J. Phys. B: At. Mol. Opt. Phys. 28 (1995) 869–877. Printed in the UK

Relativistic distorted-wave calculation of the excitation of the $^3D_3$ state of heavy noble gases

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Received 30 August 1994

Abstract. Recent measurements have been made of the integrated Stokes parameters for the light emitted following the excitation of the $np^5\,(n+1)p\,[5/2]_3\,(^3D_3)$ state of the noble gases by spin-polarized electrons. These measurements give direct information on the spin-orbit coupling effects on the scattered electron. Hence it is of considerable interest to compare these data with the results of a completely relativistic distorted-wave treatment of these collisions which includes the relativistic interactions of the scattered electron to all orders. Results in this approximation are presented for the excitation of argon, krypton and xenon.

1. Introduction

The $np^5\,(n+1)p\,[5/2]_3\,(^3D_3)$ states of the heavier noble gases ($n = 3$ for argon, $n = 4$ for krypton and $n = 5$ for xenon) can be uniquely represented in LS-coupling. In this representation the excitation of this state takes place purely via exchange scattering. Thus any spin-polarization phenomena are due to the relativistic interactions (primarily spin-orbit coupling) involving the scattered electron and these can then be identified directly.

Measurements of the integrated Stokes parameters for the light emitted following the decay of this state to the $np^5\,(n+1)s\,[3/2]_2$ state in neon, argon, krypton and xenon which have been excited by a transversely spin-polarized electron beam have been made by Furst *et al* (1992, 1993). These papers also contained the results of a semi-relativistic distorted-wave calculation (DWB1).

In the present paper we report the results of a completely relativistic distorted-wave (RDW) calculation for the Stokes parameters and other scattering parameters for the excitation process described above. In our method, the target atom is represented by Dirac–Fock wavefunctions while the wavefunction for the scattered electron is calculated via the Dirac equations. Thus relativistic effects are included to all orders. We compare our results with the experimental data and previous calculations as well as test the validity of the LS-approximation for these systems.

2. Theoretical methods

The relativistic wavefunctions for both the ground and excited $^3D_3$ states of the noble gases can be represented by single determinantal wavefunctions which we calculate in the Dirac–Fock approximation using the MCDF program of Grant *et al* (1980). The details of the RDW method used to calculate the scattering amplitudes have been published by Zuo *et al* (1991). General formulae for the Stokes parameters have been given by Bartschat *et al* (1981).

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0953-4075/95/050869+09$19.50

We take as our coordinate system the collision frame where the z-axis is in the direction of the incident electron beam which is spin polarized in the direction of the y-axis. If the scattered electron is not detected then the formulae for the integrated Stokes parameters are

$$\eta_1^y = P_2 = -\frac{2}{I_{\text{Tot}}^y}\begin{Bmatrix} 1 & 1 & 2 \\ J & J & J_f \end{Bmatrix} G_2(J)\text{Re}\left[\langle \mathcal{T}(J)_{21}^+\rangle\right] P_y \tag{1}$$

$$\eta_2^y = -P_3 = \frac{2}{I_{\text{Tot}}^y}\begin{Bmatrix} 1 & 1 & 1 \\ J & J & J_f \end{Bmatrix} G_1(J)\text{Im}\left[\langle \mathcal{T}(J)_{11}^+\rangle\right] P_y \tag{2}$$

$$\eta_3^y = P_1 = \frac{1}{I_{\text{Tot}}^y}\begin{Bmatrix} 1 & 1 & 2 \\ J & J & J_f \end{Bmatrix} \sqrt{3/2}\, G_2(J)\langle \mathcal{T}(J)_{20}^+\rangle \tag{3}$$

where

$$I_{\text{Tot}}^y = \frac{2(-1)^{J+J_f}}{3\sqrt{2J+1}} G_0(J)\langle \mathcal{T}(J)_{00}^+\rangle + \begin{Bmatrix} 1 & 1 & 2 \\ J & J & J_f \end{Bmatrix} \sqrt{1/6}\, G_2(J)\langle \mathcal{T}(J)_{20}^+\rangle \tag{4}$$

and P_y is the magnitude of the spin polarization of the incident electron beam. The superscript y in (1)–(4) indicates the direction in which the emitted light is detected. The values of the coefficients $G_K(J)$ are given in Furst *et al* (1993). The state multipoles are given by

$$\langle T(J)_{KQ}^+\rangle = \sum_M (-1)^{J-M}\sqrt{2K+1}\begin{pmatrix} J & J & K \\ M & Q-M & -Q \end{pmatrix}\langle f(M) f(M-Q)^*\rangle \tag{5}$$

while the integrated quantities $\langle \mathcal{T}(J)_{KQ}\rangle$ are just the integral of the state multipoles given in (5) over all scattering angles.

