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Inelastic scattering of electrons and protons by atoms at intermediate energies: II

Charles J Joachain and Robert Vanderpoorten

Physique Théorique et Mathématique, Faculté des Sciences, Université Libre de Bruxelles, Brussels, Belgium

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Abstract. The Eikonal distorted wave method is used to analyse the inelastic scattering of electrons and protons by helium atoms at intermediate energies. Detailed calculations are performed for the excitation of the 2^1S and 2^1P states.

1. Introduction

In a previous paper (Joachain and Vanderpoorten 1973, to be referred to as Paper I), we studied the inelastic scattering of fast charged particles by neutral atoms by means of the eikonal distorted wave method of Chen *et al* (1972). In particular, we suggested that the distorting potentials in the initial and final channels could be chosen to be Glauber optical potentials (Glauber 1959) which include absorption effects (Joachain and Mittleman 1971a, b) in addition to the usual static (charge cloud) part. We applied this method to analyse the direct excitation of the $n = 2$ levels of hydrogen by electrons and protons at intermediate energies. In the present paper we shall extend these calculations to the case of the excitation of the 2^1S and 2^1P states of helium, for which several recent experimental measurements are available. After recalling the basic features of our method in § 2, we present our calculations and discuss our results in § 3. Particular attention is paid to the choice of the helium bound state wavefunctions. As in the case of atomic hydrogen discussed in Paper I, we find that our results for the *forbidden* transition $1\,^1S \rightarrow 2\,^1S$ are only reliable at rather high colliding energies. On the contrary, for the *allowed* transition $1\,^1S \rightarrow 2\,^1P$ we obtain good agreement with the experimental data, even at relatively low energies.

2. Theory

Let us briefly recall a few basic equations of Paper I. Under the following assumptions

(1) neglect of exchange effects between the projectile and the target
(2) use of the distorted wave Born approximation (DWBA)
(3) use of eikonal wavefunctions to describe the elastic scattering in the initial and final channels

we may write the direct Eikonal distorted wave transition matrix element as [see equation (2.43) of Paper I]

$$T_{ba}^{Eik} = (2\pi)^{-3} \int d\boldsymbol{r} \exp(i\boldsymbol{K} \cdot \boldsymbol{r}) \exp\{i(\Lambda_i(\boldsymbol{b}, z) + \Lambda_f(\boldsymbol{b}, z)\} A(\boldsymbol{b}, z) \qquad (2.1)$$

where $\boldsymbol{K} = \boldsymbol{k}_i - \boldsymbol{k}_f$ is the momentum transfer ($\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ being respectively the centre of mass momenta in the initial and final channel), while

$$\Lambda_i(\boldsymbol{b}, z) = -\frac{1}{v_i}\int_{-\infty}^{z} U_i(\boldsymbol{b}, z')\,dz' \tag{2.2a}$$

$$\Lambda_f(\boldsymbol{b}, z) = -\frac{1}{v_f}\int_{z}^{\infty} U_f(\boldsymbol{b}, z')\,dz' \tag{2.2b}$$

and

$$A(\boldsymbol{b}, z) = \langle \phi_n | V | \phi_0 \rangle. \tag{2.3}$$

In these equations the objects U_i and U_f are (local) optical potentials describing respectively the elastic scattering in the initial and final channels, V is the full interaction potential between the projectile and the target and we have used a cylindrical coordinate system to write the relative coordinate $\boldsymbol{r}$ as $\boldsymbol{r} \equiv (\boldsymbol{b}, z) \equiv (b, z, \phi)$. Furthermore, the quantities $\boldsymbol{v}_i = \boldsymbol{k}_i/M$ and $\boldsymbol{v}_f = \boldsymbol{k}_f/M$ are respectively the initial and final relative velocities, M being the reduced mass of the colliding particles. The functions ϕ_m are the eigenstates of the target hamiltonian corresponding to the internal energies ϵ_m, the initial and final states of the atom being respectively $|\phi_0\rangle$ and $|\phi_n\rangle$.

In the applications which we shall consider the function $A(\boldsymbol{b}, z)$ factorizes into

$$A(\boldsymbol{b}, z) = \exp\{i(m_i - m_f)\phi\} V_{n0}(b, z) \tag{2.4}$$

where m_i and m_f denote respectively the magnetic quantum numbers of the target states $|\phi_0\rangle$ and $|\phi_n\rangle$, and the quantity V_{n0} is independent of the azimuthal angle ϕ. Moreover, since the phase functions Λ_i and Λ_f are also independent of ϕ, equations (3.2) and (3.3) of Paper I may be directly applied. We recall them here for convenience. Choosing the z-axis along $\boldsymbol{k}_i$ and denoting by θ the scattering angle between the vectors $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$, we first have

$$\begin{aligned} T_{ba}^{Eik} = (2\pi)^{-2} i^{(m_i - m_f)} \int_0^{\infty} db\, b \int_{-\infty}^{+\infty} dz \exp\{i(k_i - k_f \cos\theta)z\} J_{|m_i - m_f|}(k_f b \sin\theta) \\ \times \exp\{i[\Lambda_i(b, z) + \Lambda_f(b, z)]\} V_{n0}(b, z). \end{aligned} \tag{2.5}$$

Similarly, if the eikonal phase functions are calculated along a z-axis chosen along the bisector of the scattering angle, we have

$$\begin{aligned} \tilde{T}_{ba}^{Eik} = (2\pi)^{-2} i^{(m_i - m_f)} \int_0^{\infty} db\, b \int_{-\infty}^{+\infty} dz \exp\{i(k_i - k_f)\cos(\tfrac{1}{2}\theta)z\} \\ \times J_{|m_i - m_f|}\{(k_i + k_f) b \sin\tfrac{1}{2}\theta\} \exp\{i[\Lambda_i(b, z) + \Lambda_f(b, z)]\} V_{n0}(b, z). \end{aligned} \tag{2.6}$$

