

Total cross sections for electron scattering by molecular hydrogen

B van Wingerden[†], R W Wagenaar and F J de Heer

FOM Institute for Atomic and Molecular Physics, Kruislaan 407, Amsterdam, The Netherlands

Received 8 February 1980

Abstract. Total cross sections for electron scattering by H_2 are obtained for impact energies ranging from 25 to 750 eV. The method used is a linearisation of the Ramsauer-type experiment, which enables good angular and energy resolution; the accuracy obtained in the total cross sections is better than 4%. We compare our results with existing experimental data of other groups in this energy range. By means of analysis of theories and experiments on total cross sections for elastic scattering, excitation and ionisation we evaluate a set of semi-empirical total cross sections from 20 up to 2000 eV impact energy, which are accurate to about 5% over most of this energy interval. In the overlapping range we find excellent agreement of the data obtained thus with our experimental results.

1. Introduction

After the first measurements of the total cross section for electron scattering by molecular hydrogen in the twenties and thirties, as described in the review of Bederson and Kieffer (1971), this field has remained untouched for many years. The enhanced interest in the total cross sections of atoms and molecules during recent decades may be ascribed to various causes.

(i) The stimulating effort by the late Robert L Platzman in an extensive research programme on the delivery of energy to individual molecules by ionising particles, with the total scattering cross section being one of the main parameters (Fano 1975).

(ii) Renewed theoretical interest in electron-molecule scattering phenomena with growing computational facilities, of course focusing on the simplest molecule H_2 (e.g. Liu 1973, Gerhart 1975).

(iii) The applicability of total cross section data in astrophysics and technical sciences, for which a large impact energy range is required.

At the same time it has become clear that the most commonly used experimental technique—that of Ramsauer—has some serious shortcomings. This technique is an absorption measurement in which the electrons move along a circular path in a homogeneous magnetic field. The difficulties in angular and energy analysis at higher (above the inelastic threshold) impact energies result in total cross sections (Bederson and Kieffer 1971) which are too low.

In § 2 of this paper we describe the experimental determination of the total cross sections for electron scattering by H_2 obtained to a high accuracy (better than 4%) in the

[†] Present address: Korinklyke/Shell Laboratorium, PO Box 3003, 1003 AA Amsterdam, The Netherlands.

energy range from 25 to 750 eV. This is done in an apparatus that circumvents the above mentioned drawback of the Ramsauer technique. In § 3 we compare our data with those of other groups in the overlapping energy ranges; none of these groups corrected for the effects just described. In § 4 we describe the construction of a set of semi-empirical total cross sections over the energy range from 20 to 2000 eV, obtained by a combination of the known (experimental or theoretical) total cross sections for elastic scattering, ionisation and excitation. We compare these results with our experimental data in the overlapping energy range as a consistency check.

2. Experimental

2.1. Apparatus

An extensive discussion of the experimental set-up has already been given elsewhere (Blaauw *et al* 1980) and for convenience only a brief description will be given here. A schematic drawing of the experimental set-up is given in figure 1. The experiment is basically a linearisation of the Ramsauer absorption technique; unlike this technique, there is no magnetic field present. This modification enables one to fix the scattering geometry unambiguously, so the angular resolution can be defined exactly.

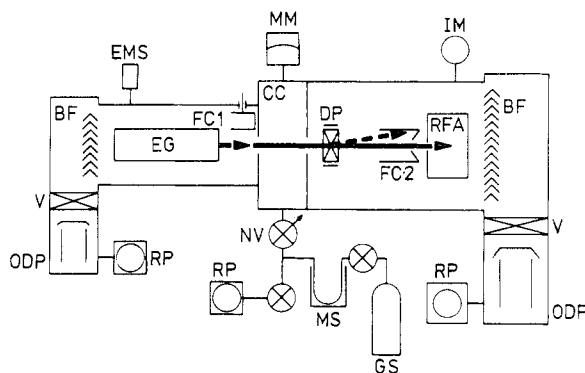


Figure 1. Scheme of the experimental set-up (not to scale). *Optics:* EG, electron gun; CC, collision chamber; DP, deflection plates; RFA, retarding field analyser; FC1, FC2, Faraday cups. *Vacuum system:* ODP, oil diffusion pump; RP, roots pump; V, valve; BF, baffle (liquid-air cooled), EMS, electric quadrupole mass spectrometer; IM, ionisation manometer. *Gas supply system:* GS, supply; MS, molecular sieve; RP, roots pump; NV, needle valve; MM, membrane manometer.