Another integrated parameter which is of interest is η_2^z which is non-zero when the electrons are longitudinally polarized, i.e. when the direction of spin polarization of the incident electron beam is along the direction of motion and, as above, the scattered electron is not detected. In this case

$$\eta_2^z = \frac{\sqrt{2}}{I_{\text{Tot}}^z}\begin{Bmatrix} 1 & 1 & 1 \\ J & J & J_f \end{Bmatrix} G_1(J)\langle \mathcal{T}(J)_{10}^+\rangle P_z \tag{6}$$

where

$$I_{\text{Tot}}^z = \frac{2(-1)^{J+J_f}}{3\sqrt{2J+1}} G_0(J)\langle \mathcal{T}(J)_{00}^+\rangle - \begin{Bmatrix} 1 & 1 & 2 \\ J & J & J_f \end{Bmatrix} \sqrt{2/3}\, G_2(J)\langle \mathcal{T}(J)_{20}^+\rangle \tag{7}$$

and P_z is the magnitude of the spin-polarization vector in this case. The relationships between the state multipoles $\langle T(J)_{KQ}\rangle$ and the T-matrix elements (scattering amplitudes) are given in Bartschat *et al* (1981). For the present case $J = 3$ and $J_f = 2$.

When the incident electrons are polarized, the differential cross section is given by (Bartschat and Madison 1988)

$$\frac{d\sigma}{d\omega} = \frac{d\sigma_u}{d\omega}(1 + S_A P_y) \tag{8}$$

where $d\sigma_u/d\omega$ is the differential cross section for the scattering of unpolarized electrons and S_A is the spin asymmetry parameter.

The integrated cross section obtained by integrating (8) over the scattering angles is the same as the integrated cross section for the scattering of unpolarized electrons since the term depending on S_A integrates to zero.

Other differential quantities depending on the scattering angles of the electron are the Sherman function S_P (Bartschat and Madison 1988) which represents the spin polarization of an initially unpolarized electron beam after scattering through a given angle and the differential form of the Stokes parameters given above as well as η_3^x ($= P_4$). The general expressions for these parameters are given in Bartschat *et al* (1981) as well as Bartschat (1988).

3. Results

We have calculated the integrated Stokes parameters η_1^y, η_2^y and η_3^y for the $^3D_3$ states of argon, krypton and xenon excited by transversely polarized electrons for the range of incident energies from threshold to 100 eV. We have taken P_y to be the experimental values of 0.30 in the case of scattering from argon, 0.35 for krypton and 0.30 for xenon (Wijayaratna 1993). η_1^y is a direct measure of the effect of spin-orbit coupling since it is zero in the absence of such forces. In the measurements of Furst *et al* (1992, 1993) this quantity was zero to within experimental error. In our calculations we find the quantity to be small ($\lesssim 10^{-3}$) in all cases with the magnitude increasing as we go from argon to xenon.

In the measurements of Furst *et al* cascades from higher lying electronically excited states contributed to the light signal used to extract the Stokes parameters. In particular, optically allowed cascading from higher lying states can lead to η_1^y values which are non-zero even in the absence of relativistic interactions. Thus valid comparisons between theory and experiment can only be made between the threshold energies for the excitation of the $^3D_3$

