MULTIPLE IONIZATION OF ATOMS AND POSITIVE IONS BY ELECTRON IMPACT

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Abstract

Semiempirical formulae for multiple-ionization (MI) cross sections σ_n of atoms and ions by electron impact for ejection three or more electrons $n \geq 3$ have been deduced on the basis of available experimental data and the assumption of the Born-Bethe dependence of σ_n on the incident electron energy E. A comparison of the semiempirical formulae suggested with experimental data for atoms and positive ions shows that the formulae can be used for estimation of MI cross sections σ_n, $n \geq 3$, from ionization threshold up to high electron-impact energies $E \approx 10^5$ eV for an arbitrary atomic or ionic target.

A double-peak srtucture of the triple-ionization cross section in neutral Ar has been also discussed.

1. INTRODUCTION

The problem of the multiple ionization (MI) arising in electron-atom and ion-atom collisions is of great interest both for our understanding of many-electron processes, e.g., multi-electron transitions, electron correlation effects [1,2] and for different physical applications [3,4] such as plasma kinetics problems, charge-state evaluation of atoms exposed to an electron beam, a contribution of Auger and shake-off processes and others .

In the case of MI of atoms and positive ions by electron impact the available experimental data on the cross sections σ_n are often not consistent and complete, and sometimes large discrepancies exist among the data, in particular for large numbers of the ejected electrons n (see a compilation [5]). The quantum mechanical calculations of MI cross sections even for $n > 2$ are still unknown, therefore analytical semiempirical formulae constitute a special interest. A semiempirical formalism to predict double- and triple-ionization cross sections in the vicinity of ionization threshold of some specific atomic targets is applied in [6]. Scaling laws of multiple-ionization cross sections and semiempirical formulae for σ_n have been recently discussed in [7].

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Our aim in this work is to investigate the multiple-ionization process in electron-atom and electron-ion collisions

$$e + A^{z+} \rightarrow e + A^{(z+n)+} + ne, \quad n > 1$$

and to obtain semiempirical formulae for MI cross sections to describe their behavior on the average in a wide range of the incident electron energy.

2. BASIC FORMULAE

The measured threshold energy E_{th} for MI cross section σ_n coincides to within 20% with the minimal ionization energy I_n required to remove n outmost target electrons (see, e.g., [7-9]):

$$E_{th} = I_n = \sum_{i=0}^{n-1} I_{i,i+1}, \tag{1}$$

where $I_{i,i+1}$ is the one-electron ionization energy from the charge i to $i+1$. The values for I_n can be estimated from the tables [10,11]. For example, the minimal energy I_6 required to ionize six electrons in Kr atom is: I_6 = I(Kr) + I(Kr^{+}) + I(Kr^{2+}) + I(Kr^{3+}) + I(Kr^{4+}) + I(Kr^{5+}) = 14.0 eV + 27.89 eV + 41.78 eV + 55.67 eV + 70.31 eV + 84.52 eV = 294.17 eV.

Each target atom or ion is characterized by its own set of the minimal ionization energies I_n, so it is natural to choose I_n as a scaling parameter for the incident electron energy E similar to the case of a single ionization:

$$u = E / I_n - 1 \tag{2}$$

Our analysis of the experimental data available on MI cross sections σ_n for atoms and ions by electron impact has shown [12-14] that the majority of the cross sections has a similar electron-impact energy dependence for all targets and all cases with $n \geq 3$ and is described by a universal Born- Bethe type formula :

$$\sigma_n = \frac{C(n,N)}{(I_n / Ry)^2} F(u)[10^{-18} cm^2], \quad n \geq 3, \tag{3}$$

$$F(u) = \left(\frac{u}{u+1}\right)^c \frac{\ln(u+1)}{u+1}, \tag{4}$$

where 1Ry = 13.6 eV; the constant $c = 1$ for neutral targets (z = 0) and $c =$ 0.75 for positive ions ($z \geq 1$). Unfortunately, the energy dependence of double-ionization cross sections (n = 2) can not be properly described by Eqs. (3,4) and will be considered separately.

We note that in [7] the shape of the MI cross sections instead of (4) is described by another semiempirical formula (see also Section 3):

$$\Phi(u)=4\frac{\ln(u+1)}{(u+1)^{1.4}}\left(1-2e^{-0.7(u+1)}\right)$$

(4a)

The analysis of the available experimental data with a fixed number of the ejected electrons n and different numbers of the target electrons N has also shown that the constant C(n, N) in Eq.(3) can be written in the form:

$$C(n,N)=a(n)\,N^{b(n)},$$

(5)

where a and b are the fitting parameters. They were obtained by fitting Eqs.(3-5) to the experimental data on σ_n at low as well as at high incident electron energies. As the references of experimental data for electron-atom collisions we used the results of [15-19] and for electron-ion ones the data in [20-27], respectively. Most of the MI cross sections for electron-ion collisions have been measured using the crossed beam technique.

Finally, the expression for MI cross section in electron-atom and electron-ion collisions can be written in the form:

$$\sigma_n=\frac{a(n)\,N^{b(n)}}{(I_n/Ry)^2}\left(\frac{u}{u+1}\right)^c\frac{\ln(u+1)}{u+1}\,[10^{-18}\,cm^2],\quad u=E/I_n-1,\qquad n\geq 3,$$

(6)

where u is the scaled incident electron energy (2) and the constant c is given in Eq. (4). Fortunately, it is possible to describe MI cross sections of atoms and ions by a single set of the fitting parameters a(n) and b(n). For ejection of $3\leq n\leq 10$ electrons the values for a(n) and b(n) are listed in Table I; for n > 10 one can use the asymptotic values:

$$a(n)\approx 1350/n^{5.7},\qquad b(n)\;=\;2.00,\quad n>10$$

(7)

We note that the parameters a(n) and b(n) given in Table I are the smooth functions of n.

