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# Elastic scattering of electrons from neon and argon

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**Abstract.** We continue our earlier study on the effects of polarisation and exchange in low-energy elastic scattering of electrons from noble gases. We present detailed results for argon and conclude, as we did for helium, that including only the dipole part of the polarisation potential and treating exchange exactly produces the most reliable results. The behaviour for neon is also very similar to that for helium. Values for phaseshifts, differential, total elastic and momentum transfer cross sections are given for neon and argon and compared with recent experimental and theoretical results.

## 1. Introduction

In a previous paper (McEachran and Stauffer 1983, to be referred to as I) we examined the effects of the polarisation potential and the treatment of exchange in elastic scattering of electrons from helium. In this paper we present a similar study for electron scattering from neon and argon.

Like helium there has been a great deal of experimental work done on neon and especially on argon. For neon recent measurements of total cross sections have been made by Gus'kov *et al* (1978), Kauppila *et al* (1981), Salop and Nakano (1970), Stein *et al* (1978) and Wagenaar and de Heer (1980). Robertson (1972) has presented results for the momentum transfer cross section. Differential cross sections have been measured by Brewer *et al* (1981), DuBois and Rudd (1976), Register and Trajmar (1983) and Williams and Crowe (1975). In some cases phaseshifts have been derived from experimental cross sections (Brewer *et al* 1981, O'Malley and Crompton 1980, Register and Trajmar 1983, Williams 1979). Theoretical calculations for this atom have been carried out by Garbaty and LaBahn (1971) and Thompson (1966, 1971) using a simplified polarised-orbital approximation, by McCarthy *et al* (1977) using an optical potential method, by Fon and Berrington (1981) in an $R$-matrix calculation and by Berg (1982) using the $X_\alpha$ method.

There has been a large number of experimental measurements on argon. The more recent ones include total cross section measurements by Charlton *et al* (1980), Golden and Bandel (1966), Gus'kov *et al* (1978), Kauppila *et al* (1976, 1977, 1981), and Wagenaar and de Heer (1980). Momentum transfer results have been given by Frost and Phelps (1964), McPherson *et al* (1976) and Milloy and Crompton (1977). Differential cross section measurements have been made by Andrick and Bitsch (1983), DuBois and Rudd (1976), Lewis *et al* (1974), Srivastava *et al* (1981), Williams and Willis (1975) and Zhou Qing *et al* (1982). Derived phaseshifts have been given by Andrick and Bitsch (1983), Srivastava *et al* (1981) and Williams (1979). Theoretical

 

calculations have been carried out by Fon *et al* (1983) using an $R$-matrix method, by Garbaty and LaBahn (1971) and Thompson (1966, 1971) in a simplified polarised orbital calculation, Amusia *et al* (1982) and McCarthy *et al* (1977) with an optical potential model and Walker (1971) using a relativistic approximation.

## 2. Theoretical method

The method we employed to calculate the scattering of electrons from neon and argon is basically the same as the one previously used for helium, the details of which are given in I. The only difference was in the choice of the integration mesh. Since several of the bound-state orbitals for neon and argon have nodes very close to the origin it was found necessary in the solution integral equations for the scattering functions (cf equation (24) of I) to have a much finer mesh near the origin. The choice of $h = 1/128$ for $0 \leq r \leq 1.0\ a_0$ and $h = 1/32$ for $r > 1.0\ a_0$ was found to be sufficient in order to achieve the same accuracy as we did with helium.

The results presented here were calculated in an adiabatic exchange (AE) approximation, which included polarisation potentials that were previously used in positron scattering from noble gases (McEachran *et al* 1978, 1979). The present calculations are also similar to the electron scattering calculations of Yau *et al* (1978, 1980) (to be referred to as II and III respectively) except that in the latter cases exchange was treated via a local approximation. In the present paper exchange is treated exactly, allowing us to evaluate the accuracy of the local exchange approximation. We also investigate the effects of the various multipole contributions to the polarisation potential. As in I we conclude that the local exchange approximation is not sufficiently accurate to yield good agreement with experimental measurements especially at low energies. We also conclude as for helium, that only the dipole part of the polarisation potential should be included for scattering by electrons.

In § 3 detailed calculations in several approximations are presented for argon. We compare our results for phaseshifts, differential, total elastic and momentum transfer cross sections with experimental measurements of these quantities and with other theoretical calculations. We also present adiabatic exchange results and their comparisons for neon.

## 3. Results

### *3.1. Argon*

We have chosen argon as the atom on which to perform detailed calculations to determine the effects of polarisation and exchange as we did for helium in I. Argon was chosen as representative of the heavier noble gases as calculations on neon produce results similar in character to those for helium (see following section).

Table 1 contains our phaseshifts calculated using the adiabatic exchange approximation with the dipole part of the polarisation potential. These are shown in figure 1 along with the adiabatic exchange results using the full polarisation potential. Also included are JSHE local exchange approximation values using both the full polarisation potential (see II and III) and the dipole polarisation potential. We also display the phaseshifts derived from the experimental differential cross sections of Andrick and

**Table 1.** Elastic scattering phaseshifts $\delta_l$ for argon

| $k$ | $\delta_0$ | $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ | $\delta_6$ |
|---|---|---|---|---|---|---|---|
| 0 | −1.505 749 | | | | | | |
| 0.1000 | 0.043 767 | 0.014 115 | 0.003 261 | 0.001 075 | 0.000 489 | 0.000 263 | 0 000 158 |
| 0.2000 | −0.045 684 | 0.024 923 | 0.013 899 | 0.004 311 | 0.001 954 | 0.001 052 | 0.000 631 |
| 0.3000 | −0.188 574 | 0.002 073 | 0.035 822 | 0.009 776 | 0.004 402 | 0.002 366 | 0.001 419 |
| 0.4000 | −0.352 898 | −0.058 178 | 0.078 074 | 0.017 657 | 0.007 846 | 0.004 210 | 0.002 524 |
| 0.4696 | −0.472 381 | −0.117 094 | 0.127 906 | 0.024 760 | 0.010 847 | 0.005 808 | 0.003 479 |
| 0.5000 | −0.525 073 | −0.145 991 | 0.157 297 | 0.028 331 | 0.012 318 | 0.006 587 | 0.003 945 |
| 0.6000 | −0.698 376 | −0.250 090 | 0.300 700 | 0.042 383 | 0.017 869 | 0.009 505 | 0.005 686 |
| 0.6063 | −0.709 236 | −0.256 980 | 0.312 701 | 0.043 397 | 0.018 257 | 0.009 707 | 0.005 806 |
| 0.7000 | −0.869 093 | −0.361 917 | 0.542 995 | 0.060 502 | 0.024 599 | 0.012 973 | 0.007 747 |
| 0.7426 | −0.940 468 | −0.410 438 | 0.680 512 | 0.069 614 | 0.027 853 | 0.014 624 | 0.008 723 |
| 0.8000 | −1.035 074 | −0.475 862 | 0.889 395 | 0 083 302 | 0.032 630 | 0.017 019 | 0.010 131 |
| 0.8575 | −1.127 867 | −0.540 954 | 1.104 919 | 0.098 695 | 0.037 886 | 0.019 618 | 0.011 653 |
| 0.9000 | −1.195 099 | −0.588 538 | 1.253 346 | 0.111 177 | 0.042 086 | 0.021 672 | 0.012 849 |
| 1.0000 | −1.348 547 | −0.698 077 | 1.531 412 | 0.144 259 | 0.053 059 | 0.026 965 | 0.015 909 |
| 1.0501 | −1.422 870 | −0.751 467 | 1.632 131 | 0.162 755 | 0.059 151 | 0.029 864 | 0.017 574 |
| 1.1000 | −1.495 193 | −0.803 571 | 1.711 710 | 0.182 388 | 0.065 624 | 0.032 920 | 0.019 319 |
| 1.2000 | −1.635 072 | −0.904 657 | 1.825 957 | 0.225 031 | 0.079 822 | 0.039 574 | 0.023 082 |
| 1.2131 | −1.652 904 | −0.917 567 | 1.837 586 | 0.230 910 | 0.081 801 | 0.040 500 | 0.023 602 |
| 1.3000 | −1.768 381 | −1.001 262 | 1.900 968 | 0.271 380 | 0.095 590 | 0.046 968 | 0.027 221 |
| 1.4000 | −1.895 413 | −1.093 501 | 1.952 239 | 0.320 623 | 0.112 784 | 0.055 076 | 0.031 772 |
| 1.4847 | −1.998 352 | −1.168 387 | 1.983 661 | 0.363 993 | 0.128 405 | 0.062 441 | 0.035 928 |
| 1.5000 | −2.016 520 | −1.181 611 | 1.988 478 | 0.371 939 | 0.131 330 | 0.063 819 | 0.036 703 |
| 1.6000 | −2.132 035 | −1.265 857 | 2.015 207 | 0.424 266 | 0.151 213 | 0.073 221 | 0.041 932 |
| 1.7000 | −2.242 354 | −1.346 470 | 2.035 787 | 0.476 511 | 0.172 218 | 0.083 393 | 0.047 526 |
| 1.7149 | −2.258 368 | −1.358 183 | 2.038 438 | 0.484 244 | 0.175 420 | 0.084 972 | 0.048 404 |
| 1.8000 | −2.347 824 | −1.423 665 | 2.051 772 | 0.528 053 | 0.183 986 | 0.094 214 | 0.053 668 |
| 1.9000 | −2.448 783 | −1.497 683 | 2.064 046 | 0.578 486 | 0.216 471 | 0.105 361 | 0.060 208 |
| 1.9197 | −2.468 168 | −1.511 911 | 2.066 117 | 0.588 228 | 0.221 007 | 0.107 591 | 0.061 500 |
| 2.0000 | −2.545 546 | −1.568 761 | 2.073 687 | 0.627 014 | 0.239 866 | 0.116 928 | 0.066 703 |