Let us now turn to the choice of the distorting optical potentials U_i and U_f. To first order in a multiple scattering expansion in powers of V (ie a Born series expansion for the optical potentials), U_i and U_f simply reduce to the *static potentials* (see for example Goldberger and Watson, 1964)

$$V_i^{(1)} = \langle \phi_0 | V | \phi_0 \rangle \tag{2.7a}$$

and

$$V_f^{(1)} = \langle \phi_n | V | \phi_n \rangle \tag{2.7b}$$

which may be obtained directly once the target eigenstates $|\phi_0\rangle$ and $|\phi_n\rangle$ are known.

We note that these static potentials are very strong at short distances. It is therefore very plausible that in the region of intermediate energies considered here the potentials $V_i^{(1)}$ and $V_f^{(1)}$ may govern elastic scattering—respectively in the initial and in the final channel—at not too small angles. This has been recently verified for the case of electron–helium elastic collisions, where an exact treatment of the static potential has been shown to yield excellent results outside the forward scattering cone. (Byron and Joachain 1973b).

In order to get higher-order contributions to the optical potentials, we use the Glauber potentials U_i^G and U_f^G such that [see equations (2.35)–(2.38) of Paper I]

$$\chi_i^G(\boldsymbol{b}) \equiv \Lambda_i^G(\boldsymbol{b}, +\infty) = -\frac{1}{v_i}\int_{-\infty}^{+\infty} U_i^G(\boldsymbol{b}, z)\,dz = -i \ln \langle \phi_0 | \exp(i\chi_i^{tot}) | \phi_0 \rangle \tag{2.8a}$$

$$\chi_f^G(\boldsymbol{b}) \equiv \Lambda_f^G(\boldsymbol{b}, -\infty) = -\frac{1}{v_f}\int_{-\infty}^{+\infty} U_f^G(\boldsymbol{b}, z)\,dz = -i \ln \langle \phi_n | \exp(i\chi_f^{tot}) | \phi_n \rangle \tag{2.8b}$$

where

$$\chi_i^{tot} = -\frac{1}{v_i}\int_{-\infty}^{+\infty} V(\boldsymbol{b}, z, \rho)\,dz, \tag{2.9a}$$

$$\chi_f^{tot} = -\frac{1}{v_f}\int_{-\infty}^{+\infty} V(\boldsymbol{b}, z, \rho)\,dz \tag{2.9b}$$

and ρ denotes the internal target coordinates. As in I we shall write the Glauber potentials U_i^G and U_f^G as

$$U_i^G = V_i^{(1)} + \tilde{U}_i^G \tag{2.10a}$$

and

$$U_f^G = V_f^{(1)} + \tilde{U}_f^G \tag{2.10b}$$

where the quantities $\tilde{U}_i^G$ and $\tilde{U}_f^G$ mainly account for *absorption effects* and play therefore an important role in the description of small angle elastic scattering (Joachain and Mittleman 1971a, b) in the initial and final channel. As we discussed in I, the functions $\tilde{U}_i^G(r)$ or $\tilde{U}_f^G(r)$ may be obtained from the knowledge of the corresponding Glauber phase shifts χ_i^G or χ_f^G, ie without introducing any phenomenological parameter.

3. Calculations and results

3.1 Helium bound state wavefunctions

Before presenting our results for differential and total cross sections, let us briefly discuss our choice of bound state wavefunctions $|\phi_0\rangle$ and $|\phi_n\rangle$ for the helium atom. Because accurate, orthogonal wavefunctions are already necessary to obtain reliable first Born values for inelastic transition matrix elements (Kim and Inokuti 1968, Bell *et al* 1968, 1969), it is also desirable to use precise, orthogonal bound state wavefunctions in distorted wave calculations. On the other hand, these wavefunctions should also be suitable for the computation of the various quantities appearing in the eikonal transition matrix elements (2.5) or (2.6). We have therefore constructed *configuration–interaction*

wavefunctions of the form (Byron and Joachain 1967, Joachain and Vanderpoorten 1970)

$$\phi_{1^1S}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{4\pi} \sum_{l=0}^{\lambda} \sum_{\substack{m,n \\ m \leqslant n}} A_{m,n}^{(l)} [r_1^{m+l} r_2^{n+l} + r_2^{m+l} r_1^{n+l}] \exp[-\tfrac{1}{2}\alpha(r_1 + r_2)] P_l(\cos\theta_{12}) \tag{3.1}$$

for the ground state,

$$\phi_{n^1S}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{4\pi} \sum_{l=0}^{\lambda} \sum_{m,n} B_{m,n}^{(l)} [r_1^{m+l} r_2^{n+l} \exp[-\tfrac{1}{2}(\beta r_1 + \gamma r_2)] + (1 \rightleftarrows 2)] P_l(\cos\theta_{12}) \tag{3.2}$$

for other ^{1}S states and

$$\phi_{n^1L}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{(4\pi)^{1/2}} \sum_{m,n} C_{m,n} [r_1^{m+L} r_2^{n} \exp[-\tfrac{1}{2}(\delta r_1 + \eta r_2)] Y_L^M(\hat{\boldsymbol{r}}_1) + (1 \rightleftarrows 2)] \tag{3.3}$$

for singlet states with $L \neq 0$. In these expressions θ_{12} denotes the angle between the vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, while the quantities $A_{m,n}^{(l)}$, $B_{m,n}^{(l)}$, $C_{m,n}$, α, β, γ, δ and η are variational parameters.