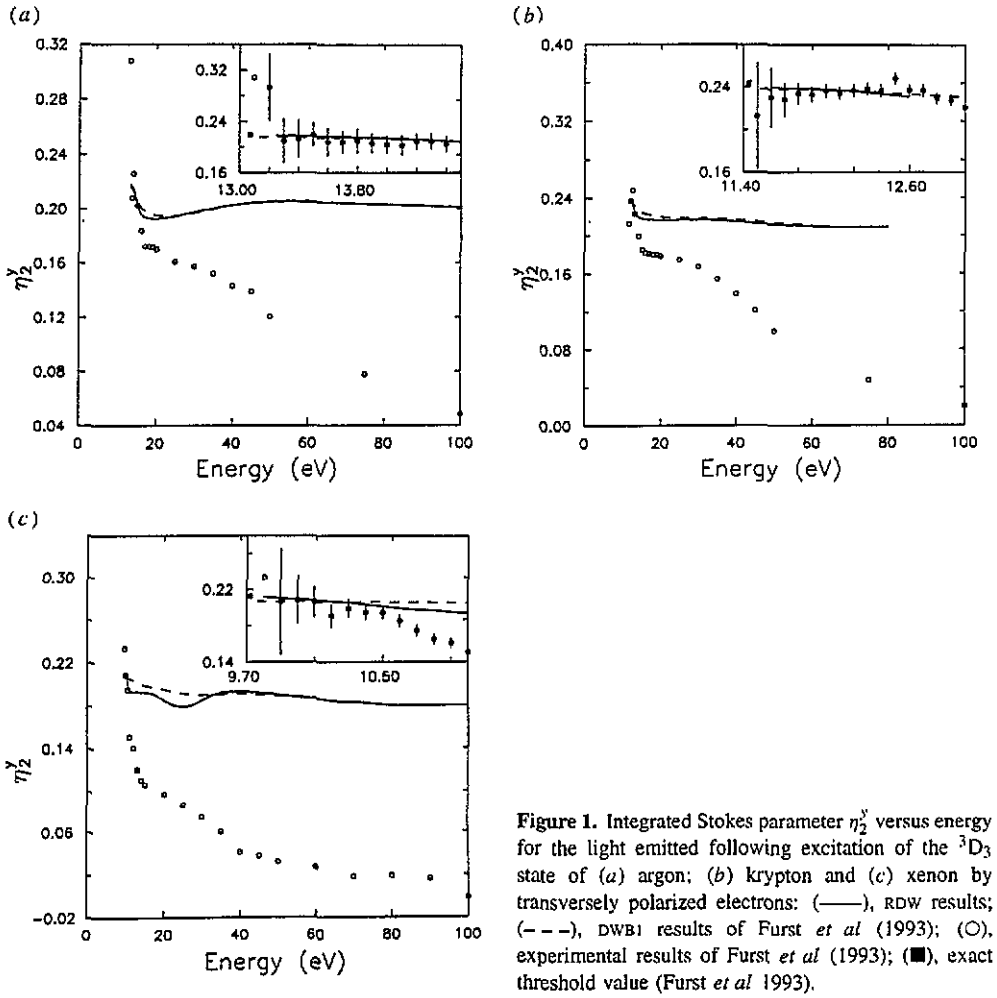

Figure 1. Integrated Stokes parameter η_2^y versus energy for the light emitted following excitation of the $^3D_3$ state of (*a*) argon; (*b*) krypton and (*c*) xenon by transversely polarized electrons: (——), RDW results; (– – –), DWB1 results of Furst *et al* (1993); (○), experimental results of Furst *et al* (1993); (■), exact threshold value (Furst *et al* 1993).

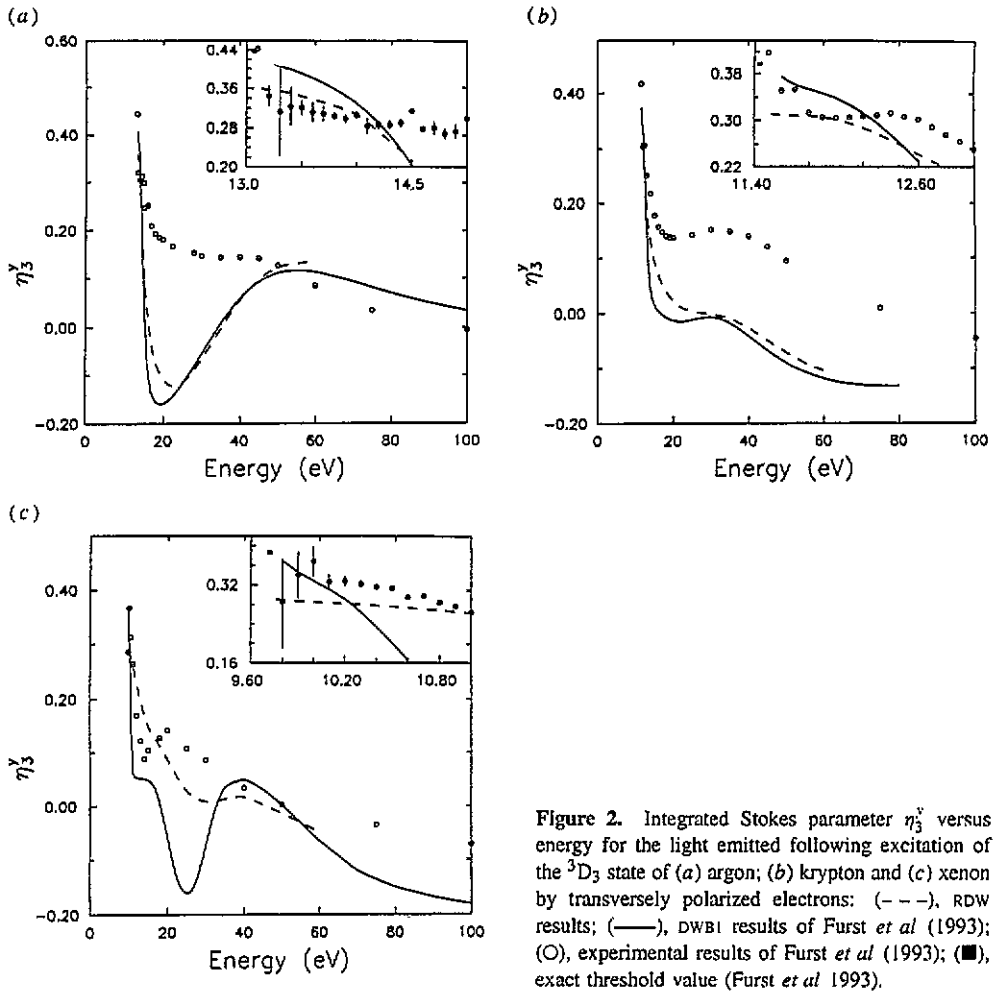

Figure 2. Integrated Stokes parameter η_3^y versus energy for the light emitted following excitation of the $^3D_3$ state of (*a*) argon; (*b*) krypton and (*c*) xenon by transversely polarized electrons: (– – –), RDW results; (——), DWBI results of Furst *et al* (1993); (O), experimental results of Furst *et al* (1993); (■), exact threshold value (Furst *et al* 1993).

state under investigation and the next higher lying state which can optically decay to this state. A table of these energy values is given in Furst *et al* (1993).

