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A high energy approximation: II. Hydrogen atom excitation by electrons

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Abstract. The cross sections for the excitation of the 2s, 2p, and 3s states of hydrogen by electron impact are evaluated at high energy in a first order approximation in which the interaction between the incident electron and target proton is represented by a Coulomb wave final state in the T matrix. In the usual Born approximation this interaction has a vanishing contribution because of the orthogonality of the atomic states. The evaluated total cross sections converge to the Born result at high energies, but the differential cross sections at large angles show marked disagreement with the Born approximation and reasonable agreement with experiment. We conclude that for non-forward inelastic scattering the Born approximation does not converge to the correct result in the high energy limit.

1. Introduction

The subject of electron–hydrogen atom scattering theory has been exhaustively studied over the past forty five years. Numerous approximations, both classical and quantum, have been proposed for use on excitation cross sections in various energy regions. There are excellent review articles (e.g. Moiseiwitsch and Smith 1968), so an extensive review here would be out of place.

The one approximation which had universal acceptance is perhaps the first one used in quantum scattering theory, the Born approximation. It is generally believed that if the incident electron energy is 'high enough' the Born approximation will give the exact result both for the differential cross section at any angle and for the total excitation cross section. The 'high energy' Born results have been used extensively to normalize experimental measurements of relative cross sections, or so called excitation functions. But the question of how high the energy should be for the Born approximation to have any desired accuracy has never been quantitatively answered.

In this paper we will examine the application of the 'Coulomb projected Born approximation' of the preceding paper (Geltman 1971, hereinafter referred to as I) to the hydrogen atom excitations 1s→2s, 1s→2p, 1s→3s. We will compare the results with the usual Born approximation and with available experimental observations.

2. Theory

The general form of the theory of I is applicable to the electron–hydrogen collision problem, with a number of small modifications needed to describe a direct process rather than a rearrangement. The direct T matrix element analogous to equation (2) in I is

$$T_{pq}{}^{\mathrm{d}} = \langle \Psi_q{}^0, V_{\mathrm{i}}\Psi_p{}^+ \rangle \tag{1}$$

where $H = H_{\mathrm{i}} + V_{\mathrm{i}}$ and $(H_{\mathrm{i}} - E)\Psi_q{}^0 = 0$. The differential cross section has the same form as equation (1) in I.

A simplification in the theory of electron–atom scattering, which does not occur in the charge transfer case, is that we may assume the nucleus to be a fixed origin. The error involved in this assumption will be of order m/M in the cross sections. Also, our primary interest in this work will be the evaluation of cross sections neglecting exchange. At lower energies (say < 100 eV) exchange will be important and any comparison with low energy experimental data must include the exchange contribution.

The scattering amplitude is more commonly used than the T matrix in electron–atom problems and they are related by

$$f(p \to q) = -\frac{m}{2\pi\hbar^2} T_{pq}{}^{\mathrm{d}}. \tag{2}$$

We make the following choices for H_{i}, V_{i}, and $\Psi_q{}^0$ to obtain our present approximation:

$$H_{\mathrm{i}} = -\frac{\hbar^2}{2m}\nabla_1{}^2 - \frac{e^2}{r_1} - \frac{\hbar^2}{2m}\nabla_2{}^2 - \frac{e^2}{r_2}$$

$$V_{\mathrm{i}} = \frac{e^2}{r_{12}}$$

$$\Psi_q{}^0 = \exp(-\tfrac{1}{2}\pi\alpha)\Gamma(1-\mathrm{i}\alpha)\exp(\mathrm{i}\boldsymbol{k}_q\cdot\boldsymbol{r}_1)\,{}_1F_1(\mathrm{i}\alpha;1;-\mathrm{i}k_q r_1 - \mathrm{i}\boldsymbol{k}_q\cdot\boldsymbol{r}_1)\psi_q(\boldsymbol{r}_2) \tag{3}$$

where the Coulomb parameter α now represents the attractive electron–proton field (in I it represented proton–proton repulsion) and is $-e^2m/\hbar^2k_q$. Making the first Born approximation for $\Psi_p{}^+$ we have

$$f_{\mathrm{P}}(p \to q) = -\frac{m}{2\pi\hbar^2}\left\langle \Psi_q{}^0, \frac{e^2}{r_{12}}\exp(\mathrm{i}\boldsymbol{k}_p\cdot\boldsymbol{r}_1)\psi_p(\boldsymbol{r}_2)\right\rangle. \tag{4}$$

