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Elastic electron scattering from helium: absolute experimental cross sections, theory and derived interaction potentials

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Abstract. We report new differential cross section measurements ($E_0 = 1.5$–50 eV) and coupled channel optical model calculations for elastic scattering of electrons from helium. The experimental results, from a crossed electron–atomic beam apparatus, are analysed via a complex phaseshift technique with the derived phaseshifts, and their associated errors, then being used to derive total elastic, total reaction, total momentum transfer and grand total cross sections, and their respective errors, in a mathematically rigorous manner. Further, we have applied fixed-energy inverse scattering techniques to the present data in order to deduce the interaction potential between the colliding particles.

1. Introduction

An increasingly reputable procedure for the determination of absolute elastic differential electron–atom (molecule) scattering cross sections is based on relative scattering intensity measurements and subsequent calibration of these scattering intensities against a supposedly well established standard collision process. The reason for this is transparent when one considers the difficulties involved in a direct measurement of an absolute differential cross section as such an experiment requires an accurate knowledge of the interacting beam fluxes, flux distributions, the scattering geometry and instrument efficiency functions. The popular choice for the standard collision process would appear to be the elastic differential cross section for helium (Nickel *et al* 1989) with the basis for this choice being that helium is amenable to experimental studies and, more importantly, it is widely believed that some of the previous theoretical and experimental investigations into the electron–helium scattering process have adequately described the problem.

There have been many investigations of elastic electron scattering from helium, particularly below the $n = 2$ inelastic threshold. Whilst it is beyond the scope of this paper to discuss these in detail, we draw the attention of the reader to the previous experimental studies of McConkey and Preston (1975), Andrick and Bitsch (1975), Jansen *et al* (1976), Milloy and Crompton (1977), Williams (1979), Register *et al*

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(1980) and Golden *et al* (1984). From a theoretical perspective we note the calculations of LaBahn and Callaway (1970), Duxler *et al* (1971), Sinfailam and Nesbet (1972), Nesbet (1979), Scott and Taylor (1979), O'Malley *et al* (1979) and Fon *et al* (1981). The clear picture which emerges from a comparison of this large body of data and calculations is that for incident electrons with energies below the 2^3S threshold the overall level of agreement between the experiments and between theory and experiment is good. Hence it could well be argued that for this case a set of recommended elastic cross sections can be compiled with a reasonable degree of confidence from the existing data and that this set would largely satisfy the criteria of providing a 'standard' scattering system.

Nonetheless, we have chosen to include our differential cross section data taken at electron energies of 1.5, 5, 10 and 18 eV, in addition to energies greater than the 2^3S inelastic threshold. This work is incorporated into the present manuscript for the following reasons. Firstly, the current relative data are placed on an absolute scale via a phaseshift analysis (Allen *et al* 1987, Allen and McCarthy 1987) which incorporates an error analysis into the fitting procedure so that the derived phaseshifts and total elastic and momentum transfer cross sections are calculated with mathematically rigorous error limits, based on the statistical uncertainties in the measured angular distribution. This particular feature was absent in both the earlier studies of Andrick and Bitsch (1975) and Register *et al* (1980). Secondly, the data at 10 and 18 eV (and at the higher energies) have been analysed using fixed-energy inverse scattering methods to derive the form of the interaction potential between the colliding particles at each of the energies of the present study, which also provides a check on the reasonableness of the measured cross sections.

Previous applications of inverse scattering techniques to electron–atom scattering have been limited. Bürger *et al* (1983) used both fully quantal Lipperheide–Fiedeldey and semiclassical WKB inversion methods to obtain local potentials for e–He scattering at incident energies of 2 and 19 eV. Subsequently, developments to the quantum mechanical inversion by Leeb *et al* (1985) allowed a fastidious analysis of the confidence intervals of the potential, taking into account the errors on the measured differential cross section. Allen (1986) then showed that, within the WKB approximation, much of this error analysis could be done analytically, with considerable savings in computational complexity and time. He next employed this formalism on the 19 eV data of Andrick and Bitsch (1975) to show that their data implies that the polarization potential behaves like $-\alpha/2r^4$ for large r, in agreement with the adiabatic polarization potential. This work was further extended by Allen and McCarthy (1987) to extract a local potential, with error bars, for e–He scattering, again using the data of Andrick and Bitsch at 19 eV. In obtaining that result, they applied the procedure of statistical regularization with equal weighting being given to the variational phaseshifts of Nesbet (1979).

A more complete discussion of the application of inverse-scattering procedures to electron–atom systems and the fundamental importance of this technique can be found in Allen (1991). We note here that it allows an extraction of the interaction potential (with confidence intervals) from the data in a rigorous way without *a priori* assumptions about the shape of the potential. This is a more fundamental approach to determining the r-dependence of the equivalent local potential than parametrized models (see, for example, Thirumalai *et al* 1982).

For electrons with incident energies above the $n = 2$ inelastic threshold it is apparent (see Register *et al* 1980) that there is appreciable uncertainty over the elastic cross sections. The available data of McConkey and Preston (1975), Register *et al* (1980) and others differ at certain combinations of energy and scattering angle by 30% or more. Similarly, there is a serious lack of agreement between the various theoretical calculations of La Bahn and Callaway (1970), Scott and Taylor (1979) and Fon *et al* (1981) and between theory and experiment. Consequently, there is a demonstrable need in this energy regime for the joint experimental and theoretical study that we outline in the present paper and, indeed, one of the principle motivations of the present study was to shed new light on this problem.

Thus we report both new measurements of elastic differential cross sections for electrons of incident energy 20, 30, 40 and 50 eV scattering off helium atoms and results from the application of the coupled channels optical model (CCOM) formalism (see, for example, McCarthy and Stelbovics 1983) to this same scattering system at incident electron energies 18, 20, 30, 40 and 50 eV. The present data, at each energy, is analysed via a regularized complex phaseshift technique to derive values for the complex phaseshifts and the uncertainties on these phaseshifts. These are then employed to calculate values for the total elastic, total reaction, total momentum transfer and grand total cross sections and their respective errors. Inverse scattering techniques are used to derive potentials for these phaseshifts.