Bitsch (1983), Srivastava *et al* (1981) and Williams (1979). Comparing our four sets of calculated data we draw similar conclusions to that of the case of helium (see I): i.e.

(i) the best general agreement with the experimental data is obtained with the adiabatic exchange approximation using the dipole part of the polarisation potential only;

(ii) there is relatively good agreement between the adiabatic exchange results with the dipole polarisation potential and the local exchange results with the full polarisation potential. This fortuitous agreement is not as close as in helium but does account for the success of the local exchange approximation reported in II and III;

(iii) the JSHE local exchange approximation is not a good approximation to the full adiabatic exchange results. Here the pattern is somewhat different from helium, the best agreement occurring for the P wave while the error in the D wave is very large.

Hence we again take the adiabatic exchange approximation with just the dipole polarisation potential to yield the most reliable results. Comparing these phaseshifts with the experimental ones shown in figure 1 we find that we agree within 4% with the S-wave values of Andrick and Bitsch (1983) and Williams (1979). Above $k = 0.5$ our P-wave results agree with Williams data within his quoted error of 2.8%. In this region we also agree well with Andrick and Bitsch. At lower values of $k$ the percentage

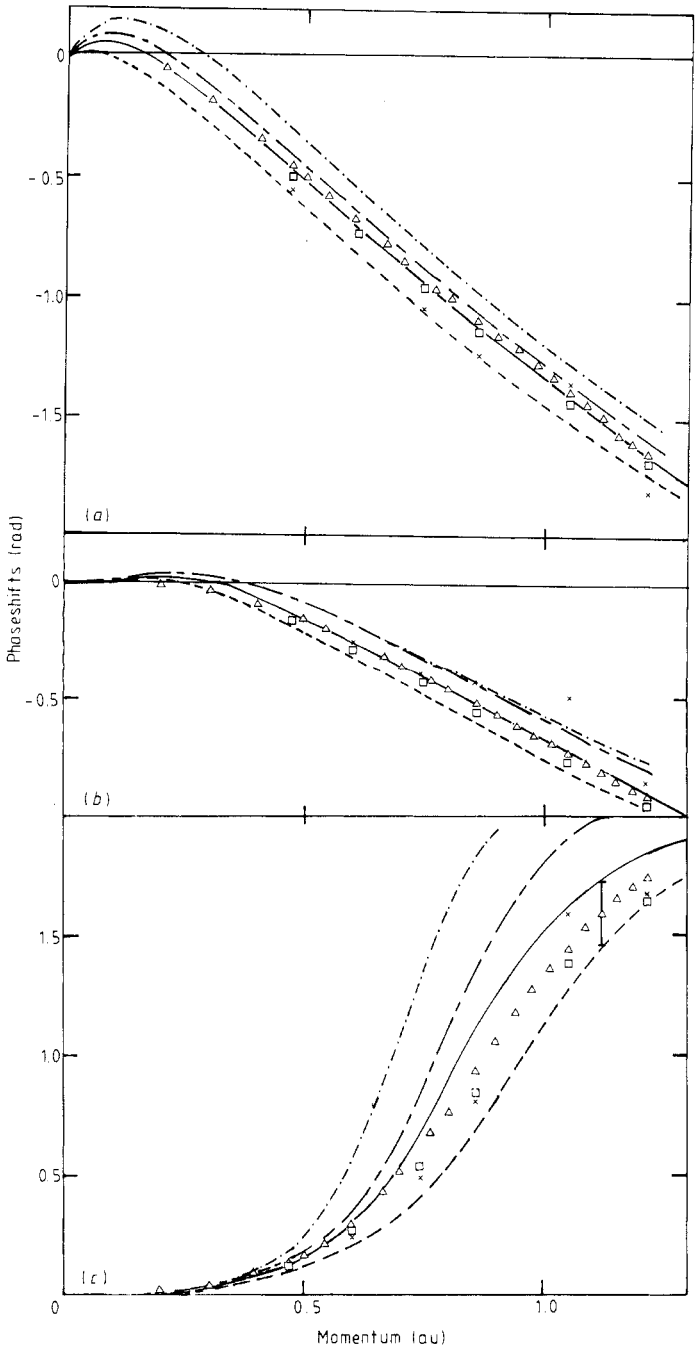


**Figure 1.** Phaseshifts for elastic scattering of electrons from argon: (*a*) S waves; (*b*) P waves; (*c*) D waves. ——, AE, dipole potential; —·—, AE, all multipoles; – – –, JSHE, dipole potential; — – —, JSHE, all multipoles; □, Andrick and Bitsch (1983); ×, Srivastava *et al* (1981); △, Williams (1979).

difference becomes large because the P-wave phaseshift goes through zero near $k = 0.3$. For the D wave the agreement is good with both sets of experimental data for $k \leqslant 0.5$ but there are systematic deviations at larger values of $k$. For all three phaseshifts the data of Srivastava *et al* (1981) exhibit rather large scatter about the other three sets of data.