We show our results for η_2^y in figure 1. Both theoretical calculations show little dependence on energy except near threshold. They agree well with one another, though the RDW results show somewhat more structure at lower energies in the case of xenon. Near threshold both sets of calculations are consistent with the experimental results with the RDW values being in slightly better agreement in the case of xenon. For all three atoms the RDW results seem to be approaching the exact threshold values (Furst *et al* 1993).

In figure 2 we present the data for η_3^y. The two theoretical calculations are in general agreement with each other for argon and krypton. For xenon, the RDW results again show considerably more structure than the DWBI calculations below about 35 eV. In the near threshold region the DWBI results agree better with the experimental data for argon while for krypton and xenon the RDW results are more consistent with experiment. Again, the RDW results seem to be approaching the exact threshold value for all three atoms.

The parameter η_2^z arises in the case where the incident electron beam is longitudinally polarized, i.e. the spin-polarization vector lies along the direction of incidence. Although

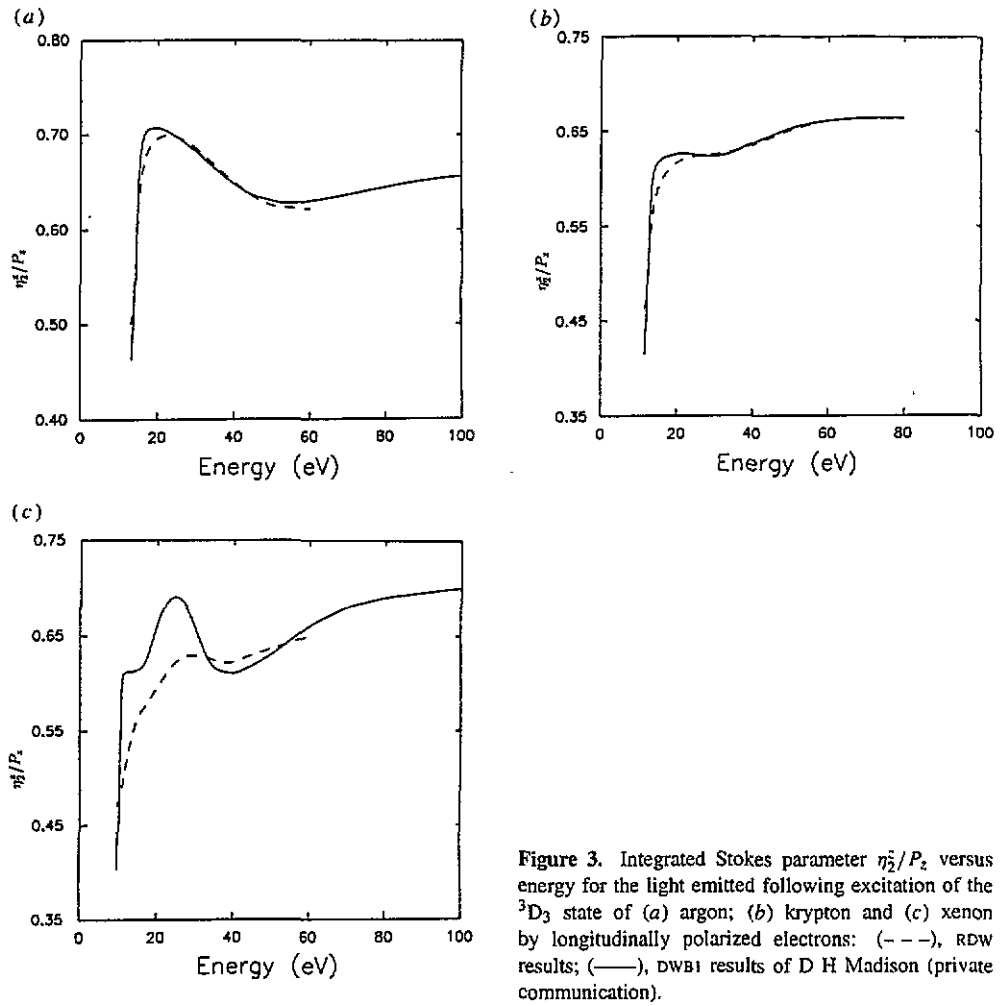

Figure 3. Integrated Stokes parameter η_2^z/P_z versus energy for the light emitted following excitation of the $^3D_3$ state of (*a*) argon; (*b*) krypton and (*c*) xenon by longitudinally polarized electrons: (– – –), RDW results; (——), DWB1 results of D H Madison (private communication).

there are no experimental measurements for this quantity for the excitation of the $^3D_3$ states, DWB1 calculations have been carried out (D H Madison, private communication). Thus we present data for the quantity η_2^z/P_z in figure 3. As for η_2^y there is little dependence of this quantity on the energy of the incident electrons except in the threshold region. Once again, the RDW results show some structure near threshold for xenon but otherwise the agreement between the two sets of calculations is good.