In the next section we discuss details of the CCOM calculation, phaseshift analysis of the data and very briefly the inversion technique used in the present study. In section 3 the experimental procedure, including the normalization of the angular distributions, employed in the current measurements is given. Our results, and a discussion of them, are presented in section 4 with conclusions being drawn in section 5.

2. The phaseshift analysis procedure and inversion technique, and the CCOM calculation

2.1. *Phaseshift analysis and inversion*

Details of the phaseshift analysis can be found in Allen (1986), Allen and McCarthy (1987) and Allen *et al* (1987). We recap the essential points here.

The differential cross section is given by

$$\sigma(\theta, \boldsymbol{a}) = \left|(2k)^{-1} \sum_{l=0}^{\infty} (2l+1)[S_l(\boldsymbol{a}) - 1] P_l(\cos\theta)\right|^2 \tag{1}$$

with

$$S_l(\boldsymbol{a}) = \exp[2\mathrm{i}\delta_l(\boldsymbol{a})] = \prod_{n=1}^{N} \frac{\lambda^2 - \beta_n^2}{\lambda^2 - \alpha_n^2} \tag{2}$$

where $\delta_l(\boldsymbol{a})$ is the phaseshift for each partial wave, $\lambda = l + \frac{1}{2}$ and $\boldsymbol{a} = \{a_n\}$ is the set of all the real and imaginary parts of the $2N$ complex parameters α_n and β_n, $n = 1, 2, \ldots, 2N$. This generally valid representation of the phaseshifts (see Leeb *et al* 1985, Allen 1991) allows the (complex) phaseshifts for many partial waves to be varied in terms of a tractable number of variational parameters. In minimizing

$$\chi_1^2 = \frac{1}{M-P} \sum_{i=1}^{M} \frac{[\sigma_i - \kappa\sigma(\theta_i, \boldsymbol{a})]}{(\Delta\sigma_i)^2} \tag{3}$$

χ_1^2 should be close to unity if the parametrization (2) is satisfactory and non-statistical errors are small. Here σ_i is the measured value of the differential cross section at the angle θ_i and $\Delta\sigma_i$ is the statistical error in σ_i. M is the number of data points and $P=2N$ for real phaseshifts (since $\beta_n=\alpha_n^*$ in that case) and $4N$ for the complex case. κ is the renormalization parameter which sets the absolute scale of the data. In this paper we apply regularization by minimizing

$$\chi^2=\chi_1^2+\gamma\sum_{n=1}^{P}(a_n-a_n^{(0)})^2+\eta(Q_{\text{tot}}-Q_{\text{tot}}^{(0)})^2 \tag{4}$$

where the *a priori* information for $\{a_n^{(0)}\}$ is taken from theory, and $Q_{\text{tot}}^{(0)}$ is the total cross section which is taken from independent measurements. Depending on the value of γ and η the phaseshifts are obtained subject to a given weighting of the *a priori* information. Below the first excitation threshold we found that $\eta=0$ and using the *ab initio* variational phaseshifts of Nesbet (1979), which many previous comparisons have shown to be highly accurate (e.g. Register *et al* 1980, Golden *et al* 1984, Buckman and Lohmann 1985), to regularize, proved satisfactory, γ was varied until the Philipps condition (Allen and McCarthy 1987) was appropriately satisfied. Above the excitation threshold we investigated two methods of regularization, one using the CCOM calculation and the other using the total cross section of Kennerly and Bonham (1978). It was found that regularizing using the present CCOM calculation was not as satisfactory as using the measured total cross section as a constraint. This we believe is a result of the small, but nonetheless significant, differences in shape between the present experiment and theory. So above the threshold, η was varied until the Philipps condition was satisfied. From the covariance of the set of variational parameters $\{a_n\}$, it is now possible to find errors on all quantities derived from the phaseshifts in a rigorous way. The details of this procedure and expressions for errors on the total elastic cross section, total momentum transfer cross section, total reaction cross section and grand total cross section can be found in Allen (1986) and Allen and McCarthy (1987).

The interaction potential (with confidence intervals) can be derived from the cross section via the phaseshifts using the inverse scattering problem at fixed energy within the WKB approximation. The details of the procedure are discussed in Allen (1986, 1991). Briefly, the quasi-potential is related to the phaseshift $\delta(\lambda)$ by

$$Q(\sigma)=\frac{4E}{\pi}\frac{1}{\sigma}\frac{\mathrm{d}}{\mathrm{d}\sigma}\int_\sigma^\infty\frac{\delta(\lambda)\,\mathrm{d}\lambda}{(\lambda^2-\sigma^2)^{1/2}} \tag{5}$$

and the potential $V(\rho)$, $\rho=kr$, is related to $Q(\sigma)$ by

$$V(\rho)=E(1-\exp\{Q(\sigma)/E\}) \tag{6a}$$

and

$$\rho=\sigma\exp\{Q(\sigma)/2E\}. \tag{6b}$$

Using the parametrization implied by (2) for $\delta(\lambda)$, $Q(\sigma)$ can be found analytically. In addition analytic expressions for the error on the potential can be derived (Allen 1986).

If the initial phaseshift analysis indicates that there are non-statistical errors present in the data (χ_1^2 significantly greater than 1) then the following smoothing procedure was used. For all θ_i for which the contribution to χ_1^2 was greater than some value, a, the value of θ_i was shifted by an amount b in the appropriate direction (b less than half the estimated uncertainty in θ_i). If the χ^2 contribution was greater than $2a$ then shift by $2b$. This procedure was then iterated using $a/2$ and $b/2$ in subsequent steps until $\chi_1^2\approx 1$. (All θ_i values are only allowed to shift within the uncertainty on θ_i.) These smoothed data were used in the subsequent statistical regularization and calculation of derived quantities and their errors. The smoothing procedure merely assists in the removal of non-statistical errors whose presence we infer as a direct result of the χ^2 per degree of freedom being too large.

