.55^{-2}	7.97^{-3}	4.51^{-3}	3.13^{-1}	2.38^{+0}	3.29^{-3}	1.99^{-4}	2.00^{-5}	2.39^{+0}
17	2.35^{-1}	3.45^{-2}	8.29^{-3}	7.64^{-3}	2.92^{-1}	2.21^{+0}	3.78^{-3}	2.88^{-4}	2.69^{-5}	2.21^{+0}
18	2.18^{-1}	3.13^{-2}	7.51^{-3}	1.07^{-2}	2.74^{-1}	2.05^{+0}	4.21^{-3}	3.78^{-4}	7.52^{-5}	2.06^{+0}
19	2.03^{-1}	2.90^{-2}	7.31^{-3}	1.31^{-2}	2.59^{-1}	1.91^{+0}	4.57^{-3}	4.65^{-4}	1.15^{-4}	1.92^{+0}
20	1.90^{-1}	2.70^{-2}	6.96^{-3}	1.55^{-2}	2.45^{-1}	1.78^{+0}	4.87^{-3}	5.47^{-4}	1.89^{-4}	1.79^{+0}
21	1.78^{-1}	2.52^{-2}	6.54^{-3}	1.68^{-2}	2.33^{-1}	1.67^{+0}	5.11^{-3}	6.22^{-4}	2.79^{-4}	1.68^{+0}
22	1.69^{-1}	2.34^{-2}	6.06^{-3}	1.81^{-2}	2.22^{-1}	1.57^{+0}	5.31^{-3}	6.90^{-4}	3.77^{-4}	1.57^{+0}
23	1.59^{-1}	2.22^{-2}	5.79^{-3}	1.91^{-2}	2.11^{-1}	1.47^{+0}	5.47^{-3}	7.50^{-4}	4.93^{-4}	1.48^{+0}
24	1.50^{-1}	2.12^{-2}	5.66^{-3}	1.93^{-2}	2.02^{-1}	1.39^{+0}	5.58^{-3}	8.02^{-4}	6.18^{-4}	1.39^{+0}
25	1.44^{-1}	1.95^{-2}	5.24^{-3}	2.10^{-2}	1.95^{-1}	1.31^{+0}	5.67^{-3}	8.47^{-4}	7.47^{-4}	1.32^{+0}
26	1.39^{-1}	1.80^{-2}	4.77^{-3}	2.11^{-2}	1.87^{-1}	1.24^{+0}	5.73^{-3}	8.85^{-4}	8.79^{-4}	1.25^{+0}
27	1.32^{-1}	1.74^{-2}	4.68^{-3}	2.15^{-2}	1.79^{-1}	1.17^{+0}	5.76^{-3}	9.18^{-4}	1.03^{-3}	1.18^{+0}
28	1.27^{-1}	1.59^{-2}	4.27^{-3}	2.24^{-2}	1.74^{-1}	1.11^{+0}	5.78^{-3}	9.45^{-4}	1.17^{-3}	1.12^{+0}
29	1.23^{-1}	1.51^{-2}	4.05^{-3}	2.18^{-2}	1.67^{-1}	1.06^{+0}	5.77^{-3}	9.65^{-4}	1.29^{-3}	1.06^{+0}
30	1.18^{-1}	1.41^{-2}	3.79^{-3}	2.29^{-2}	1.63^{-1}	1.00^{+0}	5.75^{-3}	9.82^{-4}	1.42^{-3}	1.01^{+0}
31	1.15^{-1}	1.33^{-2}	3.54^{-3}	2.17^{-2}	1.56^{-1}	9.57^{-1}	5.72^{-3}	9.94^{-4}	1.55^{-3}	9.66^{-1}
32	1.11^{-1}	1.26^{-2}	3.39^{-3}	2.25^{-2}	1.52^{-1}	9.13^{-1}	5.68^{-3}	1.00^{-3}	1.68^{-3}	9.23^{-1}
33	1.07^{-1}	1.21^{-2}	3.22^{-3}	2.14^{-2}	1.47^{-1}	8.72^{-1}	5.63^{-3}	1.01^{-3}	1.80^{-3}	8.81^{-1}
34	1.04^{-1}	1.11^{-2}	2.98^{-3}	2.25^{-2}	1.44^{-1}	8.34^{-1}	5.57^{-3}	1.01^{-3}	1.91^{-3}	8.43^{-1}
35	1.00^{-1}	1.09^{-2}	2.92^{-3}	2.14^{-2}	1.38^{-1}	7.98^{-1}	5.51^{-3}	1.02^{-3}	2.03^{-3}	8.07^{-1}