An electron gun (EG) produces a highly parallel beam with impact energies variable between 25 and 750 eV and a thermal energy spread of 0.4 eV. The energy range is limited at 750 eV because of electrical breakdown at higher energies and at 25 eV because of the divergence of the electron beam. This beam is led through the collision chamber (CC) of length 42 mm and diameter 80 mm and having entrance and exit orifices of 1 mm diameter. The detection takes place in a retarding field analyser (RFA) at 100 mm behind the centre of the collision chamber or by a Faraday cup (FC2) just in front of the analyser. This Faraday cup has a small hole in its back to let the beam pass through in the case of measurements with the analyser. Deflection plates behind the

collision chamber ensure optimal collection in both detection modes. To control the variation of the electron beam properties during the measurements another Faraday cup just in front of the collision chamber (FC1) can be moved downwards to intercept the primary beam. The cups are connected with Keithley 610 A current meters, which are read out digitally. Under vacuum conditions we have complete transmission of the electrons through the collision chamber.

After shielding the Earth's magnetic field the residual field is maximally 2 mG along the axis and 0.5 mG perpendicular to it. The scattering geometry is defined by the aperture of the RFA (diameter 1 mm) or by the aperture of FC2 (diameter 8 mm). This leads to a well defined solid angle of the analyser/collector: as seen from the centre of the collision chamber these values are 7.85×10^{-5} sr for the RFA and 1.03×10^{-2} sr for FC2.

Mathur *et al* (1975) have performed calculations on gas density profiles in cylindrical boxes as a function of the ratio of the box dimensions to the diameter of the orifices. Applying their results to our gas cell geometry we find end effects due to the large change in pressure over the orifices to be negligible. Hence one should take the geometrical length L of the gas cell as the path length of the electrons through the gas. This was confirmed in a previous experiment with varying collision cell lengths by Blaauw *et al* (1980). The pressure drop over the orifices was about 10^3 ; typical gas pressures for H_2 were 2.66 Pa inside and 1.3×10^{-3} Pa outside the collision chamber. The background pressure was 5×10^{-7} Pa. The gas pressure inside the collision chamber was determined by means of a baratron membrane manometer. We correct for the thermal transpiration effect that arises from the difference in temperature between the collision chamber (at about room temperature) and the baratron sensing head (kept at a temperature of 322 K, as prescribed by the manufacturer). This results in an increase of the apparent pressure of 2.5%, as found by Blaauw *et al* (1980). For our typical pressures the accuracy of the membrane manometer is 0.5%.

By means of a needle valve (NV2) in the electron gun region we can simulate a pressure of the same value and composition as there will be by flow through the orifices of the collision chamber when we lead gas into it. In this way we avoid variations of the primary beam during the absorption measurements, caused by the sensitivity of the indirectly heated oxide cathode for gas pressure changes; in particular for H_2 this effect can produce serious errors (mainly by water formation at the cathode).

2.2. Experimental procedure

The experimental procedure is based on the relation between the beam attenuation and the total cross section σ_{tot} and is given by

$$I_2/I_1 = \exp\left(-NL\sigma_{\text{tot}} + N(l\Delta\Omega)_{\text{eff}} \frac{d\sigma}{d\Omega}(\theta = 0^\circ)\right). \quad (1)$$

Here I_2/I_1 is the ratio of the beam intensities in the collectors behind and in front of the collision chamber. To correct for small differences in the collection efficiencies of the collectors the actual value used on the left-hand side of equation (1) is determined by dividing I_2/I_1 by its value measured without gas in the collision chamber. By choosing the appropriate gas densities we maintain an absorption of typically 10% at nearly all impact energies; only at energies above 500 eV this value was slightly smaller. On the right-hand side of equation (1) we have the usual Lambert-Beer law, supplemented with a second term to correct for small-angle scattering into the collector (RFA or FC2).

