J. Phys. B: At. Mol. Phys. **14** (1981) 1857–1873. Printed in Great Britain

Elastic scattering of electrons by atomic hydrogen and helium

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Received 5 September 1980, in final form 30 December 1980

Abstract. Two procedures, one based on the evaluation of the static part exactly and all the rest of the multiple scattering series terms on an equal footing and the other providing for a special treatment of the second-order term are investigated and applied to e^-–H and e^-–He elastic scattering. The second-order term has been evaluated with an average excitation energy and the higher order terms in the Glauber approximation. The first procedure is found to be quite good at low intermediate energies and may also be used over a wider energy region if the target polarisability is low. The second procedure is an improvement over the eikonal Born series and the modified Glauber approaches at intermediate energies.

1. Introduction

The study of the scattering of electrons by simple atomic targets such as hydrogen and helium is quite frequently used as a testing ground for various theoretical schemes to predict differential and total scattering cross sections. The experimental measurements of absolute cross sections made over the last few years for the scattering of electrons by hydrogen (Lloyd *et al* 1974, Williams 1975a, b, van Wingerden *et al* 1977) and helium (Bromberg 1969, 1974, Sethuraman *et al* 1974, McConkey and Preston 1975, Kurepa and Vuskovic 1975, Gupta and Rees 1975, Jansen *et al* 1976, Register *et al* 1980) have provided a further stimulation to this study. The theoretical approaches in the literature can be broadly grouped into three categories depending on the energy range (low, intermediate and high) over which they are applicable (see Moiseiwitsch 1977, Bransden and McDowell 1977, 1978, Burke and Williams 1977 for recent reviews). On the low-energy side, there are methods like the close-coupling approximation and its variants (correlation approximation, polarised pseudostate approximation), R-matrix methods, etc which have been shown to give very good results. As the energy increases, a large number of channels open up and some sort of approximation scheme becomes imperative to account for these. The intermediate-energy region is served by simple modifications of the basically high-energy methods. The eikonal optical model (EOM) (Joachain and Mittleman 1971, Byron and Joachain 1974b, Vanderpoorten 1975), the second-order potential (SOP) method (Winters *et al* 1974), eikonal Born series (EBS) approach (Byron and Joachain 1973, 1974c, 1977a, b), the modified Glauber (MG) (Byron and Joachain 1975a, Gien 1976) and the two-potential eikonal approximation (TPE) (Ishihara and Chen 1975) fall into this group. All these methods give a good description of e^-–H elastic scattering for energies above about 50 eV and e^-–He elastic scattering for energies above about 200 eV.

0022-3700/81/111857+17\$01.50

A common feature in all these intermediate-energy methods (EOM, SOP, EBS, MG) is that they all pay special attention to the second-order term of the multiple-scattering series and treat it more accurately than the higher order ones. This is justified as the Born series is fairly convergent and the higher order terms are relatively small. At lower energies (lower than about 50 eV in the e^-–H case and lower than about 100 eV in the e^-–He case) these methods cannot be expected to yield good results. The Born series in this energy range has poor convergence and all the multiple-scattering terms should be evaluated on the same footing. One of us (Srivastava 1980, to be referred to as I) recently looked at this aspect and found that the cancellations involved in summing the multiple-scattering series are quite delicate and any attempt to calculate any one term more accurately than the others will only worsen the result. The contribution of the static part of the interaction can, however, be included separately. The method reported in I essentially amounts to an exact evaluation of the static part and on-shell evaluation of the remaining:

$$F_1 = f(V_{st}) + (f_G(V) - f_G(V_{st})). \tag{1}$$

Here f and f_G are respectively the exact and the Glauber scattering amplitudes corresponding to the total interaction V or the static interaction V_{st}. Expression (1) is very close to that used in the TPE approximation of Ishihara and Chen and leads to very good results for e^-–H elastic scattering at low intermediate energies down to 16.5 eV (Srivastava 1980).

At higher energies the approach indicated by (1) though continuing to remain better than the TPE approximation is not expected to compete with the EBS and MG approximations. At these energies the multiple-scattering series is fairly convergent. One may attempt a better estimate of the second-order term and continue to calculate the higher order ones ($n \geqslant 3$) in the Glauber approximation to get

$$F_2 = f(V_{st}) + (f_{B2}(V) - f_{B2}(V_{st})) + (f_G(V) - f_{G2}(V) - f_G(V_{st}) + f_{G2}(V_{st})) \tag{2}$$

where f_{B2} and f_{G2} are respectively the second-order Born and Glauber scattering amplitudes. This approach is expected to be even better than the EBS and MG methods as it exactly includes, unlike them, the contribution due to the static interaction to all orders. The MG scattering amplitude is obtained by dropping terms of order higher than two in $f(V_{st})$ and its compensating term $f_G(V_{st})$:

$$F_{MG} = f_G(V) + f_{B2}(V) - f_{G2}(V). \tag{3}$$

The EBS amplitude corresponds to also dropping the higher order Glauber terms G_4, G_5, G_6, ... in a bid to obtain an expression correct to order $1/k_i^2$:

$$F_{EBS} = f_{B1}(V) + f_{B2}(V) + f_{G3}(V). \tag{4}$$

The aim of this paper is (i) to explore the energy range of reliability of the method P1, based on equation (1), proposed in I for low intermediate energies and (ii) to improve upon the existing results at intermediate energies ($\geqslant$50 eV for e^-–H scattering) through the method P2 based on equation (2). It should be pointed out that the improvement obtained in the results at higher energies by a better treatment of the second-order term (method P2 in place of method P1) depends mainly on the polarisability of the target. Therefore much less change compared with e^-–H scattering (polarisability 4.5 a_0^3) is expected in the case of targets with low polarisability (say helium, polarisability 1.39 a_0^3) by this evaluation of the second-order term. Method P1 is therefore expected to yield good results for e^-–He scattering over quite a wide energy range. Our results confirm this observation. We have calculated elastic scattering cross sections for e^-–H scattering in the energy range 12–200 eV and e^-–He scattering in the energy range 50–500 eV.

In the next section we briefly outline the details of the calculation. Sections 3 and 4 contain our results which are summarised and commented upon in § 5.