A comparison with the previous calculations of Garbaty and LaBahn (1971) and Thompson (1971) shows that our S-wave phaseshifts lie between these two sets of data. Our P-wave results are below these for $k \leqslant 0.6$ while for larger $k$ we again lie

**Table 2.** Differential cross sections $d\sigma/d\Omega(a_0^2\ sr^{-1})$ for argon.

| Angle (deg) | Energy (eV) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 3.0 | 5.0 | 7.5 | 10.0 | 15.0 | 20.0 | 30.0 | 40.0 | 50.1 |
| 0 | 1.597 | 7.707 | 29.008 | 52.275 | 54.070 | 46.039 | 42.886 | 46.251 | 50.506 |
| 5 | 1.121 | 6.356 | 26.089 | 48.990 | 51.946 | 43.614 | 38.184 | 39.054 | 41.072 |
| 10 | 0.710 | 4.907 | 22.366 | 44.217 | 48.592 | 40.424 | 33.072 | 31.563 | 31.465 |
| 15 | 0.501 | 3.772 | 18.729 | 38.880 | 44.406 | 36.885 | 28.579 | 25.490 | 23.979 |
| 20 | 0.470 | 2.969 | 15.346 | 33.234 | 39.453 | 32.903 | 24.473 | 20.521 | 18.203 |
| 25 | 0.582 | 2.476 | 12.317 | 27.526 | 33.917 | 28.504 | 20.571 | 16.325 | 13.669 |
| 30 | 0.806 | 2.268 | 9.741 | 22.039 | 28.089 | 23.847 | 16.832 | 12.719 | 10.073 |
| 35 | 1.109 | 2.315 | 7.701 | 17.043 | 22.282 | 19.128 | 13.276 | 9.600 | 7.209 |
| 40 | 1.456 | 2.566 | 6.229 | 12.752 | 16.803 | 14.574 | 9.964 | 6.909 | 4.922 |
| 45 | 1.817 | 2.968 | 5.316 | 9.314 | 11.939 | 10.423 | 7.006 | 4.647 | 3.131 |
| 50 | 2.163 | 3.459 | 4.906 | 6.788 | 7.892 | 6.867 | 4.500 | 2.826 | 1.782 |
| 55 | 2.466 | 3.966 | 4.892 | 5.134 | 4.774 | 4.033 | 2.520 | 1.456 | 0.837 |
| 60 | 2.704 | 4.427 | 5.148 | 4.245 | 2.611 | 1.991 | 1.119 | 0.551 | 0.275 |
| 65 | 2.863 | 4.783 | 5.528 | 3.948 | 1.338 | 0.732 | 0.299 | 0.095 | 0.060 |
| 70 | 2.931 | 4.983 | 5.880 | 4.027 | 0.807 | 0.170 | 0.008 | 0.034 | 0.136 |
| 75 | 2.902 | 4.995 | 6.077 | 4.262 | 0.826 | 0.170 | 0.156 | 0.291 | 0.429 |
| 80 | 2.784 | 4.810 | 6.027 | 4.456 | 1.178 | 0.557 | 0.612 | 0.757 | 0.850 |
| 85 | 2.582 | 4.432 | 5.673 | 4.452 | 1.652 | 1.144 | 1.232 | 1.317 | 1.304 |
| 90 | 2.312 | 3.886 | 5.013 | 4.157 | 2.067 | 1.755 | 1.872 | 1.857 | 1.707 |
| 95 | 1.995 | 3.222 | 4.102 | 3.560 | 2.301 | 2.245 | 2.403 | 2.275 | 1.984 |
| 100 | 1.650 | 2.497 | 3.036 | 2.721 | 2.293 | 2.520 | 2.732 | 2.502 | 2.091 |
| 105 | 1.302 | 1.778 | 1.944 | 1.765 | 2.057 | 2.544 | 2.815 | 2.509 | 2.017 |
| 110 | 0.973 | 1.135 | 0.984 | 0.872 | 1.668 | 2.336 | 2.645 | 2.298 | 1.773 |
| 115 | 0.684 | 0.636 | 0.313 | 0.243 | 1.249 | 1.963 | 2.261 | 1.909 | 1.404 |
| 120 | 0.451 | 0.334 | 0.072 | 0.078 | 0.952 | 1.532 | 1.742 | 1.414 | 0.978 |
| 125 | 0.286 | 0.269 | 0.373 | 0.548 | 0.930 | 1.162 | 1.181 | 0.897 | 0.569 |
| 130 | 0.196 | 0.460 | 1.278 | 1.767 | 1.310 | 0.967 | 0.676 | 0.442 | 0.249 |
| 135 | 0.180 | 0.904 | 2.795 | 3.776 | 2.177 | 1.039 | 0.317 | 0.127 | 0.081 |
| 140 | 0.231 | 1.575 | 4.872 | 6.533 | 3.556 | 1.432 | 0.164 | 0.004 | 0.104 |
| 145 | 0.340 | 2.427 | 7.393 | 9.905 | 5.398 | 2.147 | 0.243 | 0.093 | 0.326 |
| 150 | 0.490 | 3.395 | 10.198 | 13.688 | 7.593 | 3.139 | 0.544 | 0.381 | 0.726 |
| 155 | 0.662 | 4.406 | 13.093 | 17.621 | 9.977 | 4.315 | 1.016 | 0.820 | 1.250 |
| 160 | 0.839 | 5.377 | 15.858 | 21.402 | 12.342 | 5.553 | 1.586 | 1.345 | 1.834 |
| 165 | 0.999 | 6.228 | 18.277 | 24.727 | 14.474 | 6.713 | 2.165 | 1.874 | 2.401 |
| 170 | 1.127 | 6.895 | 20.164 | 27.331 | 16.168 | 7.657 | 2.656 | 2.319 | 2.865 |
| 175 | 1.210 | 7.319 | 21.364 | 28.990 | 17.258 | 8.273 | 2.984 | 2.615 | 3.170 |
| 180 | 1.236 | 7.457 | 21.761 | 29.547 | 17.632 | 8.491 | 3.109 | 2.733 | 3.293 |
| $\sigma_{el}(a_0^2)$ | 19.112 | 37.920 | 71.894 | 97.273 | 86.511 | 68.295 | 49.458 | 40.086 | 34.319 |
| $\sigma_{mt}(a_0^2)$ | 15.467 | 33.781 | 64.366 | 76.112 | 49.571 | 32.702 | 21.569 | 17.785 | 15.677 |