L stands for the length of the collision chamber and N for the gas density inside it. The total cross section is represented by σ_{tot} , while $d\sigma(\theta = 0^\circ)/d\Omega$ refers to elastic scattering into the solid angle of the RFA or to elastic plus inelastic scattering into the solid angle of FC2 when this is used. We use FC2 at energies below 40 eV, because it is not possible to produce a primary beam of sufficiently low divergence here for a complete transmission into the RFA.

We can estimate the contribution of small-angle scattering, as reflected in the correction term in equation (1), in the following way. By the extrapolation of differential elastic scattering data by van Wingerden *et al* (1977) at impact energies above 100 eV, we obtain an estimate of the differential elastic cross section at zero degrees, which is probably accurate to a factor of two. If we assume a constant value of $d\sigma/d\Omega$ over the entire acceptance angle of the RFA, we overestimate the contribution of small-angle scattering, since at impact energies above 100 eV $d\sigma(\theta)/d\Omega$ falls off with increasing scattering angle. We find the relative elastic scattering contribution above 100 eV always to be smaller than 2×10^{-4} and hence we can neglect it in equation (1). For energies below 60 eV we find experimentally that the total cross section is independent within the experimental error of the choice of RFA or FC2 (having different solid angles), which indicates that small-angle scattering is unimportant here.

The energy of the primary electron beam is calibrated by measuring the position of the 19.3 eV resonance in e^- -He scattering originally determined by Schulz (1973), see Blaauw *et al* (1980). As a final consistency check of the measurements we verify the independence of σ_{tot} on the gas pressure and beam current.

2.3. Error discussion

We distinguish systematical and statistical (random) errors. The latter ones originate mainly from reading errors of the baratron and Keithley's and are estimated to be together 1%. The systematical errors involved in equation (1) are as follows.

- (i) Determination of gas cell length L : 0.1%.
- (ii) Calibration of the membrane manometer: the manufacturer guarantees an accuracy of 2.0% for a pressure of 0.13 Pa and 0.2% for 1.3 Pa and so on. At the pressures used in our case this leads to an error of 0.5% below 100 eV and 0.3% above this energy.
- (iii) The linearity of the Keithley 610 A current meter connected with a digital read-out unit: better than 0.5%. This error propagates via a logarithm in the derivation of the total cross section (see equation (1)). Below 100 eV this gives a contribution of 1%, above this energy 2%.
- (iv) The energy definition: this uncertainty is estimated to be 0.2% at the highest; its effect on σ_{tot} depends on the energy. We estimate this contribution to be only significant below 100 eV impact energy, with a value of 0.5%.

Linear addition of all contributions gives a conservative error limit assigned to our data of 3.1% below 100 eV and 3.4% above this energy.

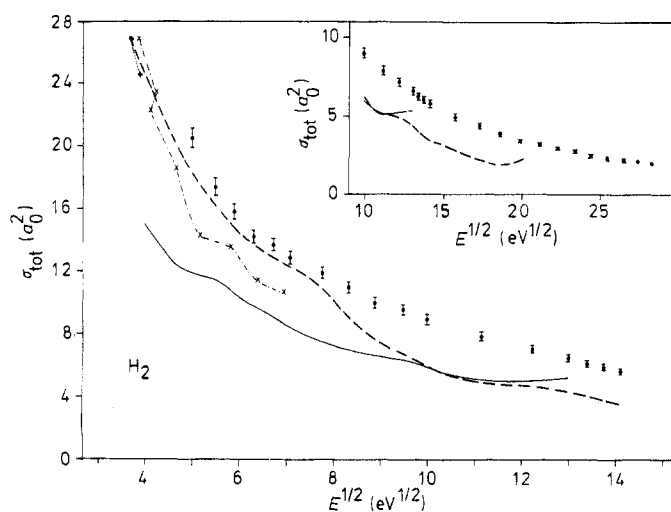
3. Results and discussion

Our experimental results between 25 and 750 eV impact energy are given in table 1 and shown in figure 2. We first compare our data with experimental results of other groups existing in this energy range.

