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Total cross sections for electron scattering by He

F J de Heer and R H J Jansen†
FOM-Institute for Atomic and Molecular Physics, Kruislaan 407, Amsterdam(Wgm), The Netherlands

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Abstract. A set of total cross sections for scattering of electrons by He has been evaluated over the energy range of zero to 3000 eV by means of the analysis of experiments and theories on total cross sections for elastic scattering, ionisation and excitation, and on differential cross sections for elastic and inelastic scattering. Between 0 and 19·8 eV, where no inelastic processes occur, the total cross sections for scattering are equal to those for elastic scattering. Above 19·8 eV we have evaluated total cross sections for scattering of electrons by adding those for ionisation, excitation and elastic scattering. The total cross sections thus obtained are probably accurate to about 5% over a large part of the energy range. They appear to be in very good agreement with the recent experimental results of Blaauw *et al.* The output of this work has already been proved to be useful for application in the dispersion relation for forward scattering in electron–helium collisions.

1. Introduction

The critical analysis and evaluation of cross sections for total scattering by noble gases over a large energy range was started by de Heer and Jansen (1975a, b) in connection with the theoretical work of Bransden and McDowell (see for instance 1970) on the phaseshift and dispersion-relation analysis of electron–atom scattering. It was clear that next to accurate differential elastic cross sections there was a great need for accurate total cross sections, including both elastic and inelastic scattering.

A set of total cross sections of electron scattering by He, σ_{tot}, has been evaluated over the energy range zero to 3000 eV by means of an analysis of experiments and theories on total cross sections for elastic scattering, σ_{el}, ionisation, σ_{ion}, and excitation, σ_{exc} and on differential cross sections for elastic and inelastic scattering.

Above 19·8 eV, total cross sections for He have been evaluated by adding experimental data for σ_{el}, σ_{ion} and σ_{exc} with the aim of getting σ_{tot} with an accuracy to about 5%. The data presented here are mostly in close agreement with those of the original work of de Heer and Jansen (1975a), but supersede those data.

Various direct measurements of total electron scattering using a simple beam-attenuation method (see Ramsauer 1921) have been carried out for several noble gases. At low energies, where the inelastic processes do not yet occur, these studies have been summarised by Andrick (1973). Another critical review of the different

† Present address: Waterloopkundig Laboratorium, Postbus 177, Delft, The Netherlands.

experiments has been given by Bederson and Kieffer (1971). From the latter review and remarks made by Inokuti and McDowell (1974) it became clear that more reliable cross sections for total scattering were needed at energies above about 30 eV. This has led to the analysis of de Heer and Jansen (1975a, b) presented in this article and to new recent experimental results on total scattering (see for instance Blaauw *et al* 1977). The data of de Heer and Jansen (1975a) have already been proved to be useful in the dispersion relation for forward scattering in electron–helium collisions (see de Heer *et al* 1976).

In the following sections, we shall show how values for σ_{ion}, σ_{exc} and σ_{el} are derived from different experimental (and theoretical) results leading to σ_{tot} accurate to about 5%. The idea of adding total cross sections for ionisation, excitation and elastic scattering to get the total cross sections for scattering has recently been carried out in an experiment by Jost and Möllenkamp (1977). They determined σ_{ion}, σ_{exc} and σ_{el} from their energy spectrum of scattered and ejected electrons in the angular range of $\theta = 3$–$160°$. Because of the similarity of their experimental method and our semi-empirical method, their values of σ_{ion}, σ_{exc} and σ_{el} will be compared separately with ours.

The error in σ_{tot} is calculated by taking the root of the sum of the squared errors in σ_{ion}, σ_{exc} and σ_{el} separately. This procedure is also applied for σ_{exc}, which is also evaluated by a sum of terms.

2. Ionisation

Absolute ionisation cross sections were derived by using the experimental results of Rapp and Englander-Golden (1965, error 7%), Smith (1930, error 5%), Harrison (1956, error 8%), Asundi and Kurepa (1963, error 8%), Schram *et al* (1964, 1966, error 7%) and Gaudin and Hagemann (1967, error 10%). The errors given here are those quoted by the authors. When no error is known, we have taken 8%. Most data are given in the compilation of Kieffer (1965) and the addendum (Kieffer 1966), see also Kieffer and Dunn (1966). The average experimental values given in table 1 have been derived by a procedure given by Langenberg and van Eck (1976). A weighted average has been derived taking into account the errors given above. Data have only been considered at the energies quoted in the table. The calculations lead to an external and internal error of which the largest is given in table 1. Mostly these two errors differ by less than a factor of 2. The data thus obtained contain errors between about 4% and 11%.

The ionisation cross sections in column 2 of table 1 are so-called gross ionisation cross sections, that is

$$\sigma_{\text{gross ion}} = \sigma_{\text{i}}(\text{He}^{+}) + 2\sigma_{\text{i}}(\text{He}^{2+}). \tag{1}$$

In fact, what we need are the counting ionisation cross sections defined by

$$\sigma_{\text{count ion}} = \sigma_{\text{i}}(\text{He}^{+}) + \sigma_{\text{i}}(\text{He}^{2+}). \tag{2}$$

However, because $\sigma_{\text{i}}(\text{He}^{2+})/\sigma_{\text{gross ion}}$ is less than 6×10^{-3} throughout the whole energy range (see for instance Van der Wiel *et al* 1969, Bleakney 1936) $\sigma_{\text{gross ion}}$ and $\sigma_{\text{count ion}}$ are nearly identical for He and have been taken to be both equal to σ_{ion} as quoted in table 1.