Table I. Fitting parameters a(n) and b(n) in Eq.(6) for MI cross sections σ_n as a function of the number of the ejected electrons n, $3\leq n\leq 10$

n	a(n)	b(n)
3	6.30	1.20
4	0.50	1.73
5	0.14	1.85
6	0.049	1.96
7	0.021	2.00
8	0.0096	2.00
9	0.0049	2.00
10	0.0027	2.00

3. COMPARISON WITH EXPERIMENTAL DATA

A comparison of the MI cross sections described by the semiempirical formula (6) with experimental data available for neutral atomic targets from Ne up to U and ejection up to 13 electrons and with those for positive ions are given in Figs. 1- 4. The figures are presented in the order of increasing of the target nuclear charge. The minimal ionization potentials I_n calculated from [10,11] for some atomic and ionic targets are given in Tables II and III.

Figs. 1-2 show typical MI cross sections of neutral atoms by electron impact. In the case of triple-ionization cross section of Ne atoms (Fig.1a), the discrepancy among different experimental data is estimated to be a factor of 2 although the energy dependence seems to be the same. Similar discrepancy is observed between the present result and experiments. The dashed curve in Fig. 1a represents the result of semiempirical formalism [6]. The present calculations of σ_4 and σ_5 have also shown the large discrepancies from experiment. For instance, in the case n = 5 the present semiempirical formula Eq. (6) gives results which overestimate experimental data [28] more than one order of magnitude. In general, the agreement for the Ne target is the "worse" one as compared to the other targets.

Fig. 1b shows the triple- and quadruple-ionization cross sections of Mg. The agreement of the semiempirical formula (6) with experiment is quite good, except for a region of maximum of σ_n where the discrepancy is estimated to be within a factor of 2.

MI cross sections (n = 3, 4 and 5) of Ar atoms are shown in Fig. 1c. It is noted that we have chosen these experimental data as the reference to obtain the constant C(n,N) in Eq.(6). The experimental triple-ionization cross section shows a double-peak structure which is later discussed in Section 4.

Experimental triple-, quadruple- and quintuple-ionization cross sections of Fe and Cu (Figs. 1d and 1e) have been also chosen as the reference. These cases is an example of non rare gas targets.

Triple-ionization cross sections of Ga are shown in Fig.1f where our results are compared with experimental data and semiempirical formalism by Deutsch et al. [6].

Fig. 2a represents MI cross sections of Kr atoms. The open circles are the experimental data [26] for n = 6, 7 and 8. The crosses represent the experimental data [33] for n = 6 - 9. All experimental data are quite consistent with each other as well as with our prediction. Fig. 2a shows also a comparison of MI cross sections (n=6-8) with semiempirical formulae recently proposed in [7] (see also Eq. (4a)). In the incident electron energy range E < 1000 eV there is a quite good agreemnent with the present results, but, in general, we think that our formula (6) is more easy to be used for estimation because it gives a systematic dependence of the proportional coefficient C(n, N) on the atomic parameters in the closed analytical form (Eq.(5) and Table 1) and therefore can be applied for an arbitrary atomic target.

Triple ionization of In is shown in Fig. 2b; the situation is similar to ionization of Ga atoms (Fig. 1f). Multiple-ionization cross sections of Xe atoms are given in Figs. 2c-2e. The observed data are quite consistent among differenet experiments. The behavior of the cross sections in the region of maximum is clearly related with autoionization processes which can not be described by the semiempirical formula (6). It is quite surprising that our prediction is in agreement with experimental data [33] in such many-electron ionization processes as with ejection n =11 and 13 electrons.

Quadruple-ionization cross sections of the heaviest metallic target U for which experimental data exist are given in Fig. 2f. The semiempirical formula (6) gives quite a good description of σ_4 in this case as well.

Table II. The number of the target electrons N and the minimal ionization energies I_n, eV, for neutral atoms

Atom	N	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	I_{12}	I_{13}
Ne	10	126	223	349	513	734	986	2109	3405			
Mg	12	103	212	353	539	788	1066	1423	1819	3490	5370	
Ar	18	84.3	144	219	310	450	602	996	1465	2006	2620	3309
Fe	26	56.2	114	197	305	438	596	836	1107	1410	1745	2117
Cu	29	77.3	153	256	386	543	728	940	1179	1446	1818	2226
Ga	31	56.7	118	214	344	508	707	942	1208	1510	1847	2218
Ge	32	58.6	105	192	317	479	678	916	1190	1504	1853	2242
Se	34	66.3	112	183	269	416	607	841	1119	1440	1805	2214
Kr	36	83.7	139	209	294	410	542	763	1030	1348	1715	1231
Ag	47	70.1	131	211	310	429	568	728	906	1105	1383	1686
In	49	51.9	106	184	284	408	555	725	919	1137	1378	1643
Sn	50	53.1	94.8	169	267	392	540	713	911	1135	1384	1658
Sb	51	52.8	98.0	155	250	372	520	695	896	1123	1378	1660
Te	52	58.1	98.4	160	233	353	500	675	878	1109	1365	1653
Xe	54	70.8	117	177	249	347	459	630	832	1064	1328	1656
Pb	82	55.3	98.9	166	254	363	492	642	813	1008	1224	1462
Bi	83	51.3	97.4	156	241	350	480	631	804	1000	1219	1462
U	92	35.7	66.6	116	185	275	380	500	633	798	979	1202