2. Calculation

We have used the Hartree–Fock wavefunction of Byron and Joachain (1966) for the ground state of helium

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = \phi_{1s}(\boldsymbol{r}_1)\phi_{1s}(\boldsymbol{r}_2) \tag{5}$$

where

$$\begin{aligned} &\phi_{1s} = (4\pi)^{-1/2}[C_1 \exp(-\alpha_1 r) + C_2 \exp(-\alpha_2 r)] \\ &C_1 = 2.60505 \qquad C_2 = 2.08144 \\ &\alpha_1 = 1.41 \qquad \alpha_2 = 2.61. \end{aligned} \tag{6}$$

The direct scattering contribution is obtained from expressions (1) or (2). $f(V_{st})$ is easily obtained by solving the radial Schrödinger equation ($l \leqslant 10$) for the static interaction (in Rydberg atomic units)

$$V_{st} = \begin{cases} -2\left(1+\dfrac{1}{r}\right) e^{-2r} \qquad \text{(e–H)} & (7) \\ -\frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} C_i C_j (\alpha_i + \alpha_j) & \\ \quad \times [1 + 2/(\alpha_1 + \alpha_j) r] \exp[-(\alpha_i + \alpha_j) r] \qquad \text{(e–He).} & (8) \end{cases}$$

The Glauber amplitudes have been obtained in the standard way (Franco 1971, Kumar and Srivastava 1975). For the second-order Glauber term f_{G2} we have used the analytic expression given by Yates (1974). The second-order Born term has been evaluated by using an average excitation energy $\bar{\omega}$ (=0.93 Ryd) and closure

$$f_{B2}(V) - f_{B2}(V_{st}) = f_{B2}(V, \bar{\omega}) - f_{B2}(V_{st}, \bar{\omega}). \tag{9}$$

Here $f_{B2}(V, \bar{\omega})$ and $f_{B2}(V_{st}, \bar{\omega})$ are, respectively, what Byron and Joachain call the simplified Born term and the off-shell contribution from the ground state. These have been evaluated by using the analytic expressions given by Byron and Joachain (1977a).

The exchange contribution has been calculated in the Glauber–Bonham–Ochkur approximation (Dewangan 1976, Khayrallah 1976).

$$g_{GBO}^{H} = 4^{1-i\eta}\Gamma(1-i\eta)[-8 + i\eta(4-q^2)]/[k_i^2(4+q^2)^{2-i\eta}] \tag{10}$$

$$g_{GBO}^{He} = \frac{-2^{1-i\eta}\Gamma(1-i\eta)}{k_i^2} \sum_{i=1}^{2} \sum_{j=1}^{2} C_i C_j \left(\frac{-2\beta_{ij}^2 + i\eta(\beta_{ij}^2 - q^2)}{-\beta_{ij}^{1+i\eta}(\beta_{ij}^2 + q^2)^{2-i\eta}} \right). \tag{11}$$

Here $\boldsymbol{k}_i$ is the momentum of the incident electron, $\eta = 1/k_i$, $\beta_{ij} = \alpha_i + \alpha_j$ and $\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f$ is the momentum transfer. It should perhaps be calculated in a better way for lower energies ($\leqslant 50$ eV) by including it with the static interaction (Ishihara and Chen 1975) or using the semiclassical local approximation (Furness and McCarthy 1973, Riley and

Truhlar 1975, Bransden and Noble 1976) or using the eikonal approach (Madan 1975, Foster and Williamson 1976b). The present approach has been adopted for all the energies considered here in the interest of simplicity and has turned out to be quite reasonable.

The differential cross sections for the elastic scattering of electrons by atomic hydrogen and helium are, respectively, given by

$$\frac{d\sigma^{H}}{d\Omega}=\tfrac{1}{4}|F^{H}+g^{H}|^{2}+\tfrac{3}{4}|F^{H}-g^{H}|^{2} \tag{12}$$

and

$$\frac{d\sigma^{He}}{d\Omega}=|F^{He}-g^{He}|^{2}. \tag{13}$$

3. Electron scattering by hydrogen

The scattering of electrons by hydrogen in the energy range 16.5 to 30 eV has been considered in I using F_1 for the direct scattering amplitude and the GBO exchange. Better overall agreement with the experimental data compared with the results of other calculations is found. Here we wish to explore the energy range of reliability of F_1. This, of course, is governed by the inadequacy of the Glauber approximation on the low-energy side and the convergence of the Born series on the high. In figure 1 we

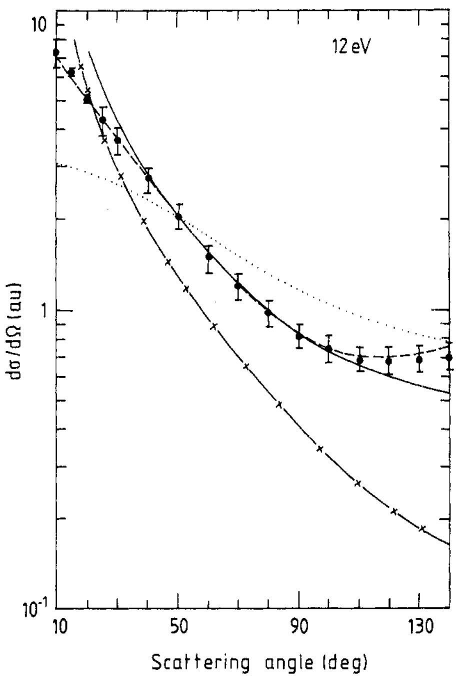

Figure 1. Differential cross sections for elastic e$^-$–H scattering in units of a_0^2 sr^{-1} at 12 eV. Present PI results: ——; static contribution including GBO exchange: · · · ·; Glauber approximation including GBO exchange: –×–; Callaway and Williams (1975): – – –; experimental data of Williams (1975b) ⧳.

present our P1 results (obtained from F_1) at 12 eV. They are compared with the results of a pseudostate calculation of Callaway and Williams (1975) and the experimental data of Williams (1975b). The results in the Glauber approximation and with only the static interaction are also indicated. Our results overestimate the cross section near the forward direction due to the unphysical logarithmic divergence of the Glauber amplitude (right-hand side of equation (1)). The underestimation at large scattering angles is also typical of the Glauber approximation. This is too low an energy for the method P1 to work. The interesting feature which we would like to point out is that the static contribution and Glauber approximation, which separately give widely differing results, when combined together as in equation (1) yield reasonable results except near the forward direction and for large scattering angles. As the energy increases both the undesirable features indicated above (overestimation near the forward direction and underestimation at large scattering angles) get reduced leading to an overall improvement in the results. The good agreement with the experimental data in the energy region 16.5 to 30 eV has been shown in I.

Figure 2 shows our PI results for 30 eV along with the TPE results of Ishihara and Chen (1975) and pseudostate results of Callaway and Williams (1975) and Fon *et al*

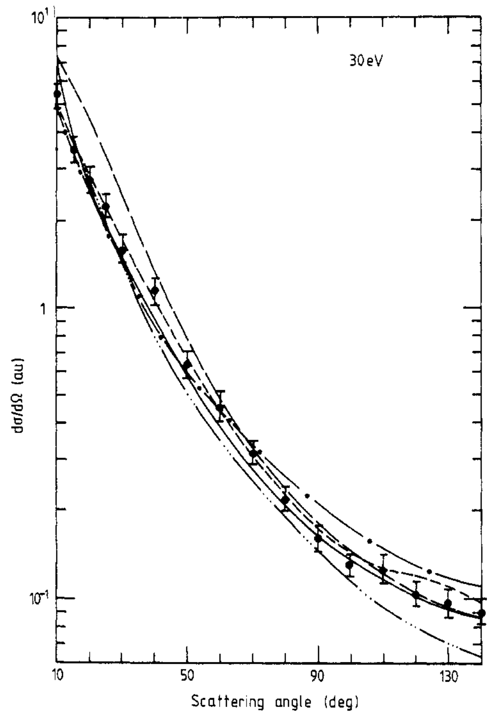

Figure 2. Differential cross sections for elastic e^-–H scattering in units of $a_0^2\ sr^{-1}$ at 30 eV. Present P1 results: ——; present P2 results: — —; Callaway and Williams (1975): - - -; Fon *et al* (1978): – · –; Ishihara and Chen (1975): – · · · –; experimental data of Williams (1975b): ⧫.