The *ground state* wavefunction ϕ_{1^1S} was determined by using 108 linear parameters $A_{m,n}^{(l)}$ with $\lambda = 3$ in equation (3.1). The nonlinear parameter α was chosen to be

$$\alpha = 3{\cdot}41 \simeq 2(|\epsilon_{1^1S}|)^{1/2},$$

ϵ_{1^1S} being the binding energy of the helium ground state. This choice, suggested by Pekeris (1958), yields reliable values of first Born excitation cross sections at small momentum transfers. The value of the ground state energy ϵ_{1^1S} corresponding to our wavefunction ϕ_{1^1S} is $\epsilon_{1^1S} = -2{\cdot}90267$ au. This is to be compared with the Hartree–Fock energy $\epsilon_{HF} = -2{\cdot}86167$ au (Roothaan *et al* 1960) and the 'exact' (non-relativistic) value $\epsilon_{ex} = -2{\cdot}90372$ au (Pekeris 1958). Hence we see that our wavefunction ϕ_{1^1S} includes 98·5 % of the correlation energy $\Delta\epsilon = \epsilon_{ex} - \epsilon_{HF}$.

Wavefunctions for *excited states* of helium having the form (3.2) and (3.3) have been determined for states n^1L with $n = 2, 3, 4, 5$ and L $= 0, 1, 2, 3, 4$. Since better computing facilities were available, these wavefunctions are more accurate (particularly for ^{1}S states) than the ones we obtained previously (Joachain and Vanderpoorten 1970). Various sets of such wavefunctions, with the corresponding eigenvalues, will be presented elsewhere. For the analysis of the $1\,^1S \rightarrow 2\,^1S$ transition considered below we have used a $2\,^1S$ wavefunction of the form (3.2) with 84 linear parameters and $\lambda = 2$, orthogonal to our ground state wavefunction ϕ_{1^1S}. The corresponding eigenvalue is given by $\epsilon_{2^1S} = -2{\cdot}14577$ au, the 'exact' value being $\epsilon_{ex} = -2{\cdot}14597$ au (Acad *et al* 1971). To study the transition $1\,^1S \rightarrow 2\,^1P$ we used a $2\,^1P$ wavefunction of the form (3.3) with 15 linear parameters. The corresponding energy is $\epsilon_{2^1P} = -2{\cdot}12256$ au, the 'exact' value being $\epsilon_{ex} = -2{\cdot}12384$ au (Acad *et al* 1971).

As a check on the accuracy of our wavefunctions, we present in table 1 the values of the *generalized oscillator strengths*

$$f_{n0}(K) = \frac{2(\epsilon_n - \epsilon_0)}{K^2} |\langle \phi_n | \sum_{i=1}^{Z} \exp(\mathrm{i}Kz_i) | \phi_0 \rangle|^2 \tag{3.4}$$

for the $1\,^1S \rightarrow 2\,^1S$ and $1\,^1S \rightarrow 2\,^1P$ transitions in helium, and we compare them with the accurate values of Kim and Inokuti (1968). We note that the agreement is excellent for the $1\,^1S \rightarrow 2\,^1S$ transition, the two sets of values differing only by about 1 %. In the case of the $1\,^1S \rightarrow 2\,^1P$ transition the agreement is poorer, the differences being of the order of 5 % at small momentum transfers and 15 % at $K^2 = 10$. This is related to our

Table 1. Generalized oscillator strengths for the transitions $1^1S \rightarrow 2^1S$ and $1^1S \rightarrow 2^1P$ in helium

K^2 (in au)	$f_{2^1S,1^1S}$		$f_{2^1P,1^1S}$	
	Our values	Kim and Inokuti	Our values	Kim and Inokuti
0·1	$7{\cdot}26^{-3}$†	$7{\cdot}32^{-3}$	$2{\cdot}25^{-1}$	$2{\cdot}35^{-1}$
0·2	$1{\cdot}26^{-2}$	$1{\cdot}26^{-2}$	$1{\cdot}91^{-1}$	$2{\cdot}01^{-1}$
0·5	$2{\cdot}09^{-2}$	$2{\cdot}09^{-2}$	$1{\cdot}22^{-1}$	$1{\cdot}29^{-1}$
1	$2{\cdot}24^{-2}$	$2{\cdot}25^{-2}$	$6{\cdot}14^{-2}$	$6{\cdot}54^{-2}$
2	$1{\cdot}55^{-2}$	$1{\cdot}56^{-2}$	$1{\cdot}91^{-2}$	$2{\cdot}07^{-2}$
4	$5{\cdot}91^{-3}$	$5{\cdot}95^{-3}$	$3{\cdot}10^{-3}$	$3{\cdot}45^{-3}$
6	$2{\cdot}49^{-3}$	$2{\cdot}51^{-3}$	$7{\cdot}69^{-4}$	$8{\cdot}70^{-4}$
8	$1{\cdot}19^{-3}$	$1{\cdot}20^{-3}$	$2{\cdot}47^{-4}$	$2{\cdot}83^{-4}$
10	$6{\cdot}25^{-4}$	$6{\cdot}33^{-4}$	$9{\cdot}45^{-5}$	$1{\cdot}10^{-4}$
20	$6{\cdot}40^{-5}$	$6{\cdot}49^{-5}$	$3{\cdot}34^{-6}$	$4{\cdot}20^{-6}$

† The superscript in this and the following tables denotes the power of ten by which the entry should be multiplied.

choice of the $2\,^1P$ wavefunction (see equation (3.3)), which is cruder than the one adopted for the $2\,^1S$ state.