We note here that the smoothing procedure described immediately above needed to be employed only sparingly and the quality of the fit to the present data, at each energy studied, was in general excellent. This latter point is reinforced by the values of χ^2 (as defined in equation (4)) quoted in table 1.

2.2. *The coupled-channels-optical calculation*

The coupled-channels-optical method involves the solution of the set of coupled momentum-space integral equations for a set (P space) of channels including the ones of experimental interest.

$$\langle \boldsymbol{k}_i i|T|0\boldsymbol{k}_0\rangle = \langle \boldsymbol{k}_i i|V^{(Q)}|0\boldsymbol{k}_0\rangle + \sum_{j\in P}\int \mathrm{d}^3q \langle \boldsymbol{k}_i i|V^{(Q)}|j\boldsymbol{q}\rangle \frac{1}{E^{(+)}-\varepsilon_j-\frac{1}{2}q^2}\langle \boldsymbol{q}j|T|0\boldsymbol{k}_0\rangle. \tag{7}$$

The Schrödinger equation for the target, whose Hamiltonian is H_{T}, is

$$(\varepsilon_j - H_{\mathrm{T}})|j\rangle = 0. \tag{8}$$

The ground state is $|0\rangle$. The coupling potential is the optical potential $V^{(Q)}$ in which a complex polarization potential $W^{(Q)}$, describing excitations of all channels outside P space (Q space), is added to the first-order potential V which governs the projectile–target interaction.

$$V^{(Q)} = V + W^{(Q)}. \tag{9}$$

In the present calculation P space consists of ten channels: $1,2,3\,^1$S; $2,3\,^3$S; $2,3\,^1$P; $2,3\,^3$P; $3\,^1$D. Q space includes only the ionization continuum. The target states are calculated in a configuration-interaction representation where the basis consists of all allowed excitations of electrons in the 1,2,3,4s; 2,3,4p and 3d Hartree–Fock orbitals with higher excitations allowed for by $\bar{\mathrm{s}}$, $\bar{\mathrm{p}}$ and $\bar{\mathrm{d}}$ pseudo-orbitals.

The polarization potential is based on the approximation

$$\langle \boldsymbol{k}'i|W^{(Q)}|j\boldsymbol{k}\rangle = \int \mathrm{d}^3q' \int \mathrm{d}^3q (a_s + b_s P_r)\langle \boldsymbol{k}'i|V|\psi^{(-)}(\boldsymbol{q}_<)\boldsymbol{q}_>\rangle$$
$$\times \frac{1}{E^{(+)}-\frac{1}{2}(q^2+q'^2)}\langle \boldsymbol{q}_>\psi^{(-)}(\boldsymbol{q}_<)|V|j\boldsymbol{k}\rangle. \tag{10}$$

Here $q_>$, $q_<$ represent the greater or lesser of q', q respectively. $\psi^{(-)}(q)$ is a Coulomb wave orthogonalized to the orbital containing the electron that is excited in the Hartree–Fock approximation to the target state in the same amplitude. The coefficients a_s and b_s, which depend on the spin s of the target state, are given by McCarthy *et al* (1988). The space-exchange operator is P_r.

For computational feasibility it is necessary to calculate the polarization potential in the form of a function $U_{l''l'l}(K)$ of the single variable K, where

$$\boldsymbol{K} = \boldsymbol{k} - \boldsymbol{k}'. \tag{11}$$

This function is determined by the angular-momentum projection

$$\langle \boldsymbol{k}'i|W^{(Q)}|j\boldsymbol{k}\rangle = \sum_{l''m''} i^{l''} C^{m''m'm}_{l''l'l} U_{l''l'l}(K) Y_{l''m''}(\hat{\boldsymbol{K}}). \tag{12}$$

It is also necessary to use the equivalent local approximation for exchange amplitudes in (10) and to make the half-on-shell approximation

$$\tfrac{1}{2}\kappa^2 = E - \varepsilon_j. \tag{13}$$

Polarization potentials are calculated for the channel couplings 1^1S to 2^1S, 2^3S, 2^1P and 2^3P and the corresponding diagonal couplings.

Finally, the differential cross section for elastic scattering is

$$\sigma(\hat{\boldsymbol{k}}') = (2\pi)^4 |\langle \boldsymbol{k}'0|T|0\boldsymbol{k}_0\rangle|^2. \tag{14}$$

3. Experimental apparatus and procedures

The experimental measurements presented here were carried out on a crossed electron-atom beam apparatus which has been described in detail elsewhere (Brunger *et al* 1990a, 1991) and so only a brief description will be given here.

The electron spectrometer is housed in a vacuum vessel with a base pressure of typically 5×10^{-9} Torr. The spectrometer is of a conventional design, employing electrostatic hemispherical energy analysers and electron optics. The energy resolution of the incident beam for the present measurements is about 100 meV, and beam currents varied between 0.5 and 10 nA. The scattered electron analyser can be rotated about the atomic beam allowing access to an angular range of $-20°$ to 130°. The true 0° position was determined from the symmetry of measurements of the scattered electron intensity at both positive and negative scattering angles and is uncertain by $\pm 1°$. The angular resolution is estimated to be 1.5°.

The atomic beam is formed by effusive flow of helium through a multichannel capillary array. Extensive investigations have been made of the scattering volume as a function of the gas driving pressure behind the capillary in order to determine the range of driving pressures over which the measurements can be made free of any angular correction factors (see Brunger *et al* 1990a, 1991). The excellent agreement which is obtained between the shape of the present results and those resulting from the variational calculations of Nesbet (1979), at low energies, provides strong support for the integrity of the present relative angular distributions.