Table 1. Experimental total cross sections for electron scattering by H_2 in units of a_0^2 , as a function of impact energy E in eV.

E (eV)	σ_{tot}	E (eV)	σ_{tot}
25	20.54	180	6.23
30	17.45	190	6.03
35	15.85	200	5.76
40	14.27	250	4.93
45	13.79	300	4.36
50	12.94	350	3.87
60	11.99	400	3.45
70	11.05	450	3.21
80	10.02	500	2.97
90	9.64	550	2.81
100	9.01	600	2.55
125	7.91	650	2.41
150	7.13	700	2.28
170	6.58	750	2.15

Unfortunately for H_2 in this range all existing data have been obtained in Ramsauer-type experiments before 1940, and they do not contain a full quantitative error discussion. The one recent experiment (Golden *et al* 1966) only extends up to 15 eV. The older experiments are those of Brode (1925, 2–270 eV), Brüche (1927, 2.25–49 eV) and Normand (1930, 0.5–400 eV). Due to insufficient energy and angular resolution all their data show a tendency to be too low at higher energies (see § 2.2), and there is not too much agreement between the three sets of data either, as can be seen in figure 2.

**Figure 2.** Total cross sections for electron scattering by H_2 in a_0^2 , as a function of energy in eV. Since the semi-empirical data obtained almost coincide with our experimental results they have been omitted. ●, present experiment; -·-·-, Brüche (1927); --, Normand (1930); +····+, Golden *et al* (1966); —, Brode (1925).

In addition to our direct cross section measurements, we also determined these data in a semi-empirical way, obtained by the addition of known total cross sections for elastic scattering, ionisation and excitation. This will be discussed extensively in the next section. In any case we show that there is excellent agreement between these semi-empirical data and our experimental ones above 40 eV, as can be seen from tables 2(a) and 2(b), columns (d) and (e).

4. Semi-empirical cross sections

4.1. Introduction

In analogy with the work of de Heer and co-workers (de Heer *et al* 1977, 1979, de Heer and Jansen 1977) on H, He and other noble gases we evaluate a set of total cross sections for electron scattering by molecular hydrogen in the impact energy range from 20 to 2000 eV. We do this by the analysis of experiments and theories on total cross sections for elastic scattering, ionisation and excitation, including dissociative processes. Because of the lack of data concerning dissociative excitation via optically forbidden triplet excitation, which is of importance below 100 eV according to Corrigan (1965), we treat the energy ranges above and below this energy separately. The upper limit of the energy range is dictated by the availability of total elastic scattering cross sections.

4.2. $E \geq 100$ eV

Here we add the separate total cross sections, as mentioned above:

$$\sigma_{\text{tot}} = \sigma_{\text{ion}} + \sigma_{\text{exc}} + \sigma_{\text{el}}. \quad (2)$$

We treat these contributions in the following subsections.

4.2.1. Ionisation. From threshold up to 1000 eV the experimental total (gross) ionisation cross section σ_{ion} has been measured by Rapp and Englander-Golden (1965), who quote an uncertainty of 6%. These measurements are in excellent agreement with those of Cowling and Fletcher (1973) and older data by Tate and Smith (1932), but are about 20% larger than values obtained by Schram *et al* (1964). However, analysis of the data of Schram *et al* (1964) on helium and neon reveals a tendency of these data to be too low as well (de Heer and Jansen 1977, de Heer *et al* 1979). Hence we reject these. Above 1000 eV we assume that the ionisation cross section can be represented by the Bethe formula, given in equation (3):

$$\sigma_{\text{ion}} = 4\pi(R/E)M_{\text{ion}}^2 \ln(4c_{\text{ion}}E/R). \quad (3)$$