The averaged ionisation data of table 1 are in reasonable agreement with the

Table 1. Total cross sections for ionisation, σ_{ion}, excitation, σ_{exc}, and inelastic scattering, σ_{inel}, in units of a_0^2. The numbers in parentheses are the total errors in the last significant digits.

E(eV)	σ_{ion} experimental average	σ_{ion} exp (Jost and Möllenkamp 1977)	σ_{ion} theory (Kim and Inokuti 1977)	σ_{exc} semi-empirical	σ_{inel} semi-empirical	σ_{ion} exp (Jost and Möllenkamp 1977)	σ_{inel} theory (Kim and Inokuti 1977)
30	0·2321 (86)			0·5830 (1084)	0·8151 (1090)		
40	0·6030 (225)			0·6680 (617)	1·271 (69)		
50	0·8445 (315)			0·6825 (475)	1·527 (57)		
60	0·9984 (421)			0·7314 (406)	1·730 (58)		
70	1·097 (46)			0·6941 (346)	1·791 (58)		
80	1·175 (44)			0·6740 (322)	1·849 (55)		
90	1·211 (56)			0·6683 (312)	1·879 (64)		
100	1·229 (58)		1·417	0·6603 (306)	1·889 (66)		2·230
150	1·191 (92)	1·24	1·414	0·5886 (273)	1·780 (96)	1·85	2·077
200	1·126 (51)	1·20	1·286	0·5217 (244)	1·648 (57)	1·73	1·848
300	0·9437 (468)	0·99	1·049	0·4307 (204)	1·374 (51)	1·39	1·484
400	0·8084 (461)	0·83	0·8796	0·3547 (187)	1·163 (50)	1·16	1·239
500	0·7023 (399)	0·73	0·7580	0·3085 (145)	1·011 (43)	0·98	1·065
600	0·6210 (362)		0·6671	0·2683 (128)	0·8893 (384)		0·9368
700	0·5592 (330)		0·5966	0·2427 (116)	0·8019 (350)		0·8377
800	0·5102 (308)		0·5403	0·2169 (104)	0·7271 (325)		0·7588
900	0·4681 (513)		0·4943	0·1990 (95)	0·6671 (522)		0·6944
1000	0·4300 (273)		0·4559	0·1839 (88)	0·6139 (287)		0·6408
2000	0·2403 (192)		0·2625	0·1079†	0·3482 (199)		0·3705
3000	0·1710 (124)		0·1877	0·07798†	0·2490 (130)		0·2658
4000	0·1346 (85)		0·1473	0·06172†	0·1963 (90)		0·2092

† Theoretical.

experimental data of Jost and Möllenkamp (1977), which were not obtained by the usual condenser method, but from the energy spectrum of scattered and ejected electrons (see §1). They quote the accuracy to be about 5%. Theoretical Born exchange cross sections are given according to Kim and Inokuti (1971). We see that above 500 eV the averaged experimental and theoretical cross sections differ from 5·8 to 10·9%, which is very close to the error limits.

3. Excitation

In order to get the cross sections for excitation to discrete states of the neutral He atom, the procedures followed above and below 100 eV differed because above 100 eV we can neglect the contribution due to triplet states as only the excitation to singlet states is important. A simplified level scheme of He is given in figure 1. For the determination of the total cross section for singlet excitation we use the formula

$$\sigma(\text{singlet}) = \sigma \sum_n (n^1\text{P} + \text{cascade}) + \sigma(2^1\text{S}). \tag{3}$$

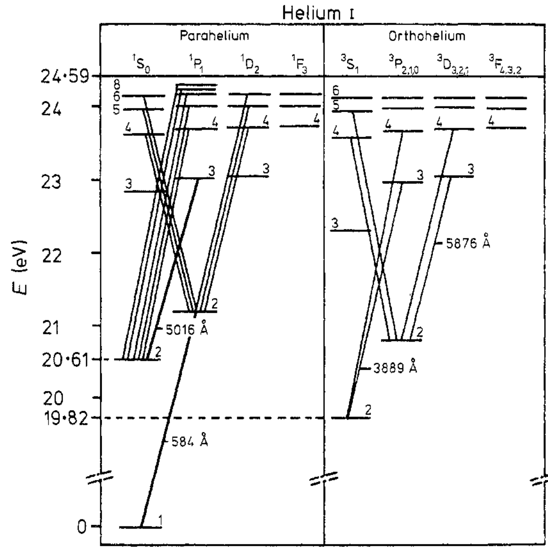

Figure 1. Level scheme of He I.

Here (n^1P + cascade) denotes the sum of the direct and the indirect excitation of n^1P levels, the latter via cascade from higher lying m^1S and m^1D levels ($m > n$). In this formula excitation of ^{1}S, ^{1}D, ^{1}F, etc, states is included, since these levels all decay in one or more steps to the ^{1}P levels and are therefore accounted for by cascade. We shall not consider cascade contributions from higher to lower ^{1}P states via double transitions for instance $4^1\mathrm{P} \rightarrow 3^1\mathrm{S} \rightarrow 2^1\mathrm{P}$. These contributions are of negligible importance.

The cross sections for (n^1P + cascade) have been obtained by de Jongh (1971) or de Jongh and van Eck (1971) and by Donaldson *et al* (1972), who measured the intensity of the n^1P–1^1S resonance lines ($n = 2, 3, 4$) in electron–helium collisions between threshold and 2000 eV. Because the data of both groups are in very close agreement with each other and consistent with the theoretical Born–Bethe calculations of Kim and Inokuti (1968), we decided not to average the great number of optical measurements on resonance levels and instead to use the representative data of de Jongh and van Eck (1971). The justification of this choice is discussed later. The only exception is at 30 eV where the cross section of de Jongh is probably too small; here we used the data of Donaldson *et al* (1972) for pure 2^1P, 3^1P and 4^1P cross sections and the cascade contributions as given by de Jongh (1971).