Let us compare this with the usual Born amplitude,

$$f_{\mathrm{B}}(p \to q) = -\frac{m}{2\pi\hbar^2}\left\langle \exp(\mathrm{i}\boldsymbol{k}_q\cdot\boldsymbol{r}_1)\psi_q(\boldsymbol{r}_2), e^2\left(-\frac{1}{r_1}+\frac{1}{r_{12}}\right)\exp(\mathrm{i}\boldsymbol{k}_p\cdot\boldsymbol{r}_1)\psi_p(\boldsymbol{r}_2)\right\rangle. \tag{5}$$

There is a vanishing contribution from the electron–proton interaction term in f_{B} because of the orthogonality of the atomic states ($q \neq p$). This means that no matter what form or strength the electron–proton interaction has, it will not affect the first Born approximation result. It will enter into second and higher Born approximations. From the point of view of impact parameter theory, as discussed in I, the electron–proton interaction will certainly be important in the determination of the trajectory of the scattered electron. Whatever the magnitude of this effect is on the cross sections, it is not included in the first Born approximation because f_{B} is independent of the electron–proton interaction. In f_{P} the effect of the electron–proton interaction is contained in $\Psi_q{}^0$, so the electron–proton interaction is felt in the lowest order approximation.

The exact value for the magnitude of the direct scattering amplitude is given by either of the forms,

$$|f(p\to q)| = \left|\frac{m}{2\pi\hbar^2}\left\langle \exp(\mathrm{i}\boldsymbol{k}_q\cdot\boldsymbol{r}_1)\psi_q(\boldsymbol{r}_2), e^2\left(-\frac{1}{r_1}+\frac{1}{r_{12}}\right)\Psi_p{}^+\right\rangle\right|$$

$$= \left|\frac{m}{2\pi\hbar^2}\left\langle \Psi_q{}^0, \left(\frac{e^2}{r_{12}}\right)\Psi_p{}^+\right\rangle\right| \tag{6}$$

where Ψ_q^0 is given by (3) and Ψ_p^+ is the exact solution of $(H-E)\Psi_p^+ = 0$ with proper scattering boundary conditions. If a particular approximate value of Ψ_p^+ is inserted into the expressions in (6), they will no longer be equal. We will adopt the point of view that an indication of the correctness of the approximate Ψ_p^+ used in (6) is given by the closeness of the resulting two values for the magnitudes of the scattering amplitude. This is not a rigorous criterion since one could easily construct a fictitious function as an approximation to Ψ_p^+ so as to assure equality of the two forms in (6). However with a physically reasonable approximation such as $\Psi_p^+ \simeq \exp(\mathrm{i}\boldsymbol{k}_p \cdot \boldsymbol{r}_1)\psi_p(\boldsymbol{r}_2)$ we believe the above criterion is meaningful.