Other interesting quantities where the accuracy of the bound state wavefunctions may be tested are the *total first Born cross sections*. Our Born values for the excitation of the $2\,^1S$ and $2\,^1P$ states by electrons of energies ranging between 30 eV and 1 keV are given in table 2, where they are compared with the accurate Born results of Bell *et al* (1969). For convenience and further reference we also list in table 3 our first Born total cross section for the excitation of the $2\,^1S$ and $2\,^1P$ states by protons having laboratory energies between 30 keV and 1 MeV. These values are compared with the accurate ones of Bell *et al* (1968). It can be seen from tables 2 and 3 that our total Born cross section for the excitation of the $2\,^1S$ state agree very closely (about one part in a thousand) with the 'exact' values of Bell *et al*. For the case of the excitation of the $2\,^1P$ state the agreement is again poorer, the difference between the two sets of values being about 5%.

Table 2. Total first Born cross sections (in units of πa_0^2) for the excitation of the states 2^1S and 2^1P of helium by electron impact

Electron energy (eV)	σ_{2^1S}		σ_{2^1P}	
	Our values	Bell *et al*	Our values	Bell *et al*
30	$4{\cdot}886^{-2}$	$4{\cdot}887^{-2}$	$1{\cdot}226^{-1}$	$1{\cdot}307^{-1}$
50	$3{\cdot}892^{-2}$	$3{\cdot}890^{-2}$	$1{\cdot}599^{-1}$	$1{\cdot}694^{-1}$
100	$2{\cdot}226^{-2}$	$2{\cdot}225^{-2}$	$1{\cdot}408^{-1}$	$1{\cdot}485^{-1}$
150	$1{\cdot}541^{-2}$	$1{\cdot}540^{-2}$	$1{\cdot}182^{-1}$	$1{\cdot}244^{-1}$
200	$1{\cdot}177^{-2}$	$1{\cdot}175^{-2}$	$1{\cdot}006^{-1}$	$1{\cdot}069^{-1}$
300	$7{\cdot}981^{-3}$	$7{\cdot}972^{-3}$	$8{\cdot}008^{-2}$	$8{\cdot}411^{-2}$
400	$6{\cdot}036^{-3}$	$6{\cdot}029^{-3}$	$6{\cdot}663^{-2}$	$6{\cdot}994^{-2}$
500	$4{\cdot}853^{-3}$	$4{\cdot}847^{-3}$	$5{\cdot}739^{-2}$	$6{\cdot}022^{-2}$
1000	$2{\cdot}450^{-3}$	$2{\cdot}447^{-3}$	$3{\cdot}503^{-2}$	$3{\cdot}675^{-2}$

Table 3. Total first Born cross sections (in units of πa_0^2) for the excitation of the states 2^1S and 2^1P of helium by proton impact

Proton energy (keV)	σ_{2^1S}		σ_{2^1P}	
	Our values	Bell *et al*	Our values	Bell *et al*
30	8.749^{-2}	8.752^{-2}	2.029^{-1}	2.165^{-1}
50	6.485^{-2}	6.485^{-2}	2.148^{-1}	2.281^{-1}
100	3.852^{-2}	3.824^{-2}	1.901^{-1}	2.009^{-1}
150	2.698^{-2}	2.696^{-2}	1.641^{-1}	1.730^{-1}
200	2.082^{-2}	2.081^{-2}	1.441^{-1}	1.519^{-1}
300	1.429^{-2}	1.428^{-2}	1.167^{-1}	1.228^{-1}
400	1.087^{-2}	1.086^{-2}	9.889^{-2}	1.039^{-1}
500	8.773^{-3}	8.764^{-3}	8.626^{-2}	9.062^{-2}
1000	4.464^{-3}	4.458^{-3}	5.445^{-2}	5.713^{-2}

3.2 Excitation of the 2 ^1S and 2 ^1P states of helium by electron impact

We display in figures 1, 2 and 3 the differential cross sections for the excitation of the 2 ^1S state of helium by incident electrons having an energy of 100, 200 and 400 eV, respectively. Our distorted-wave results, obtained with both static and Glauber optical potentials, are compared with the original experimental data of Vriens *et al* (1968), the same data as renormalized by Chamberlain *et al* (1970) and the measurements of Crooks and Rudd

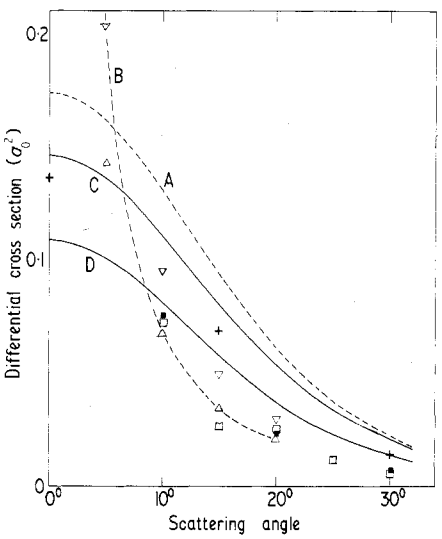

Figure 1. Differential cross sections for excitation of the 2^1S state of helium by 100 eV electrons. Curves A first Born approximation; B Berrington *et al* (1973); C present work, first order potentials $V^{(1)}$; D present work, Glauber potentials U^G. + Hidalgo and Geltman (1972); ▽ Vriens *et al* (1968); △ Vriens *et al*, renormalized by Chamberlain *et al* (1970); ■ Crooks and Rudd (1972); □ Suzuki and Takayanagi (1973).