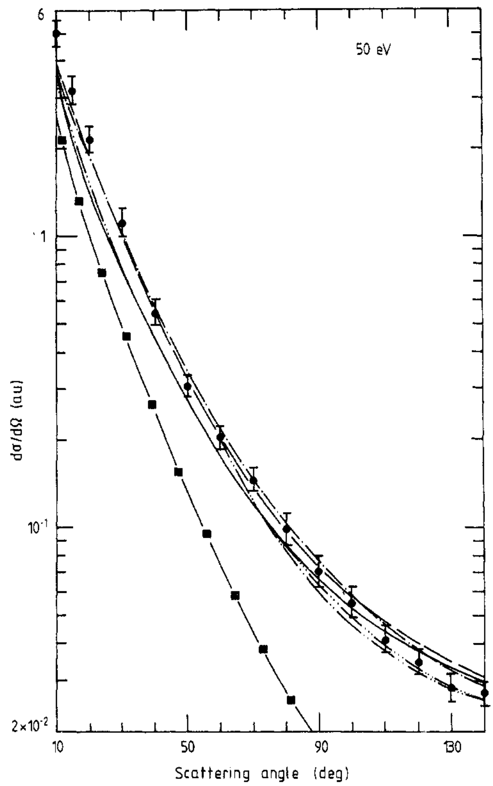

Figure 3. Differential cross sections for elastic e^-–H scattering in units of $a_0^2\ sr^{-1}$ at 50 eV. Present P1 results: ——; present P2 results: — —; MG: – · –; EBS: – · · –; TPE: – · · · –; Foster and Williamson (1976a): -■-; experimetal data of Williams (1975b): ⧫.

(1978). They are compared with the experimental data of Williams (1975b). Our results are in better agreement with the experimental data than those of Ishihara and Chen and Fon *et al* and are of the same degree of overall agreement as those of Callaway and Williams (table 1).

Table 1. Differential cross sections (in a_0^2 sr^{-1}) for the elastic scattering of electrons by atomic hydrogen at 30 eV. The superscripts to the numbers denote the power of 10 by which they should be multiplied.

Angle (deg)	Experimental values (Williams 1975b)	Present P1	Present P2	Callaway and Williams (1975)†	Fon *et al* (1978)†
10	5.32 ±0.57	6.60	7.21	—	4.74
15	3.51 ±0.37	—	—	—	—
20	2.74 ±0.28	2.48	4.34	—	2.44
25	2.25 ±0.21	—	—	—	—
30	1.60 ±0.18	1.43	2.43	1.69	1.36
40	1.15 ±0.12	8.99^{-1}	1.34	—	8.53^{-1}
50	0.641±0.065	5.77^{-1}	7.72^{-1}	—	5.91^{-1}
60	0.461±0.052	3.88^{-1}	4.82^{-1}	0.44	4.37^{-1}
70	0.316±0.028	2.75^{-1}	3.22^{-1}	—	3.36^{-1}
80	0.221±0.021	2.07^{-1}	2.36^{-1}	—	2.65^{-1}
90	0.162±0.017	1.63^{-1}	1.80^{-1}	0.17	2.13^{-1}
100	0.131±0.011	1.35^{-1}	1.45^{-1}	—	1.76^{-1}
110	0.128±0.014	1.16^{-1}	1.22^{-1}	—	1.49^{-1}
120	0.105±0.01	1.02^{-1}	1.05^{-1}	0.12	1.31^{-1}
130	0.098±0.01	9.22^{-2}	9.40^{-2}	—	1.19^{-1}
140	0.091±0.009	8.52^{-2}	8.60^{-2}	—	1.11^{-1}

† At 30.6 eV.

In figure 2 we have also shown the P2 results obtained from F_2 (equation (2)). It is found that at 30 eV this attempt at including a more accurate second-order term and leaving the rest of the multiple-scattering series unaltered, destroys the good agreement obtained earlier (from F_1). This feature confirms our observation that at lower energies all the higher order terms ($n \geqslant 2$) should be evaluated on the same footing so as not to disturb the delicate cancellations between them. The situation changes at 50 eV (figure 3). Here the results have been compared with the full eikonal results of Foster and Williamson (1976a) (without exchange), TPE results of Ishihara and Chen (1975), EBS results of Bryon and Joachain (1977b) and MG results of Gien (1977a). The full eikonal results are quite poor. Our P1 results continue to remain better than the TPE results but are not able to compete with the EBS and MG results which contain a better estimate of the second-order term. Our P2 results, on the other hand, show a better agreement with the experimental data (see table 2). This is expected since in the P2 results, the second-order term is evaluated in an identical way to that in the EBS and MG calculations and the static contribution is exactly included to all orders. Main thing to be noted is that, at 50 eV, the Born series is fairly convergent with the result that a better evaluation of the second-order term improves the overall agreement. The EBS and MG methods are not expected to work for energies lower than about 50 eV.

Table 2. Differential cross sections (in a_0^2 sr^{-1}) for the elastic scattering of electrons by atomic hydrogen at 50 eV.

Angle (deg)	Experimental values (Williams 1975b)	Present P1	Present P2	MG (Gien 1977a)
10	5.04 ±0.51	3.83	3.98	5.60
15	3.18 ±0.37	—	—	3.63
20	2.17 ±0.23	1.31	1.96	2.38
25	—	—	—	1.60
30	1.12 ±0.12	7.97^{-1}	1.05	1.11
40	0.551 ±0.059	4.54^{-1}	5.58^{-1}	5.74^{-1}
50	0.308 ±0.027	2.67^{-1}	3.19^{-1}	3.22^{-1}
60	0.205 ±0.019	1.70^{-1}	1.99^{-1}	1.95^{-1}
70	0.146 ±0.014	1.17^{-1}	1.34^{-1}	1.26^{-1}
80	0.0993±0.0121	8.55^{-2}	9.66^{-2}	8.77^{-2}
90	0.0716±0.0082	6.58^{-2}	7.30^{-2}	6.44^{-2}
100	0.0558±0.0066	5.27^{-2}	5.76^{-2}	4.92^{-2}
110	0.0421±0.0043	4.37^{-2}	4.72^{-2}	3.84^{-2}
120	0.0349±0.0033	3.74^{-2}	3.99^{-2}	3.04^{-2}
130	0.0288±0.0030	3.29^{-2}	3.47^{-2}	2.44^{-2}
140	0.0273±0.0026	2.96^{-2}	3.11^{-2}	2.07^{-2}

Figures 4 and 5 display our P2 results at 100 and 200 eV. They are found to be in better agreement with the experimental data of Williams (1975b) than the full eikonal results of Foster and Williamson (1976a), SOP results of Winters *et al* (1974), TPE results of Ishihara and Chen (1975) and EBS results of Bryon and Joachain (1977b). The P2, MG (not shown) (Gien 1977a) and the pseudostate results of Fon *et al* (1978) show the same degree of overall agreement with the experimental data (tables 3 and 4).