The major experimental development in this work over that reported earlier from this apparatus (Brunger *et al* 1990, 1991) is that in the present case the data accumulation is performed entirely under computer control. Briefly, this involved the control of the routing of the gas to the capillary array for the signal plus background measurement, and then to an alternate capillary at the periphery of the chamber for the background measurement; the regular measurement of the electron beam current and gas pressure; the angular position of the spectrometer and the counting and analysis of the scattered intensity levels. This development was essential for the present measurements as at the higher energies ($E_0 > 20$ eV) we have recorded data at angular intervals as small as 2° in an attempt to provide a tighter constraint on the shape of the angular distribution to aid the phaseshift analysis procedure. Typical measurement times varied between 2 and 5 h with the stability of the apparatus being excellent throughout. This latter point is emphasized in that while each final angular distribution at a given energy represents the weighted average of a number, in some cases as many as 20, individual distributions, the standard deviation of the combined data set, at any given angle, never exceeded 0.15%.

The energy of the incident electrons was calibrated against the well known 2^2S resonance in helium at 19.367 eV and is accurate to ±50 meV.

3.1. Normalization of the angular distributions

The renormalization parameter κ, given in equation (3) of the preceding section, can, in principle, unambiguously determine the absolute scale of the angular distributions irrespective of whether the incident electron beam energy was below or above the onset for the $n=2$ inelastic states. In the present case for $E_0 = 1.5$, 5, 10 and 18 eV the relative data were initially normalized to Nesbets' (1979) variational calculation at some arbitrary point, and then the phaseshift analysis performed. It was found that for all of the above energies κ did not vary from unity by more than 2%. This result thereby confirmed the original normalization and hence the absolute scale of the data. We note that the essence of this approach is not novel and is in fact very similar to that followed by Andrick and Bitsch (1975), Register *et al* (1980) and Golden *et al* (1984) in their earlier work below the threshold of the $n=2$ inelastic states.

For our measurements at $E_0 = 20, 30, 40$ and 50 eV we originally attempted to follow this same procedure in conjunction with the complex phaseshift analysis and the CCOM theory. However, due to α_n and β_n now being complex, thereby effectively doubling the number of parameters in the χ^2-minimization procedure, and because the total absorption cross section is small in this energy regime, the renormalization parameter κ was not well determined at any of the above energies. Consequently, we needed to adopt an alternative approach to normalize the current angular distribution measurements at these energies. One possibility was to follow the procedure of Register *et al* (1980) where the total reaction and grand total cross sections are assumed known thereby allowing the total elastic cross section to be calculated. Having obtained the total elastic cross section, a fit to the angular distribution is made enabling extrapolation of this data to 0° and 180°. The measured and extrapolated data are then integrated and normalized in the usual manner to the previously calculated total elastic cross section to set the absolute scale. Provided the fit to the angular distribution is done properly then this is a perfectly reasonable approach to the problem as, although the total reaction cross section is not well known, it is sufficiently small compared to the total elastic cross section. Nonetheless, we have chosen to normalize the current angular distributions at 20, 30, 40 and 50 eV to our coupled channels optical model calculation, at an angle where the present calculation was found to be in agreement with the measurement of Register *et al* (1980) to better than 2%. The regularized phaseshift analysis was then performed and the physical reasonableness of the present data, and hence the normalization, is checked *a posteriori* by the values determined for the derived phaseshifts, total elastic, total momentum transfer, total reaction and grand total cross sections as well as the interaction potential obtained by inversion. This point is discussed in more detail in the next section.

4. Results and discussion

The experimental results, and their discussion, are presented on a number of levels. Firstly, we present examples of the differential cross section measurements at a number of energies both below and above the first inelastic threshold. The parameters derived from the statistically regularized phaseshift analysis of these cross sections (in some cases smoothed as described in section 2) are then used to calculate the integral elastic and momentum transfer cross sections and the grand total cross section and their errors. Finally at selected energies we present and discuss both the real and imaginary parts of the interaction potential which have been derived from the data using inverse scattering techniques. Although we have chosen to make the comparison with other data and theory at the cross section level (both differential and integral) our phaseshifts are available on request. We also note that we have not attempted to re-analyse previous experimental data in this paper. As has been pointed out in the introduction, the present technique has been used to re-analyse the data of Andrick and Bitsch (1975) and the results obtained are in good agreement with the re-analysis of this data performed by Golden *et al* (1984).

The present experimental differential cross sections and their respective total (statistical and non-statistical) errors (one standard deviation) are given in table 1. This total absolute error varies between 3.5 and 5.0% and is comprised of the addition, in quadrature, of components arising from the normalization procedure (13%), counting statistics (<1%) and the variation in electron beam current and gas pressure during

Table 1. Present experimental differential cross sections ($\times 10^{-17}\,\mathrm{cm}^2\,\mathrm{sr}^{-1}$) for elastic electron scattering from helium at energies between 1.5 and 50 eV.

