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Elastic scattering of electrons from atomic helium in the second-order eikonal approximation

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Abstract. We use Wallace's second-order eikonal approximation (WSOEA) to analyse the elastic scattering of electrons from helium at incident energies of 100, 200 and 500 eV. The predictions of the WSOEA are compared with those of the partial-wave calculation and the results obtained in Glauber's first-order eikonal approximation.

In a previous paper (Roy and Sil 1978, to be referred to as I) we have reported elastic differential cross sections (DCS) for the scattering of electrons and positrons from H and Ar at 100 and 500 eV in the angular range 0–20°. The calculation of I was based upon Wallace's second-order eikonal approximation (WSOEA) (Wallace 1973, Roy and Sil 1978). In I we pointed out that the WSOEA is capable of reproducing the results of partial-wave calculations within a few per cent in the case of static potentials represented by a sum of Yukawa terms. In addition, we remarked that the WSOEA yields cross sections which are a substantial improvement on those of the first Born approximation and Glauber's first-order eikonal approximation (FOEA) (Glauber 1959, Gerjuoy and Thomas 1974).

In this note we extend the WSOEA to the case of elastic scattering of electrons by a potential which in addition to the static interaction also accounts for long-range polarisation and non-local dynamic effects. The purpose of the present work is to examine the validity and usefulness of the WSOEA by direct comparison with the results of the partial-wave calculation (PWC) of Paikeday (1976). Atomic units are used throughout.

For spherically symmetric potentials having continuous derivatives the elastic scattering amplitude for electron–atom collisions in the WSOEA is given by (Roy and Sil 1978)

$$f(\theta)=\frac{K}{\mathrm{i}}\int_0^\infty b\,\mathrm{d}b\,J_0(qb)\{\exp[\mathrm{i}(\chi(b)+\tau(b))]-1\} \tag{1}$$

where

$$\chi(b)=-\frac{1}{K}\int_{-\infty}^{\infty}\mathrm{d}z\,V(b,z) \tag{2}$$

$$\tau(b)=-\frac{1}{K^3}\int_0^\infty \mathrm{d}z\left(1+b\frac{\partial}{\partial b}\right)V^2(b,z) \tag{3}$$

and

$$q = 2K \sin(\tfrac{1}{2}\theta).$$

Here, K is the wavenumber of the incident electron, J_0 is the zeroth-order Bessel function of the first kind, q is the magnitude of momentum transfer and V is the atomic potential. Equation (3) is derived assuming a small momentum transfer approximation. In order to ensure the invariance of $f(\theta)$ under time reversal the direction of z is chosen along the direction of average momentum so that q is exactly perpendicular to the incident wavevector K. The FOEA amplitude is obtained by ignoring the second-order correction τ in equation (1).

In this work the atomic target has been represented by the potential due to Paikeday (1976) which is of the following form:

$$V(r) = \sum_{i=1}^{4} \sum_{\rho=1}^{4} A_{i\rho} \exp(-\lambda_i r) r^{\rho-2} + P \frac{1+\exp(-C_3 r)}{r^4 + d^4}. \tag{4}$$

This model potential has been constructed from the Hartree–Fock wavefunction of

Table 1. Elastic differential cross sections for the scattering of electrons from helium

Energy (eV)	θ (deg)	DCS ($a_0^2\,\mathrm{sr}^{-1}$) FOEA	WSOEA	PWC
500	0	1·68	1·74	—
	2	1·23	1·28	1·41
	4	0·917	0·960	1·08
	6	0·717	0·756	0·849
	10	0·494	0·525	0·555
	14	0·360	0·385	0·377
	16	0·304	0·327	0·313
	20	0·205	0·222	0·218
	30	0·066	0·074	0·091
	40	0·034	0·038	0·040
200	0	2·21	2·33	—
	2	1·84	1·96	—
	4	1·53	1·64	—
	8	1·08	1·17	—
	10	0·924	1·01	—
	14	0·691	0·762	—
	16	0·605	0·670	—
	20	0·467	0·523	—
	25	0·336	0·381	—
	30	0·233	0·269	—
100	0	2·44	2·59	—
	2	2·17	2·31	—
	4	1·91	2·04	—
	8	1·49	1·62	—
	10	1·32	1·44	—
	14	1·05	1·16	—
	16	0·945	1·05	—
	20	0·766	0·854	—
	25	0·594	0·670	—
	30	0·462	0·526	—

Roothaan *et al* (1960) and a three-parameter phenomenological potential to account for long-range polarisation and non-local dynamic effects.