3. Evaluation of amplitudes and cross sections

Starting with form (4) we have for the excitation amplitudes from the ground state,

$$f_{\mathrm{P}}(10 \to nlm) = -\frac{me^2}{2\pi\hbar^2}\frac{2\sqrt{\pi}}{(2l+1)}\exp(-\tfrac{1}{2}\pi\alpha)\Gamma(1+\mathrm{i}\alpha)$$

$$\times \int \mathrm{d}\boldsymbol{r}\, \exp\{\mathrm{i}(\boldsymbol{k}_p-\boldsymbol{k}_q)\cdot\boldsymbol{r}\}\,{}_1F_1(-\mathrm{i}\alpha;\, 1;\, \mathrm{i}k_q r+\mathrm{i}\boldsymbol{k}_q\cdot\boldsymbol{r})\, Y_{lm}^*(\hat{\boldsymbol{r}})y_l(10, nl|r). \tag{7}$$

Here $y_l(10, nl|r)$ is the electrostatic overlap integral

$$y_l(10, nl|r) = \int_0^\infty \mathrm{d}r' R_{10}(r')R_{nl}(r')\frac{r_<^{\,l}}{r_>^{\,l+1}}$$

where R_{nl} is r times the radial wavefunction and $r_{<,>}$ is the lesser or greater of r, r'. The y_l frequently arise in atomic structure and scattering problems and a convenient tabulation of some of them is given by Burgess *et al.* (1970). Since the y_l are products of polynomials and exponentials in r the typical term occurring in integral (7) is of the form

$$\int \mathrm{d}\boldsymbol{r}\, \exp(-\lambda r)r^n Y_{lm}^*(\hat{\boldsymbol{r}})\exp(\mathrm{i}\boldsymbol{K}\cdot\boldsymbol{r})\,{}_1F_1(\mathrm{i}a;\, 1;\, \mathrm{i}kr+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}).$$

Thus each term in (7) can be expressed in terms of a special case of the general integral given by Nordsieck (1954),

$$G(\lambda, \boldsymbol{K}, \boldsymbol{k}) = \int \mathrm{d}\boldsymbol{r}\, \exp(-\lambda r)\frac{\exp(\mathrm{i}\boldsymbol{K}\cdot\boldsymbol{r})}{r}\,{}_1F_1(\mathrm{i}a;\, 1;\, \mathrm{i}kr+\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})$$

$$= \frac{4\pi}{(K^2+\lambda^2)}\left(\frac{\frac{1}{2}(K^2+\lambda^2)}{\boldsymbol{k}\cdot\boldsymbol{K}-\mathrm{i}\lambda k+\frac{1}{2}(K^2+\lambda^2)}\right)^{\mathrm{i}a}. \tag{8}$$

The required powers of r under the integral are attained by differentiating or integrating G with respect to λ and the angular dependence in $Y_{lm}^*(\hat{\boldsymbol{r}})$ is achieved by appropriate linear combinations of differentiations with respect to the components of $\boldsymbol{K}$.

The resulting expression for $f_{\mathrm{P}}(10\to 21m)$ is

$$f_{\mathrm{P}}(10\to 21m) = -\frac{me^2}{2\pi\hbar^2}\frac{2\sqrt{\pi}}{3}\left(\frac{2}{3}\right)^{3/2}\exp(-\tfrac{1}{2}\pi\alpha)\Gamma(1+\mathrm{i}\alpha)$$

$$\times D_m\left[\frac{64}{27}\int_0^\lambda \mathrm{d}\lambda'\int_{\lambda'}^\infty \mathrm{d}\lambda'' G(\lambda'') - \frac{32}{9}\int_\lambda^\infty \mathrm{d}\lambda' G(\lambda') - \frac{8}{3}G + \frac{\partial G}{\partial\lambda}\right)_{\lambda=3/2} \tag{9}$$

where

$$a = -\alpha \qquad \boldsymbol{K} = \boldsymbol{k}_p - \boldsymbol{k}_q \qquad \boldsymbol{k} = \boldsymbol{k}_q$$

and

$$D_0 = -\mathrm{i}\left(\frac{3}{4\pi}\right)^{1/2} \frac{\partial}{\partial K_z} \quad \text{and} \quad D_{\pm 1} = \pm\mathrm{i}\left(\frac{3}{8\pi}\right)^{1/2} \left(\frac{\partial}{\partial K_x} \mp \mathrm{i}\frac{\partial}{\partial K_y}\right).$$

The integrations over λ are performed numerically.