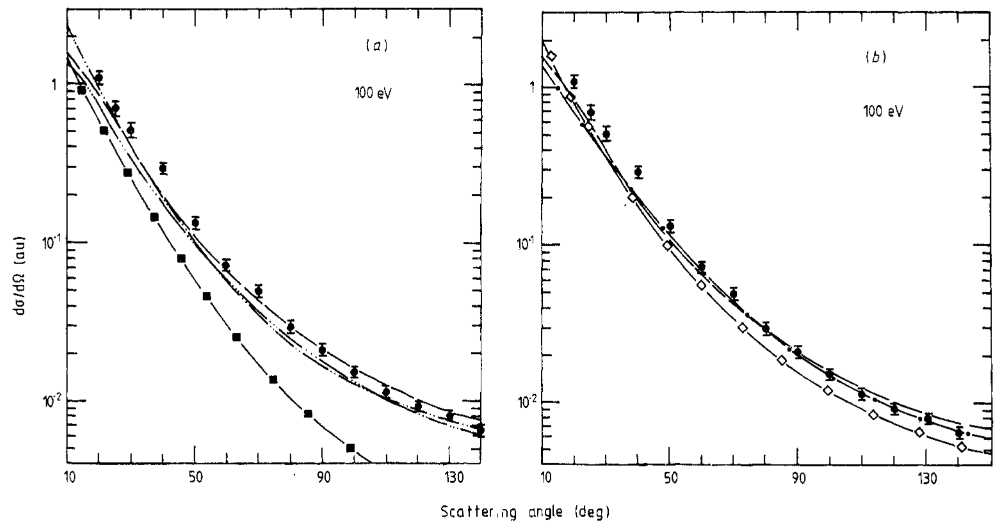

Figure 4. Differential cross sections for elastic e$^-$–H scattering in units of a_0^2 sr^{-1} at 100 eV. Present P2 results: ——; experimental data of Williams (1975b) (●). (*a*) EBS: –···–; TPE: –····–; Foster and Williamson (1976a): –■–; (*b*) SOP: –◇–; Fon *et al* (1978): –·–·–·–.

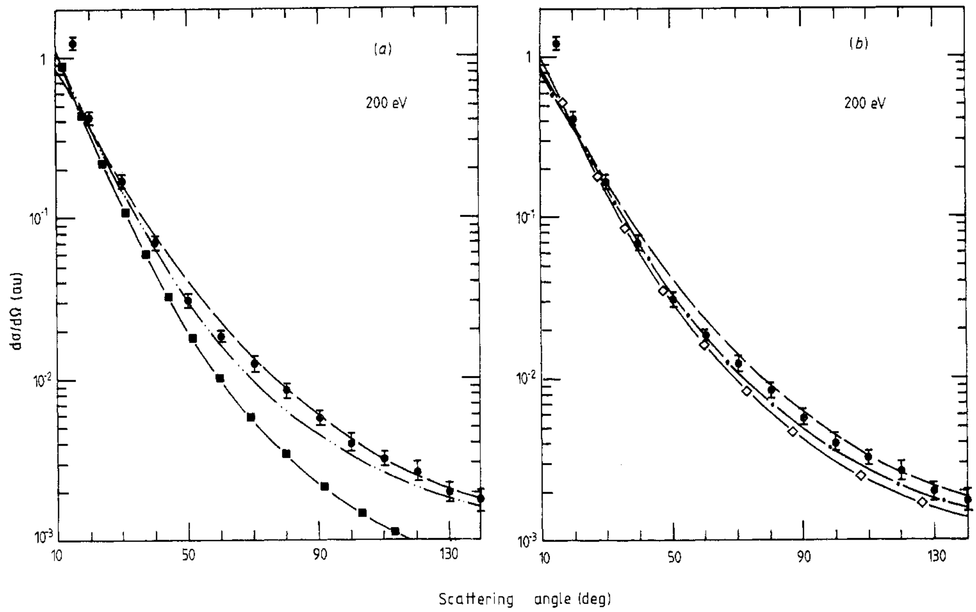

Figure 5. As figure 4 but at 200 eV.

Table 3. Differential cross sections (in a_0^2 sr^{-1}) for the elastic scattering of electrons by atomic hydrogen at 100 eV.

Angle (deg)	Experimental values (Williams 1975b)	Present P2	EO (Byron and Joachain 1974b, c)	Fon *et al* (1978)	SOP (Winters *et al* 1974)	EBS (Byron and Joachain 1977b)	Jhanwar and Khare (1976)†
10	—	1.53	2.41	1.41	2.10	—	—
20	1.10 ± 0.10	8.35^{-1}	9.03^{-1}	6.71^{-1}	8.12^{-1}	9.15^{-1}	8.19^{-1}
30	0.509 ± 0.049	4.04^{-1}	4.04^{-1}	3.71^{-1}	3.66^{-1}	4.07^{-1}	3.93^{-1}
40	0.288 ± 0.027	1.97^{-1}	1.98^{-1}	2.07^{-1}	1.80^{-1}	1.99^{-1}	2.01^{-1}
50	0.132 ± 0.012	1.09^{-1}	1.06^{-1}	1.18^{-1}	9.66^{-1}	1.06^{-1}	1.09^{-1}
60	0.0722 ± 0.0071	6.64^{-2}	6.11^{-2}	7.08^{-2}	5.72^{-2}	6.13^{-2}	6.40^{-2}
70	0.0491 ± 0.0046	4.35^{-2}	3.84^{-2}	4.47^{-2}	3.50^{-2}	3.85^{-2}	3.99^{-2}
80	0.0295 ± 0.0030	3.01^{-2}	2.60^{-2}	2.97^{-2}	2.28^{-2}	2.60^{-2}	2.64^{-2}
90	0.0209 ± 0.0020	2.16^{-2}	1.87^{-2}	2.09^{-2}	1.61^{-2}	1.88^{-2}	1.85^{-2}
100	0.0155 ± 0.0015	1.62^{-2}	1.43^{-2}	1.54^{-2}	1.21^{-2}	1.43^{-2}	—
110	0.0115 ± 0.0012	1.28^{-2}	1.14^{-2}	1.18^{-2}	—	1.14^{-2}	1.05^{-2}
120	0.0092 ± 0.0009	1.03^{-2}	9.46^{-3}	9.46^{-3}	7.53^{-3}	9.48^{-3}	—
130	0.0078 ± 0.0007	8.65^{-3}	8.15^{-3}	7.88^{-3}	—	8.16^{-3}	7.05^{-3}
140	0.0065 ± 0.0007	7.56^{-3}	7.25^{-3}	6.79^{-3}	5.41^{-3}	—	—

† B L Jhanwar and S P Khare (1976) unpublished results.

In table 5 we compare our total elastic cross section with the results obtained by various other approaches and the experimental data of Williams (1975b) and van Wingerden *et al* (1977). The Glauber results remain poor at energies up to 200 eV. The

Table 4. Differential cross sections (in a_0^2 sr^{-1}) for the elastic scattering of electrons by atomic hydrogen at 200 eV.