Table III. The number of the target electrons N and the minimal ionization energies I_n, eV, for positive ions

Ion	N	I_3	I_4	Ion	N	I_3	I_4
Ar^+	17	128	203	Mo^+	41	87	145
Ar^{2+}	16	176	267	Mo^{2+}	40	130	202
Fe^+	25	106	189	Mo^{3+}	39	173	297
Fe^{2+}	24	173	281	Mo^{4+}	38	253	400
Fe^{3+}	23	248	381	Xe^+	53	105	165
Fe^{4+}	22	324	482	Xe^{2+}	52	141	213
Fe^{5+}	21	400	640	Xe^{3+}	51	178	276
Ni^+	27	117	209	Xe^{6+}	48	381	583
Ni^{2+}	26	192	312	Cs^+	54	106	167
Ni^{3+}	25	276	424	La^{2+}	55	130	208
Ni^{4+}	24	360	536	W^+	73	80	133
Kr^+	35	125	196	W^{2+}	72	118	185
Kr^{2+}	34	168	252	W^{3+}	71	160	280
Rb^+	36	127	200	W^{4+}	70	240	381

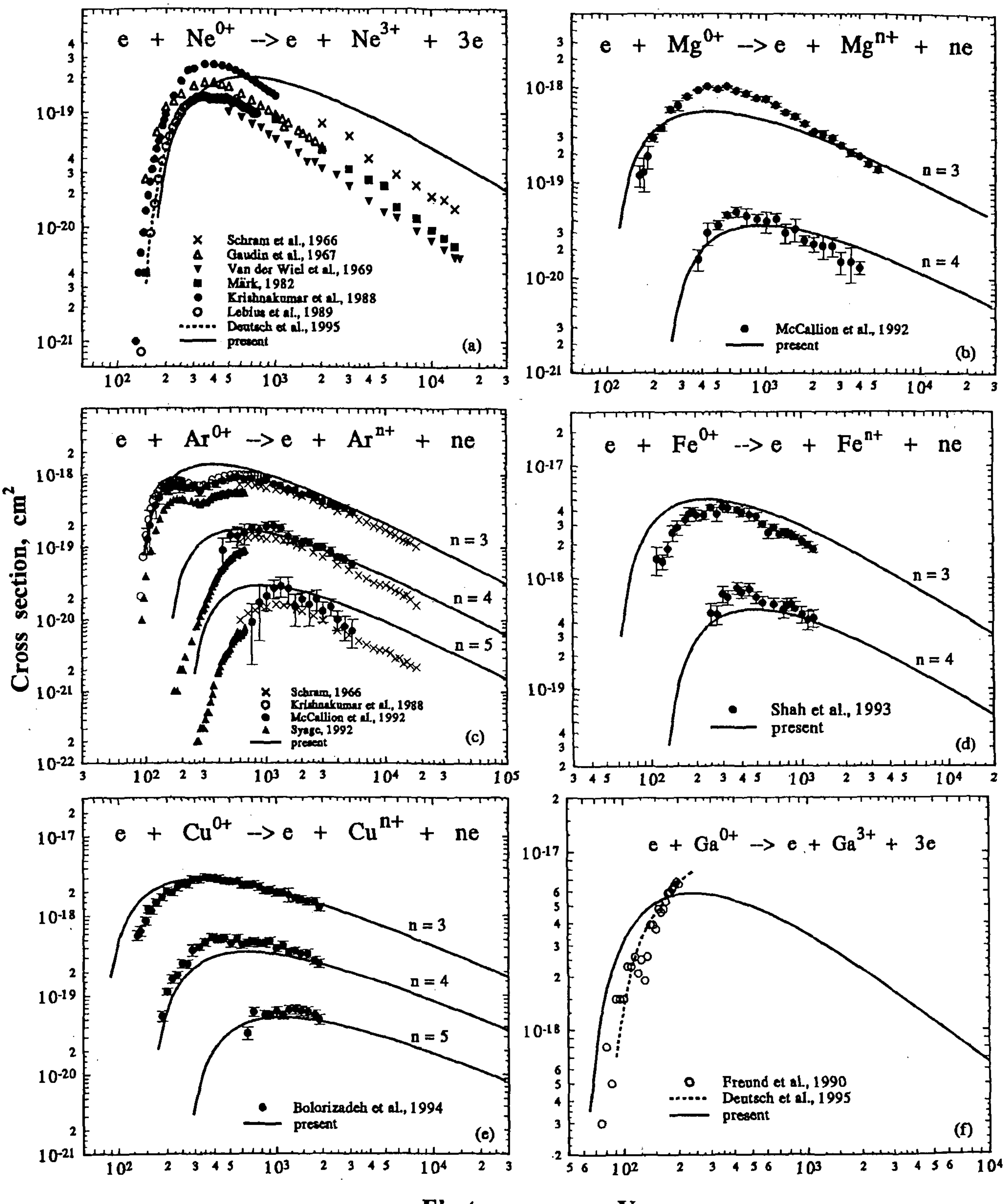

Fig. 1. *MI cross sections of* **Ne, Mg, Ar, Fe, Cu** *and* **Ga** *atoms. Symbols correspond to the experimental data, the dashed curves in Figs. 1a and 1f represent the semiempirical formalism [6] and the solid curves are the present result, Eq. (6).* **(a)** *: crosses - [28], open triangles - [29], solid triangles - [30], boxes - [31], solid circles - [32], open circles - [26];* **(b)** *: solid circles - [15];* **(c)** *: crosses - [33], open circles - [32], solid circles - [16], triangles - [34];* **(d)***: circles - [17];* **(e)** *: circles - [18];* **(f)** *: circles - [19].*