We have evaluated the DCS in the FOEA and the WSOEA for the e^-–He scattering process at 100, 200 and 500 eV using the numerical technique of I. Table 1 shows a comparison of present DCS results at 500 eV in the angular range 0–40° with the corresponding PWC values. In addition, we have listed in the same table our DCS values at electron energies of 100 and 200 eV in the angular range 0–30°. For comparison we also display in figures 1 and 2 the present DCS results along with the corresponding PWC values at incident energies of 200 and 100 eV respectively. We see that the Wallace second-order correction to the Glauber form of the eikonal approximation is about 3·6% in the forward direction at 500 eV, increasing to about 11% at 40° while at lower energies it is somewhat larger, varying from 6·1% in the forward direction to about 14% at 30° at 100 eV. However, there exists a considerable discrepancy between the present results and the 'exact' values.

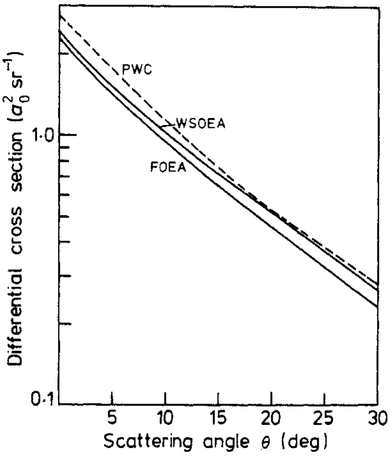

Figure 1. Differential cross section plotted against angle for the elastic scattering of electrons from helium for incident electron energy $E = 200$ eV. FOEA, Glauber's first-order eikonal approximation; WSOEA, Wallace's second-order eikonal approximation; PWC, partial-wave calculation of Paikeday (1976).

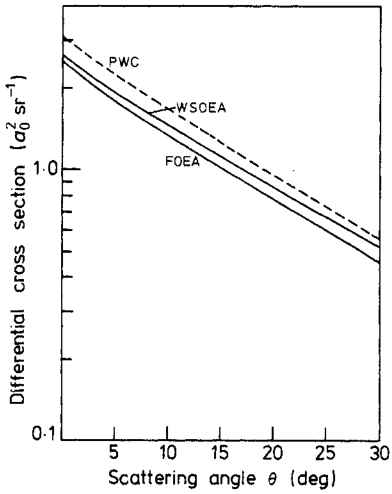

Figure 2. Differential cross section plotted against angle for the elastic scattering of electrons from helium for incident electron energy $E = 100$ eV. FOEA, Glauber's first-order eikonal approximation; WSOEA, Wallace's second-order eikonal approximation; PWC, partial-wave calculation of Paikeday (1976).

We now turn to an analysis of total cross sections. Since both the FOEA and the WSOEA satisfy the optical theorem within their ranges of validity we have evaluated the total cross section σ_T using the relation

$$\sigma_T = \frac{4\pi}{K} \operatorname{Im} f(0) \tag{5}$$

where $f(0)$ is the imaginary part of the forward elastic scattering amplitude. Table 2 shows σ_T calculated both in the FOEA and the WSOEA together with the corresponding PWC results for incident energies ranging from 100 to 1000 eV. We notice that the WSOEA is in better accord with the PWC than the FOEA.

Table 2. Total cross sections for electron–helium scattering.

Energy (eV)	FOEA	WSOEA	PWC
100	1·62	2·00	2·125
200	0·800	0·945	0·970
300	0·516	0·588	0·609
400	0·378	0·420	0·440
500	0·298	0·326	0·343
1000	0·144	0·152	0·164

In conclusion, we may state that while the WSOEA produces some improvement on the FOEA it still differs from the exact calculation by a considerable margin. This difference may in part be attributed to the neglect of Wallace's higher order correction terms.

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