The data of de Jongh (1971) and of Donaldson *et al* (1972) are not absolute, but have been normalised by means of the Bethe theory. According to this theory, the cross section for an optically allowed transition is given by

$$\sigma_n = \frac{4\pi a_0^2}{E/R} M_n^2 \ln\left(4c_n \frac{E}{R}\right) \tag{4}$$

where a_0 is the first Bohr radius, R the Rydberg energy, E the kinetic energy of the incident electrons corrected for relativistic effects, and c_n a constant dependent on the properties of the excited state. For excitation to a discrete state with energy

E_n, M_n^2 is related to the optical oscillator strength f_n given by

$$M_n^2 = f_n R/E_n. \tag{5}$$

For symmetry-forbidden excitation processes, the excitation cross section is given by

$$\sigma_n = \frac{4\pi a_0^2}{E/R} b_n. \tag{6}$$

Combining equations (4) and (6) it is clear that the cross section for (n^1P excitation + cascade) can be presented by

$$\sigma_n = \frac{4\pi a_0^2}{E/R} M_n^2 \ln\left(4c'_n \frac{E}{R}\right) \tag{7}$$

where c_n in equation (4) is replaced by c'_n.

When plotting the experimental σ_n values of equation (7) in the form ($\sigma_n E$–$\ln E$), as suggested by Fano (1954), a linear relation is found at sufficiently high impact energies. The slope of the straight line is connected to M_n^2 or f_n. Both de Jongh (1971) and Donaldson *et al* (1972) normalised their experimental slope to that corresponding to the theoretical f_n values, which are very well known. This procedure leads, within the experimental error, to the same values as for 3^1P and 4^1P cross sections derived from absolute light-intensity measurements of the 3^1P–2^1S and 4^1P–2^1S lines (see for instance Moustafa Moussa *et al* 1969) and derived by integration over angle of the differential inelastic scattering for the 2^1P level of helium (see, for instance, Dillon and Lassettre 1975). When we consider the total excitation cross section to n^1P levels including cascade (see below) then the difference between the data of de Jongh (1971) and of Donaldson *et al* (1972) vary between $-4{\cdot}2\%$ and $+5\%$, excluding the measurement at 30 eV. We give some preference to the data of the Jongh (1971) because for 2^1P, 3^1P and 4^1P their experimental c_n values (see equation (4)) agree better with theory than those of Donaldson *et al* (1972). Besides when considering 2^1P, the cross sections of de Jongh (1971) agree very well with the Born–Bethe calculations of Kim and Inokuti (1968) above 300 eV impact energy (within 2%) while those cross sections of Donaldson *et al* are more than 5% different from these calculations even at 2000 eV. Further the 2^1P cross sections of de Jongh (1971) agree better than 2% with the integrated differential cross sections of Dillon and Lassettre (1975) in the overlapping energy region of 200 to 700 eV.

Because de Jongh (1971) only provides data for $n = 2, 3, 4$ levels of ^{1}P, we had to estimate the contribution from higher levels. This was done by assuming the relevant cross sections above $n = 4$ to be proportional to n^{-3} and to exhibit the same shapes as the 4^1P cross section. For our purpose this approximation is sufficiently good. In a more accurate calculation n^{-3} should be replaced by $(n - \delta)^{-3}$ where δ is the quantum defect (see, for instance, Gabriel and Heddle 1960) and the excitation cross section should be taken to be uniform in shape as a function of E/E_n.

The total excitation cross section to n^1P levels, including cascade is now given by

$$\sigma \sum_n (n^1\mathrm{P} + \text{cascade})$$
$$= \sigma \sum_{2,3} (n^1\mathrm{P} + \text{cascade}) + 4^3\left(\sum_1^\infty \frac{1}{n^3} - \sum_{1,2,3} \frac{1}{n^3}\right)\sigma(4^1\mathrm{P} + \text{cascade}) \tag{8}$$

Table 2. Cross sections for excitation processes in units of a_0^2. The numbers in parentheses are the total errors in the last significant digits.

E(eV)	$\sum(n^1\mathrm{P}$ + cascade)	$2^1\mathrm{S}$	Singlet exc semi-empirical	Singlet exc theory (Kim and Inokuti 1971)	Triplet exc semi-empirical	Singlet + triplet exc semi-empirical	Singlet + triplet exc, experimental (Jost and Möllenkamp 1977)
30	0·2520 (756)	0·07730 (1546)	0·3293 (772)		0·2537 (761)	0·5830 (1084)	
40	0·4051 (202)	0·06965 (697)	0·4748 (214)		0·1932 (579)	0·6680 (617)	
50	0·4850 (242)	0·06243 (561)	0·5474 (248)		0·1351 (405)	0·6825 (475)	
60	0·5811 (290)	0·05691 (455)	0·6380 (294)		0·0934 (280)	0·7314 (406)	
70	0·5804 (290)	0·05181 (414)	0·6322 (292)		0·0619 (186)	0·6941 (346)	
80	0·5802 (290)	0·04884 (342)	0·6290 (292)		0·0450 (135)	0·6740 (322)	
90	0·5872 (294)	0·04757 (318)	0·6348 (296)		0·0336 (101)	0·6683 (312)	
100	0·5877 (294)	0·04587 (275)	0·6336 (295)	0·8117	0·02671 (801)	0·6603 (306)	
150	0·5426 (271)	0·03801 (228)	0·5806 (272)	0·6625	0·007994 (2398)	0·5886 (273)	0·61
200	0·4883 (244)	0·03000 (150)	0·5183 (244)	0·5615	0·003373 (1012)	0·5217 (244)	0·53
300	0·4081 (204)	0·02260 (113)	0·4307 (204)	0·4351			0·40
400	0·3372 (187)	0·01750 (88)	0·3547 (187)	0·3586			0·33
500	0·2907 (145)	0·01510 (76)	0·3085 (145)	0·3070			0·25
600	0·2567 (128)	0·01157 (58)	0·2683 (128)	0·2695			
700	0·2320 (116)	0·01070 (54)	0·2427 (116)	0·2409			
800	0·2074 (104)	0·009548 (477)	0·2169 (104)	0·2183			
900	0·1901 (95)	0·008944 (447)	0·1990 (95)	0·1999			
1000	0·1762 (88)	0·007737 (387)	0·1839 (88)	0·1847			
2000				0·1079			
3000				0·07804			
4000				0·06177			