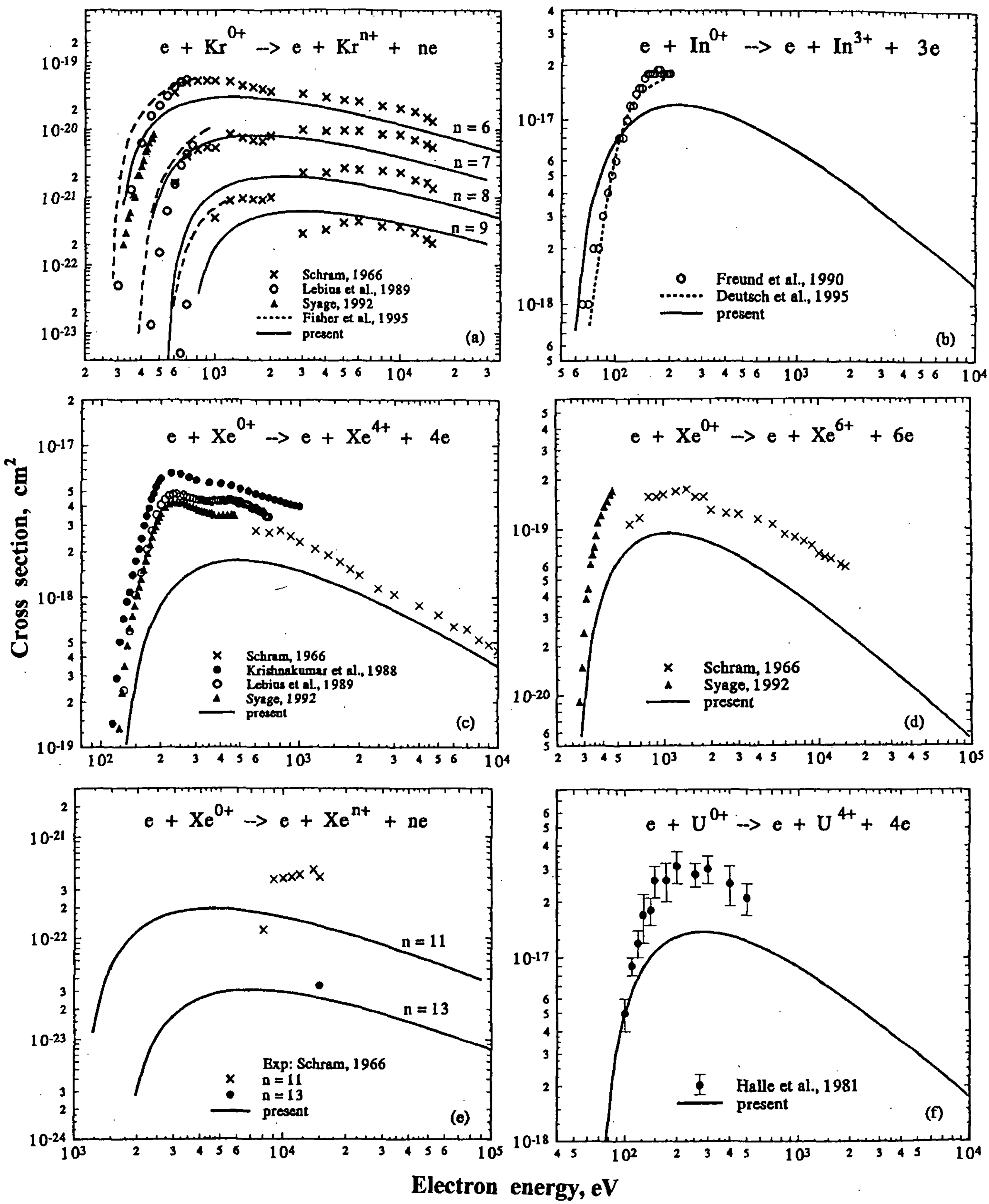

Fig. 2. *MI cross sections of* **Kr, In, Xe** *and* **U** *atoms. Symbols correspond to the experimental data, the solid curves are the present result, Eq. (6).* **(a)** *: crosses - [33], open circles - [26], triangles - [34]; theory: dashed curves - semiempirical formula [7] for n = 6, 7 and 8;* **(b)** *: open circles - [19]; theory - dashed curve - semiempirical formalism [6];* **(c)** *: crosses - [33], solid circles - [32], open circles - [26], triangles - [34];* **(d)** *: crosses - [33], triangles - [34];* **(e)** *: crosses and solid circle - [33];* **(f)** *: circles - [35].*

A similar relation between experiment and the semiempirical formula (6) is observed in the case of multiple-ionization of ions by electron impact (Figs. 3-4). We underline once more that in both cases considered (neutral atoms and positive ions) the same set of the fitting parameters *a*(n) and *b*(n) is used to estimate MI cross sections with a slightly different parameter *c* in Eq. (6).

It is interesting to note that one can estimate the maximum values of σ_n. According to Eq.(6) the cross section σ_n reaches its maximum at $u_n^{max} \approx 3.2$, namely at $E_n^{max} \approx 4.2\, I_n$, i.e.

$$\sigma_n^{max} \approx 0.27\, a(n)\, N^{b(n)} \left(\frac{Ry}{I_n}\right)^2 [10^{-18} cm^2] \tag{8}$$

Eq.(8) gives quite a good estimate for the cross section maximum σ_n^{max} and the corresponding electron energy E_n^{max}. As an example, a comparison of the estimate (8) for the maximum values of triple- and quadruple-ionization cross sections and corresponding incident electron energies with experimental data for neutral targets are given in Table IV; one can see quite a good agreement for elements from Mg to U.