Here E is the impact energy in eV, R the Rydberg energy, c_{ion} is a specific constant and σ_{ion} is in atomic units, which will be used for any cross section throughout the article. M_{ion}^2 is related to the optical oscillator strength density df/dE for excitation to the continuum by

$$M_{\text{ion}}^2 = \int_{\text{threshold}}^{\infty} \frac{R}{E} \frac{df}{dE} \eta_{\text{ion}}(E) dE \quad (4)$$

where $\eta_{\text{ion}}(E)$ is the ionisation efficiency. M_{ion}^2 and c_{ion} can be derived from photoab-

sorption measurements, which are tested for consistency by subjecting them to various constraints of sum rules (e.g. Inokuti *et al* 1967, Inokuti 1971). For ionisation the most recent determination of the Bethe formula constants stems from Douthat (1979), whose calculations extended over a wider energy range than previous ones (e.g. Liu 1973, Gerhart 1975). He finds $M_{\text{ion}}^2 = 0.721$ and $c_{\text{ion}} = 3.825$. The cross sections obtained thus produce a smooth junction with the data of Rapp and Englander-Golden (1965) at 1000 eV. We assume that the Bethe results above 1000 eV also have an accuracy of 6%. The values of σ_{ion} are given in table 2(b), column (a).

Table 2. Semi-empirical total cross sections (in a_0^2) for separate processes giving together the total cross sections, as described in § 4. In column (d) we include the final error in the semi-empirical total cross sections (in per cent) in brackets.

2(a) E	(a) σ_{el}	+	(b) $\sigma_{\text{L}} + \sigma_{\text{W}}$	+	(c) $1.1\sigma_{\text{corr}}$	=	(d) σ_{tot}	(e) $\sigma_{\text{tot}}^{\text{exp}}$
20	16.39		1.80		3.99		22.18 (14)	
25			1.96		4.22			20.54
30	9.84		2.12		4.40		16.36 (12)	17.45
40	7.29		2.44		4.73		14.46 (11)	14.27
50			2.53		4.68			12.94
60	5.24		2.49		4.51		12.24 (9)	11.99
75	3.78		2.37		4.29		10.44 (8)	10.5
80			2.33		4.20			10.02
90					4.10			9.64
100	3.17		2.16		3.95		9.28 (8)	9.01

2(b) E	σ_{ion}	+	σ_{el}	+	σ_{exc}	=	σ_{tot}	$\sigma_{\text{tot}}^{\text{exp}}$
100	3.30		3.17		2.78		9.25 (4)	9.01
150	2.90		1.85		2.35		7.10 (4)	7.13
200	2.55		1.28		2.06		5.89 (4)	5.76
300	2.05		0.809		1.60		4.48 (4)	4.36
400	1.70		0.543		1.30		3.56 (4)	3.45
500	1.45		0.420		1.11		2.98 (4)	2.97
600	1.27		0.340		0.965		2.57 (4)	2.55
700	1.13		0.260		0.859		2.24 (4)	2.28
750	1.07		0.248		0.814		2.13 (4)	2.15
800	1.01		0.235		0.775		2.00 (4)	
900	0.930		0.211		0.707		1.85 (4)	
1000	0.858		0.186		0.651		1.70 (4)	
2000	0.475		0.097		0.375		0.947 (4)	