Angle (deg)	Experimental values (Williams 1975b)		Present P2	EO (Byron and Joachain 1974b, c)	Fon *et al* (1978)	SOP (Winters *et al* 1974)	EBS (Byron and Joachain 1977b)	Jhanwar and Khare (1976)†
10	—		5.46^{-1}	1.11	8.42^{-1}	9.75^{-1}	—	—
20	0.419	±0.040	4.32^{-1}	3.99^{-1}	3.76^{-1}	3.85^{-1}	4.00^{-1}	3.79^{-1}
30	0.172	±0.017	1.57^{-1}	1.54^{-1}	1.59^{-1}	1.50^{-1}	1.54^{-1}	1.56^{-1}
40	0.0706	±0.0068	7.45^{-2}	6.57^{-2}	7.09^{-2}	6.21^{-2}	6.57^{-2}	6.80^{-2}
50	0.0314	±0.0032	4.03^{-2}	3.18^{-2}	3.49^{-2}	2.98^{-2}	3.18^{-2}	3.29^{-2}
60	0.0187	±0.0019	2.19^{-2}	1.74^{-2}	1.89^{-2}	1.64^{-2}	1.74^{-2}	1.77^{-2}
70	0.0125	±0.0014	1.35^{-2}	1.05^{-2}	1.13^{-2}	9.69^{-3}	1.05^{-2}	1.05^{-2}
80	0.00859	±0.00092	9.02^{-3}	6.89^{-3}	7.25^{-3}	6.25^{-3}	6.89^{-3}	6.74^{-3}
90	0.00584	±0.00061	5.83^{-3}	4.85^{-3}	4.97^{-3}	4.20^{-3}	4.85^{-3}	4.63^{-3}
100	0.00412	±0.0042	4.15^{-3}	3.61^{-3}	3.62^{-3}	3.12^{-3}	3.61^{-3}	—
110	0.00323	±0.00031	3.31^{-3}	2.83^{-3}	2.76^{-3}	—	2.82^{-3}	2.59^{-3}
120	0.00272	±0.00035	2.50^{-3}	2.31^{-3}	2.20^{-3}	1.93^{-3}	2.30^{-3}	—
130	0.000199	±0.00025	1.99^{-3}	1.96^{-3}	1.83^{-3}	—	1.95^{-3}	—
140	0.00178	±0.00026	1.84^{-3}	1.72^{-3}	1.58^{-3}	1.39^{-3}	—	—

† Unpublished results.

full eikonal results though relatively better underestimate the data. The method P2 predicts results within the experimental errors just as is the case with the EBS, MG and EO approaches.

4. Electron scattering by helium

In the literature results of electron–helium elastic scattering calculations at intermediate energies have been given in a wide variety of different theoretical models. Apart from the first Born approximation (FBA) (Bell *et al* 1968, Kim and Inokuti 1968), optical model results have been obtained by Byron and Joachain (1974a, 1974b, 1977c), McCarthy *et al* (1977) and Vanderpoorten (1975), a static exchange corrected simplified second Born approximation technique has been used by Buckley and Walters (1974), the EBS approach has been used by Bryon and Joachain (1973, 1974c), the MG approach has been followed by Gien (1977b), the SOP method has been applied by Winters *et al* (1974), a two-potential treatment has been used by Singhal and Srivastava (1977) and Glauber calculations have been carried out by Franco (1970) and Thomas and Chan (1973). Recently this process has been studied by Dewangan and Walters (1977) in a distorted-wave second Born approximation, by Bonham and Konaka (1978) in a second-order approximation including coupling to all channels and by Tayal *et al* (1980) in the two-potential eikonal approximation of Ishihara and Chen (1975). The many-body theory (MBT) applied earlier by Yarlagadda *et al* (1973) at low energies has recently been extended by Scott and Taylor (1979a) to the intermediate-energy range.

We have used F_1 (equations (1) and (13)) to obtain differential cross sections for this process over quite a wide energy range. The results are displayed in figures 6–10. We

Table 5. Total elastic cross sections in πa_0^2 for e$^-$–H scattering in different approximations.

Approximation	Energy (eV) 50	100	200	Reference
Present P2	1.5	0.48	0.24	
CC	0.94†	—	—	Callaway *et al* (1976)
FBA	0.51	0.29	0.15	Franco (1968)
G	0.64	0.29	0.15	Franco (1968)
E	0.84	0.43	0.24	Foster and Williamson (1976b)
EBS	1.39	0.50	0.20	Byron and Joachain (1977b)
MG	1.26	0.53	0.21	Gien (1977a)
SOP	1.03†	0.45	0.19	Winters *et al* (1974)
EQ	0.98	0.42	0.18	Vanderpoorten (1975)
BEP	—	0.44	0.18	Jhanwar and Khare (1975)
Experiment	—	0.58	0.25	van Wingerden *et al* (1977)
Experiment	1.2 ± 0.2	0.56 ± 0.1	0.21 ± 0.04	Williams (1975b)

† At 54.4 eV.

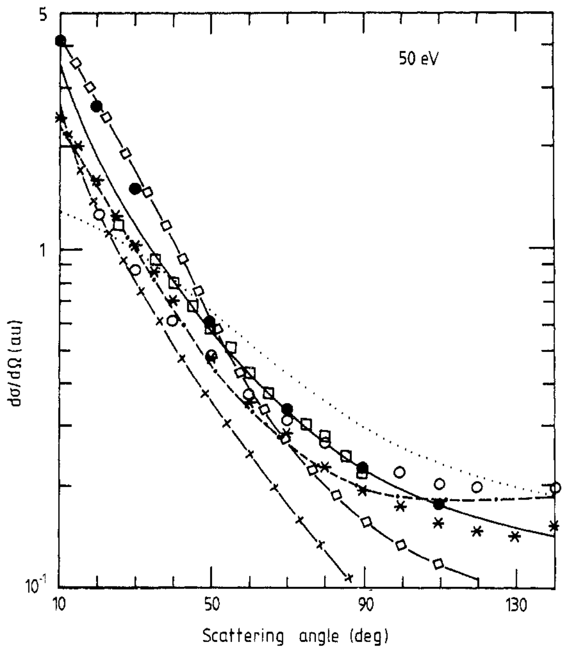

Figure 6. Differential cross sections for elastic e$^-$–He scattering in units of a_0^2 sr^{-1} at 50 eV. Present P1 results: ——; static contribution including GBO exchange: · · · ·; Glauber approximation including GBO exchange: –×–; SOP: –◇–; Scott and Taylor (1979b): – – – · – – –. Experimental data of Crooks and Rudd (1972): ●; Sethuraman *et al* 1974: ○; McConkey and Preston 1975: □; Register *et al* (1980): ∗ with ±6% error.

expect this simple approach to give quite good results. Figure 6 and table 6 at 50 eV show that our P1 results are in better agreement with the experimental data than the SOP results of Winters *et al* (1974). The MBT results of Scott and Taylor (1979b) exhibit a

Table 6. Differential cross sections (in a_0^2 sr^{-1}) for the elastic scattering of electrons by helium at 50 eV.