Similar expressions are obtained for the 1s→2s and 1s→3s amplitudes but they are simpler in that there is no $Y_{lm}^*(\hat{\boldsymbol{r}})$ angular term present. The analytic form for the 1s→2s amplitude in terms of the Born amplitude is

$$\begin{aligned} f_{\mathrm{P}}(10 \to 20) = f_{\mathrm{B}}(10 \to 20)\left(\frac{N}{D}\right)^{-\mathrm{i}\alpha} & \exp(-\tfrac{1}{2}\pi\alpha)\Gamma(1+\mathrm{i}\alpha) \\ & \times\left\{\frac{1}{2}(\mathrm{i}\alpha+1)(\mathrm{i}\alpha+2) + \frac{2}{9}\mathrm{i}\alpha(\mathrm{i}\alpha-1)\left(\mathrm{i}k_q a_0 - \frac{3}{2}\right)^2 D^{-2}N^2 \right. \\ & \left. - \frac{4}{27}\alpha k_q a_0 D^{-1}N^2 + \frac{2}{3}\mathrm{i}\alpha(\mathrm{i}\alpha+1)\left(\mathrm{i}k_q a_0 - \frac{3}{2}\right)D^{-1}N\right\} \end{aligned} \tag{10}$$

where

$$N = \frac{1}{2}\left[\frac{9}{4} + |\boldsymbol{k}_q - \boldsymbol{k}_p|^2 a_0^2\right] \quad \text{and} \quad D = \boldsymbol{k}_q\cdot(\boldsymbol{k}_q - \boldsymbol{k}_p)a_0^2\,\frac{3}{2} - \mathrm{i}k_q a_0 + N.$$

Note that if α is set equal to zero above by making the charge parameter vanish, $f_{\mathrm{P}} \to f_{\mathrm{B}}$. However if α is allowed to approach zero as $1/k_q$ in the high energy limit, the limiting behaviour is

$$f_{\mathrm{P}} \underset{E\to\infty}{\to} \alpha k_q a_0 D^{-1} N^2 f_{\mathrm{B}} \to (1-\cos\theta)^2 E^{3/2} f_{\mathrm{B}}$$

for $\theta \neq 0$. For $\theta = 0$ it can be seen from (10) that $f_{\mathrm{P}} \underset{E\to\infty}{\to} f_{\mathrm{B}}$.

4. Results

The differential cross sections at a few scattering angles and the total cross sections obtained in the present approximation are compared with the Born approximation in table 1. We note that the two approximations converge to each other in the high energy limit for the differential cross section at zero scattering angle. At all non-vanishing scattering angles they do not come together. Since the major contribution to the total cross section comes from the forward peak, there is an apparent high energy convergence of the total cross sections given by the two approximations.

This may be qualitatively understood by noting that the forward Born cross section ($\theta = 0$) goes to a constant in the high energy limit for an s→s transition and it increases as E for an s→p transition. For these cases the magnitude of the spatial integrals containing a final state Coulomb wave approach the magnitude of the corresponding integral containing a final state plane wave. For any nonzero scattering angle the Born cross section for s→s transitions goes asymptotically as E^{-6} and for s→p transitions goes as E^{-7}. For these cases the integral with a Coulomb wave

Table 1. Comparison of present (P) and Born (B) approximations for differential and total cross sections