	Energy (eV)							
θ_i (deg)	1.5	5	10	18	20	30	40	50
10	2.23	—	4.32	6.12	—	7.49	7.77	8.47
15	2.40	2.34	4.07	5.53	5.49	6.54	6.33	6.85
20	2.55	2.33	3.81	5.00	4.90	5.78	5.32	5.63
25	2.70	2.41	3.52	4.53	4.55	4.98	4.61	4.52
30	2.80	2.46	3.34	3.97	3.94	4.27	3.75	3.58
35	3.01	2.54	3.15	3.64	3.64	3.66	3.13	2.88
40	3.16	2.63	3.02	3.25	3.27	3.18	2.59	2.34
45	3.32	2.67	2.96	2.98	3.00	2.69	2.11	1.86
50	3.47	2.75	2.88	2.72	2.76	2.37	1.78	1.55
55	3.63	2.89	2.80	2.49	2.51	2.09	1.54	1.31
60	3.81	3.09	2.73	2.32	2.26	1.80	1.30	1.10
65	3.98	3.26	2.76	2.21	2.10	1.63	1.11	0.912
70	4.15	3.39	2.77	2.11	2.02	1.45	0.985	0.805
75	4.39	3.54	2.86	2.04	1.91	1.34	0.883	0.709
80	4.57	3.68	2.90	1.98	1.85	1.24	0.798	0.641
85	4.80	3.82	2.95	1.96	1.77	1.18	0.740	0.590
90	5.00	4.00	3.04	1.96	1.77	1.13	0.705	0.542
95	5.20	4.23	3.19	2.00	1.77	1.11	0.674	0.519
100	5.37	4.47	3.32	2.04	1.82	1.09	0.657	0.499
105	5.53	4.72	3.55	2.14	1.86	1.11	0.643	0.482
110	5.65	4.86	3.71	2.20	1.89	1.12	0.641	0.471
115	5.79	5.16	3.81	2.29	1.93	1.14	0.637	0.467
120	5.95	5.48	3.91	2.41	1.98	1.16	0.640	0.458
125	6.11	5.63	4.08	2.47	2.06	1.19	0.638	0.447
128	—	—	—	—	—	—	0.640	0.448
Stat. error (%)	<2.0	<1.0	<0.6	<0.3	<0.6	<0.6	<0.8	<0.8
Total error (%)	±3.5	±5.0	±3.5	±3.5	±4.0	±4.5	±4.5	±4.5
χ^2	0.91	1.65	1.35	1.19	1.70	1.43	1.30	1.27

data acquisition (<2%). At 30, 40 and 50 eV some of the data taken at smaller angular intervals has been omitted in the interest of brevity but it may be viewed in the relevant figures referred to later in the text. The cross section at 10 eV is shown in figure 1(*a*) together with the variational calculation of Nesbet (1979). The level of agreement, which is also representative of that found at 1.5 and 5 eV, is clearly very good with the theoretical curve overlapping the total experimental error bars (±3.5%) at all but a few of the forward angle points. At 18 eV (figure 1(*b*)) comparison is also made with the present CCOM calculation and the experimental data of Register *et al* (1980). At forward angles (<60°) the data are in good agreement with both Nesbet and Register *et al* but marginally higher in absolute magnitude than the CCOM calculation. At higher angles there is a small but distinct difference in shape between the two experimental results, although at all times they lie within combined experimental uncertainties. These shape differences are also mirrored by the calculations with the present theory and experiment being in excellent agreement in this region, as are Nesbet's calculation and the data of Register *et al* (1980).

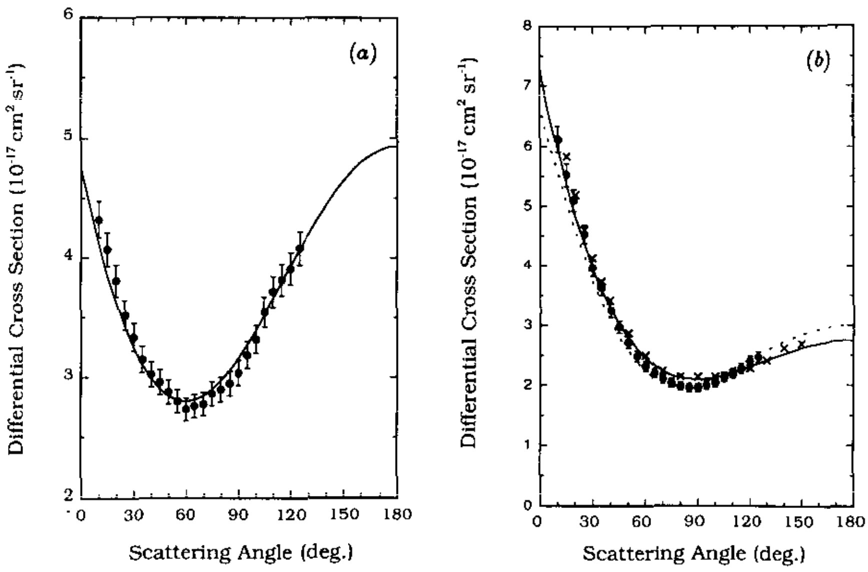

Figure 1. Differential cross sections for elastic electron scattering from helium at (*a*) $E_0 = 10$ eV and (*b*) $E_0 = 18$ eV. The present experimental data (●) and calculation (- - - - -) are compared to the experiment of Register *et al* (1980) (×) and the calculation of Nesbet (——).

Examples of the differential cross sections at energies above the first inelastic threshold are shown in figure 2. We recall that at these energies the present experimental cross sections have been placed on an absolute scale by normalizing to the CCOM calculation as the phaseshift analysis did not provide a unique value for the renormalization parameter. At 30 eV (figure 2(*a*)) the present data and theory are again compared with the measurements of Register *et al* and with the *R*-matrix calculation of Fon *et al* (1981). The present data, which have been taken at 2° intervals, are in good agreement (within combined uncertainties) with Register *et al* over the whole angular range. At forward angles both experiments lie a little higher than the *R*-matrix calculation and a little below the CCOM. There are also small differences at backward angles (>110°)