4.2.2. Excitation. Since various processes (including dissociative excitation) contribute to σ_{exc} and there are no experimental data for all these processes covering our total energy range we take the following approach to obtain an estimate of σ_{exc} . For electron impact on H_2 Liu (1973) obtained the Bethe formula constants for total inelastic (excitation plus ionisation) scattering for optically allowed transitions, using the very accurate configuration interaction wavefunctions by Kolos and Wolniewicz (1965) and applying a sum rule as suggested by Inokuti *et al* (1967) and Inokuti (1971). The Bethe

cross section for total inelastic scattering is given by:

$$\sigma_{\text{inel}} = 4\pi(R/E)[1.5487 \ln(E/R) + 2.2212]. \quad (5)$$

The estimated accuracy above 1 keV is better than 1%. We now assume that the excitation cross section for optically allowed transitions can be represented by the Bethe approximation above 300 eV and construct its values by taking the difference of σ_{inel} and σ_{ion} given respectively in equations (5) and (4). This assumption is reasonable, because experimental excitation cross sections (see, for example, Möhlmann 1977) all show a Bethe behaviour as far as the energy dependence is concerned above 300 eV. Below 300 eV we assume an energy dependence for all excitation processes similar to the one for the Werner band system and extrapolate σ_{exc} from 300 eV downwards according to this energy dependence. This dependence is given for example by Gerhart (1975) according to the data of Stone and Zipf (1972) whose results are consistent with measurements of de Heer and Carrière (1971). The obtained values of σ_{exc} are shown in table 2(b) column (c).

We check the validity of this approach by comparison with the sum of the most important separate excitation cross sections. These are excitation to the Werner band (measured by de Heer and Carrière 1971), the Lyman band and dissociative excitation leading to H(2p) and H(2s), including cascades. The sum of these last two contributions, $\sigma_{2p+2s \rightarrow 1s}$ has been measured by Möhlmann *et al* (1978). For the Lyman band, investigated by Malcolm *et al* (1979), we take, as above, a similar energy dependence to that for the cross section of the Werner band, with a ratio $\sigma_{\text{W}}/\sigma_{\text{L}} = M_{\text{W}}^2/M_{\text{L}}^2 = 0.410/0.339$ (Gerhart 1975). The values obtained are shown in table 3. There is good agreement between the summed and Bethe-type excitation cross sections, as seen in table 3. Therefore, we assign an error of 6% to σ_{exc} in this energy range.

Table 3. Comparison of total excitation cross sections in a_0^2 constructed from Bethe formula with summed experimental cross sections for the Lyman and Werner bands plus dissociative excitation leading to Ly α radiation.

E	$\sigma_{(2s+2p) \rightarrow 1s}^\dagger$	$+ 1.827\sigma_{\text{W}}^\ddagger = \sigma_{\text{exc}}$	$\sigma_{\text{inel}} - \sigma_{\text{ion}}$
100	0.686	2.083	2.77
150	0.550	1.744	2.29
200	0.461	1.502	1.96
300	0.373	1.218	1.59
400	0.299	0.990	1.29
500	0.247	0.865	1.11
600	0.213	0.741	0.954
800	0.169	0.619	0.788
1000	0.142	0.527	0.669

† From Möhlmann *et al* (1978).

‡ de Heer and Carrière (1971).

In the calculation of σ_{inel} by Liu (1973) and in our experimental estimate of σ_{exc} optically forbidden transitions have not been taken into account. Since, to our knowledge, no reliable measurements on Ly β , γ or δ emission have been done, we can only look at the relative contribution of Balmer β, γ radiation to σ_{tot} . The total emission cross sections between 20 and 2000 eV impact energy obtained by Möhlmann *et al*

(1977) show that this contribution never exceeds 0.5% of σ_{tot} . Not knowing the values for other forbidden transitions we assume these also to be negligible.

4.2.3. Elastic scattering. The total elastic scattering cross section is generally obtained from measurements of the differential elastic cross section $d\sigma(\theta)/d\Omega$

$$\sigma_{\text{el}} = 2\pi \int_0^\pi \frac{d\sigma}{d\Omega}(\theta) \sin \theta \, d\theta. \quad (6)$$

In the energy range from 100 to 2000 eV we use the integrated absolute data of van Wingerden *et al* (1977), with quoted errors of 10%. In their measurements the energy resolution was insufficient to resolve vibrational and rotational excitation. The used values are given in table 2(b), column (b).