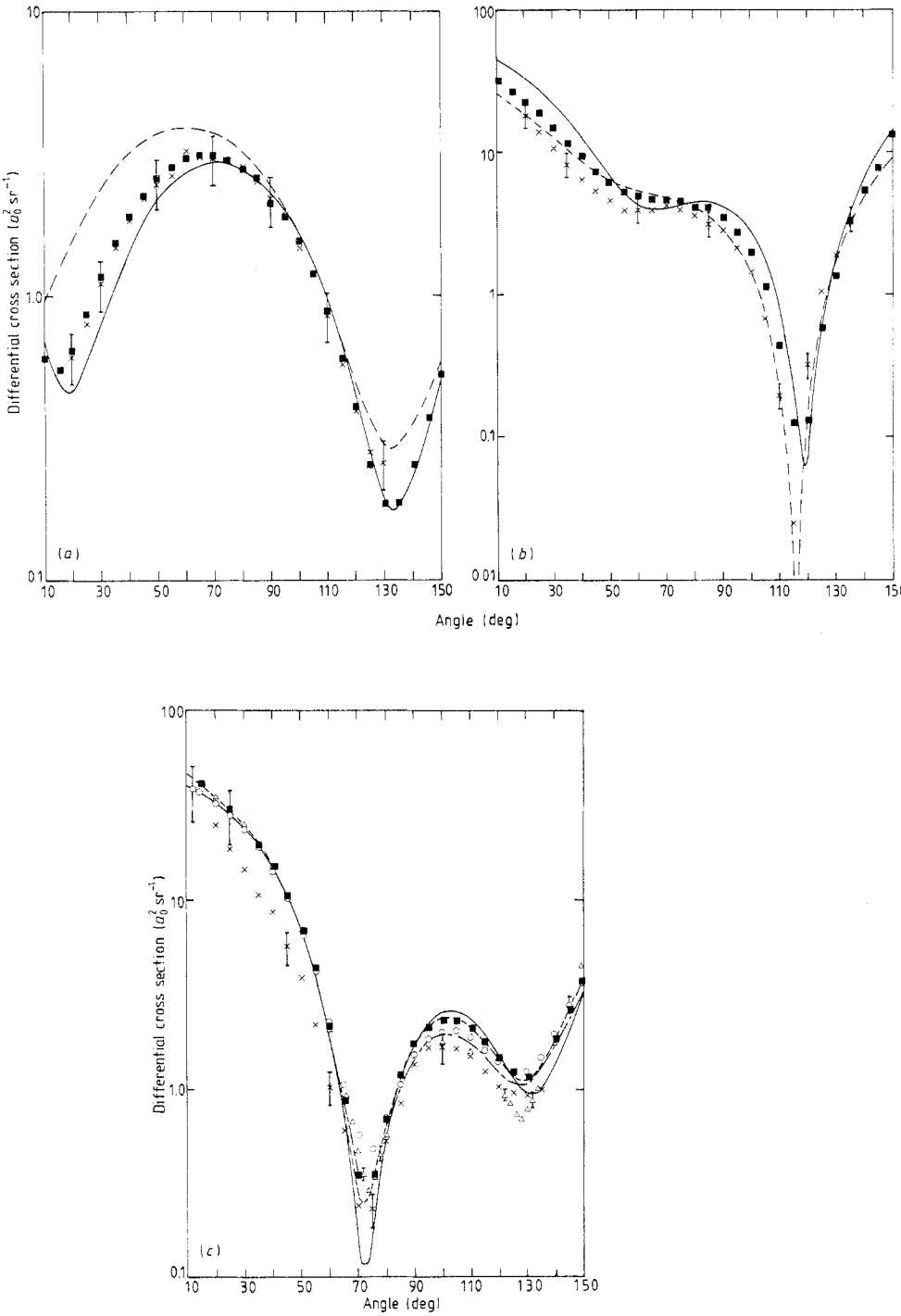


**Figure 2.** Differential cross sections for elastic scattering of electrons from argon: (*a*) 3 eV; (*b*) 10 eV; (*c*) 20 eV. ——, AE, dipole potential; – – –, JHSE, dipole potential; — – —, McCarthy *et al* (1977); ■, Andrick and Bitsch (1983); ○, DuBois and Rudd (1976); ⧗, Srivastava *et al* (1981); △, Williams and Willis (1975).

between the two sets of data. In the case of D waves our results are above Thompson's and for $k > 0.3$ between the two sets of values given by Garbaty and LaBahn.

Recently Fon *et al* (1983) have carried out an $R$-matrix calculation for this system. Their S-wave results lie some 3% below ours but for P and D waves their values are substantially below ours. In fact for D waves there is relatively good agreement between our JSHE results with the dipole polarisation potential and those of Fon *et al*.

In table 2 we present our results for differential, total elastic and momentum transfer cross sections. Figure 2 displays our differential cross sections at 3, 10 and 20 eV, calculated in the adiabatic exchange and JSHE approximations, both using the dipole polarisation potential. We also show the experimental results of Andrick and Bitsch (1983), DuBois and Rudd (1976), Srivastava *et al* (1981), Williams and Willis (1975) and the optical potential calculations of McCarthy *et al* (1977).

At 3 eV the two sets of experimental data shown agree reasonably well with each other. Our adiabatic exchange results lie within the error bars of the data of Srivastava *et al* for $40° \leqslant \theta \leqslant 125°$ and agree closely with Andrick and Bitsch for $\theta > 125°$. For $\theta < 40°$ the agreement is less good although the shapes are very similar. Our JSHE results differ substantially from the others for $\theta < 75°$ and $\theta > 125°$.

At 10 eV there is considerably more variation among all four sets of data shown. Both our AE results and the data of Srivastava *et al* exhibit two minima in the cross section while the other two sets show only one. However, no two sets of data agree with one another better than any other two.

At 20 eV there is substantially more agreement between all the data sets shown. All exhibit two distinct minima and the positions of these occur within a few degrees of each other in all the data shown. In spite of the differences in phaseshifts there is surprisingly good agreement between our two sets of calculations except in the region of the first minimum. For angles less than that at which the first minimum occurs there is good agreement among all data shown except for those of Srivastava *et al* which are considerably below the others.

Our total elastic cross sections are shown in figure 3 along with the experimental results of Charlton *et al* (1980), Golden and Bandel (1966), Gus'kov *et al* (1978), Kauppila *et al* (1976, 1981), Wagenaar and de Heer (1980), Andrick and Bitsch (1983), DuBois and Rudd (1976) and Srivastava *et al* (1981). The results of the first five groups were obtained by direct measurement and are total cross section measurements (i.e. they include inelastic effects above 15.8 eV). The remaining data were obtained from differential cross section measurements.

Below 5 eV our AE results are in very good agreement with experiment except in the neighbourhood of the minimum. In this energy range our JHSE results are very poor. Between 5 and 15 eV our AE results are considerably higher than the experimental results. There is considerable spread among the various experimental measurements in this region but our JSHE results agree with these data rather well. Above 15 eV our two calculations yield similar values which are in fair agreement with the experimental data for total cross sections. Subtracting the contribution to inelastic scattering estimated by de Heer *et al* (1979) makes the agreement with experiment worse.

It is apparent from figure 1 that the overestimation of the AE results of the 5–15 eV region is largely due to our D-wave phaseshift being too large. On the other hand, the success of the JSHE approximation here can be attributed to a fortuitous combination of overestimating the magnitudes of the S- and P-wave phaseshifts and underestimating the D wave.

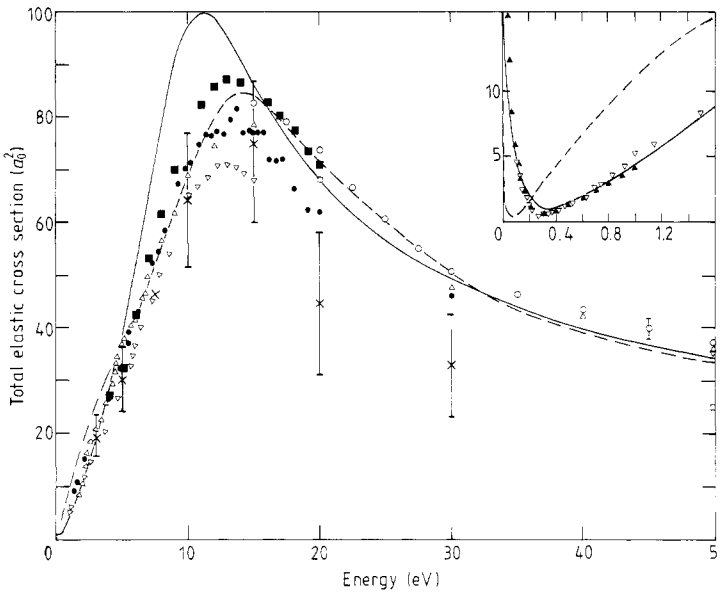


**Figure 3.** Total cross section for elastic scattering of electrons from argon: ——, AE, dipole potential; – – –, JHSE, dipole potential; ■, Andrick and Bitsch (1983); △, Charlton *et al* (1980); □, DuBois and Rudd (1976); ▽, Golden and Bandel (1966); ▲, Gus'kov *et al* (1978); ●, Kauppila *et al* (1976, 1981); ⧖, Srivastava *et al* (1981); ○, Wagenaar and de Heer (1980).