where

$$4^3\left(\sum_1^\infty \frac{1}{n^3} - \sum_{1,2,3} \frac{1}{n^3}\right) = 4^3(1{\cdot}202 - 1{\cdot}162) = 2{\cdot}56. \tag{9}$$

The numbers thus obtained for $\sigma\Sigma_n(n^1\mathrm{P}$ + cascade) are given in table 2; the error is estimated to be generally smaller than 5%.

To get the total singlet cross section (see equation (3)) we still need the 2^1S excitation cross section.

Reasonably accurate cross sections for 2^1S excitation (error 5%) have been obtained by Dillon and Lassettre (1975). They measured inelastic differential cross sections between 200 and 700 eV over an angular range of 7·5 to 35°. By fitting a polynomial to their differential scattering data for extrapolation to smaller and larger angles, they were able to obtain total cross sections for 2^1S excitation. These cross sections were shown to be in good agreement with Born exchange calculations. Cross section estimates below 200 eV and above 700 eV were obtained by assuming that the energy dependence of all the n^1S cross sections is the same as a function of E_{el}/E_n, where E_n is the excitation energy for the n^1S level. In this case we normalised the 4^1S data of van Raan *et al* (1971) to the 2^1S data of Dillon and Lassettre (1975). The numbers thus obtained are given in table 2 down to 30 eV. In table 3 we make a comparison of these extrapolated 2^1S cross sections and those obtained experimentally at about 30, 40, 50 and 100 eV by Trajmar (1973), Crooks (1972) and Hall *et al* (1973). The latter experimental 2^1S cross sections were obtained by integrating differential inelastic cross sections over all angles. This table demonstrates the reliability of our procedure to get the extrapolated 2^1S cross sections. Also included in table 3 is an experimental value of Brongersma *et al* (1972) at 23 eV from a direct measurement of the total excitation cross section by means of the RPD method. The 2^1S data of Rice *et al* (1972), not given in the table, show an energy dependence below 50 eV, which is not consistent with the other data. Their cross sections are about a factor of two lower in this region. When we add the second and third column in table 2 we get the cross sections for singlet excitation (see equation (3)), which can be compared with the theoretical values from the Born approximation (see Kim and Inokuti 1971, equation (50b)).

Table 3. 2^1S cross sections in a_0^2.

	Extrapolation of the integrated differential cross sections of Dillon and Lassettre (1975)	Integrated differential cross sections			RPD method
E(eV)		Trajmar (1973)	Hall *et al* (1973)	Crooks (1972)	Brongersma *et al* (1972)
23					0·06254
29·2			0·08613		
29·6		0·1040			
30	0·07730				
39·2			0·07505		
40	0·06965				
40·1		0·07541			
48·2			0·07005		
50	0·06243			0·06782	
100	0·04587			0·04710	

We see in table 2 that between 300 and 1000 eV the agreement between experiment and theory is excellent.

The total excitation cross section, σ_{exc}, is given by

$$\sigma_{exc} = \sigma(\text{singlet}) + \sigma(\text{triplet}) \tag{10}$$

and so we still need the cross section for excitations to triplet levels. For this purpose we apply the formula

$$\begin{aligned}\sigma(\text{triplet}) &= \sigma(2^3\text{S}) + \sum_{n\geqslant 2} \sigma(n^3\text{P}-2^3\text{S}) \\ &= \sigma(2^3\text{S}) + \sum_{n\geqslant 3} \sigma(n^3\text{P}-2^3\text{S}) + \sigma(2^3\text{P}-2^3\text{S}) \\ &= \sigma(2^3\text{S}) + \sum_{n\geqslant 3} \sigma(n^3\text{P}-2^3\text{S}) + \sigma(2^3\text{P} + \text{cascade}).\end{aligned} \tag{11}$$

This equation can be understood by considering the level scheme in figure 1. It is based on the fact that all levels excited decay to 2^3S via n^3P ($n \geqslant 2$). Hence the second term on the right-hand side of the first equation (11) is a sum of emission cross sections for n^3P–2^3S transitions including cascade transitions from higher ^{3}S and ^{3}D levels to the relevant ^{3}P levels.