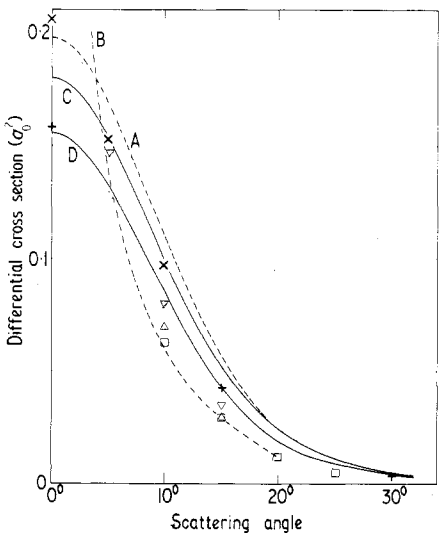

Figure 2. Differential cross sections for excitation of the 2^1S state of helium by 200 eV electrons. × simplified second Born approximation of Woollings and McDowell (1972); other symbols as for figure 1.

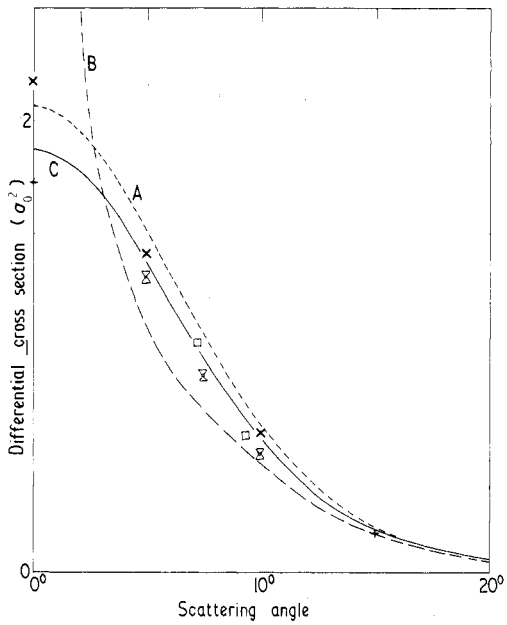

Figure 3. Differential cross sections for excitation of the 2^1S state of helium by 400 eV electrons. Curves A first Born approximation; B Berrington *et al* (1973); C present work, Glauber potentials U^G. + Hidalgo and Geltman (1972); × Woollings and McDowell (1972); ▽ Vriens *et al* (1968); △ Vriens *et al*, renormalized by Chamberlain *et al* (1970); □ Lassettre *et al* (1964) at 417 eV.

(1972). Also shown on figures 1 and 2 are the experimental points of Suzuki and Takayanagi (1973) while we have also indicated in figure 3 the results of Lassettre *et al* (1964) obtained at an incident electron energy of 417 eV. The other theoretical calculations shown on figures 1, 2 and 3 are second Born results of Woollings and McDowell (1972), the Coulomb-projected Born calculations of Hidalgo and Geltman (1972) and the recent 4-channel calculations of Berrington *et al* (1973). We see that the agreement between our distorted wave calculations and experiment is poor at 100 eV, but improves steadily as the energy increases. The 4-channel results of Berrington *et al* agree better with the experimental data, especially at the lower energies. This is particularly striking at 100 eV (see figure 1) where the calculations of Berrington *et al* are in very good agreement with experiment. It is worth noting that near the forward direction the discrepancy between our results and those of Berrington *et al* is always important. This is due to the fact that the $2\,^1S$–$2\,^1P$ coupling, which produces an important peak at small angles, is properly taken into account in the work of Berrington *et al* (see their figure 5). We note that the Glauber approximation also leads to a pronounced forward peak due to the 'nearby singularity' of the Glauber amplitude at the unphysical momentum transfer $K = 0$. Glauber calculations for the transition $1\,^1S \rightarrow 2\,^1S$ have recently been performed by Yates and Tenney (1972) who have obtained some encouraging results at surprisingly low energies (26·5 eV–82 eV). However, one should consider these Glauber calculations with considerable care since (i) exchange effects, which play an important role at such low energies, have been neglected, (ii) rather crude helium bound state wavefunctions have been used and (iii) in addition to the divergence mentioned above

the Glauber amplitude suffers from serious deficiencies which have recently been analysed in detail for the case of elastic scattering (Byron and Joachain 1973a).

Let us now examine the differential cross sections for excitation of the $2\,^1P$ state by electron impact. Before turning to the discussion of our results, we should mention a particularity of this calculation. Since for large b the functions V_{n0} [see equation (2.4)] decrease respectively like z/r^3 and b/r^3 for the $2\,^1P_0$ and $2\,^1P_1$ states, the numerical integration necessary to compute T_{ba}^{Eik} or $\tilde{T}_{ba}^{Eik}$ [see equations (2.5) or (2.6)] must be performed up to large values of b. This difficulty may be avoided if we compute first $T_{ba}^{Eik} - T_{ba}^{Born}$ (or $\tilde{T}_{ba}^{Eik} - T_{ba}^{Born}$) by replacing the quantity

$$\exp\{i[(\Lambda_i + \Lambda_f)]\}$$

by

$$\{\exp[i(\Lambda_i + \Lambda_f)] - 1\}$$

in equations (2.5) or (2.6) and then add subsequently the first Born value T_{ba}^{Born}. Moreover, since T_{ba}^{Born} constitutes the main contribution to T_{ba}^{Eik} (or $\tilde{T}_{ba}^{Eik}$) for small scattering angles, we used accurate values of T_{ba}^{Born} obtained from the generalized oscillator strengths computed by Kim and Inokuti (1968) and Bell *et al* (1969). Had we used our own set of bound state wavefunctions to calculate T_{ba}^{Born}, we would have obtained differential cross sections lower by about 5% at small scattering angles.