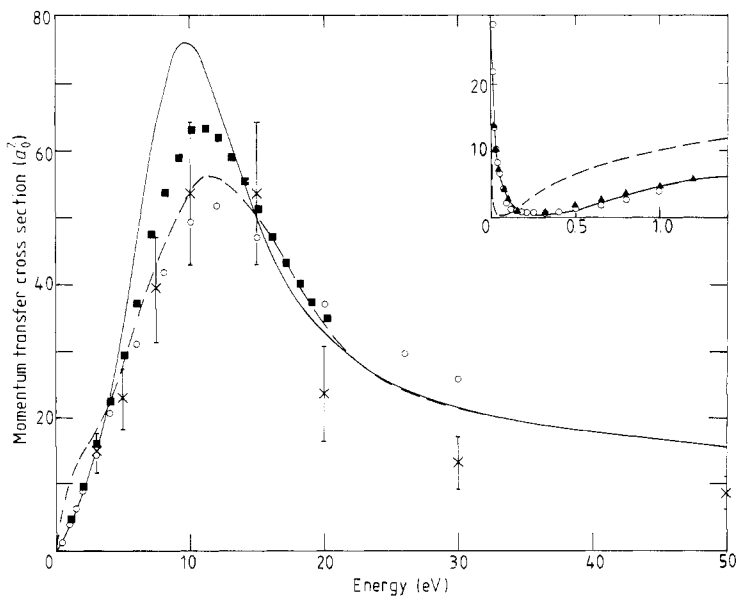


**Figure 4.** Momentum transfer cross section for scattering of electrons from argon: ——, AE, dipole potential; – – –, JHSE, dipole potential; ■, Andrick and Bitsch (1983); ○, Frost and Phelps (1969); ▲, Milloy and Crompton (1977); ⧖, Srivastava *et al* (1981).

**Table 3.** Elastic scattering phaseshifts $\delta_l$ for neon.

| $k$ | $\delta_0$ | $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ | $\delta_6$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.308 190 | | | | | | |
| 0.1000 | −0.050 715 | 0.003 116 | 0.000 714 | 0.000 237 | 0.000 108 | 0.000 058 | 0.000 035 |
| 0.2000 | −0.128 321 | 0.005 463 | 0.002 903 | 0.000 949 | 0.000 431 | 0.000 232 | 0.000 139 |
| 0.3000 | −0.221 047 | −0.001 106 | 0.006 780 | 0.002 140 | 0.000 971 | 0.000 523 | 0.000 314 |
| 0.4000 | −0.322 469 | −0.020 394 | 0.012 785 | 0.003 821 | 0.001 726 | 0.000 929 | 0.000 557 |
| 0.4696 | −0.396 075 | −0.041 387 | 0.018 551 | 0.005 296 | 0.002 381 | 0.001 280 | 0.000 768 |
| 0.5000 | −0.428 713 | −0.052 313 | 0.021 548 | 0.006 023 | 0.002 700 | 0.001 452 | 0.000 871 |
| 0.5422 | −0.474 346 | −0.069 050 | 0.026 243 | 0.007 119 | 0.003 177 | 0.001 707 | 0.001 024 |
| 0.6000 | −0.537 227 | −0.094 551 | 0.033 763 | 0.008 795 | 0.003 896 | 0.002 091 | 0.001 254 |
| 0.6062 | −0.543 986 | −0.097 442 | 0.034 649 | 0.008 987 | 0.003 977 | 0.002 134 | 0.001 280 |
| 0.6641 | −0.607 126 | −0.125 657 | 0.043 701 | 0.010 905 | 0.004 782 | 0.002 562 | 0.001 536 |
| 0.7000 | −0.646 217 | −0.144 096 | 0.050 044 | 0.012 210 | 0.005 321 | 0.002 848 | 0.001 707 |
| 0.7426 | −0.692 458 | −0.166 707 | 0.058 325 | 0.013 880 | 0.006 001 | 0.003 207 | 0.001 921 |
| 0.7668 | −0.718 627 | −0.179 837 | 0.063 399 | 0.014 889 | 0.006 408 | 0.003 420 | 0.002 048 |
| 0.8000 | −0.754 380 | −0.198 111 | 0.070 801 | 0.016 348 | 0.006 989 | 0.003 725 | 0.002 230 |
| 0.8573 | −0.815 605 | −0.230 160 | 0.084 773 | 0.019 074 | 0.008 058 | 0.004 282 | 0.002 562 |
| 0.9000 | −0.860 764 | −0.254 289 | 0.096 160 | 0.021 285 | 0.008 911 | 0.004 724 | 0.002 825 |
| 0.9391 | −0.901 720 | −0.276 451 | 0.107 297 | 0.023 448 | 0.009 735 | 0.005 149 | 0.003 077 |
| 0.9587 | −0.922 098 | −0.287 558 | 0.113 127 | 0.024 584 | 0.010 164 | 0.005 368 | 0.003 208 |
| 0.9775 | −0.941 543 | −0.298 199 | 0.118 871 | 0.025 705 | 0.010 585 | 0.005 584 | 0.003 336 |
| 1.0000 | −0.964 683 | −0.310 907 | 0.125 935 | 0.027 090 | 0.011 103 | 0.005 847 | 0.003 492 |
| 1.0144 | −0.979 415 | −0.319 019 | 0.130 562 | 0.028 000 | 0.011 443 | 0.006 019 | 0.003 594 |
| 1.0500 | −1.015 564 | −0.338 987 | 0.142 338 | 0.030 332 | 0.012 308 | 0.006 455 | 0.003 852 |
| 1.0844 | −1.050 120 | −0.358 134 | 0.154 152 | 0.032 696 | 0.013 181 | 0.006 891 | 0.004 110 |
| 1.1000 | −1.065 666 | −0.366 761 | 0.159 641 | 0.033 804 | 0.013 589 | 0.007 094 | 0.004 229 |
| 1.1178 | −1.083 308 | −0.376 558 | 0.166 001 | 0.035 095 | 0.014 064 | 0.007 330 | 0.004 367 |
| 1.1344 | −1.099 666 | −0.385 647 | 0.172 019 | 0.036 326 | 0.014 517 | 0.007 553 | 0.004 498 |
| 1.1502 | −1.115 151 | −0.394 253 | 0.177 823 | 0.037 520 | 0.014 956 | 0.007 770 | 0.004 624 |
| 1.1817 | −1.145 772 | −0.411 272 | 0.189 599 | 0.039 968 | 0.015 855 | 0.008 212 | 0.004 881 |
| 1.2000 | −1.163 407 | −0.421 071 | 0.196 557 | 0.041 431 | 0.016 393 | 0.008 475 | 0.005 033 |
| 1.2124 | −1.175 291 | −0.427 672 | 0.201 317 | 0.042 439 | 0.016 764 | 0.008 657 | 0.005 137 |
| 1.4000 | −1.348 560 | −0.523 414 | 0.276 592 | 0.059 259 | 0.022 981 | 0.011 703 | 0.006 876 |
| 1.4849 | −1.422 933 | −0.564 049 | 0.311 715 | 0.067 780 | 0.026 152 | 0.013 256 | 0.007 781 |
| 1.6000 | −1.519 799 | −0.616 411 | 0.359 162 | 0.080 155 | 0.030 810 | 0.015 495 | 0.009 098 |
| 1.7146 | −1.611 844 | −0.665 510 | 0.405 261 | 0.093 294 | 0.035 897 | 0.017 898 | 0.010 441 |
| 1.8000 | −1.677 685 | −0.700 225 | 0.438 413 | 0.103 506 | 0.039 974 | 0.019 863 | 0.011 471 |
| 1.9170 | −1.764 246 | −0.745 348 | 0.481 758 | 0.117 952 | 0.045 862 | 0.022 831 | 0.013 079 |
| 2.0000 | −1.823 212 | −0.775 751 | 0.510 877 | 0.128 485 | 0.050 189 | 0.025 051 | 0.014 429 |