E(eV)		$\frac{d\sigma}{d\Omega}(a_0^2)$ 0°	45°	90°	180°	$\sigma(\pi a_0^2)$
				$1s \rightarrow 2s$		
100	P	9·10(−1)	2·61(−3)	2·24(−4)	5·85(−5)	5·82(−2)
	B	8·86(−1)	1·85(−3)	6·70(−6)	1·62(−7)	5·77(−2)
200	P	9·48(−1)	3·19(−4)	2·74(−5)	6·76(−6)	2·96(−2)
	B	9·36(−1)	8·54(−5)	1·43(−7)	2·79(−9)	2·95(−2)
300	P	9·61(−1)	9·37(−5)	7·93(−6)	1·94(−6)	1·99(−2)
	B	9·53(−1)	1·12(−5)	1·40(−8)	2·54(−10)	1·98(−2)
400	P	9·67(−1)	3·94(−5)	3·29(−6)	8·07(−7)	1·50(−2)
	B	9·61(−1)	2·47(−6)	2·62(−9)	4·59(−11)	1·49(−2)
500	P	9·71(−1)	2·01(−5)	1·67(−6)	4·09(−7)	1·20(−2)
	B	9·66(−1)	7·39(−7)	7·10(−10)	1·22(−11)	1·20(−2)
700	P	9·75(−1)	7·24(−6)	5·99(−7)	1·47(−7)	8·58(−3)
	B	9·72(−1)	1·14(−7)	9·79(−11)	1·63(−12)	8·57(−3)
900	P	9·78(−1)	3·37(−6)	2·79(−7)	6·89(−8)	6·68(−3)
	B	9·75(−1)	2·76(−8)	2·21(−11)	3·64(−13)	6·68(−3)
1100	P	9·79(−1)	1·83(−6)	1·52(−7)	3·76(−8)	5·47(−3)
	B	9·77(−1)	8·75(−9)	6·73(−12)	1·10(−13)	5·47(−3)
1300	P	9·81(−1)	1·10(−6)	9·17(−8)	2·27(−8)	4·63(−3)
	B	9·79(−1)	3·34(−9)	2·49(−12)	4·03(−14)	4·63(−3)
1500	P	9·81(−1)	7·14(−7)	5·95(−8)	1·48(−8)	4·01(−3)
	B	9·80(−1)	1·46(−9)	1·06(−12)	1·71(−14)	4·01(−3)
				$1s \rightarrow 2p$		
100	P	9·56(1)	4·99(−3)	3·87(−4)	9·48(−5)	8·21(−1)
	B	9·89(1)	1·01(−3)	1·08(−6)	1·30(−8)	7·50(−1)
200	P	2·10(2)	5·82(−4)	4·77(−5)	1·17(−5)	5·09(−1)
	B	2·15(2)	2·29(−5)	1·12(−8)	1·10(−10)	4·80(−1)
300	P	3·25(2)	1·72(−4)	1·42(−5)	3·49(−6)	3·76(−1)
	B	3·31(2)	1·99(−6)	7·24(−10)	6·58(−12)	3·61(−1)
400	P	4·40(2)	7·24(−5)	6·00(−6)	1·48(−6)	3·02(−1)
	B	4·47(2)	3·27(−7)	1·02(−10)	8·89(−13)	2·92(−1)
500	P	5·56(2)	3·71(−5)	3·08(−6)	7·64(−7)	2·58(−1)
	B	5·63(2)	7·80(−8)	2·20(−11)	1·88(−13)	2·47(−1)
700	P	7·88(2)	1·36(−5)	1·13(−6)	2·81(−7)	1·95(−1)
	B	7·95(2)	8·60(−9)	2·16(−12)	1·80(−14)	1·91(−1)

Table 1 (cont.)