The 2^3S scattering data of Crooks *et al* (1972), Trajmar (1973), Hall *et al* (1973) and Brongersma *et al* (1972) are compiled in table 4. By a graphical averaging procedure, we arrive at the 2^3S cross sections up to 70 eV given in tables 4 and 5. The numbers above 70 eV have been obtained by assuming that the energy dependence is the same function of E_{el}/E_n for 2^3S and 4^3S cross sections, where E_n is the excitation energy of the relevant level. For 4^3S, the cross sections of van Raan *et al* (1971) (marked by II in table 4 of their article) have been taken. The second term of the last line in equation (11) is derived as follows:

$$\sum_{n\geqslant 3} \sigma(n^3\text{P}-2^3\text{S}) = \sigma(^3\text{P}-2^3\text{S}) + 2{\cdot}56\sigma(4^3\text{P}-2^3\text{S}). \tag{12}$$

Here we have made a similar approximation to that in equations (8) and (9) and assumed that the emission cross sections with $n = 4, 5, 6, \ldots$ are proportional to n^{-3}.

Table 4. 2^3S cross sections in a_0^2.

	Integrated differential cross sections					RPD method	
E(eV)	Trajmar (1973)	Hall *et al* (1973)	Crooks *et al* (1972)	Crooks (1972)	Vriens *et al* (1968a)	Brongersma *et al* (1972)	Estimated
24						0·08399	
29·2		0·06076					
29·6	0·06934						
30							0·064
39·2		0·05004					
40			0·03656				0·044
40·1	0·04360						
48·2		0·02859					
50			0·02971	0·02983			0·028
60			0·01451				0·0185
70			0·007130				0·0125
80							0·0093
90							0·0073
100				0·004961	0·002842		0·0060

Table 5. Triplet cross sections in a_0^2.

E(eV)	2^3S	$1\cdot842 \times 3^3$P	2^3P + cascade	Triplet
30	0·064	0·0587	0·131	0·2537
40	0·044	0·05010	0·09907	0·1932
50	0·028	0·03358	0·07355	0·1351
60	0·0185	0·02304	0·05182	0·0934
70	0·0125	0·01613	0·03326	0·0619
80	0·0093	0·01191	0·02381	0·0450
90	0·0073	0·008757	0·01751	0·0336
100	0·0060	0·007701	0·01400	0·0277

Subsequently we replace both terms on the right-hand side of equation (12):

$$\begin{aligned}\sigma(3^3\mathrm{P}\text{–}2^3\mathrm{S}) &= \frac{A(3^3\mathrm{P}\text{–}2^3\mathrm{S})}{A(3^3\mathrm{P}\text{–}2^3\mathrm{S}) + A(3^3\mathrm{P}\text{–}3^3\mathrm{S})}\,\sigma(3^3\mathrm{P} + \text{cascade}) \\ \sigma(4^3\mathrm{P}\text{–}2^3\mathrm{S}) &= \frac{A(4^3\mathrm{P}\text{–}2^3\mathrm{S})}{\sum_{n=2}^{4} A(4^3\mathrm{P}\text{–}n^3\mathrm{S}) + A(4^3\mathrm{P}\text{–}3^3\mathrm{D})}\,\sigma(4^3\mathrm{P} + \text{cascade}).\end{aligned} \tag{13}$$

Using the transition probabilities of Gabriel and Heddle (1960), we find from equations (12) and (13):

$$\sum_{n\geqslant 3} \sigma(n^3\mathrm{P}\text{–}2^3\mathrm{S}) = 0\cdot8966\sigma(3^3\mathrm{P} + \text{cascade}) + 0\cdot7821 \times 2\cdot56\sigma(4^3\mathrm{P} + \text{cascade}). \tag{14}$$

Furthermore, by assuming that 3^3P and 4^3P cross sections have the same energy dependence (see de Heer *et al* 1969) and taking their cross section values, we get

$$\sum_{n\geqslant 3} \sigma(n^3\mathrm{P}\text{–}2^3\mathrm{S}) = 1\cdot818 \times \sigma(3^3\mathrm{P} + \text{cascade}) \tag{15}$$

which constitutes the basis for the numbers given in the third column of table 5.

For the third term in the last line of equation (11) we use the experimental emission cross sections of Jobe and St John (1967) (fourth column of table 5). The triplet cross sections between 30 and 100 eV are then calculated by application of equation (11) and (12) and are shown in the fifth column of table 5 and the sixth column of table 2.

The errors in the latter values are less than about 30%. This is not a serious deficiency in this treatment since $\sigma_{\text{triplet}} \ll \sigma_{\text{tot}}$. It remains to show that the optical (2^3P + cascade) data of Jobe and St John (1967), used in our calculation, are consistent with the inelastic scattering data of Crooks (1972), Trajmar (1973) and Hall *et al* (1973), noting that cascade is not included in the scattering experiments.

The correction due to cascade is calculated from

$$\sigma(2^3\mathrm{P} + \text{cascade}) = \sigma(2^3\mathrm{P}) + 1\cdot665\sigma(3^3\mathrm{S} + \text{cascade}) + 1\cdot963\sigma(3^3\mathrm{D} + \text{cascade}). \tag{16}$$

Using the data of de Heer *et al* (1969) for the last two terms on the right-hand side of equation (16), we find the cascade contribution as given in the ninth column of table 6. This contribution has been subtracted from the (2^3P + cascade) data of Jobe and St John to get the pure 2^3P cross sections in the fifth column of table 6. These values agree very well with the inelastic scattering data in the overlapping energy region. This is not the case when we apply the cascade correction as given

Table 6. 2^3P cross sections in a_0^2 and cascade contributions.