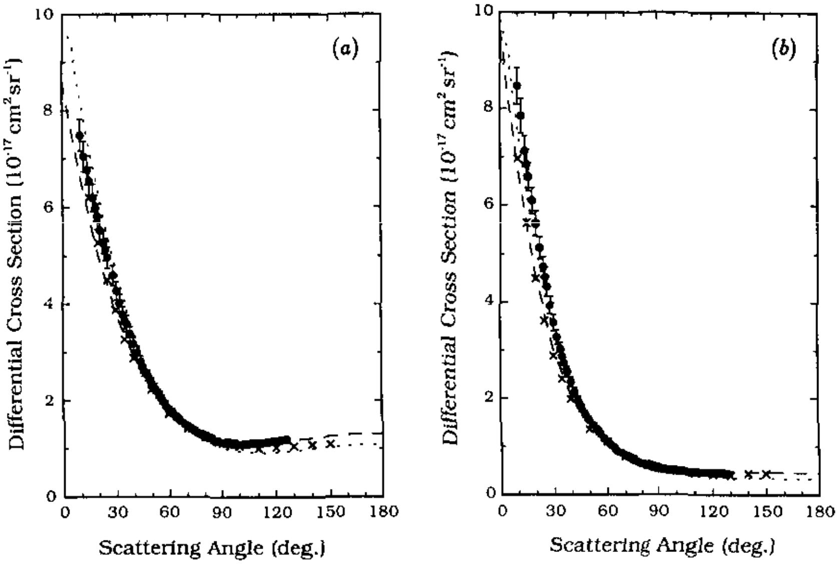

Figure 2. Differential cross sections for elastic electron scattering from helium at (*a*) $E_0 = 30$ eV and (*b*) $E_0 = 50$ eV. The present experimental data (●) and calculation (- - - - -) are compared to the experiment of Register *et al* (1980) (×); and the *R*-matrix calculation of Fon *et al* (1981) (– – –).

but overall the agreement can be classified as good. At 50 eV (figure 2(*b*)) the level of agreement between the two experiments is not as good, particularly at forward scattering angles (<40°). For example at 10° the present result is about 20% higher than that of Register *et al.* At all angles greater than 40° the agreement between the two experiments is excellent. There are also similar discrepancies between the two calculations, with the present results lying closer to the present theory in the forward direction while the *R*-matrix calculation is in good agreement with Register *et al.*

Figures 3–5 and tables 2–5 enable a comparison to be made between the present experiment and theory and various other determinations of integral cross sections. In

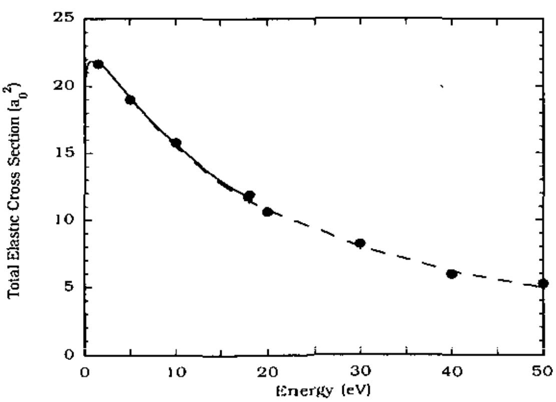

Figure 3. Total elastic cross section. The present data (●) and calculation (+) are compared to the earlier calculations of Nesbet (——) and the *R*-matrix method (– – –) of Fon *et al* (1981).

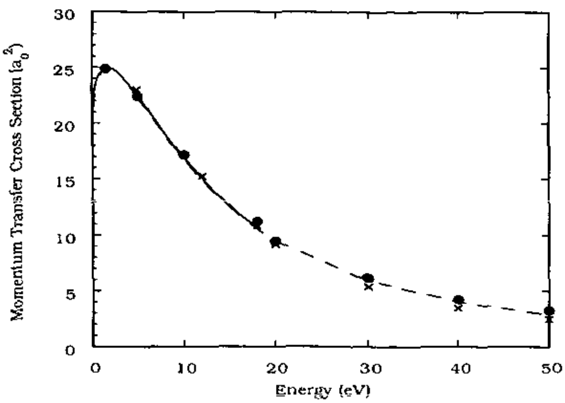

Figure 4. Elastic momentum transfer cross section. The present data (●) are compared to the experiments of Register *et al* (1980) (×); the swarm derived values of Crompton (1970) (- - - - -); the calculations of Nesbet (——) and the *R*-matrix method (– – –) of Fon *et al* (1981).

figure 3 the total elastic cross section derived from the phaseshift analysis of the present experimental data is compared with the present CCOM calculation and the calculations of Nesbet and Fon *et al.* The agreement across the entire energy range is excellent. In figure 4 the present experimental momentum transfer cross section is compared with the experimental values of Register *et al* and those derived from a swarm experiment (Crompton *et al* 1970) as well as the variational and *R*-matrix calculations. Once again the agreement across the entire energy range is excellent.

In figure 5 and table 5 comparison is made of the present grand total cross sections, that is the sum of elastic plus all energetically possible inelastic events, with various other determinations. At energies below 19.8 eV this of course represents the total elastic cross section. In general there is excellent agreement between the present experimentally derived values and those from the time-of-flight attenuation experiments of Kennerly and Bonham (1976) and Buckman and Lohmann (1985). The agreement

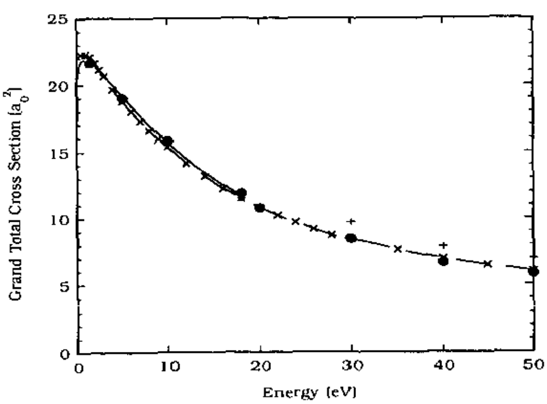

Figure 5. Grand total cross section. The present data (●) and calculation (+) are compared to the experiment of Kennerly and Bonham (1976) (–×–) and the calculation of Nesbet (1979) (——).

Table 2. Present experimental and theoretical (CCOM) total elastic cross sections (Å^2). The present data are compared with the earlier calculation of Nesbet (1979), the R-matrix calculation of Fon *et al* (1981) and the calculation of Callaway and Unnikrishnan (1989). The errors quoted are the statistical errors on the derived total elastic cross sections given the relevant statistical errors on the respective elastic differential cross sections.