Angle (deg)	Experimental values: McConkey and Preston (1975)	Experimental values: Register *et al* (1980)	Present P1	Scott and Taylor (1979b)	Winters *et al* (1974)
5	—	—	9.36	2.75	4.99
10	—	2.42	3.37	2.29	4.08
15	—	1.96	2.22	1.88	3.32
20	1.28	1.56	1.82	1.53	2.66
25	1.18	1.26	1.44	1.23	2.12
30	1.02	1.00	1.13	9.91^{-1}	1.67
35	9.18^{-1}	8.34	9.68^{-1}	7.97^{-1}	—
40	7.79^{-1}	6.90	7.91^{-1}	6.46^{-1}	1.04
50	5.86^{-1}	4.70	5.69^{-1}	4.46^{-1}	6.32^{-1}
60	4.18^{-1}	3.52	4.29^{-1}	3.33^{-1}	3.97^{-1}
70	3.14^{-1}	2.76	3.32^{-1}	2.65^{-1}	2.70^{-1}
80	2.75^{-1}	2.23	2.67^{-1}	2.22^{-1}	2.01^{-1}
90	2.21^{-1}	1.93	2.23^{-1}	1.95^{-1}	1.59^{-1}
100	—	1.69	1.93^{-1}	1.83^{-1}	1.32^{-1}
110	—	1.53	1.72^{-1}	1.80^{-1}	1.16^{-1}
120	—	1.44	1.57^{-1}	1.81^{-1}	1.08^{-1}
130	—	1.39	1.46^{-1}	1.83^{-1}	—
140	—	1.52	1.39^{-1}	1.84^{-1}	9.85^{-2}

more favourable agreement with the recent absolute data of Register *et al* (1980). Our results, on the other hand, show better agreement with the data of McConkey and Preston (1975) at the intermediate scattering angles ($20° \leq \theta \leq 90°$) for which the data exist. At 100 eV (figure 7 and table 7) our results are in fair agreement with the data of Register *et al* and are comparable with the MBT results. The agreement of our results with the absolute measurements of Jansen *et al* (1976) in the angular range 10 to 50° (for which data are available) and with the data of Gupta and Rees (1975) for larger scattering angles is very good. The absolute measurements of Sethuraman *et al* (1974) lie slightly below our results. The data of Crooks and Rudd (1972) and Kurepa and Vuskovic (1975) stand higher while those of McConkey and Preston (1975) stand lower than our results at all scattering angles. If the results of Kurepa and Vuskovic (1975) for large angles ($\geq 50°$) are normalised to data of Jansen *et al* at 50°, they fall very close to those of Gupta and Rees and our results are found to be in agreement with them. Among the various other theoretical results the EBS values overestimate the differential cross sections compared with our results below 40° and are in agreement with the measurements of Kurepa and Vuskovic (1975), while at larger scattering angles they, along with the results of Winters *et al* (1974) and Dewangan and Walters (1977), underestimate the results. The MG results of Gien (1977b) overestimate the cross sections in comparison with the present ones in the entire angular region.

At 200 eV (figure 8 and table 8) the different results tend to merge into each other as expected. Our results are found to be in good agreement with the data of Jansen *et al* (1976), Bromberg (1969, 1974) and Sethuraman *et al* (1974) up to $\theta = 120°$. Beyond

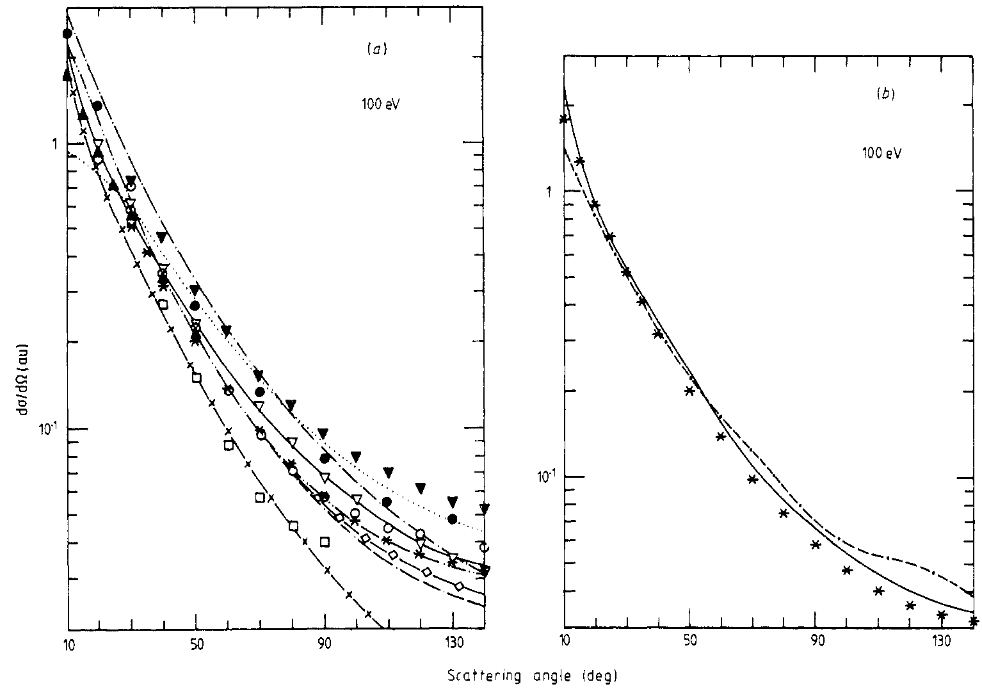

Figure 7. Differential cross sections for elastic e^-–He scattering in units of $a_0^2\ sr^{-1}$ at 100 eV. (*a*) Present P1 results: ——; static contribution including GBO exchange: · · · ·; Glauber approximation including GBO exchange: –×–; MG: – · –; EBS: – · · – , SOP: –◇–; Dewangan and Walters (1977): · – – · – – · – ·. Experimental data of Crooks and Rudd (1972): ●; Sethuraman *et al* (1974): ○; Jansen *et al* (1976): ▲; Kurepa and Vuskopic (1975): ▼; McConkey and Preston (1975): □; Gupta and Rees (1975): ▽; Register *et al* (1980): * with ±7.5% error. The results of Jansen *et al* and Register *et al* are indistinguishable up to 25°. (*b*) Present P1 results: ——; Scott and Taylor (1979): - - - · - - -. Experimental data: Register *et al* (1980) * with ±7.5 error.

$\theta = 120°$ our results underestimate the cross sections. The results of Gien, Winters *et al* and Dewangan and Walters underestimate the cross sections still more. The data of Crooks and Rudd (1972) lie above, while those of Kurepa and Vuskovic (1975) lie lower than our results at all scattering angles. The overall agreement of our results with the experimental data is comparable with that of the EBS and MBT results and is better than that of other theoretical results. At intermediate and large scattering angles (⩾60°) the best agreement with the data of Register *et al* is provided by the EBS results. Figures 9 and 10 show the situations at 400 and 700 eV. Some of the theoretical results have been omitted for clarity. Our results are found to be better than the SOP results of Winters *et al* (1974) but are slightly inferior to the EBS ones of Byron and Joachain (1973, 1974c). In this energy range the present P1 results may be improved and made even better than the EBS results by switching over to equation (2). However this would make the calculations rather difficult due to the complexities involved in evaluating f_{B2} and mar the simplicity of the approach followed here.