We show in figures 4, 5 and 6 the differential cross sections corresponding to the excitation of the $2\,^1P$ state, and for incident electron energies of 81·6 eV, 100 eV and 200 eV respectively. Our eikonal distorted wave calculations are seen to agree well with the experimental results of Chamberlain *et al* (1970), Truhlar *et al* (1970), Crooks

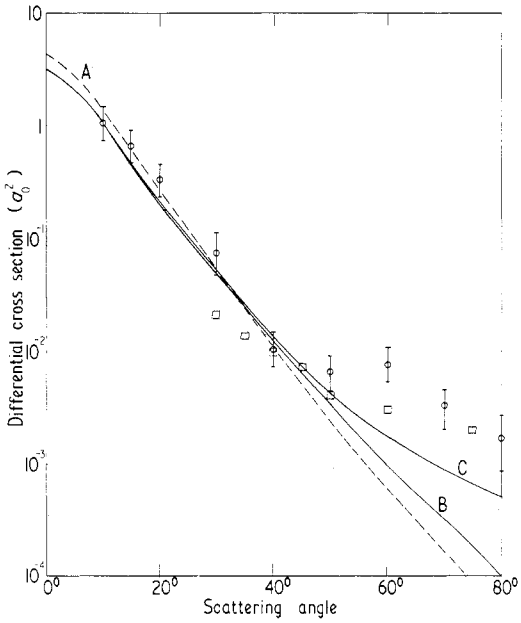

Figure 4. Differential cross sections for excitation of the 2^1P state of helium by 81·6 eV electrons. Curves A first Born approximation; B present work, Glauber potentials U^G, equation (2.5); C present work, Glauber potentials U^G, equation (2.6). ○ Truhlar *et al* (1970); □ Opal and Beaty (1972).

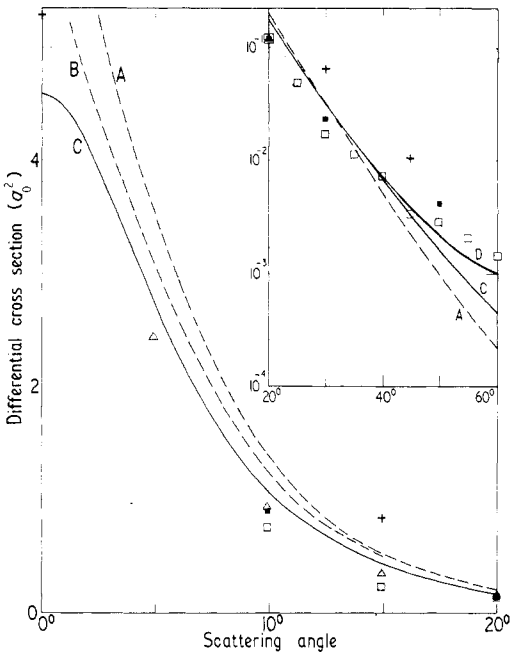

Figure 5. Differential cross sections for excitation of the $2\,^1P$ state of helium by 100 eV electrons. Curves A first Born approximation; B Berrington *et al* (1973); C present work, Glauber potentials U^G, equation (2.5); D present work, Glauber potentials U^G, equation (2.6). + Hidalgo and Geltman (1972); △ Vriens *et al* (1968), renormalized by Chamberlain *et al* (1970); ■ Crooks and Rudd (1972); □ Suzuki and Takayanagi (1973).

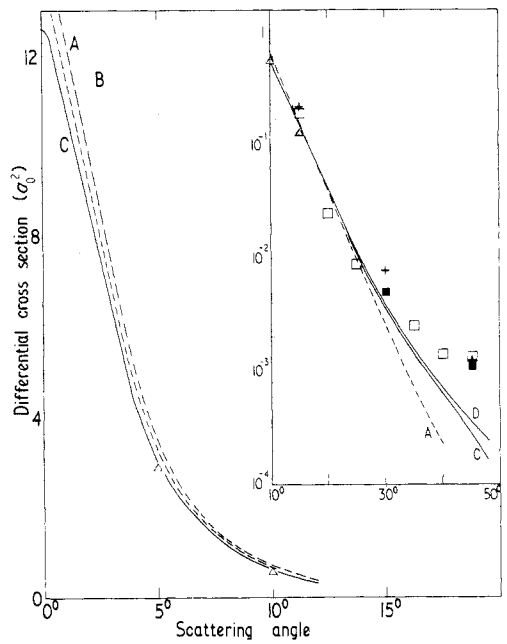

Figure 6. Differential cross sections for excitation of the $2\,^1P$ state of helium by 200 eV electrons. ■ Opal and Beaty (1972); other symbols as for figure 5.

and Rudd (1972), Opal and Beaty (1972) and Suzuki and Takayanagi (1973). Figures 4, 5 and 6 also show the differences between the results obtained by using the eikonal DWBA T-matrix element (2.5) [with the z-axis along $\boldsymbol{k}_i$] and those given by equation (2.6) [with the z-axis chosen along the bisector of the scattering angle]. As expected, the values obtained from equation (2.6) are more accurate at larger angles. Also shown for comparison in figures 4, 5 and 6 are the first Born results of Kim and Inokuti (1968), the Coulomb-projected Born values of Hidalgo and Geltman (1972) and the 4-channel calculations of Berrington *et al* (1973).