Table IV. Maximum values of triple- and quadruple-ionization cross sections σ_n, cm^2, and the corresponding incident electron energies E_n^{max}, eV, Eq. (8) in comparison with experimental data for neutral targets

ATOM	Exp. σ3max	Exp. E_3^{max}	Eq. (8) σ_3^{max}	Eq.(8) E_3^{max}	Exp. σ_4^{max}	Exp. E_4^{max}	Eq. (8) σ_4^{max}	Eq.(8) E_4^{max}
Mg	1.0×10^{-18}	430	5.85×10^{-19}	433	5.0×10^{-20}	650	4.00×10^{-20}	890
Ar	9.12×10^{-19}	570	1.42×10^{-18}	354	1.93×10^{-19}	1150	1.79×10^{-19}	605
Fe	4.3×10^{-18}	130	4.97×10^{-18}	236	8.0×10^{-19}	450	5.39×10^{-19}	479
Cu	3.04×10^{-18}	350	2.99×10^{-18}	325	5.3×10^{-19}	770	3.61×10^{-19}	643
Ga	6.8×10^{-18}	195	6.03×10^{-18}	238			6.82×10^{-19}	496
Ge	3.5×10^{-18}	185	5.86×10^{-18}	246			7.29×10^{-19}	441
Se	5.1×10^{-18}	195	4.93×10^{-18}	278			8.88×10^{-19}	470
Kr	4.9×10^{-18}	400	3.31×10^{-18}	352	1.5×10^{-18}	500	6.36×10^{-19}	584
Ag	9.4×10^{-18}	180	6.50×10^{-18}	294			1.14×10^{-18}	584
In	9.6×10^{-18}	130	1.25×10^{-17}	218			1.87×10^{-18}	550
Sn	1.7×10^{-17}	165	1.22×10^{-17}	223			2.41×10^{-18}	445
Sb	2.2×10^{-17}	150	1.26×10^{-17}	222			2.34×10^{-18}	398
Te	2.1×10^{-17}	150	1.07×10^{-17}	244			2.40×10^{-18}	412
Xe	2.0×10^{-17}	140	7.53×10^{-18}	297	5.0×10^{-18}	225	1.81×10^{-18}	491
Pb	2.1×10^{-17}	185	2.04×10^{-17}	232			5.22×10^{-18}	413
Bi	2.7×10^{-17}	170	2.40×10^{-17}	215			5.30×10^{-18}	409
U	8.0×10^{-17}	180	5.60×10^{-17}	150			1.41×10^{-17}	280

In general, a comparison of the cross sections described by the semiempirical formula (6) with experimental data available for neutral atomic targets from Ne up to U and ejection up to 13 electrons and with those for positive ions from Ar^+ up to W^{4+} with ejection up to four electrons has shown that in the most cases considered the accuracy of the present formula is within