In table 9 we compare our total elastic cross sections with other results. The results P1 are found to be almost as good as the SOP, EO1, semiempirical and MBT ones. At 50 eV our results are a bit on the high side but improve as the energy increases.

Table 7. Differential cross sections (in a_0^2 sr^{-1}) for the elastic scattering of electrons by helium at 100 eV.

	Experimental values				EBS	EO	MBT
Angles (deg)	Register *et al* (1980)	Jansen *et al* (1976)	Sethuraman *et al* (1974)	Present P1	Byron and Joachain (1975b)	Byron and Joachain (1977c)	Scott and Taylor (1979b)
5	—	2.32	—	3.93	3.48	3.22	1.88
10	1.81	1.70	—	2.30	2.51	2.43	1.42
15	1.27	1.24	—	1.34	1.78	1.78	1.09
20	9.03^{-1}	9.32^{-1}	—	8.84^{-1}	1.26	1.29	8.36^{-1}
25	6.91^{-1}	7.09^{-1}	—	6.82^{-1}	9.03^{-1}	9.51^{-1}	6.51^{-1}
30	5.20^{-1}	5.49^{-1}	5.61^{-1}	5.72^{-1}	6.55^{-1}	7.06^{-1}	5.21^{-1}
35	4.11^{-1}	4.25^{-1}	—	4.34^{-1}	—	5.30^{-1}	4.08^{-1}
40	3.14^{-1}	3.30^{-1}	3.35^{-1}	3.50^{-1}	3.64^{-1}	4.04^{-1}	3.29^{-1}
50	1.99^{-1}	2.16^{-1}	2.26^{-1}	2.35^{-1}	2.18^{-1}	2.44^{-1}	2.23^{-1}
60	1.37^{-1}	1.56^{-1}	1.36^{-1}	1.58^{-1}	1.40^{-1}	1.57^{-1}	1.62^{-1}
70	9.83^{-2}	1.13^{-1}	9.54^{-2}	1.12^{-1}	9.71^{-2}	1.08^{-1}	1.24^{-1}
80	7.47^{-2}	8.56^{-2}	7.11^{-2}	8.51^{-2}	7.16^{-2}	7.89^{-2}	9.38^{-2}
90	5.79^{-2}	6.90^{-2}	5.82^{-2}	6.70^{-2}	5.62^{-2}	6.11^{-2}	7.12^{-2}
100	4.72^{-2}	5.73^{-2}	5.04^{-2}	5.50^{-2}	4.64^{-2}	4.98^{-2}	5.82^{-2}
110	4.00^{-2}	5.10^{-2}	4.43^{-2}	4.66^{-2}	4.00^{-2}	—	5.32^{-2}
120	3.61^{-2}	4.48^{-2}	4.18^{-2}	4.08^{-2}	3.57^{-2}	3.74^{-2}	5.05^{-2}
130	3.36^{-2}	4.03^{-2}	3.93^{-2}	3.67^{-2}	—	—	4.54^{-2}
140	3.14^{-2}	3.72^{-2}	3.82^{-2}	3.38^{-2}	3.07^{-2}	3.17^{-2}	3.89^{-2}

Table 8. Differential cross sections (in a_0^2 sr^{-1}) for the elastic scattering of electrons by helium at 200 eV.

	Experimental values				EBS	EO	
Angle (deg)	Register *et al* (1980)	Jansen *et al* (1976)	Sethuraman *et al* (1974)	Present P1	Byron and Joachain (1975b)	Byron and Joachain (1977c)	Scott and Taylor (1979b)
5	—	1.68	—	1.60	1.99	1.98	1.33
10	1.10	1.08	—	9.60^{-1}	1.25	1.25	8.69^{-1}
15	7.34^{-1}	7.39^{-1}	—	6.92^{-1}	8.23^{-1}	8.33^{-1}	6.03^{-1}
20	4.87^{-1}	5.28^{-1}	—	5.00^{-1}	5.62^{-1}	5.75^{-1}	4.43^{-1}
25	—	3.85^{-1}	—	3.68^{-1}	3.94^{-1}	4.06^{-1}	3.41^{-1}
30	2.48^{-1}	2.81^{-1}	2.63^{-1}	2.71^{-1}	2.82^{-1}	2.91^{-1}	2.68^{-1}
35	—	2.05^{-1}	—	2.00^{-1}	—	2.11^{-1}	2.11^{-1}
40	1.35^{-1}	1.51^{-1}	1.57^{-1}	1.50^{-1}	1.52^{-1}	1.55^{-1}	1.64^{-1}
50	7.50^{-2}	8.85^{-2}	9.29^{-2}	8.74^{-2}	8.71^{-2}	8.86^{-2}	9.71^{-2}
60	4.65^{-2}	5.37^{-2}	5.36^{-2}	5.41^{-2}	5.40^{-2}	5.43^{-2}	6.07^{-2}
70	2.97^{-2}	3.23^{-2}	3.57^{-2}	3.58^{-2}	3.60^{-2}	3.57^{-2}	4.05^{-2}
80	2.07^{-2}	2.43^{-2}	2.47^{-2}	2.52^{-2}	2.58^{-2}	2.49^{-2}	2.78^{-2}
90	1.54^{-2}	1.89^{-2}	1.75^{-2}	1.86^{-2}	1.96^{-2}	1.84^{-2}	2.04^{-2}
100	1.25^{-2}	1.4^{-2}	1.39^{-2}	1.44^{-2}	1.57^{-2}	1.42^{-2}	1.67^{-2}
110	9.29^{-3}	1.2^{-2}	1.14^{-2}	1.16^{-2}	1.31^{-2}	—	1.39^{-2}
120	8.22^{-3}	1.1^{-2}	9.6^{-3}	9.72^{-3}	1.13^{-2}	9.51^{-3}	1.06^{-2}
130	7.15^{-3}	1.0^{-2}	7.86^{-3}	8.41^{-3}	—	—	8.44^{-3}
140	6.79^{-3}	9.00^{-3}	6.8^{-3}	7.49^{-3}	9.14^{-3}	7.28^{-3}	8.17^{-3}