4.3. $E < 100$ eV

Since we have to consider triplet excitation in this range and there is some controversial information about dissociative excitation leading to $\text{H}(2p)$ and $\text{H}(2s)$ (see, for example, Mumma and Zipf 1977, Gerhart 1975), we take a different approach here to obtain σ_{tot} . We add the separate contributions from excitation to the Lyman and Werner bands, total ionisation, total dissociative excitation, and total elastic scattering, according to:

$$\sigma_{\text{tot}} = \sigma_{\text{L}} + \sigma_{\text{W}} + \sigma_{\text{ion}} + \sigma_{\text{diss}} + \sigma_{\text{el}}. \quad (7)$$

4.3.1. Lyman and Werner band excitation. Just as we have done between 100 and 300 eV we use the energy dependence given by Gerhart (1975) and normalising to our excitation cross sections at 300 eV we construct the combined Werner and Lyman band contribution down to threshold. Due to the uncertainty in reading Gerhart's graph, especially below 40 eV where the energy dependence varies drastically, we estimate the accuracy of the data to be somewhat lower than between 100 and 300 eV. The estimated errors are 10% between 100 and 50 eV and about 20% below. The data are given in table 2(a), column (b).

4.3.2. Dissociative excitation and ionisation. The total electron impact dissociative cross section has been measured below 100 eV by Corrigan (1965). However, his experimental cross sections also include total ionisation. At 100 eV we can compare his result with the sum of total ionisation (Rapp and Englander-Golden 1965) and total singlet excitation (Möhlmann 1977), whereas triplet excitation can be neglected here (Corrigan 1965). In order to obtain a smooth junction we have to multiply Corrigan's result with a correction factor of 1.10, in accordance with the findings of Gerhart (1975). In table 2(a), column (c) we give the obtained values, according to

$$\sigma_{\text{diss}} + \sigma_{\text{ion}} = 1.10\sigma_{\text{Corrigan}}. \quad (8)$$

The relative accuracy in Corrigan's data is quoted to be 10%.

4.3.3. Elastic scattering. Here we use integrated data of Srivastava *et al* (1975, 20–75 eV) who normalised their results to the differential helium cross sections by McConkey and Preston (1975). They claim an accuracy in their integrated cross sections of 18%. The energy resolution is good enough to resolve vibrational excitation, but rotational excitation is summed over. The influence of vibrational excitation

below 100 eV was investigated by Phelps and Schulz (see for references Laborie *et al* 1968) and in particular by Trajmar *et al* (1970). They show, that the vibrational excitation above 20 eV is always smaller than 1% of the corresponding elastic cross sections and hence we can neglect it.

Comparison of the elastic data above and below 100 eV shows that there is no smooth junction. We decided to increase the Srivastava data by a factor of 1.23, which is the mean multiplication factor needed to obtain good agreement for these energies where we can compare empirical data according to equation (7) with the experimental data of σ_{tot} . The multiplication factor found apparently leads to a smooth junction with the data by van Wingerden *et al* (1977). The values of σ_{el} used are shown in table 2(a), column (a).

4.4. Total cross sections

The total cross sections are obtained by the addition of the values in the first three columns in table 2 and given in column (d).

The error bar in σ_{tot} is calculated by the addition of the squared absolute error bars for the separate contributions and taking the square root of this sum. The resulting error bars (in per cent) are given in brackets in column (d). We see that there is excellent agreement between these semi-empirical total cross sections and our directly measured ones above 100 eV. This confirms our assumptions with regard to the relative contribution from optically forbidden transitions in this energy range (see § 4.2.2). Below 100 eV, we find satisfactory agreement between the two sets of data after renormalisation of the available experimental total elastic scattering data (see § 4.3.3).

Acknowledgments

We are indebted for critical remarks on the manuscript by Professor Dr J C Kluiver and Professor Dr M J van der Wiel. This work is part of the research programme of the Stichting voor Fundamenteel Onderzoek der Materie (Foundation for Fundamental Research on Matter) and was made possible by financial support from the Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek (Netherlands Organisation for the Advancement of Pure Research).