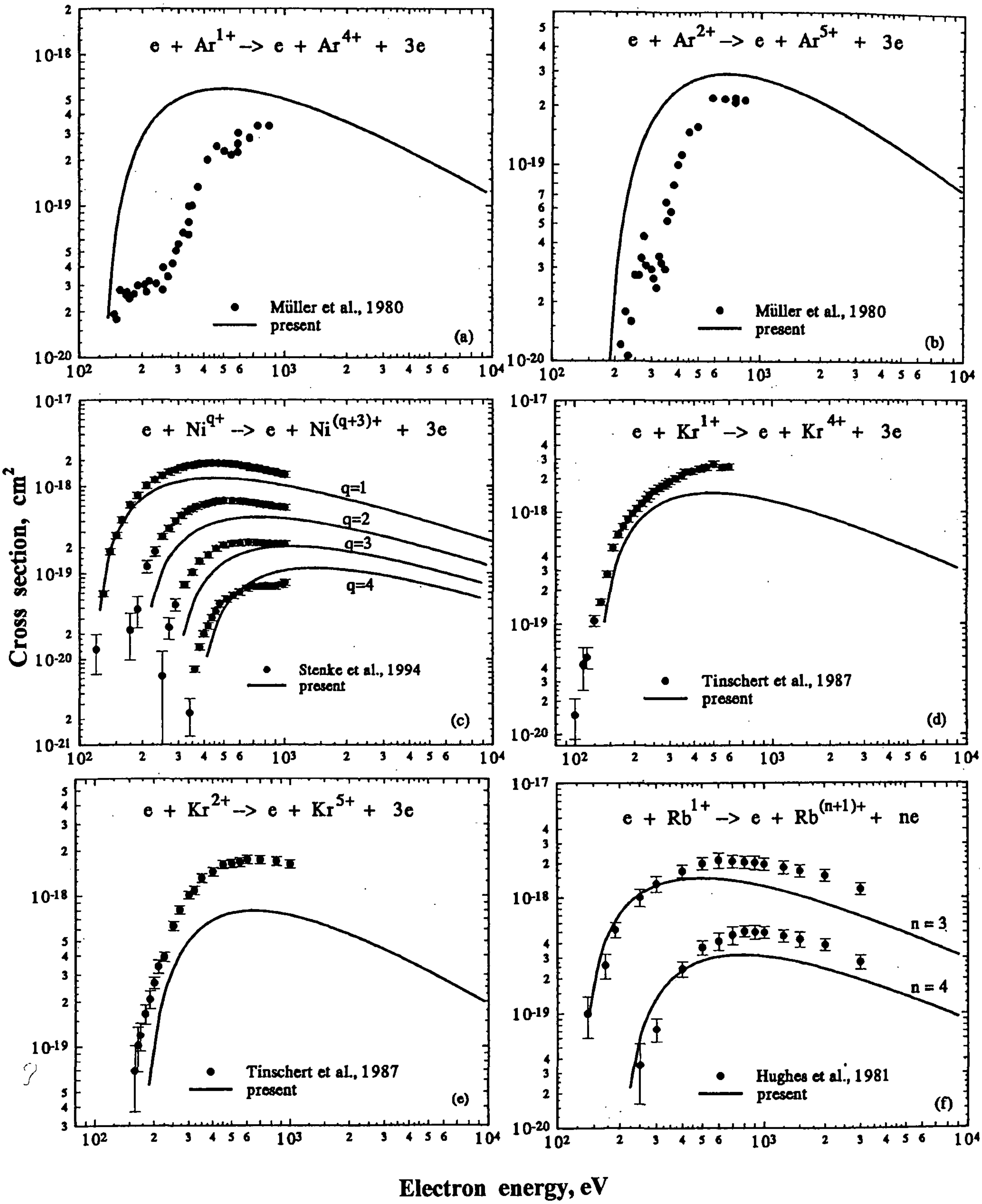

Fig. 3. *MI cross sections of positive ions. Circles correspond to the experimental data, the solid curves are the present result, Eq. (6).* **(a)** : *[20];* **(b)** : *[20];* **(c)** : *[27];* **(d)** : *[9];* **(e)** : *[9];* **(f)** : *[36].*

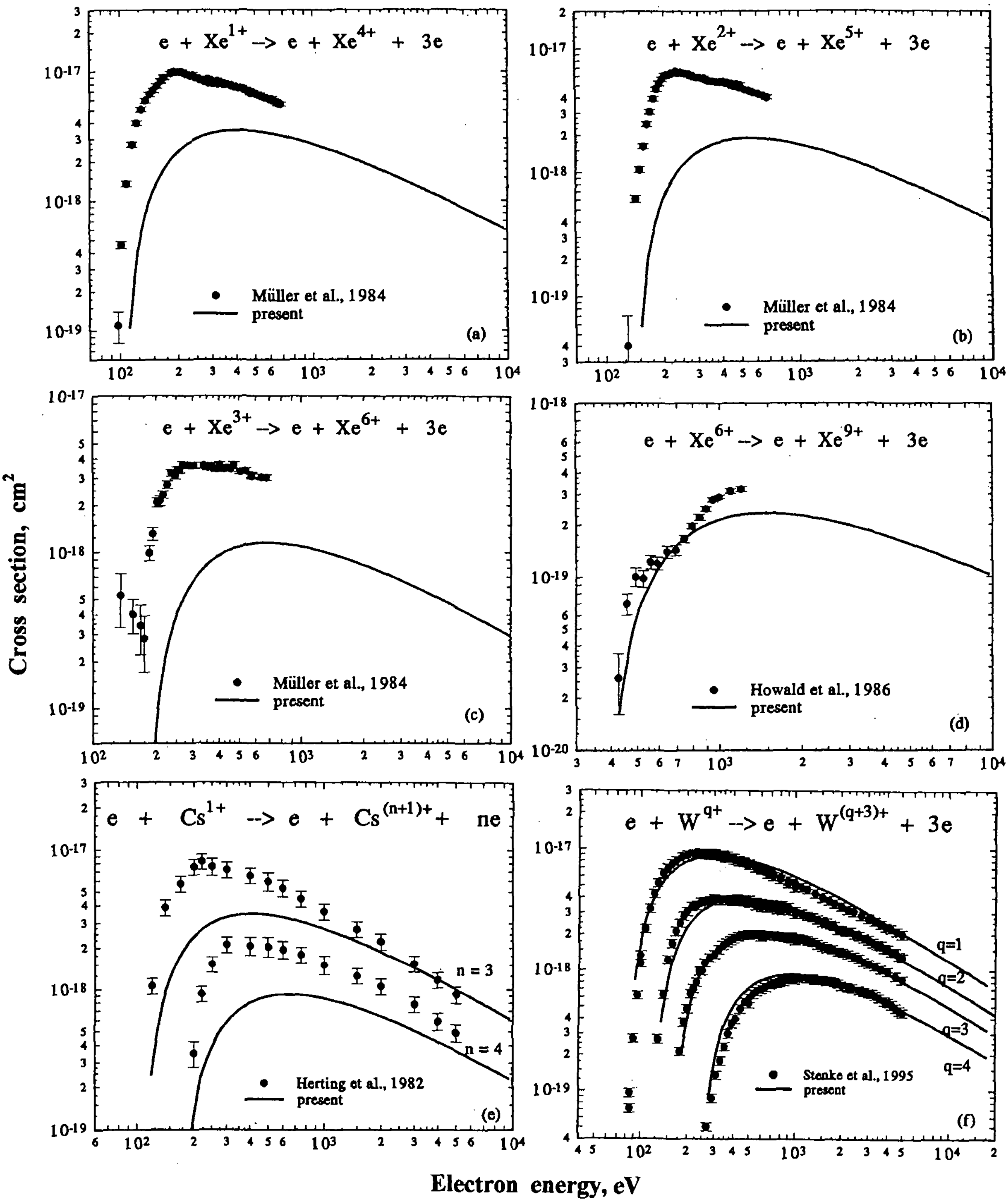

Fig. 4. *MI cross sections of positive ions. Circles correspond to the experimental data, the solid curves are the present results, Eq. (6).* **(a)** : *[22]* ; **(b)**: *[22]*; **(c)**: *[22]*; **(d)** : *[25]*; **(e)** : *[38]*; **(f)** : *[4]*.

a factor of 2 or even better. Two cases are exceptional: quintuple ionization of Ne atoms and triple ionization of La^{2+} ions where the discrepancy between the semiempirical formula (6) and experimental data (Refs. [28] and [37], respectively) is more than one order of magnitude. To make more general conclusion about cross section behavior it is necessary to perform further experimental investigations with better accuracies.