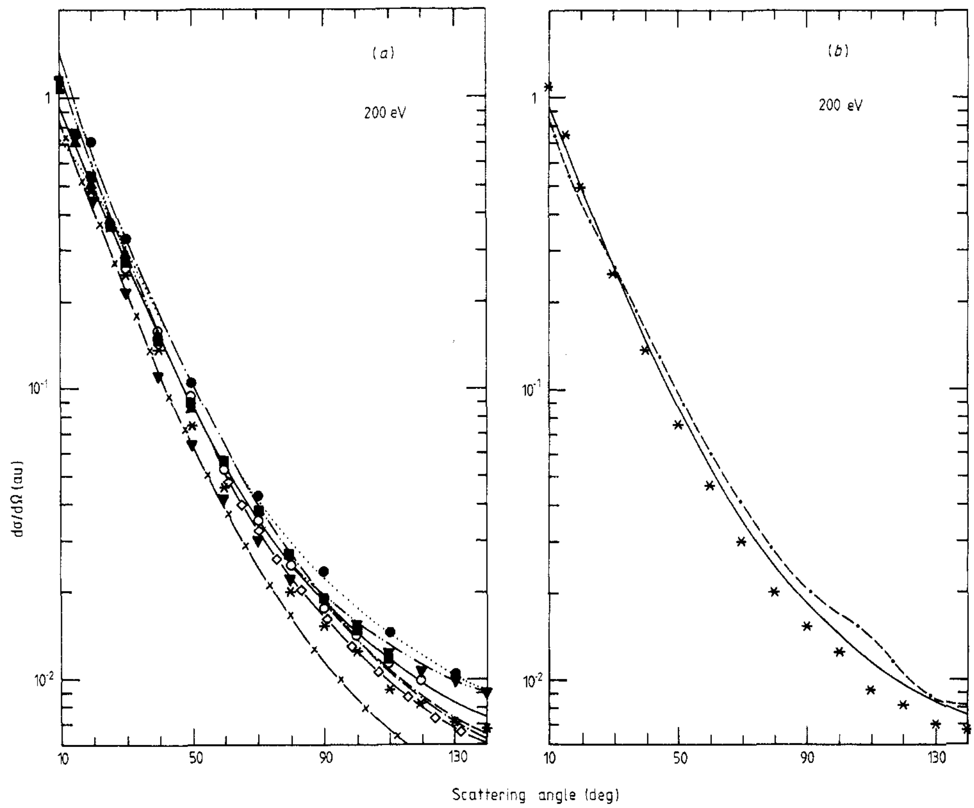

Figure 8. As figure 7 but at 200 eV. Experimental data of Bromberg (1969, 1974): ■. Results of Register *et al* have ±9% error.

5. Summary and comments

The comparison of our P1 results with the experimental data and other theoretical results shows that the approach indicated by equation (1) is capable of yielding good results for electron scattering by helium over quite a wide energy range. A performance of the same order is expected for other low-polarisability targets. Computationally the method is very simple.

In the case where the target polarisability is not low, the comparison of electron–hydrogen results with experimental data shows that the method based on equation (1) can still be used for low intermediate energies. For higher energies this method is not able to compete with the EBS approach which leads to good agreement with the measured data. In this energy region which corresponds to the situation where the Born series is fairly convergent, our method P2, based on equation (2), gives very good results. Computationally it is as difficult as the EBS and MG approaches, but it leads to better results.

In spite of the overall good results, our method P1 suffers from certain basic drawbacks. Since the second term in equation (1) is evaluated in the Glauber approximation, it does not include the long-range polarisation contribution and suffers from a

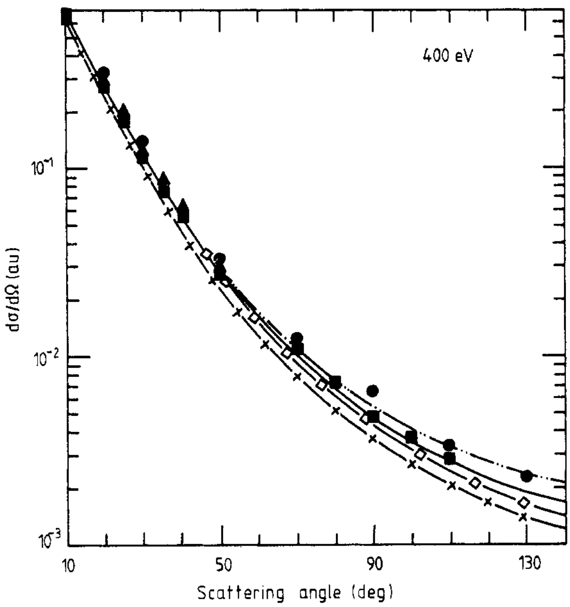

Figure 9. Differential cross sections for elastic e^--He scattering in units of $a_0^2\ sr^{-1}$ at 400 eV. Present P1 results: ——; Glauber approximation including GBO exchange: –×–; EBS: –···–; SOP: –◇–. Experimental data of Crooks and Rudd (1972): ●; Jansen *et al* 1976): ▲; Bromberg (1969, 1974): ■.

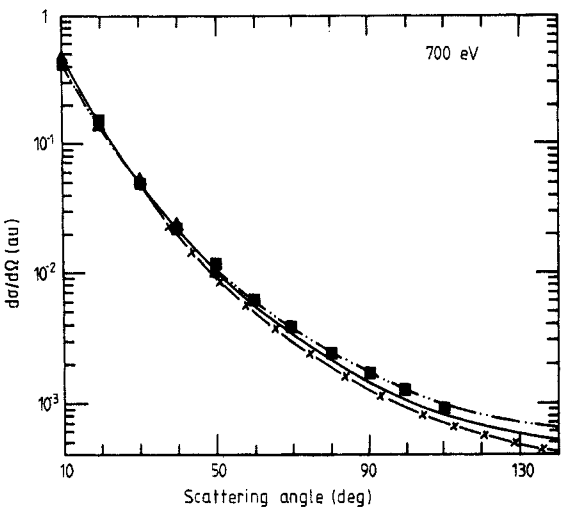

Figure 10. As figure 9 but at 700 eV.

logarithmic divergence in the forward direction. The method P2 is, however, free from these shortcomings.

In all our calculations the exchange contribution has been evaluated in the GBO approximation. A better approach could have been followed but the present one was adopted to keep the calculations simple. It turned out to be quite reasonable.

Table 9. Total elastic cross sections in πa_0^2 for e^-–He scattering in different approximations.

Approximation	Energy (eV) 50	100	200	400	Reference
Present P1	1.76	7.27^{-1}	2.98^{-1}	1.28^{-1}	
FBA	7.13^{-1}	4.12^{-1}	2.22^{-1}	1.15^{-1}	Bell *et al* (1968)
SOP	1.82	7.53^{-1}	3.19^{-1}	1.38^{-1}	Winters *et al* (1974)
EO1	—	6.33^{-1}	2.90^{-1}	1.35^{-1}	Vanderpoorten (1975)
EO2	—	8.02^{-1}	3.31^{-1}	1.41^{-1}	Byron and Joachain (1977c)
EBS	—	7.67^{-1}	3.31^{-1}	1.44^{-1}	Byron and Joachain (1977b)
Semiempirical	1.58 ± 0.10	$6.94^{-1} \pm 0.042$	$3.12^{-1} \pm 0.007$	1.41^{-1} ± 0.0033	de Heer and Jansen (1975, 1977)
MBT	1.44	6.54^{-1}	2.92^{-1}	—	Scott and Taylor (1979b)
Experiment	—	6.78^{-1}	3.11^{-1}	1.44^{-1}	Jansen *et al* (1976)
Experiment	1.43 ± 0.06	$6.37^{-1} \pm 0.034$	$2.84^{-1} \pm 0.023$	—	Register *et al* (1980)

The comparable agreements exhibited by our results and the MBT results of Scott and Taylor with the e^-–He data should perhaps not be taken too seriously. The motivation behind our prescription is the simplicity of approach and the method certainly lacks in the sophistication of the MBT procedure.

Acknowledgment

One of us (ML) would like to thank the Council of Scientific and Industrial Research, Government of India for the award of a post-doctoral research fellowship.

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