Equation (7) of Furst *et al* (1993) is a relation between η_2^y and η_3^y when spin-orbit effects are negligible. We have investigated how well our calculated results satisfy this relationship. In fact, it is satisfied very well with the largest differences occurring in xenon where at some energies it is approximately 1%. Along with the very small value for η_1^y, this indicates that the $^3D_3$ state is well represented in LS-coupling.

Although the above results show little evidence of the effect of spin-orbit coupling, it must be recalled that these quantities represent the case where the scattered electron is not detected, i.e. these quantities are integrated over all angles of the scattered electrons. On the other hand, the differential parameters are much more sensitive to relativistic effects. Thus we present results for $d\sigma_u/d\omega$, and the differential forms of S_A, S_P, η_1^y, η_2^y, η_3^y and η_2^z

Table 1. Differential scattering parameters for the excitation of the $^3D_3$ state of argon by 30 eV electrons.

Angle	σ	S_A	S_P	P_1	P_2	P_3	P_4
0	1.12, −2	0.00, −0	0.00, −0	4.39, −1	0.00, −0	0.00, −0	4.39, −1
5	1.11, −2	−1.36, −1	1.36, −1	4.34, −1	1.62, −2	−2.32, −1	4.38, −1
10	1.07, −2	−2.50, −1	2.50, −1	4.16, −1	2.81, −2	−4.26, −1	4.34, −1
15	1.01, −2	−3.23, −1	3.24, −1	3.76, −1	3.27, −2	−5.49, −1	4.30, −1
20	9.54, −3	−3.46, −1	3.47, −1	3.13, −1	2.84, −2	−5.83, −1	4.25, −1
25	8.92, −3	−3.18, −1	3.19, −1	2.30, −1	1.55, −2	−5.31, −1	4.21, −1
30	8.26, −3	−2.50, −1	2.51, −1	1.36, −1	−4.86, −3	−4.11, −1	4.16, −1
35	7.59, −3	−1.54, −1	1.55, −1	3.83, −2	−3.25, −2	−2.50, −1	4.07, −1
40	6.92, −3	−4.39, −2	4.53, −2	−5.64, −2	−6.80, −2	−7.11, −2	3.87, −1
45	6.31, −3	6.55, −2	−6.39, −2	−1.44, −1	−1.12, −1	1.05, −1	3.44, −1
50	5.79, −3	1.59, −1	−1.57, −1	−2.21, −1	−1.65, −1	2.59, −1	2.67, −1
55	5.38, −3	2.23, −1	−2.22, −1	−2.85, −1	−2.24, −1	3.74, −1	1.55, −1
60	5.08, −3	2.53, −1	−2.51, −1	−3.33, −1	−2.86, −1	4.43, −1	1.90, −2
65	4.86, −3	2.49, −1	−2.47, −1	−3.65, −1	−3.46, −1	4.63, −1	−1.19, −1
70	4.70, −3	2.20, −1	−2.18, −1	−3.78, −1	−4.00, −1	4.39, −1	−2.39, −1
75	4.59, −3	1.78, −1	−1.76, −1	−3.69, −1	−4.45, −1	3.81, −1	−3.29, −1
80	4.53, −3	1.33, −1	−1.31, −1	−3.35, −1	−4.77, −1	3.05, −1	−3.84, −1
85	4.48, −3	9.34, −2	−9.11, −2	−2.80, −1	−4.95, −1	2.27, −1	−4.09, −1
90	4.43, −3	6.43, −2	−6.19, −2	−2.08, −1	−5.00, −1	1.62, −1	−4.11, −1
95	4.34, −3	4.71, −2	−4.45, −2	−1.30, −1	−4.95, −1	1.21, −1	−3.99, −1
100	4.19, −3	4.11, −2	−3.81, −2	−5.91, −2	−4.84, −1	1.06, −1	−3.79, −1
105	3.96, −3	4.43, −2	−4.11, −2	−3.86, −3	−4.71, −1	1.15, −1	−3.58, −1
110	3.63, −3	5.47, −2	−5.10, −2	2.90, −2	−4.57, −1	1.42, −1	−3.38, −1
115	3.21, −3	6.95, −2	−6.55, −2	3.53, −2	−4.42, −1	1.78, −1	−3.20, −1
120	2.74, −3	8.54, −2	−8.11, −2	1.26, −2	−4.23, −1	2.14, −1	−3.02, −1
125	2.26, −3	9.64, −2	−9.18, −2	−3.82, −2	−3.94, −1	2.30, −1	−2.78, −1
130	1.80, −3	9.17, −2	−8.71, −2	−1.08, −1	−3.47, −1	2.02, −1	−2.32, −1
135	1.43, −3	5.44, −2	−5.04, −2	−1.72, −1	−2.74, −1	1.06, −1	−1.35, −1
140	1.17, −3	−3.20, −2	3.47, −2	−1.95, −1	−1.76, −1	−6.23, −2	5.67, −2
145	1.03, −3	−1.59, −1	1.60, −1	−1.50, −1	−7.27, −2	−2.59, −1	3.32, −1
150	1.02, −3	−2.76, −1	2.75, −1	−4.47, −2	1.18, −2	−4.19, −1	5.54, −1
155	1.11, −3	−3.32, −1	3.30, −1	8.61, −2	6.34, −2	−4.98, −1	6.19, −1
160	1.27, −3	−3.20, −1	3.17, −1	2.10, −1	8.32, −2	−4.93, −1	5.84, −1
165	1.45, −3	−2.63, −1	2.61, −1	3.11, −1	7.93, −2	−4.20, −1	5.27, −1
170	1.61, −3	−1.83, −1	1.82, −1	3.83, −1	6.01, −2	−3.03, −1	4.79, −1
175	1.72, −3	−9.35, −2	9.26, −2	4.25, −1	3.21, −2	−1.58, −1	4.49, −1
180	1.76, −3	0.00, −0	0.00, −0	4.39, −1	0.00, −0	0.00, −0	4.39, −1