4. DIIRECT MULTIPLE IONIZATION OF Ar ATOMS BY ELECTRON IMPACT

For relatively light atoms some experimental MI cross sections have a multiple peak structure. For example, the first peak of σ_3 in Ar atoms is: $\sigma_3^{max1} \approx 7\text{x}10^{-19}$ cm^2 at E $\approx$ 200 eV, and the second one $\sigma_3^{max2} \approx 9\text{x}10^{-19}$ cm^2 at E $\approx$ 600 eV [16] (Fig. 1c).

The physical explanation of a few maximum structure in MI cross sections is given in [20] where σ_n have been measured for collisions of electrons with Ar^{q+} ions (q = 1, 2 and 3; n = 2, 3 and 4). In the case of triple ionization of Ar^+ ions the authors of [20] assumed that two-maxima structure of σ_3 is connected with two different processes. At relatively low electron-impact energies E < 250 eV, the direct ionization of three outer 3p-electrons dominates and leads to the first maximum of the total cross section. The second (high energy) peak is caused by the single inner 2s- and 2p-ionization (L shell) followed by Auger decay processes. The contribution of the inner-electron ionization to the total double- and triple-ionization cross sections was considered in various papers (see, e.g., [39]).

Let us estimate the direct triple ionization cross section of neutral Ar by electron impact. Following [20] the triple-ionization cross section σ_3 in Ar can be presented in the form:

$$\sigma_3 = \sigma_3^{dir} + a_{2s}\sigma_{2s} + a_{2p}\sigma_{2p}, \tag{9}$$

where σ_3^{dir} is the direct ionization cross section of three outer 3p-electrons. Coefficients a_{2s} and a_{2p} correspond to the rates of Auger and Coster-Kronig transitions contributing to the triple ionization; here we use the same values as in [20]: a_{2s} = 0.84 and a_{2p} = 0.26.

In this work, one-electron inner-shell ionization cross sections σ_{2s} and σ_{2p} for Ar atom have been calculated in the normalized Born approximation by the ATOM code described in [40]; the values of cross sections are turned to be in a good agreement with those calculated in a modified plane-wave Born approximation [39] and available experimental data for 2p-electron ionization [41].

Thus, the direct cross section σ_3^{dir} for Ar was obtained from (9) by subtraction of the sum $a_{2s}\sigma_{2s} + a_{2p}\sigma_{2p}$ from the experimental triple ionization cross section σ_3 [16]. The result is shown in Fig. 5 together with other related cross sections. At electron-impact energies E < 250 eV only the direct triple ionization of three outer 3p-electrons contributes to the processes because the channels for single ionization of the inner 2p- and 2s-electrons are closed (the binding energies I_{2p} = 250 eV, I_{2s} = 320 eV). Therefore, the first maximum in σ_3 at E $\approx$ 200 eV corresponds to the direct ionization of three outer electrons. At higher energies E > 250 eV, ionization of the L-shell electrons is possible producing inner-shell vacancies, i.e., Ar ions in autoionizing states. The corresponding Auger decays of these states yield the second maximum of the total cross section σ_3. From this point of view it is clear why experimental multiple-ionization cross sections of heavy atoms by electron impact [17-19] show no distinguished structure: many closely-lying subshell vacancies of the target are involved in MI processes as was clearly indicated in [18].