36	9.87^{-2}	1.00^{-2}	2.66^{-3}	2.11^{-2}	1.35^{-1}	7.65^{-1}	5.45^{-3}	1.01^{-3}	2.13^{-3}	7.74^{-1}
40	8.79^{-2}	8.37^{-3}	2.23^{-3}	2.02^{-2}	1.21^{-1}	6.51^{-1}	5.14^{-3}	9.94^{-4}	2.50^{-3}	6.61^{-1}
45	7.88^{-2}	6.60^{-3}	1.74^{-3}	1.80^{-2}	1.07^{-1}	5.43^{-1}	4.75^{-3}	9.48^{-4}	2.83^{-3}	5.52^{-1}
50	7.05^{-2}	5.53^{-3}	1.45^{-3}	1.60^{-2}	9.48^{-2}	4.61^{-1}	4.36^{-3}	8.91^{-4}	3.05^{-3}	4.70^{-1}
55	6.38^{-2}	4.54^{-3}	1.18^{-3}	1.48^{-2}	8.54^{-2}	3.97^{-1}	4.01^{-3}	8.28^{-4}	3.21^{-3}	4.06^{-1}
60	5.80^{-2}	3.87^{-3}	9.97^{-4}	1.30^{-2}	7.68^{-2}	3.47^{-1}	3.65^{-3}	7.75^{-4}	3.27^{-3}	3.55^{-1}
70	4.98^{-2}	2.80^{-3}	7.16^{-4}	1.08^{-2}	6.48^{-2}	2.72^{-1}	3.09^{-3}	6.66^{-4}	3.27^{-3}	2.79^{-1}
80	4.28^{-2}	2.17^{-3}	5.53^{-4}	8.73^{-3}	5.48^{-2}	2.20^{-1}	2.64^{-3}	5.76^{-4}	3.21^{-3}	2.27^{-1}
90	3.74^{-2}	1.73^{-3}	4.37^{-4}	7.23^{-3}	4.72^{-2}	1.82^{-1}	2.27^{-3}	5.01^{-4}	3.01^{-3}	1.89^{-1}
100	3.31^{-2}	1.41^{-3}	3.54^{-4}	5.96^{-3}	4.12^{-2}	1.54^{-1}	1.97^{-3}	4.38^{-4}	2.85^{-3}	1.60^{-1}
120	2.67^{-2}	9.70^{-4}	2.41^{-4}	4.48^{-3}	3.26^{-2}	1.16^{-1}	1.52^{-3}	3.42^{-4}	2.50^{-3}	1.20^{-1}
140	2.19^{-2}	7.23^{-4}	1.78^{-4}	3.36^{-3}	2.63^{-2}	9.06^{-2}	1.21^{-3}	2.74^{-4}	2.18^{-3}	9.45^{-2}
160	1.85^{-2}	5.55^{-4}	1.36^{-4}	2.62^{-3}	2.19^{-2}	7.33^{-2}	9.87^{-4}	2.25^{-4}	1.91^{-3}	7.66^{-2}
180	1.59^{-2}	4.40^{-4}	1.05^{-4}	2.06^{-3}	1.86^{-2}	6.08^{-2}	8.20^{-4}	1.87^{-4}	1.67^{-3}	6.37^{-2}
200	1.38^{-2}	3.55^{-4}	8.62^{-5}	1.69^{-3}	1.60^{-2}	5.15^{-2}	6.91^{-4}	1.58^{-4}	1.48^{-3}	5.39^{-2}
250	1.01^{-2}	2.28^{-4}	5.51^{-5}	1.10^{-3}	1.15^{-2}	3.61^{-2}	4.78^{-4}	1.10^{-4}	1.11^{-3}	3.79^{-2}
300	7.79^{-3}	1.59^{-4}	3.83^{-5}	7.54^{-4}	8.78^{-3}	2.71^{-2}	3.49^{-4}	8.07^{-5}	8.66^{-4}	2.84^{-2}
350	6.24^{-3}	1.17^{-4}	2.80^{-5}	5.45^{-4}	6.96^{-3}	2.12^{-2}	2.68^{-4}	6.20^{-5}	6.96^{-4}	2.23^{-2}
400	5.13^{-3}	8.95^{-5}	2.15^{-5}	4.07^{-4}	5.66^{-3}	1.71^{-2}	2.11^{-4}	4.88^{-5}	5.66^{-4}	1.80^{-2}