E(eV)		$\frac{d\sigma}{d\Omega}\,\sigma(a_0^2)$ 0°	45°	90°	180°	$\sigma(\pi a_0^2)$
				1s→2p		
900	P	1·02(3)	6·40(−6)	3·37(−7)	1·34(−7)	1·59(−1)
	B	1·03(3)	1·61(−9)	3·78(−13)	3·11(−15)	1·57(−1)
1100	P	1·25(3)	3·52(−6)	2·96(−7)	7·38(−8)	1·35(−1)
	B	1·26(3)	4·18(−10)	9·40(−14)	7·66(−16)	1·34(−1)
1300	P	1·48(3)	2·14(−6)	1·80(−7)	4·50(−8)	1·18(−1)
	B	1·49(3)	1·35(−10)	2·94(−14)	2·38(−16)	1·17(−1)
1500	P	1·71(3)	1·39(−6)	1·18(−7)	2·95(−8)	1·05(−1)
	B	1·72(3)	5·09(−11)	1·09(−14)	8·76(−17)	1·05(−1)
				1s→3s		
100	P	1·26(−1)	7·13(−4)	5·50(−5)	1·44(−5)	1·16(−2)
	B	1·21(−1)	5·67(−4)	2·11(−6)	5·09(−8)	1·15(−2)
200	P	1·30(−1)	8·03(−5)	6·60(−6)	1·62(−6)	5·91(−3)
	B	1·27(−1)	2·64(−5)	4·39(−8)	8·53(−10)	5·87(−3)
300	P	1·31(−1)	2·29(−5)	1·90(−6)	4·63(−7)	3·96(−3)
	B	1·29(−1)	3·45(−6)	4·24(−9)	7·67(−11)	3·94(−3)
400	P	1·32(−1)	9·50(−6)	7·84(−7)	1·92(−7)	2·98(−3)
	B	1·30(−1)	7·54(−7)	7·92(−10)	1·38(−11)	2·97(−3)
500	P	1·32(−1)	4·81(−6)	3·96(−7)	9·70(−8)	2·38(−3)
	B	1·31(−1)	2·24(−7)	2·14(−10)	3·65(−12)	2·38(−3)
700	P	1·32(−1)	1·72(−6)	1·42(−7)	3·48(−8)	1·70(−3)
	B	1·32(−1)	3·45(−8)	2·93(−11)	4·88(−13)	1·70(−3)
900	P	1·32(−1)	8·02(−7)	6·60(−8)	1·63(−8)	1·33(−3)
	B	1·32(−1)	8·30(−9)	6·61(−12)	1·09(−13)	1·33(−3)
1100	P	1·33(−1)	4·35(−7)	3·59(−8)	8·86(−9)	1·09(−3)
	B	1·33(−1)	2·63(−9)	2·01(−12)	3·27(−14)	1·09(−3)
1300	P	1·33(−1)	2·61(−7)	2·16(−8)	5·35(−9)	9·19(−4)
	B	1·33(−1)	1·00(−9)	7·43(−13)	1·20(−14)	9·19(−4)
1500	P	1·33(−1)	1·69(−7)	1·40(−8)	3·47(−9)	7·97(−4)
	B	1·33(−1)	4·36(−10)	3·17(−13)	5·10(−15)	7·97(−4)

never 'catches up' with the plane wave integral. In fact, while in I the ratio

$$\frac{d\sigma_P/d\Omega}{d\sigma_{BK}/d\Omega}$$

appears to approach asymptotically a quantity which is a function of angle only (for zero or nonzero angles), in the electron excitation case for nonzero angles the present approximation has the limiting energy dependence

$$\frac{d\sigma_P}{d\Omega} \to a(\theta)E^{-3}$$

for both 1s→2s and 1s→2p cases. This is in sharp distinction to the predictions of the Born approximation.

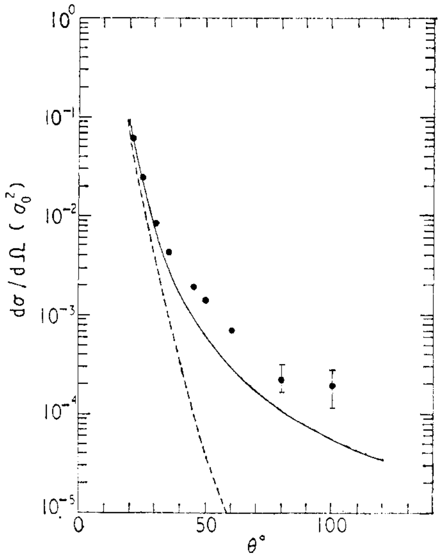

Figure 1. The differential cross section for the $n = 1 \to 2$ excitation of hydrogen by electron impact at 200 eV. Broken curve, Born approximation; full curve, present approximation; full circles, experiment of Williams (1969) at 200 eV, normalized at 21° to the present approximation.