for 30 eV electrons scattering from argon, krypton and xenon in tables 1–3, respectively.

We note that for all three atoms there is very little variation of the differential cross section with angle. This is characteristic of exchange scattering. There is an approximate relationship $S_P \approx -S_A$ which is evident from the data in the tables. It holds quite well for argon but not as well for krypton and xenon, especially in regions where there are nodes in these functions. Bartschat and Madison (1988) noted the same relationship for the excitation of the $^3P_2$ state of xenon and mercury. All the η parameters for the three atoms take on significant values over much of the angular range. In particular, it is interesting to note that η_1^y, which had a nearly zero value for the integrated parameter, approaches its maximum magnitude of unity for large angle scattering in xenon. Results for these parameters at other values of the incident energy are available from the authors upon request.

In table 4, we give our values for the integrated cross sections at various low and medium energy values.

Table 2. Differential scattering parameters for the excitation of the $^3D_3$ state of krypton by 30 eV electrons.

Angle	σ	S_A	S_P	P_1	P_2	P_3	P_4
0	1.30, −3	0.00, −0	0.00, −0	4.05, −1	0.00, −0	0.00, −0	4.05, −1
5	1.72, −3	−2.44, −1	2.44, −1	2.87, −1	1.43, −1	−3.77, −1	4.45, −1
10	2.86, −3	−1.83, −1	1.82, −1	1.38, −1	1.15, −1	−2.69, −1	4.90, −1
15	4.44, −3	−4.70, −2	4.63, −2	4.90, −2	5.42, −2	−6.62, −2	4.99, −1
20	6.01, −3	6.49, −2	−6.57, −2	−6.74, −3	8.13, −3	9.71, −2	4.85, −1
25	7.12, −3	1.45, −1	−1.46, −1	−4.46, −2	−2.04, −2	2.14, −1	4.65, −1
30	7.55, −3	2.01, −1	−2.02, −1	−7.19, −2	−3.56, −2	2.97, −1	4.44, −1
35	7.28, −3	2.37, −1	−2.38, −1	−9.24, −2	−4.15, −2	3.52, −1	4.23, −1
40	6.54, −3	2.57, −1	−2.57, −1	−1.09, −1	−4.20, −2	3.81, −1	4.03, −1
45	5.57, −3	2.58, −1	−2.56, −1	−1.25, −1	−4.13, −2	3.84, −1	3.78, −1
50	4.60, −3	2.38, −1	−2.35, −1	−1.42, −1	−4.46, −2	3.56, −1	3.44, −1
55	3.75, −3	1.97, −1	−1.93, −1	−1.67, −1	−5.75, −2	2.97, −1	2.92, −1
60	3.07, −3	1.40, −1	−1.34, −1	−2.02, −1	−8.61, −2	2.11, −1	2.13, −1
65	2.58, −3	7.74, −2	−6.78, −2	−2.47, −1	−1.37, −1	1.12, −1	9.89, −2
70	2.26, −3	2.28, −2	−1.01, −2	−2.92, −1	−2.13, −1	1.93, −2	−4.52, −2