A source of error in some of the experimental measurements is the lack of ability to measure the contribution to the total cross section from small angle scattering. For instance, Kauppila *et al* (1981) estimate that at 20 eV they do not measure electrons scattered through less than 10°. From our differential cross section results we have calculated that such an error would produce a cross section 6% below the true value. Correcting for this error would bring the results of Kauppila *et al* into good agreement with the other total cross section measurements above 20 eV. Below 20 eV the small angle scattering error is not given. However, such an error cannot explain the discrepancy between our AE results and experiment at 10 eV, for example, where the angular discrimination would have to exced 30°.

Figure 4 shows our momentum transfer cross sections along with the experimental measurements of Frost and Phelps (1964), Milloy and Crompton (1977), Andrick and

Bitsch (1983) and Srivastava *et al* (1981). As for the total cross section the last two sets of data are obtained from differential cross section measurements. The agreement between our results and experiment follows very closely the pattern of the total elastic cross section shown in the previous figure.

### *3.2. Neon*

The behaviour for neon is very similar to that of helium (see I) and we present here just our adiabatic exchange results using the dipole polarisation potential only along with a comparison with experimental results and other theoretical calculations.

Our calculated phaseshifts are given in table 3 and shown graphically in figure 5 along with the $R$-matrix calculations of Fon and Berrington (1981), the phaseshifts derived from experimentally measured differential cross sections by Brewer *et al* (1981), Register and Trajmar (1983) and Williams (1979) as well as the results of the modified effective range analysis of the data of Robertson (1972) and Williams (1979) by O'Malley and Crompton (1980).

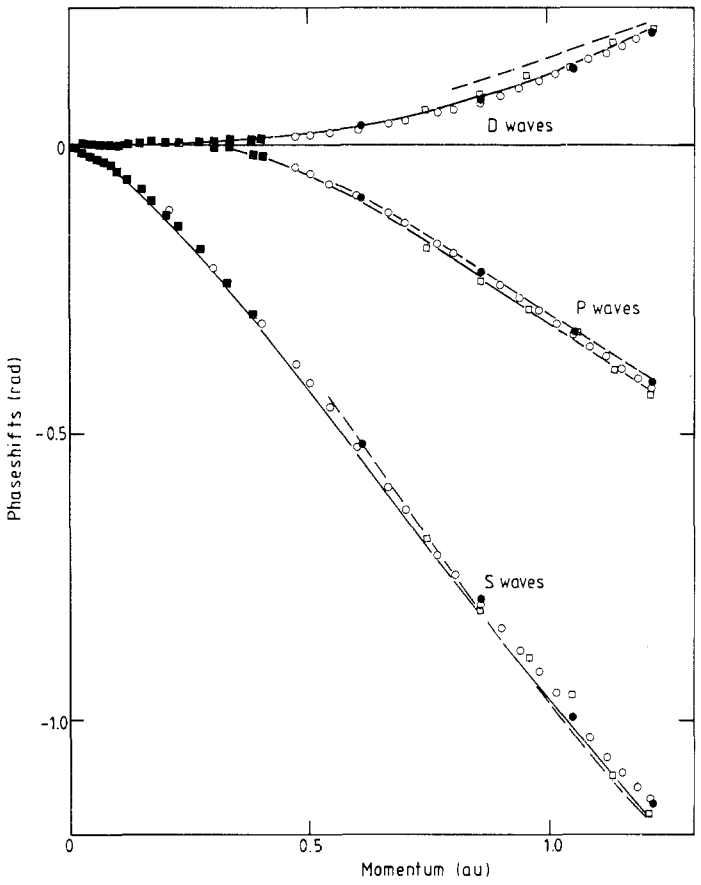


**Figure 5.** Phaseshifts for elastic scattering of electrons from neon: ——, AE, dipole potential; – – –, Fon and Berrington (1981); □, Brewer *et al* (1981); ■, O'Malley and Crompton (1980); ●, Register and Trajmar (1983); ○, Williams (1979).

Comparing our phaseshifts with those of Fon and Berrington (1981) we find reasonable agreement for $\delta_0$ where the maximum difference is 8% over the energy range shown. A similar situation exists for $\delta_1$ but for $\delta_2$ the maximum difference is 38%. There is quite satisfactory agreement between our calculations and the experimental results shown although the results of Brewer *et al* at 15 eV seem somewhat out of line. The scattering length derived by O'Malley and Crompton is 30% below ours but the agreement between the phaseshifts is superior to that. We have not shown the experimental results of Andrick and Bitsch but their results are comparable