	2^3P					2^3P + cascade (Jobe and St John 1967)	Cascade (Jobe and St John‡)	Cascade (this work†)
E(eV)	Trajmar (1973)	Hall *et al* (1973)	Crooks (1972)	Jobe and St John†	Jobe and St John‡			
29·2		0·09864						
29·6	0·08363							
30				0·08542	0·07405	0·1306	0·05654	0·04861
39·2		0·07648						
40				0·06619	0·05618	0·09907	0·04289	0·03288
40·1	0·06326							
48·2		0·05003	0·06154					
50				0·05400	0·04171	0·07355	0·03184	0·01956
60				0·03942	0·02939	0·05182	0·02244	0·01242
70				0·02499	0·01887	0·03326	0·01440	0·008277
80				0·01747	0·01350	0·02381	0·01030	0·006333
90				0·01249	0·009929	0·01751	0·007577	0·005018
100				0·009857	0·007941	0·01400	0·006147	0·004146

Columns 2–4 contain integrated differential cross sections.
† Cascade correction according this work.
‡ Cascade correction according Jobe and St John, who measured 2^3P–2^3S emission.

by Jobe and St John themselves, shown in the seventh column of table 6, leading to their 2^3P cross sections in the sixth column.

In table 2 we have the cross section for excitation, σ_{exc}, calculated by adding the triplet excitation to the singlet excitation. The triplet cross sections at 150 and 200 eV for triplet excitation were derived by extrapolation of the triplet cross sections according to σ(triplet) $\propto E^{-3}$ (see Ochkur 1964, van Raan *et al* 1974) where E is the electron impact energy. Above 200 eV we can neglect σ(triplet) with respect to σ(singlet).

Our semi-empirical data for σ_{exc} are in good agreement with the experimental cross sections of Jost and Möllenkamp (1977), who quote an accuracy of about 5%.

4. Inelastic scattering

Adding the data for ionisation and excitation we directly obtain the cross section for total inelastic scattering, σ_{inel}. The results are shown in table 1, where σ_{inel} is compared with the experimental data of Jost and Möllenkamp (1977) and with the theoretical calculations of Kim and Inokuti (1971) who used the Born exchange approximation and only included singlet excitation. Within the error limit there is agreement between our data and those of Jost and Möllenkamp (1977) and our data are close to the theoretical values above about 600 eV. For energies above 1000 eV, we have taken theoretical values for σ_{exc} because the relevant experimental data of de Jongh (1971) and Donaldson *et al* (1972) did not extend further. This procedure is certainly justified because σ_{exc} agrees within a few per cent semi-empirically with the theoretical calculations of Kim and Inokuti (1971) between 300 and 1000 eV (see table 2).

5. Elastic scattering

The only term we still need for calculations of σ_{tot} is the total elastic scattering cross section, σ_{el}. This cross section has been calculated using the formula

$$\sigma_{el} = 2\pi \int_0^{\pi} \sigma_{el}(\theta) \sin\theta \, d\theta \tag{17}$$

where $\sigma_{el}(\theta)$ is the differential elastic scattering cross section. We have used the data of different groups and carried out the integration of $\sigma_{el}(\theta)\sin\theta$ for each of these groups when this had not previously been done. For an explanation of our procedure (see also Jansen 1975) we first discuss how we handle the data of Jansen *et al* (1976) who have measured σ_{el} only between $5° < \theta < 55°$ in the energy range of 100–3000 eV. In order to cover a larger angular range, these data were extended by absolute data of other authors normalised to the data of Jansen *et al*. The data used for the extension of the angular range were the following.

At 100 and 150 eV, for $55° < \theta < 150°$: Kurepa and Vusković (1975). At 200 eV, for $2° < \theta < 5°$: Bromberg (1974); for $55° < \theta < 110°$: Bromberg (1975 private communication), supplemented by Kurepa and Vusković (1975), and for $110° < \theta < 150°$: Kurepa and Vusković (1975). Between 300 and 700 eV, for $2° < \theta < 5°$: Bromberg (1974) and for $55° < \theta < 110°$: Bromberg (1975 private communication). At 1000 eV, for $55° < \theta < 150°$: N Oda, F Nishimura and S Tahita (1973 private communication).

Table 7. Total elastic cross sections (in units of a_0^2) obtained from experimental $\sigma_{el}(\theta)$ data. The numbers in parentheses are the total errors in the last significant digits.

E(eV)	Jansen *et al* (1976) 7%	Bromberg (1974, 1975†) 3%	Gupta and Rees (1975) Gupta (1975) 10%	Oda *et al* (1973)‡ 10%	Sethuraman *et al* (1972, 1974) 20%	McConkey and Preston (1975) 12%	Kurepa and Vusković (1975) 17·5%	Crooks (1972) 15%	Jost and Möllenkamp (1977) 5%	Vriens *et al* (1968b) 10%	Semi-empirical (average)
15						12·24					12·84
20						10·04					10·52
25					8·806	7·833					8·240
30						7·589					7·987
35						6·377					6·724
40						5·699					5·998
45						5·733					6·030
50			4·920		4·647	4·699		6·22			4·946 (326)
60						4·354					4·578
80						2·699					2·841
100	2·118		2·249		2·779		2·967	2·60		2·394	2·180 (131)
150	1·378		1·428				1·493		1·37	1·392	1·369 (51)
200	0·9771	0·9819	1·026		1·233	1·62	1·076	1·31	0·96	0·9676	0·9775 (223)
300	0·6046	0·6136			0·6565				0·59	0·5969	0·6029 (145)
400	0·4550	0·4430	0·4463					0·496	0·45	0·4461	0·4433 (103)
500	0·3481	0·3470	0·3425	0·3524	0·3712				0·35		0·3467 (79)
700	0·2321	0·2386		0·2285							0·2357 (63)
1000	0·1545			0·1385							0·1487 (85)
2000	0·07377										
3000	0·04777										

† J P Bromberg 1975 private communication.
‡ N Oda, F Nishimura and S Tahira 1973 private communication.