75	2.13, −3	−9.15, −3	2.49, −2	−3.14, −1	−3.11, −1	−4.31, −2	−1.84, −1
80	2.18, −3	−1.38, −2	3.21, −2	−2.85, −1	−4.09, −1	−5.88, −2	−2.73, −1
85	2.38, −3	1.86, −4	1.97, −2	−2.04, −1	−4.77, −1	−3.30, −2	−3.01, −1
90	2.70, −3	1.98, −2	1.02, −3	−9.69, −2	−5.01, −1	1.10, −2	−2.90, −1
95	3.05, −3	3.72, −2	−1.60, −2	2.75, −3	−4.90, −1	5.33, −2	−2.66, −1
100	3.37, −3	5.10, −2	−2.95, −2	7.75, −2	−4.58, −1	8.72, −2	−2.43, −1
105	3.56, −3	6.24, −2	−4.06, −2	1.23, −1	−4.18, −1	1.15, −1	−2.27, −1
110	3.57, −3	7.37, −2	−5.15, −2	1.39, −1	−3.76, −1	1.41, −1	−2.19, −1
115	3.39, −3	8.72, −2	−6.45, −2	1.26, −1	−3.34, −1	1.71, −1	−2.19, −1
120	3.03, −3	1.05, −1	−8.17, −2	8.30, −2	−2.93, −1	2.09, −1	−2.24, −1
125	2.55, −3	1.28, −1	−1.05, −1	8.95, −3	−2.51, −1	2.55, −1	−2.30, −1
130	2.05, −3	1.57, −1	−1.35, −1	−8.93, −2	−2.04, −1	3.04, −1	−2.21, −1
135	1.61, −3	1.84, −1	−1.67, −1	−1.81, −1	−1.51, −1	3.36, −1	−1.59, −1
140	1.33, −3	1.92, −1	−1.84, −1	−2.07, −1	−9.25, −2	3.17, −1	2.37, −2
145	1.25, −3	1.59, −1	−1.63, −1	−1.28, −1	−4.27, −2	2.42, −1	3.18, −1
150	1.39, −3	9.55, −2	−1.08, −1	1.60, −2	−1.15, −2	1.45, −1	5.20, −1
155	1.72, −3	3.99, −2	−5.45, −2	1.57, −1	3.14, −3	6.85, −2	5.54, −1
160	2.19, −3	7.49, −3	−2.05, −2	2.62, −1	8.08, −3	2.15, −2	5.16, −1
165	2.69, −3	−6.28, −3	−3.61, −3	3.33, −1	8.35, −3	−1.29, −3	4.70, −1
170	3.13, −3	−8.92, −3	2.38, −3	3.75, −1	6.43, −3	−8.30, −3	4.34, −1
175	3.44, −3	−5.70, −3	2.47, −3	3.98, −1	3.44, −3	−6.22, −3	4.13, −1
180	3.55, −3	0.00, −0	0.00, −0	4.05, −1	0.00, −0	0.00, −0	4.05, −1

Table 3. Differential scattering parameters for the excitation of the $^3D_3$ state of xenon by 30 eV electrons.

Angle	σ	S_A	S_P	P_1	P_2	P_3	P_4
0	2.53, −3	0.00, −0	0.00, −0	3.87, −1	0.00, −0	0.00, −0	3.87, −1
5	2.72, −3	−2.90, −1	2.88, −1	3.22, −1	6.83, −2	−4.62, −1	4.08, −1
10	3.33, −3	−3.11, −1	3.08, −1	1.67, −1	4.78, −2	−4.73, −1	4.39, −1
15	4.32, −3	−1.39, −1	1.36, −1	1.15, −2	−1.84, −2	−2.00, −1	4.39, −1
20	5.36, −3	4.55, −2	−4.82, −2	−9.92, −2	−7.50, −2	7.93, −2	4.06, −1
25	6.05, −3	1.78, −1	−1.78, −1	−1.67, −1	−1.11, −1	2.78, −1	3.58, −1
30	6.24, −3	2.50, −1	−2.48, −1	−2.00, −1	−1.29, −1	3.90, −1	3.02, −1
35	6.01, −3	2.62, −1	−2.55, −1	−2.07, −1	−1.31, −1	4.13, −1	2.41, −1
40	5.57, −3	2.16, −1	−2.02, −1	−1.90, −1	−1.16, −1	3.42, −1	1.78, −1

Table 3. Continued.