**Table 4.** Differential cross sections $d\sigma/d\Omega(a_0^2\ sr^{-1})$ for neon.

| Angle (deg) | Energy (eV) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 5.0 | 7.5 | 10.0 | 12.5 | 15.0 | 17.5 | 20.0 | 30.0 | 40.0 | 50.0 |
| 0 | 0.558 | 0.696 | 0.829 | 1.001 | 1.227 | 1.510 | 1.840 | 3.418 | 4.983 | 6.283 |
| 5 | 0.643 | 0.786 | 0.906 | 1.049 | 1.235 | 1.469 | 1.744 | 3.081 | 4.411 | 5.501 |
| 10 | 0.755 | 0.912 | 1.022 | 1.136 | 1.277 | 1.456 | 1.668 | 2.719 | 3.768 | 4.611 |
| 15 | 0.869 | 1.044 | 1.151 | 1.243 | 1.349 | 1.481 | 1.637 | 2.427 | 3.218 | 3.839 |
| 20 | 0.977 | 1.172 | 1.282 | 1.361 | 1.439 | 1.532 | 1.642 | 2.203 | 2.767 | 3.196 |
| 25 | 1.076 | 1.292 | 1.408 | 1.479 | 1.538 | 1.600 | 1.671 | 2.037 | 2.402 | 2.668 |
| 30 | 1.164 | 1.399 | 1.524 | 1.592 | 1.637 | 1.675 | 1.716 | 1.917 | 2.114 | 2.242 |
| 35 | 1.237 | 1.489 | 1.622 | 1.691 | 1.727 | 1.748 | 1.764 | 1.832 | 1.890 | 1.907 |
| 40 | 1.292 | 1.556 | 1.698 | 1.769 | 1.799 | 1.808 | 1.806 | 1.768 | 1.716 | 1.645 |
| 45 | 1.330 | 1.601 | 1.748 | 1.821 | 1.848 | 1.848 | 1.833 | 1.714 | 1.576 | 1.439 |
| 50 | 1.349 | 1.620 | 1.770 | 1.843 | 1.868 | 1.862 | 1.837 | 1.658 | 1.456 | 1.272 |
| 55 | 1.348 | 1.613 | 1.760 | 1.832 | 1.853 | 1.842 | 1.810 | 1.587 | 1.341 | 1.126 |
| 60 | 1.329 | 1.580 | 1.720 | 1.786 | 1.804 | 1.788 | 1.750 | 1.497 | 1.223 | 0.991 |
| 65 | 1.293 | 1.523 | 1.651 | 1.709 | 1.720 | 1.699 | 1.655 | 1.382 | 1.094 | 0.857 |
| 70 | 1.242 | 1.445 | 1.555 | 1.602 | 1.605 | 1.577 | 1.528 | 1.242 | 0.953 | 0.721 |
| 75 | 1.176 | 1.348 | 1.437 | 1.469 | 1.462 | 1.427 | 1.373 | 1.081 | 0.799 | 0.581 |
| 80 | 1.100 | 1.237 | 1.302 | 1.318 | 1.299 | 1.256 | 1.198 | 0.905 | 0.639 | 0.443 |
| 85 | 1.015 | 1.117 | 1.156 | 1.154 | 1.124 | 1.074 | 1.011 | 0.724 | 0.480 | 0.310 |
| 90 | 0.925 | 0.990 | 1.005 | 0.986 | 0.945 | 0.889 | 0.824 | 0.548 | 0.331 | 0.192 |
| 95 | 0.833 | 0.863 | 0.855 | 0.822 | 0.772 | 0.712 | 0.647 | 0.390 | 0.205 | 0.097 |
| 100 | 0.741 | 0.741 | 0.713 | 0.669 | 0.615 | 0.554 | 0.491 | 0.262 | 0.111 | 0.035 |
| 105 | 0.652 | 0.626 | 0.585 | 0.535 | 0.480 | 0.422 | 0.366 | 0.174 | 0.061 | 0.012 |
| 110 | 0.568 | 0.522 | 0.474 | 0.424 | 0.374 | 0.326 | 0.280 | 0.135 | 0.060 | 0.038 |
| 115 | 0.490 | 0.432 | 0.384 | 0.342 | 0.304 | 0.269 | 0.239 | 0.151 | 0.116 | 0.114 |
| 120 | 0.421 | 0.358 | 0.318 | 0.290 | 0.271 | 0.256 | 0.245 | 0.225 | 0.229 | 0.242 |
| 125 | 0.361 | 0.301 | 0.276 | 0.270 | 0.275 | 0.286 | 0.299 | 0.356 | 0.398 | 0.421 |
| 130 | 0.310 | 0.260 | 0.257 | 0.280 | 0.315 | 0.356 | 0.398 | 0.540 | 0.618 | 0.644 |
| 135 | 0.269 | 0.235 | 0.260 | 0.317 | 0.387 | 0.463 | 0.536 | 0.768 | 0.880 | 0.905 |
| 140 | 0.236 | 0.223 | 0.281 | 0.376 | 0.485 | 0.598 | 0.705 | 1.030 | 1.174 | 1.192 |
| 145 | 0.211 | 0.223 | 0.317 | 0.451 | 0.601 | 0.752 | 0.894 | 1.312 | 1.484 | 1.492 |
| 150 | 0.193 | 0.232 | 0.362 | 0.537 | 0.727 | 0.916 | 1.092 | 1.600 | 1.797 | 1.791 |
| 155 | 0.181 | 0.246 | 0.411 | 0.625 | 0.853 | 1.078 | 1.286 | 1.877 | 2.096 | 2.076 |
| 160 | 0.173 | 0.262 | 0.460 | 0.709 | 0.971 | 1.229 | 1.465 | 2.129 | 2.367 | 2.332 |
| 165 | 0.169 | 0.278 | 0.503 | 0.782 | 1.073 | 1.357 | 1.616 | 2.341 | 2.593 | 2.545 |
| 170 | 0.166 | 0.292 | 0.538 | 0.838 | 1.151 | 1.455 | 1.733 | 2.503 | 2.765 | 2.706 |
| 175 | 0.165 | 0.301 | 0.559 | 0.874 | 1.200 | 1.517 | 1.805 | 2.604 | 2.872 | 2.807 |
| 180 | 0.164 | 0.304 | 0.566 | 0.885 | 1.216 | 1.537 | 1.829 | 2.635 | 2.906 | 2.838 |
| $\sigma_{el}(a_0^2)$ | 10.363 | 11.601 | 12.408 | 12.931 | 13.258 | 13.449 | 13.544 | 13.381 | 12.816 | 12.114 |
| $\sigma_{mt}(a_0^2)$ | 7.636 | 8.151 | 8.733 | 9.299 | 9.799 | 10.215 | 10.538 | 11.034 | 10.671 | 9.924 |