	Present				
E_0 (eV)	Experiment	CCOM	Nesbet (1979)	R-matrix	Callaway and Unnikrishnan (1989)
1.5	6.07 ± 0.02	—	6.09	—	—
5.0	5.32 ± 0.04	—	5.38	5.37	—
10.0	4.43 ± 0.15	—	4.39	4.35	—
18.0	3.34 ± 0.06	3.22	3.27	3.21	—
20.0	2.98 ± 0.01	3.45	—	3.03	—
30.0	2.30 ± 0.01	2.24	—	2.23	2.39
40.0	1.67 ± 0.08	1.69	—	1.73	1.89
50.0	1.47 ± 0.01	1.40	—	1.38	1.47

Table 3. Present elastic momentum transfer cross sections (Å^2). The present data are compared with the earlier experimental results of Crompton *et al* (1970) and Register *et al* (1980) and with the theoretical work of Nesbet (1979) and the R-matrix calculation of Fon *et al* (1981). The errors quoted are the statistical errors on the derived total momentum transfer cross sections given the relevant statistical errors on the respective elastic differential cross sections.

E_0 (eV)	Present data	Register *et al* (1980)	Crompton *et al* (1970)	Nesbet (1979)	R-matrix
1.5	6.98 ± 0.04	—	6.96	6.98	—
5.0	6.28 ± 0.07	6.45	6.26	6.32	6.28
10.0	4.81 ± 0.04	—	4.72	4.75	4.71
18.0	3.15 ± 0.03	3.04	—	3.00	2.96
20.0	2.67 ± 0.01	2.58	—	—	2.66
30.0	1.73 ± 0.01	1.51	—	—	1.67
40.0	1.19 ± 0.10	0.98	—	—	1.13
50.0	0.93 ± 0.01	0.70	—	—	0.80

Table 4. Present total reaction cross sections (Å^2).

E_0 (eV)	Present experiment
20	0.039
30	0.064
40	0.20
50	0.16

Table 5. Present experimental and theoretical (CCOM) grand total cross sections (Å^2) compared with the earlier experimental data of Golden and Bandel (1965), Kennerly and Bonham (1978), Register *et al* (1980), Golden *et al* (1984), Buckman and Lohmann (1985) and the calculations of Nesbet (1979) and Fon *et al* (1981). The errors quoted are the statistical errors on the derived grand total cross sections given the relevant statistical errors on the respective elastic differential cross sections.

	Grand total cross section (Å^2)								
	Present								
E_0 (eV)	Data	CCOM	Kennerly and Bonham	Buckman and Lohmann	Register *et al*	Golden and Bandel	Golden *et al*	Nesbet	*R*-matrix
1.5	6.07 ± 0.02	—	6.18	6.02	—	5.59	—	6.09	—
5.0	5.32 ± 0.04	—	5.25	5.26	5.48	4.95	4.89	5.38	5.37
10.0	4.43 ± 0.15	—	4.30	4.30	—	3.89	—	4.39	4.35
18.0	3.34 ± 0.06	3.22	3.22	3.17	3.35	—	—	3.27	3.21
20.0	3.02 ± 0.05	3.46	3.03	—	—	—	—	—	—
30.0	2.37 ± 0.07	2.71	2.36	—	—	—	—	—	—
40.0	1.87 ± 0.28	2.20	1.95	—	—	—	—	—	—
50.0	1.63 ± 0.17	1.94	1.68	—	—	—	—	—	—

with the attenuation measurement of Golden and Bandel (1965) is not as good, with the present results being uniformly higher and either just on or outside the combined error estimates. There is also excellent agreement with the values derived from the phaseshift analysis of Register *et al* at 5.0 and 18 eV whilst the phaseshift-derived result of Golden *et al* at 5.0 eV is lower than the present result and outside the combined uncertainties. There is also excellent agreement with the *R*-matrix calculation (Fon *et al*) at 5.0, 10 and 18 eV. At energies above 30 eV the agreement between the present CCOM calculation and experiment is not as good. Given the good agreement that exists between the present results for both differential and integral elastic scattering below the first inelastic threshold this may be indicative of an overestimation of the effects of excitation and/or ionization in the CCOM. Indeed, it has been shown that the CCOM somewhat overestimates contributions from inelastic channels, particularly triplet states (Brunger *et al* 1990b) and also the strength of the total ionization cross section (McCarthy *et al* 1988). The former problem is believed to be due to inadequacies in the exchange approximation used in the calculation.

The real and imaginary parts of the optical potential for e–He scattering obtained from the 10, 20 and 50 eV data are shown in figure 6. No *a priori* assumptions have been made about the shape of these potentials. A systematic energy dependence is apparent in the real part of the potential, $V(r)$, with $V(r)$ becoming less attractive with increasing energy. This is a manifestation of the energy dependence of the underlying exchange interaction and polarization effects. The imaginary part of the potential is small at 20 eV, as one would expect just above the first excitation threshold, increasing as loss of flux from the incident channel (absorption) increases at higher energies. A potential for e–He scattering at 30 eV obtained by inversion has been given by Allen (1991).