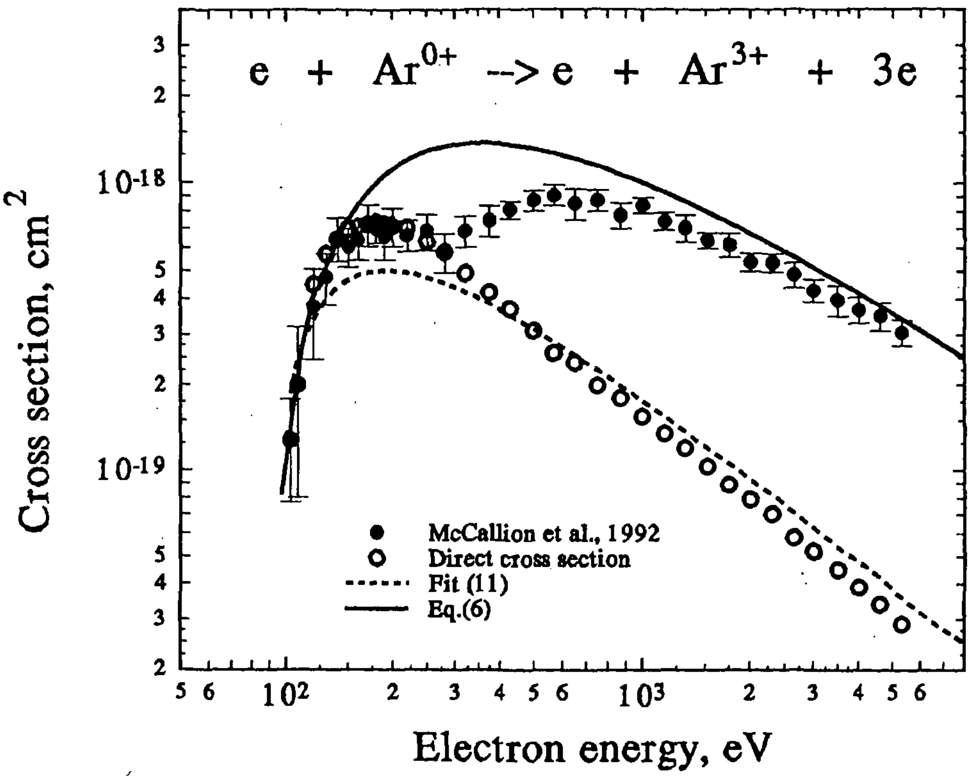

Fig. 5. *Triple-ionization cross sections of* **Ar** *atoms. Experiment: solid circles - [16]. Theory: open circles - direct ionization cross section of threee outer 3p-electrons, dashed curve - fitting (11), solid cirve - total cross section, Eq. (6).*

The analysis of the cross section σ_3^{dir} obtained this way shows that in the energy range $E \approx 10^3 - 10^4$ eV ($u \approx 10 - 10^2$) the cross section σ_3^{dir} falls of approximately as

$$\sigma_3^{dir} \sim u^{-1}, \tag{10}$$

i.e. decsreases more rapidly than the total triple-ionization cross section $\sigma_3 \sim (\ln u)/u$.

It is interesting to note that experimental double-ionization cross section σ_2 in He [42] shows an asymptotic behaviour similar to σ_3^{dir} in Ar^{3+} production. So it was possible to describe σ_3^{dir} for Ar and σ_2 in He by a single analytical formula:

$$\sigma = C\left(\frac{Ry}{I_n}\right)^2 \frac{1}{u+\varphi}\left(\frac{u}{u+1}\right)^2 [10^{-17}\,cm^2], \tag{11}$$

where C and φ are the fitting parameters. For σ_2 in He and σ_3^{dir} in Ar the following values for C and φ were obtained, respectively:

$$\text{He:}\quad C = 2.22,\quad \varphi = 0.87,\quad I_2 = 79 \text{ eV}, \tag{12}$$

$$\text{Ar:}\quad C = 8.91,\quad \varphi = 0.18,\quad I_3 = 84 \text{ eV} \tag{13}$$

Table V. Double-ionization cross sections of He, 10^{-19} cm^2, *vs* incident electron energy E, eV

E, eV	Exp. [42]	Eqs. (11, 12)
90.2	0.063 ±0.012	0.098
100	0.172 ±0.025	0.26
110	0.302 ±0.015	0.41
120	0.439 ±0.031	0.55
150	0.806 ±0.036	0.83
205	1.16 ±0.05	1.01
280	1.35 ±0.06	0.92
375	1.33 ±0.04	0.89
500	1.16 ±0.05	0.75
750	0.836 ±0.025	0.56
1000	0.597 ±0.015	0.45
2010	0.313 ±0.017	0.24
3000	0.187 ±0.006	0.16
4000	0.142 ±0.003	0.13
5300	0.102 ±0.006	0.095
7000	0.073 ±0.002	0.073
9000	0.059 ±0.003	0.068
10000	0.051 ±0.003	0.051

Table VI. Direct triple-ionization cross sections of Ar, 10^{-19} cm^2, *vs* incident electron energy E, eV, below the ionization thresholds of the inner 2p- and 2s-electrons (I_{2p} = 250 eV, I_{2s} = 320 eV)

E, eV	Exp. [16]	Eqs. (11, 13)
90.2	-	0.40
103	1.28 ±0.5	1.87
108	2.00 ±1.20	2.43
120	3.76 ±1.31	3.41
130	4.77 ±0.95	3.96
140	6.48 ±1.1	4.36
150	6.05 ±0.91	4.64
160	6.41 ±0.96	4.82
170	7.23 ±1.16	4.93
180	6.95 ±0.76	4.98
190	6.58 ±1.12	5.01
200	7.11 ±1.07	5.00
220	6.62 ±0.80	4.93
250	6.84 ±0.96	4.75

The maximum error of the fitting by Eqs. (11-13) is estimated to be about 30% (Tables V and VI). It is worth noting here that the experimental double-ionization cross section in He (as well as the direct ionization cross section of three outer electrons in Ar) decreases more rapidly than the Bethe asymptotic (lnu)/u that is probably related with the fact that in both cases the correlation effects between the ionizing target electrons play an important role and close encounter collisions mainly contribute to the simultaneous ionization of two in He (or three in Ar case) electrons. However, this question would require future consideration.

5. CONCLUSION

It has been found semiempirically that the MI cross section σ_n is given as a function of the three atomic parameters: the minimal ionization energy I_n, the total number of the target electrons N and the number of the ejected electrons n. For large $n \gg 1$, one has the following asymptotic behavior of the MI cross sections:

$$\sigma_n \propto \frac{N^2}{n^6\, I_n^2},$$

where F(u) is the universal function Eq. (4) for all cases with $n \geq 3$.

Of course, the present semiempirical formula (6) based on the Born-Bethe approximation is unable to describe properly the energy dependence below the cross section maximum where the indirect processes are known to play a significant role. However, the formula suggested is very simple and can be used for estimation of multiple-ionization cross sections for an arbitrary atomic or ionic target in a wide range of the incident electron energy.

In this work it was also possible to estimate the direct triple-ionization cross section of Ar and to describe it by the analytical formula with two fitting parameters. The double-peak structure of σ_3 was explained to be due to the direct ionization of three outer electrons at low energies (E < 250 eV) and Auger processes caused by single ionization of the L-shell electrons.

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