At larger energies there are no experimental results to extend the cross sections of Jansen *et al* (1976). However, they showed that their results at 2000 and 3000 eV practically coincide with the first Born approximation and the eikonal–Born series (EBS) calculations of Byron and Joachain (1975). We used these EBS results at 2000 and 3000 eV for $55° < \theta < 180°$.

Because it is experimentally difficult to measure at angles close to zero and 180°, $\sigma_{el}(\theta)$ is only available experimentally in an angular range between a lower limit $\theta_m \neq 0$ and an upper limit $\theta_M \neq \pi$. This leads to a splitting up of equation (17) into three terms

$$\sigma_{el} = 2\pi[I(0, \theta_m) + I(\theta_m, \theta_M) + I(\theta_M, \pi)] \tag{18}$$

where

$$I(x, y) = \int_x^y \sigma_{el}(\theta) \sin\theta \, d\theta \qquad 0 < \theta_m < \theta_M < \pi. \tag{19}$$

$I(\theta_m, \theta_M)$ was calculated by numerical integration. $I(0, \theta_m)$ and $I(\theta_M, \pi)$ were obtained by extrapolation of the integrand $\sigma_{el}(\theta)\sin\theta$. The latter is zero at $\theta = 0, \pi$. At small θ the integrand shows a steep increase as θ increases, followed by a maximum, whereas it is a slowly varying function of θ at large angles. Therefore we have extrapolated the integrand for $0 \leqslant \theta < \theta_m$ by a parabolic function and for $\theta_M < \theta \leqslant \pi$ by a linear function of θ.

The total elastic cross sections thus obtained, corresponding to Jansen *et al* (1976), are given in the second column of table 7. The uncertainty in the total elastic cross sections due to the extrapolation procedure will generally be very small. The contribution of $I(0, \theta_m)$ to σ_{el} varied from 3% at 100 eV, through 10% at 1000 eV to 24% at 3000 eV. Although the contribution at large energies is rather large, the parabolic extrapolation is quite accurate due to the steep, almost linear, rise of $\sigma_{el}(\theta)\sin\theta$ between zero and $\theta_m (< 5°)$. The contribution of $I(\theta_M, \pi)$ to σ_{el} was always smaller than 3%. The σ_{el} cross sections corresponding to McConkey and Preston (1975) (see table 7), who measured $\sigma_{el}(\theta)$ between 20° and 90°, have been obtained by extension of the angular range with the theoretical data of La Bahn and Callaway (1970). Their cross sections were normalised to those of McConkey and Preston at 20° and 90°. Then the relevant integration was carried out.

In table 7 only for the data of Crooks (1972), Vriens *et al* (1968b) and Jost and Möllenkamp (1977) have their own integrated values been used.

For the σ_{el} data at electron impact energies of 50 eV and above we have evaluated the average experimental values in the same way as for σ_{ion} in §2. The results are given in the last column of table 7. In addition the data below 50 eV in that column have been obtained by fitting the data of McConkey and Preston (1975) to the averaged values at 50 eV down to 15 eV.

The elastic scattering data below about 20 eV are equivalent to total scattering, because the inelastic channels are no longer open. We shall treat these data in detail in a separate article on total cross section measurements.

In table 8 we compare the averaged semi-expirical σ_{el} data for He with the theoretical results. We first consider data for energies of 100 eV and above. The first Born approximation seriously underestimates the cross section at low energies; at higher energies it tends to the semi-empirical values but the discrepancy still remains. The second Born calculation of Buckley and Walters (1974) overestimates the cross sections. The eikonal–Born series (EBS) theory of Byron and Joachain (1975, 1977b)

Table 8. Total elastic cross sections (in units of a_0^2). Comparison of semi-empirical and theoretical data. The numbers in parentheses are the total errors in the last significant digits.

		Theory											
		First Born	Second Born	DWSB	PWP	EBS	Phaseshift methods					SOP	OM
E(eV)	Semi-empirical	Buckley and Walters (1974)	Buckley and Walters (1974)	Dewangan and Walters (1977)	Jhanwar and Khare (1976)	Byron and Joachain (1975, 1977b)	Dewangan and Walters (1977)	La Bahn and Callaway (1969)	La Bahn and Callaway (1970)	Purcell *et al* (1970)	Ganas *et al* (1970)	Winters *et al* (1974)	Byron and Joachain (1977a)
15	12·84								12·66				
20	10·52								10·62				
25	8·240								9·04				
30	7·987								7·79				
35	6·724								6·80				
40	5·998								6·00				
45	6·030								5·35				
50	4·946(326)		8·15						4·81			5·72	
60	4·578								3·97				
70									3·35				
80	2·841								2·89				
90									2·52				
100	2·180(131)	1·29	3·24	2·38		2·41	2·01	2·23		2·51	2·26	2·37	2·52
150	1·369(51)	0·905	1·83					1·38		1·47	1·41		
200	0·9775(223)	0·697	1·23	1·00	1·02	1·04	0·911	0·977		1·00	0·995	1·00	1·04
300	0·6029(145)	0·478	0·723	0·616	0·613	0·636	0·578	0·609		0·615	0·608	0·613	0·623
400	0·4433(103)	0·361	0·500	0·443	0·435	0·453	0·421	0·441		0·431	0·430	0·434	0·443
500	0·3467(79)	0·292	0·380	0·346	0·337	0·351	0·330			0·345	0·331	0·336	0·343
700	0·2357(63)	0·211	0·256	0·237	0·232	0·241	0·231						0·235
1000	0·1487(85)			0·161	0·160	0·160	0·158						
2000				0·0779			0·0773						
3000				0·0515			0·0512						

DWSB Distorted-wave second Born.
PWP Plane-wave polarisation.
EBS Eikonal–Born series.
SOP Second-order potential.
OM Optical model.

and the distorted-wave second Born calculation of Dewangan and Walters (1977) generally agree with the present work between 300 and 1000 eV within the quoted error limit. There is also a good agreement with the plane wave + energy dependent polarisation calculation of Jhanwar and Khare (1976).