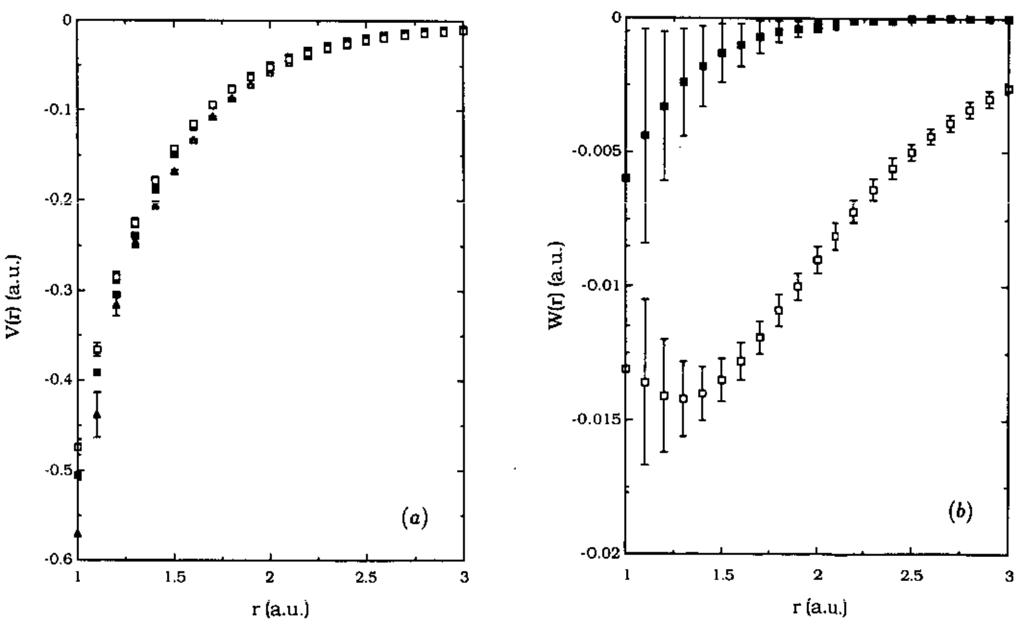

Figure 6. (*a*) The real part of the interaction potential, $V(r)$, derived from the present differential cross section data at $E_0 = 10, 20$ and 50 eV (▲, ■, □). (*b*) The imaginary part of the interaction potential, $W(r)$, derived from the present differential cross section data at $E_0 = 20$, and 50 eV (■, □).

No attempt has been made to compare our potentials with those of Thirumalai *et al* (1982). This is because whilst, by construction, the fits to the present elastic differential cross sections are to a high degree of accuracy, those of Thirumalai *et al* (1982) are in considerably worse agreement with their data which makes a comparison of our potentials with theirs a doubtful exercise.

5. Conclusions

The present experiments and CCOM calculation, together with the experiments of Kennerly and Bonham (1978), Williams (1979), Register *et al* (1980), Golden *et al* (1984) and Buckman and Lohmann (1985) (to mention just a few), and the calculations of Nesbet (1979), Fon *et al* (1981) provides a volume of data for electrons of incident energy $\leqslant 50$ eV from which workers in the field can draw upon to employ as a so-called 'standard collision system'. The importance of this result lies in its application in conjunction with the relative flow technique (Srivastava *et al* 1976, Nickel *et al* 1989) to determine absolute elastic collision cross sections for other, more exotic, atoms and molecules.

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References

Allen L J 1986 *Phys. Rev.* A **34** 2706
—— 1991 *Aust. J. Phys.* **44** 231
Allen L J, Brunger M J, McCarthy I E and Teubner P J O 1987 *J. Phys. B: At. Mol. Phys.* **20** 4861
Allen L J and McCarthy I E 1987 *Phys. Rev.* A **36** 2570
Andrick D and Bitsch A 1975 *J. Phys. B: At. Mol. Phys.* **8** 393
Brunger M J, Buckman S J and Newman D S 1990a *Aust. J. Phys.* **43** 665
Brunger M J, Buckman S J, Newman D S and Alle D T 1991 *J. Phys. B: At. Mol. Opt. Phys.* **24** 1435
Brunger M J, McCarthy I E, Ratnavelu K, Teubner P J O, Weigold A M, Zhou Y and Allen L J 1990b *J. Phys. B: At. Mol. Opt. Phys.* **23** 1325
Buckman S J and Lohmann B 1986 *J. Phys. B: At. Mol. Phys.* **19** 2547
Bürger H, Allen L J, Fiedeldey H, Sofianos S A and Lipperheide R 1983 *Phys. Lett.* **97A** 39
Callaway J and Unnikrishnan K 1989 *Proc. 16th Int. Conf. on the Physics of Electronic and Atomic Collisions* (Amsterdam: North-Holland) Abstracts p 136
Crompton R W, Elford M T and Robertson A G 1970 *Aust. J. Phys.* **23** 667
Duxler W M, Poe R T and LaBahn R W 1971 *Phys. Rev.* A **4** 1935
Fon W C, Berrington K A and Hibbert A 1981 *J. Phys. B: At. Mol. Phys.* **14** 307
Golden D E and Bandel H W 1965 *Phys. Rev.* **138** A14
Golden D E, Furst J E and Magherefteh M 1984 *Phys. Rev.* A **30** 1247
Jansen R H J, DeHeer F J, Luyken H J, van Wingerden B and Blaauw H J 1976 *J. Phys. B: At. Mol. Phys.* **9** 185
Kennerly R E and Bonham R A 1978 *Phys. Rev.* A **17** 1844

LaBahn R W and Callaway J 1970 *Phys. Rev.* A **2** 366
Leeb H, Fiedeldey H and Lipperheide R 1985 *Phys. Rev.* C **32** 1223
McCarthy I E, Ratnavelu K and Weigold A M 1988 *J. Phys. B: At. Mol. Opt. Phys.* **21** 3999
McCarthy I E and Stelbovics A T 1983 *Phys. Rev.* A **28** 2693
McConkey J W and Preston J A 1975 *J. Phys. B: At. Mol. Phys.* **8** 63
Milloy H B and Crompton R W 1977 *Aust. J. Phys.* **30** 51
Nesbet R K 1979 *Phys. Rev.* A **20** 58
Nickel J C, Zetner P W, Shen G and Trajmar S 1989 *J. Phys. E: Sci. Instrum.* **22** 730
O'Malley T F, Burke P G and Berrington K A 1979 *J. Phys. B: At. Mol. Phys.* **12** 953
Register D F, Trajmar S and Srivastava S K 1980 *Phys. Rev.* A **21** 1134
Sinfailam A L and Nesbet R K 1972 *Phys. Rev.* A **6** 2118
Scott T and Taylor H S 1979 *J. Phys. B: At. Mol. Phys.* **12** 3385
Thirumalai D, Truhlar D G, Brandt M A, Eades R A and Dixon D A 1982 *Phys. Rev.* A **25** 2946
Williams J F 1979 *J. Phys. B: At. Mol. Phys.* **12** 265