Other interesting theoretical investigations of the transition $1\,^1S \rightarrow 2\,^1P$ have been made by several authors. Byron (1971) has used a modified version of the Glauber approximation which includes the effect of the longitudinal momentum transfer. We compare in figure 7 his results with our eikonal DWBA values, the quantity shown being the ratio of the differential cross section to the corresponding first Born value at a scattering angle $\theta = 5°$ and for energies ranging from 50 eV to 400 eV. Also shown on figure 7 are the experimental data of Chamberlain *et al* (1970). We note from figure 7 that our results agree very well with those of Byron (the 'theoretical error bars' of Byron being due to the use of the Monte-Carlo method to perform multi-dimensional integrals). Both theoretical approaches are seen to be in good agreement with the experimental data.

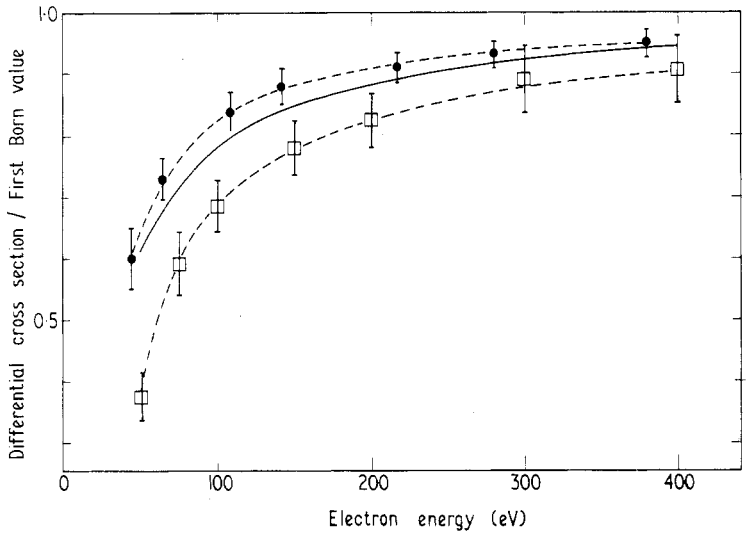

Figure 7. Ratio of differential cross section to the corresponding first Born approximation value, at a scattering angle of 5°, for electron excitation of the 2^1P state of helium. —— present work, Glauber potentials U^G; ● Byron (1971); □ Chamberlain *et al* (1970).

More recently, distorted wave calculations with static potentials and including approximately exchange effects (within the framework of the DWBA) have been performed for the $1\,^1S \rightarrow 2\,^1P$ transition by Madison and Shelton (1973). These calculations, done with a variety of rather crude bound state wavefunctions, show a marked improvement over first Born values at large angles. This result is not surprising in view of the properties of the potentials $V_i^{(1)}$ and $V_f^{(1)}$, which we discussed at the end of § 2.

Let us now consider the total cross sections. We compare in figure 8 our eikonal DWBA results (obtained by using Glauber optical potentials) with the first Born values of Bell *et al* (see table 2) and the recent 4-channel calculations of Berrington *et al* (1973) for the transition $1\,^1S \rightarrow 2\,^1S$. Also shown in figure 8 are the experimental results of Lassettre (1965), Vriens *et al* (1968), Chamberlain *et al* (1970) and Rice *et al* (1972). Because of the large spreading in the experimental results it is difficult to make a significant comparison between theory and experiment. We note, however, that our values agree well with those of Berrington *et al* above an incident electron energy of 100 eV. At lower energies an important part of the discrepancy between our values and those of Berrington *et al* is probably due to the small angle approximation used by these authors to compute total cross sections.

Our total cross sections for the excitation of the $2\,^1P$ state of helium by electron impact (obtained by using Glauber distorting potentials) are given in figure 9, where they are compared with the experimental data of Jobe and St John (1967), Vriens *et al* (1968) (as renormalized by Chamberlain *et al* (1970)), Moustafa Moussa *et al* (1969), de Jongh and van Eck (1971), Donaldson *et al* (1972) and Crooks and Rudd (1972). Also shown on figure 9 are the first Born values of Bell *et al* (1969) (see table 2), the eikonal results of Byron (1971) and the 4-channel calculations of Berrington *et al* (1973). It is apparent from figure 9 that our results and those of Byron agree well with the experimental data. As in the case of the excitation of the 2p state of hydrogen by

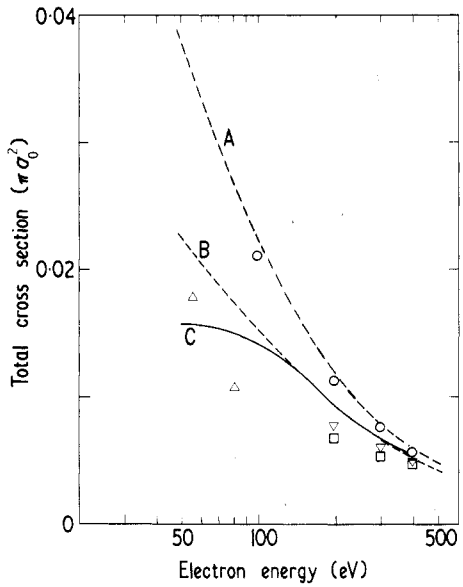

Figure 8. Total cross sections for excitation of the 2^1S state of helium by electrons. Curves A first Born approximation; B Berrington *et al* (1973); C present work, Glauber potentials U^G; ▽ Vriens *et al* (1968); □ Vriens *et al*, renormalized by Chamberlain *et al* (1970); ○ Lassettre (1965); △ Rice *et al* (1972).