The simplest phaseshift method, the so-called pure static exchange calculation (see, for instance, Dewangan and Walters 1977), does not deviate more than about 7% from the semi-empirical cross sections between 200 and 1000 eV. There is also good agreement with the extended polarisation calculation of La Bahn and Callaway (1969). They have used a phaseshift method with second-order potentials accounting for adiabatic and non-adiabatic distortion of the atom and also with exchange effects taken into account. The theories of Purcell *et al* (1970) and Ganas *et al* (1970) are also based on a phaseshift method accounting for polarisation and provide data which are also in good agreement with our semi-empirical data. However, although the agreement is good, these three phaseshift calculations do not fulfill unitarity conditions. These conditions are met in the optical-model (OM) calculations of Byron and Joachain (1977a) and in the second-order potential approximation of Winters *et al* (1974). Above 100 eV again these theoretical cross sections are close to the present semi-empirical ones.

Between 15 and 100 eV the semi-empirical σ_{el} data are determined to a large part by the σ_{el} values of McConkey and Preston (1975). These values compare very well with the extended polarisation calculations of La Bahn and Callaway (1970). However, at some energies, where large differences are present, it appears that there must be some irregularities in the data of McConkey and Preston (namely at 45, 60 and 100 eV). For the evaluation of semi-empirical cross sections for total scattering we use the theoretical σ_{el} data of La Bahn and Callaway (1970) at 60 eV, and at 70 and 90 eV where no σ_{el} data can be derived from experiment.

6. Total scattering

As is clear from the introduction and the previous sections, the cross section for total scattering can now be calculated by adding σ_{el} and σ_{inel} and using semi-empirical values from tables 1 and 8 (only at 60, 70 and 90 eV are theoretical σ_{el} values used, see §5).

The values for σ_{tot} thus obtained are given in the second column of table 9 and the error varies between about 3 and 6%. The semi-empirical values are compared with some recent results of total cross section measurements and theory. A more complete comparison is postponed to the completion of an article on total scattering measurements by Blaauw *et al* (1977).

Our semi-empirical cross sections appear to be in very good agreement with the experimental ones of Jost and Möllenkamp (1977) and of Blaauw *et al* (1977).

The Born values in table 9 have been obtained by adding the three-term Bethe cross sections for total inelastic scattering according to Kim and Inokuti (1971) and the Born cross sections for total elastic scattering from Buckley and Walters (1974). We see that by 150 eV there is very good agreement between these Born data and the semi-empirical data. This agreement may be accidental and means that exchange and higher-order effects cancel each other out in the sum of the total elastic and the total inelastic cross sections. Other theoretical values obtained with higher-order approximations than the first Born are much higher than the present semi-empirical

Table 9. Total scattering cross sections (in units of a_0^2). The numbers in parentheses are the total errors in the last significant digits.

		Experimental			Theory					
E(eV)	Semi-empirical	Jost and Möllenkamp (1977)	Blaauw *et al* (1977)	Kauppila *et al* (1977)	Born	Second Born (Buckley and Walters 1974)	DWSB (Dewangan and Walters 1977)	EBS (Byron and Joachain 1975, 1977b)	OM (Byron and Joachain 1977a)	SOP (Winters *et al* 1974)
15				12·89						
20			10·90	10·71						
30	8·802		8·520							
40	7·269		7·009							
50	6·473 (330)		6·202			7·72				9·61
60	5·700		5·486							
70	5·141		4·945							
80	4·690		4·510							
90	4·399		4·225							
100	4·069 (146)		3·991		3·71	5·31	5·47	4·68	6·16	6·03
150	3·149 (109)	3·22	3·125		3·15	3·93		3·54		
200	2·626 (61)	2·69	2·592		2·67	3·13	3·20	2·92	3·37	3·55
300	1·977 (74)	1·98	1·990		2·04	2·26	2·32	2·15	2·38	2·54
400	1·606 (51)	1·61	1·646		1·65	1·79	1·83	1·71	1·86	2·00
500	1·358 (44)	1·33	1·355		1·39	1·49	1·52	1·43	1·54	1·67
700	1·038 (36)		1·053		1·07	1·12	1·15	1·09	1·16	
1000	0·7626 (415)				0·817		0·855	0·82		
2000	0·439						0·474			
3000	0·308						0·333			

DWSB Distorted-wave second Born.
EBS Eikonal–Born series.
OM Optical model.
SOP Second-order potential.

data between 100 and 200 eV but approach them at higher energies. The data of Winters *et al* (1974), however, still remain much larger.

The calculations of σ_{tot} by Inokuti and McDowell (1974), presented numerically by Bransden and Hutt (1975), are done by adding the integrated first Born $\sigma_{el}(\theta)$ and the two-term Bethe cross sections σ_{inel} of Kim and Inokuti (1968). These data are not given in table 9 but instead we give the better fitting Born σ_{el} + three-term Bethe σ_{inel} cross sections mentioned previously (see also de Heer *et al* 1976).

Summarising we can conclude that we have obtained semi-empirical cross sections for total scattering to an accuracy of about 3–6% which are consistent with the (recent) most reliable experiments on total scattering and the theoretical results at low and high energies. They have been proved to be useful for the study of the forward dispersion relations (see de Heer *et al* 1976).

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