Angle	σ	S_A	S_P	P_1	P_2	P_3	P_4
45	5.07, −3	1.25, −1	−1.04, −1	−1.60, −1	−8.11, −2	1.91, −1	1.19, −1
50	4.58, −3	1.80, −2	1.10, −2	−1.29, −1	−3.08, −2	−8.84, −4	6.95, −2
55	4.11, −3	−7.05, −2	1.07, −1	−1.10, −1	1.98, −2	−1.72, −1	3.07, −2
60	3.70, −3	−1.14, −1	1.55, −1	−1.07, −1	4.34, −2	−2.64, −1	4.85, −3
65	3.38, −3	−1.02, −1	1.44, −1	−1.14, −1	1.21, −2	−2.48, −1	−5.39, −3
70	3.15, −3	−4.63, −2	8.66, −2	−1.17, −1	−8.29, −2	−1.45, −1	4.52, −4
75	3.01, −3	2.25, −2	1.17, −2	−1.06, −1	−2.20, −1	−1.36, −2	1.62, −2
80	2.91, −3	7.59, −2	−5.03, −2	−7.94, −2	−3.61, −1	9.39, −2	2.95, −2
85	2.82, −3	1.00, −1	−8.45, −2	−4.41, −2	−4.70, −1	1.53, −1	2.79, −2
90	2.74, −3	9.73, −2	−9.18, −2	−9.51, −3	−5.18, −1	1.66, −1	3.42, −3
95	2.68, −3	7.77, −2	−8.24, −2	1.33, −2	−4.86, −1	1.50, −1	−4.57, −2
100	2.65, −3	5.41, −2	−6.82, −2	1.33, −2	−3.67, −1	1.25, −1	−1.15, −1
105	2.66, −3	3.45, −2	−5.66, −2	−1.69, −2	−1.81, −1	1.04, −1	−1.94, −1
110	2.71, −3	1.92, −2	−4.75, −2	−7.72, −2	2.52, −2	8.70, −2	−2.74, −1
115	2.75, −3	2.28, −3	−3.47, −2	−1.59, −1	1.99, −1	5.97, −2	−3.45, −1
120	2.76, −3	−2.47, −2	−1.03, −2	−2.50, −1	3.05, −1	3.52, −3	−4.00, −1
125	2.70, −3	−6.89, −2	3.28, −2	−3.33, −1	3.30, −1	−9.49, −2	−4.34, −1
130	2.58, −3	−1.35, −1	9.93, −2	−3.95, −1	2.83, −1	−2.37, −1	−4.33, −1
135	2.40, −3	−2.25, −1	1.91, −1	−4.19, −1	1.89, −1	−4.09, −1	−3.80, −1
140	2.19, −3	−3.34, −1	3.04, −1	−3.97, −1	7.95, −2	−5.87, −1	−2.54, −1
145	1.99, −3	−4.49, −1	4.26, −1	−3.30, −1	−1.53, −2	−7.43, −1	−5.31, −2
150	1.81, −3	−5.51, −1	5.36, −1	−2.31, −1	−7.62, −2	−8.57, −1	1.84, −1
155	1.66, −3	−6.15, −1	6.09, −1	−1.12, −1	−9.98, −2	−9.21, −1	3.82, −1
160	1.54, −3	−6.23, −1	6.25, −1	1.51, −2	−9.34, −2	−9.26, −1	4.85, −1
165	1.44, −3	−5.62, −1	5.69, −1	1.43, −1	−6.93, −2	−8.55, −1	4.95, −1
170	1.36, −3	−4.30, −1	4.37, −1	2.62, −1	−4.06, −2	−6.81, −1	4.53, −1
175	1.31, −3	−2.34, −1	2.39, −1	3.52, −1	−1.69, −2	−3.87, −1	4.06, −1
180	1.30, −3	0.00, −0	0.00, −0	3.87, −1	0.00, −0	0.00, −0	3.87, −1

Table 4. Values of the integrated cross sections for the excitation of the $^3D_3$ states of argon, krypton and xenon.

Energy (eV)	Total cross section (a_0^2)		
	Argon	Krypton	Xenon
15	3.038, −1	6.825, −1	3.734, −1
20	4.033, −1	1.973, −1	1.580, −1
25	1.252, −1	7.889, −2	9.605, −2
30	5.407, −2	4.108, −2	4.064, −2
40	1.729, −2	1.756, −2	1.164, −2
50	8.169, −3	9.643, −3	5.906, −3
60	4.778, −3	5.682, −3	3.809, −3
80	2.207, −3	2.224, −3	1.968, −3
100	1.222, −3	1.038, −3	1.056, −3

4. Conclusions

We have presented results of our calculations in the RDW approximation for the integrated Stokes parameters and integrated cross sections for the excitation of the $^3D_3$ state of argon, krypton and xenon. We have also given a set of differential parameters for incident electrons at 30 eV. Our results for the integrated Stokes parameters agree quite well with the DWBI calculations reported by Furst *et al* (1993) for argon and krypton, but there are greater

differences in xenon, especially for η_3^γ. This is not unexpected since xenon, being the heaviest of the three atoms, would show enhanced relativistic effects. For all three atoms our results seem to approach the exact threshold values of η_2^γ and η_3^γ.

In the near threshold region, where comparison with the experimental data is meaningful, both sets of calculations are in reasonable agreement with the experimental results of Furst *et al*, with the DWB1 approximation yielding better agreement for argon, and the RDW approximation being closer for krypton and xenon.

We have included some differential values of easily measured scattering quantities to indicate the rich structure they possess and to encourage experimental measurements of them.

Acknowledgments

We would like to thank Professor Bartschat for providing us with details of his calculations of the Stokes parameters. We are grateful to Professors T J Gay and D H Madison for providing us with numerical tables of their data and for certain unpublished results. This research was supported by the Natural Sciences and Engineering Research Council of Canada. One of us (RS) is grateful for financial support which made this collaboration possible.

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