Note added in proof. Professor S Trajmar has kindly pointed out to us that the helium results of McConkey and Preston (1975), which were used in the normalisation of the elastic data below 100 eV by Srivastava *et al* (1975), have recently been remeasured (Register *et al* 1980 to be published). This results in an increase of the Srivastava data in accordance with the renormalisation deduced from our analysis.

References

- Bederson B and Kieffer L J 1971 *Rev. Mod. Phys.* **43** 601
- Blaauw H J, Wagenaar R W, Barends D H and de Heer F J 1980 *J. Phys. B: Atom. Molec. Phys.* **13** 359
- Brode R B 1925 *Phys. Rev.* **25** 636
- Brüche E 1927 *Ann. Phys., Lpz.* **82** 912
- Corrigan S J B 1965 *J. Chem. Phys.* **43** 4381
- Cowling I and Fletcher J 1973 *J. Phys. B: Atom. Molec. Phys.* **6** 665

- Douthat D A 1979 *J. Phys. B: Atom. Molec. Phys.* **12** 663
Fano U 1975 *Radiat. Res.* **64** 217
Gerhart D E 1975 *J. Chem. Phys.* **62** 821
Golden D E, Bandel H W and Salerno J A 1966 *Phys. Rev.* **146** 40
de Heer F J and Carrière J D 1971 *J. Chem. Phys.* **55** 3829
de Heer F J and Jansen R H J 1977 *J. Phys. B: Atom. Molec. Phys.* **10** 3741
de Heer F J, Jansen R H J and van der Kaay W 1979 *J. Phys. B: Atom. Molec. Phys.* **12** 979
de Heer F J, McDowell M R C and Wagenaar R W 1977 *J. Phys. B: Atom. Molec. Phys.* **10** 1945
Inokuti M 1971 *Rev. Mod. Phys.* **43** 297
Inokuti M, Kim Y K and Platzman R L 1967 *Phys. Rev.* **164** 55
Kolos W A and Wolniewicz L 1965 *J. Chem. Phys.* **43** 2429
Laborie P, Rocard J M, Rees J A, Delcroix J L and Craggs J D 1968 *Sections Efficaces Électroniques, Coefficients Macroscopiques*, (Paris: Dunod) p 23
Liu J W 1973 *Phys. Rev. A* **7** 103
McConkey J W and Preston J A 1975 *J. Phys. B: Atom. Molec. Phys.* **8** 63
Malcolm, I C, Dassen H W and McConkey J W 1979 *J. Phys. B: Atom. Molec. Phys.* **12** 1003
Mathur B P, Field J E and Colgate S O 1975 *Phys. Rev. A* **11** 830
Möhlmann G R, de Heer F J and Los J 1977 *Chem. Phys.* **25** 103
Möhlmann G R, Shima K H and de Heer F J 1978 *Chem. Phys.* **28** 331
Mumma M J and Zipf E C 1971 *J. Chem. Phys.* **55** 1671
Normand C E 1930 *Phys. Rev.* **35** 1217
Rapp D and Englander-Golden P 1965 *J. Chem. Phys.* **43**, 4381
Schram B L, de Heer F J, van der Wiel M J and Kistemaker J 1964 *Physica* **31** 94
Schulz G J 1973 *Rev. Mod. Phys.* **45** 378
Srivastava S K, Chutjian A and Trajmar S 1975 *J. Chem. Phys.* **63** 2659
Stone E J and Zipf E C 1972 *J. Chem. Phys.* **56** 4646
Tate J T and Smith P T 1932 *Phys. Rev.* **39** 270
Trajmar S, Truhlar D G, Rice J K and Kupperman A 1970 *J. Chem. Phys.* **63** 2659
van Wingerden B, Weigold E, de Heer F J and Nygaard K J 1977 *J. Phys. B: Atom. Molec. Phys.* **10** 1345