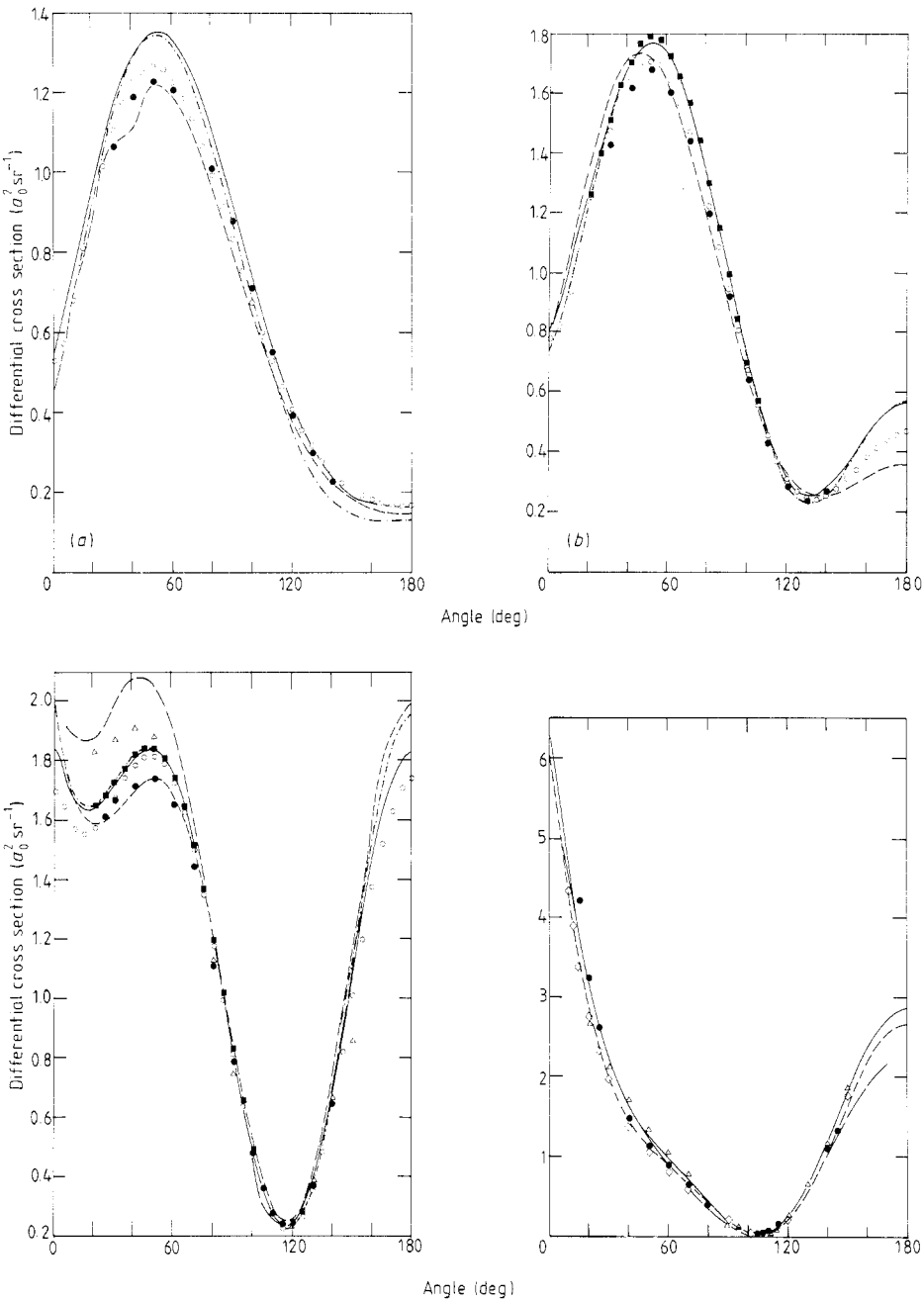


**Figure 6.** Differential cross sections for elastic scattering of electrons from neon: (*a*) 5 eV; (*b*) 10 eV; (*c*) 20 eV; (*d*) 50 eV. ——, AE, dipole potential; — · —, Andrick and Bitsch (1983); – – –, Fon and Berrington (1981); — —, McCarthy *et al* (1977); ■, Brewer *et al* (1981); ◇, DuBois and Rudd (1976); ●, Register and Trajmar (1983); ○, Williams (1979); △, Williams and Crowe (1975).

to the others shown. Garbaty and LaBahn (1971) and Thompson (1971) have also calculated phaseshifts for electron scattering from neon. In the former publication, the authors used two different approximations and our results lie between them. We agree well with Thompson's results at higher energies but there are systematic deviations at lower energies for the S- and P-wave phaseshifts.

Table 4 contains our differential cross sections as well as total elastic and momentum transfer cross sections. Figure 6 shows a selection of the differential cross sections along with theoretical calculations of Fon and Berrington (1981) and McCarthy *et al* (1977) and the experimental results of Andrick and Bitsch (1983), Brewer *et al* (1981), DuBois and Rudd (1976), Register and Trajmar (1983), Williams (1979) and Williams and Crowe (1975). The results attributed to Williams are calculated from his published phaseshifts.

At 5 eV there is some variation in magnitude among the various sets of data but general agreement as to the angular position of the peak of the cross section. We agree extremely well with Andrick and Bitsch and Brewer *et al* at 10 eV. Again there is general agreement about the positions of the maximum and minimum excepting the calculations of Fon and Berrington.

At 20 eV there is remarkably good agreement among the eight data sets shown over the angular range of 70° to 140°. There is considerable variation at low energies but again we have very good agreement with both Andrick and Bitsch and Brewer *et al* in that region. Finally at 50 eV there is substantial agreement among the three theoretical calculations and three experiments over the whole angular range. It is somewhat surprising that our present approximation produces such good results at this high energy.

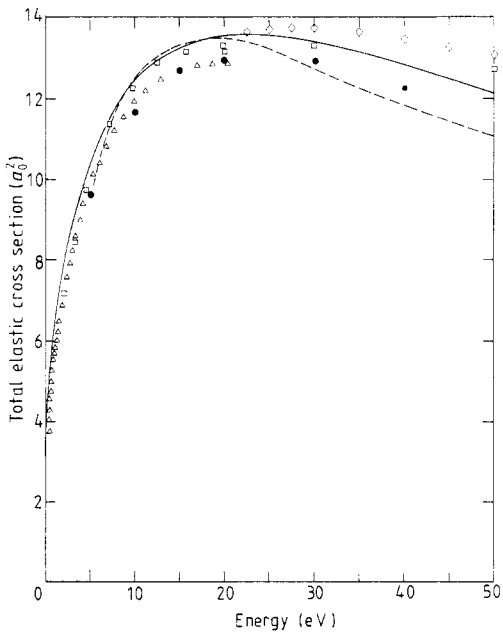


**Figure 7.** Total cross section for elastic scattering of electrons from neon: ——, AE, dipole potential; – – –, Fon and Berrington (1981); □, Kauppila *et al* (1981) and Stein *et al* (1978); ●, Register and Trajmar (1983); △, Salop and Nakano (1970); ◇, Wagenaar and de Heer (1980).

In figure 7 we present the results for our total elastic cross section as well as those of Fon and Berrington (1981) and the experimental results of Kauppila *et al* (1981), Register and Trajmar (1983), Salop and Nakano (1970), Stein *et al* (1978) and Wagenaar and de Heer (1980). The results of Register and Trajmar are calculated from the phaseshifts derived from their measured differential cross sections. The other experimental results are total cross section measurements i.e. they include inelastic contributions above 21.5 eV. We agree well with the results of Stein *et al* and with those of Salop and Nakano at lower energies. Near the peak of the cross section the latter results are systematically below ours. The measurements of Wagenaar and de Heer between 20 and 50 eV are above our results. However correcting for inelastic contributions using the semi-empirical results of de Heer *et al* (1979) brings them into close agreement with Register and Trajmar. The theoretical calculations of Fon and Berrington have a somewhat different shape than ours and predict a maximum cross section at a lower energy.

Figure 8 shows our momentum transfer cross section, the theoretical results of Fon and Berrington (1981), and the experimental results of Register and Trajmar

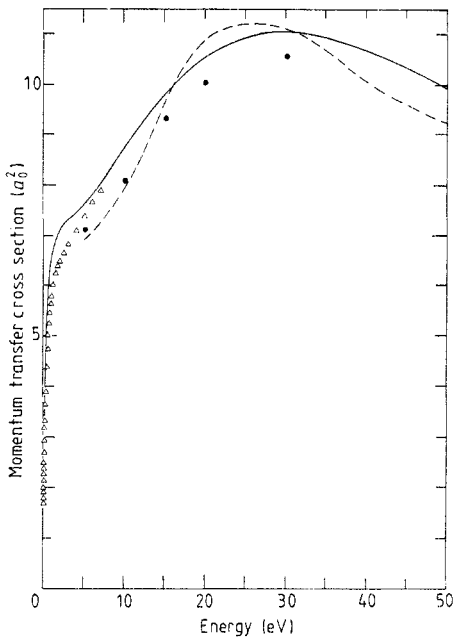


**Figure 8.** Momentum transfer cross section for scattering of electrons from neon: ——, AE, dipole potential; – – –, Fon and Berrington (1981); ●, Register and Trajmar (1983); △, Robertson (1972).

(1983) and Robertson (1972). The latter are derived from drift velocity measurements and we agree well with them. The results of Register and Trajmar are systematically below ours as was the case for the total cross section. Again the results of Fon and Berrington have a somewhat different shape.

## 4. Conclusions

Our calculations for neon and argon support the conclusions of I, viz that at these energies exchange must be treated exactly and that only the dipole part of the polarisation potential should be included.

The behaviour of neon is quite similar to that of helium and in general there is good agreement between our calculations and various experimental calculations. Argon is a heavier atom and exhibits rather different behaviour from the other two. The easiest way to quantify this difference is to point to the large D-wave phaseshift for argon above $k = 0.5$. In helium and neon this phaseshift remains relatively small. It is the large size of this phaseshift that is responsible for the peak in the total and momentum transfer cross sections. Our adiabatic exchange calculations overestimate this phase at higher energies and lead to peak total and momentum transfer cross sections which are larger than experimental results. In the differential cross section this overestimate leads to results which are too large for small angles. However below 3 eV our results give the good agreement with experiment that we saw in helium and neon. Also above 20 eV the adiabatic exchange approximation yields good results although the reason why the approximation works at higher energies is not entirely clear.

It would be of considerable interest to see whether the inclusion of the polarised exchange terms would improve the results in the region of the peak of the total cross section.

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