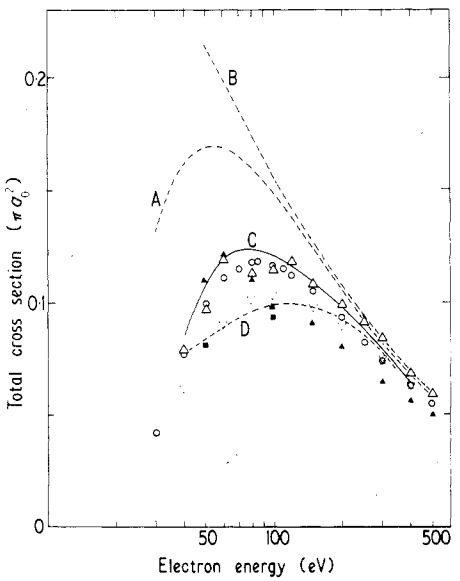

Figure 9. Total cross sections for excitation of the 2^1P state of helium by electrons. Curves A first Born approximation; B Berrington *et al* (1973); C present work, Glauber potentials U^G; D Byron (1971); ○ Donaldson *et al* (1972); △ de Jongh and van Eck (1971); ■ Crooks and Rudd (1972); ▲ Moustafa Moussa *et al* (1969); □ Vriens *et al* (1968); renormalized by Chamberlain *et al* (1970); ▽ Jobe and St John (1967).

electron impact (see Paper I) the values of Byron are slightly lower than ours. Here again the results of Berrington *et al* are probably affected at low incident energies by the small-angle approximation which they used to obtain total cross sections.

3.3 Excitation of the 2 ^1S and 2 ^1P states of helium by proton impact

We show in figure 10 the eikonal DWBA total cross sections which we have obtained for the excitation of the 2 ^{1}S state of helium by proton impact, using respectively the static potentials $V_i^{(1)}$, $V_f^{(1)}$ and the Glauber optical potentials U_i^G, U_f^G as distorting potentials in the initial and final channels. Our results are compared with the first Born values of Bell *et al* (1968) [see table 3] and with various other theoretical approaches: 4-state close coupling calculations of Flannery (1970), 4-channel results of Begum *et al* (1973), 9-state calculations of Vanden Bos (1969) and first-order results of Baye (1970), who used the 'diagonalisation method' to perform approximate close-coupling calculations.

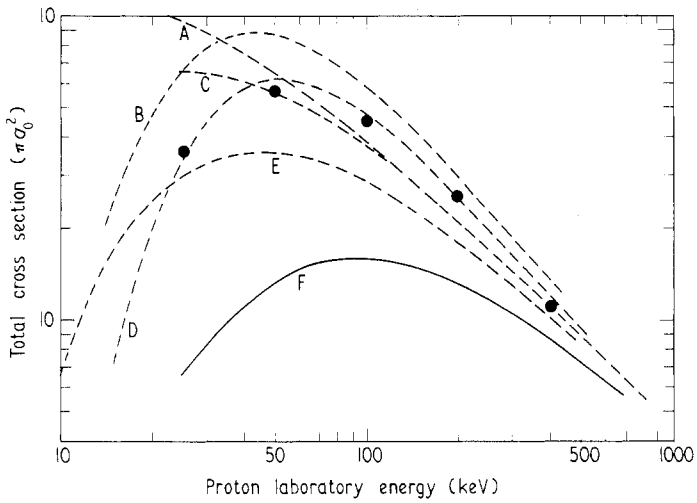

Figure 10. Total cross sections for excitation of the 2^{1}S state of helium by protons. Curves A first Born approximation; B Flannery (1970); C Baye (1970); D van den Bos (1969); E present work, first order potentials $V^{(1)}$; F present work, Glauber potentials U^G. ● 4-channel approximation of Begum *et al* (1973).

The second order results given by this method, which have been obtained recently by Baye and Heenen (1973), are not shown on figure 10 since they are similar to the values of Flannery. As in figure 6 of Paper I (corresponding to the excitation of the 2s state of hydrogen by proton impact), our distorted wave results differ substantially from those given by the methods which include explicitly the coupling between the states 1 ^{1}S, 2 ^{1}S and 2 ^{1}P.

Our total cross sections for the excitation of the 2 ^{1}P state of helium by proton impact (using both static potentials and Glauber optical potentials) are shown in figure 11. We note that in this case our values agree well with the 4-state close coupling calculations of Flannery (1970), the second-order diagonalization results of Baye and Heenen (1973) (undistinguishable on the graph from the values of Flannery), and the 4-channel approximation of Begum *et al* (1973); they are also in good agreement with the experimental data

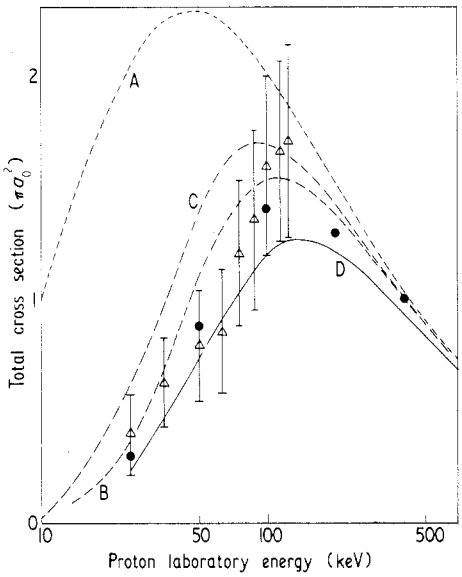

Figure 11. Total cross sections for excitation of the 2^1P state of helium by protons. Curves A first Born approximation; B Flannery (1970); C present work, first order potentials $V^{(1)}$; D present work, Glauber potentials U^G. ● 4-channel approximation of Begum *et al* (1973); △ Park and Schowengerdt (1969).

of Park and Schowengerdt (1969). This result, together with the electron impact calculations discussed above, confirms the fact (already noted in Paper I for the case of hydrogen) that our eikonal DWBA calculations are most reliable for allowed transitions.

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