In figure 1 we compare theoretical and experimental values for the $n = 1 \to n = 2$ differential cross sections at 200 eV, that is,

$$\frac{d\sigma}{d\Omega}(1s \to 2s) + \frac{d\sigma}{d\Omega}(1s \to 2p).$$

The measurement (Williams 1969) is a relative one and we have normalized it to $d\sigma_P/d\Omega$ at the smallest angle for which data was taken (21 °). We estimate on the basis of Ochkur type approximations that the exchange contribution at this energy will contribute an increase of ~ 5% to the direct excitation at 90 ° and decreases sharply at smaller angles. Thus the exchange contribution is insignificant in the comparison of experiment with the two theoretical curves. The present approximation is in far better agreement with experiment at all measured angles than is the Born approximation, with the latter falling orders of magnitude too low at the larger angles. Recent work using the Glauber approximation (Tai *et al.* 1970) has also been successful in giving a much better inelastic angular distribution for this cross section than the Born approximation. Further evidence of the insufficiency of the Born approximation at large scattering angles occurs in the measurements of Truhlar *et al.* (1970) on the $1^1S \to 2^1P$ excitation of helium at 55·5 and 81·63 eV.

As mentioned above, the total excitation cross sections in the present approximation approach those of the Born approximation at high energies because of the strong weighting given to directly forward scattering. The two approximations give total cross sections within 1% of each other at the following energies:

$\sigma(1s \to 2s)$	$E = 80$ eV
$\sigma(1s \to 2p)$	$E = 1200$ eV
$\sigma(1s \to 3s)$	$E = 150$ eV

and thus we can assume the Born approximation for total cross sections are correct to 1% or better (for the direct scattering) above these energies.

A quantity which depends less on directly forward scattering than does the total cross section is the momentum transport cross section

$$\sigma_{\mathrm{M}}(p \to q) = 2\pi \int \mathrm{d}(\cos\theta)(1-\cos\theta)|f(p\to q)|^2. \tag{11}$$

We have evaluated this for the $1s \to 2s$ transition and find that at 1000 eV the present and Born approximations for σ_{M} are still 6·5% apart. The convergence of the two approximations for σ_{M} occurs at a much higher energy than for σ because the factor $1-\cos\theta$ in (11) gives greater weight to larger scattering angles.

A consequence of the present approximation is that it yields inelastic scattering amplitudes and generalized oscillator strengths which are no longer a function of the magnitude of the momentum transfer vector only ($K = |\boldsymbol{k}_p - \boldsymbol{k}_q|$). At low K there is a confluence with the Born generalized oscillator strength but as K is increased an additional E dependence enters. The differential cross section measurements of Lassettre *et al.* (1969) and Chamberlain *et al.* (1970) were done for scattering angles of less than 15 °, where departures of the present approximation from the Born approximation are insignificant except at energies above 2500 eV. The present approximation also of course would not affect the correlation between the $K = 0$ limit of the generalized oscillator strength and the optical oscillator strength.

5. Conclusions

On the basis of the present results and those of the preceding paper we conclude that the high energy limit for the angular distribution of inelastic scattering of charged particles by an atomic target is not correctly given by the Born approximation. The reason for this is that the Born approximation takes no account of the interaction between the incident particle and the atomic nucleus. We have presented here an alternative treatment that explicitly includes this interaction, which may be called a 'Coulomb projected Born approximation'. Of course we have no assurance that this approximation gives rigorously correct cross sections in the high energy limit, but comparison with available experimental observations (for electron impact excitation) suggests our results are close to the correct results.

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