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Elastic scattering of fast electrons by atoms
I. Helium to neon

Mito Inokuti†‡ and M R C McDowell§

† Argonne National Laboratory, Argonne, Illinois 60439, USA

§ Mathematics Department, Royal Holloway College, Englefield Green TW20 0EX, UK

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Abstract. The total elastic-scattering cross section for electrons on neutral atoms is evaluated in the first Born approximation via tabulated values of the form factors. The cross section at energy k^2 rydbergs is expressed as

$$\sum_{j=1}^{3} a_j(Z)k^{-2j}$$

values of the coefficients $a_j(Z)$ being given for $1 \leqslant Z \leqslant 10$. The theoretical results are compared with experiment for helium, neon and lithium. For lithium, comparisons are also made with Glauber and polarized-Glauber models. A simple model is presented for low-energy total cross sections in lithium, and its predictions compared with experiment. Elastic differential cross sections for helium and neon are briefly discussed.

1. Introduction

There has been relatively little interest in total cross sections for electron impact on atoms since the early 1930's, except at low energies. Application of the dispersion-relation technique to atomic scattering (Gerjuoy 1958) does however require a knowledge of the total cross section at all energies if the Gerjuoy–Krall dispersion relation (Gerjuoy and Krall 1960, 1962) is to be adequately tested. The present status in this regard has been briefly reviewed elsewhere (McDowell 1974). It suffices to remark here that the dispersion relation expresses the real part of the forward elastic-scattering amplitude at k^2 rydbergs in terms of, among other quantities, a principal value integral over the total cross section σ at all energies,

$$I_\lambda = \frac{P}{4\pi}\int_0^\infty \frac{s^\lambda \sigma(s^2)\,\mathrm{d}s^2}{s^2-k^2}$$

where $\lambda = -1$ or $+1$ depending on whether a subtracted dispersion relation is used or not.

A full bibliography on theoretical and experimental determination of σ is available (Kieffer 1967). The experimental measurements have been critically reviewed by Bederson and Kieffer (1971).

We shall be largely concerned with evaluating σ at high energies, that is, in the region of validity of the first Born approximation. We may consider the total cross section as made up of elastic and inelastic contributions

$$\sigma = \sigma_{\mathrm{el}} + \sigma_{\mathrm{inel}}.$$

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Very recently Bethe–Born evaluations of $\sigma_{\rm inel}$ for the ground states of all atoms from hydrogen to argon have been reported (Inokuti *et al* 1974), so that our task is to evaluate the corresponding elastic contributions. We give results in this paper for the atoms from hydrogen to neon and intend to discuss the second-row atoms in a subsequent paper.

The Born elastic-scattering cross section for hydrogen is well known. Accurate results for helium have been given by Kennedy (1968) and lithium has been treated in this approximation by Walters (1973) and Sarkar *et al* (1973).

The Glauber approximation was applied to elastic scattering by atomic hydrogen and helium (Franco 1968, 1970). It has also been used by Walters (1973) for lithium, while Sarkar *et al* (1973) have considered a polarized-Glauber approximation for the same atom. LaBahn and Callaway (1969) and Purcell *et al* (1970) have discussed polarization effects for helium. All these calculations, together with experimental data, give a guide in assessing the range of validity of the first Born approximation to elastic scattering.

The theoretical formulation is outlined in § 2 and our detailed results presented in § 3. These are discussed and compared with experiment and other theoretical calculations in § 4, in which we also present an alternative simple model for atoms of high polarizability.

2. Theory

Within the first Born approximation, the differential cross section $\mathrm{d}\sigma_{\rm el}$ for elastic scattering of an electron (or positron) into solid-angle element $\mathrm{d}\omega$ near angle θ upon collision with a neutral atom with nuclear charge Z is (Bethe 1930, 1933, Bethe and Jackiw 1968)

$$\mathrm{d}\sigma_{\rm el} = 4K^{-4}|Z-F(K)|^2\,\mathrm{d}\omega. \tag{1}$$

Here the momentum transfer K is given in terms of θ and the incident-electron momentum k as

$$K = 2k\sin(\theta/2) \tag{2}$$

and the atomic form factor $F(K)$ is the ground-state expectation value

$$F(K) = \left\langle \sum_{j=1}^{Z} \exp(\mathrm{i}Kx_j) \right\rangle \tag{3}$$

where x_j is a Cartesian component of the jth electron position vector $\boldsymbol{r}_j$. Atomic units are used throughout the paper, unless otherwise specified, energies being given in rydbergs (13·6 eV).

At a fixed incident momentum k (ie at fixed incident energy k^2) equation (2) enables one to express $\mathrm{d}\sigma_{\rm el}$ in terms of $\mathrm{d}K$ as

$$\mathrm{d}\sigma_{\rm el} = 8\pi k^{-2}|Z-F(K)|^2K^{-3}\,\mathrm{d}K. \tag{4}$$

The total cross section σ for elastic scattering then is

$$\sigma_{\rm el} = 8\pi k^{-2}\int_0^{K_{\max}} |Z-F(K)|^2K^{-3}\,\mathrm{d}K \tag{5}$$

where

$$K_{\max} = 2k \tag{6}$$

occurs for $\theta = \pi$.

We shall summarize below general characteristics of the crucial atomic property $F(K)$. First of all, we note that equation (3) can be put in an alternative form

$$F(K) = \int \rho(\boldsymbol{r}) \exp(\mathrm{i}Kx)\, \mathrm{d}^3\boldsymbol{r} \tag{7}$$

where $\rho(\boldsymbol{r})$ is the electron density defined as

$$\rho(\boldsymbol{r}) = \left\langle \sum_{j=1}^{Z} \delta(\boldsymbol{r}_j - \boldsymbol{r}) \right\rangle. \tag{8}$$

Here and in what follows, we presume that the atoms are randomly oriented and imply that all expectation values (denoted by $\langle\ \rangle$) are averages over the magnetic sublevels and hence we may replace $\rho(\boldsymbol{r})$ by $\rho(r)$.

First of all, $F(K)$ is even in K because $\rho(r)$ is even in $\boldsymbol{r}$ owing to the definite parity of any atomic state. Second, for small K one may expand the exponential in equation (7) and obtain

$$F(K) = Z - K^2\langle x^2\rangle/2 + K^4\langle x^4\rangle/24 + \dots \tag{9}$$

a property that guarantees the analyticity of the integrand of equation (5) near $K = 0$. The range of convergence of the above power series is limited (Vriens 1967, Vriens *et al* 1968). Third, an asymptotic expansion for large K,

$$F(K) = 4\pi[-2\rho'(0)K^{-4} + 4\rho'''(0)K^{-6} - 6\rho'''''(0)K^{-8} + \dots] \tag{10}$$

where the prime denotes differentiation with respect to r, can be obtained by repeated partial integrations (Goscinski and Lindner 1970, Sahni and Krieger 1972). Finally, the treatment by Lassettre (1965) of the inelastic-scattering form factor, which can be readily adapted to $F(K)$ (Vriens 1967), leads to an analytic representation

$$F(K) = Z(1+\zeta)^{-2}\left[1 + \sum_{\nu=1}^{\infty} c_\nu \zeta^\nu (1+\zeta)^{-\nu}\right] \tag{11}$$

where ζ equals K^2 multiplied by a constant and c_ν are constants to be determined so that equation (11) may be consistent with equations (9) and (10).

We have reviewed major current tabulations of $F(K)$ for atoms He through Ne (Tavard *et al* 1967, Kim and Inokuti 1968, Tanaka and Sasaki 1971, Brown 1970, 1971, Naon and Cornille 1972). Comparison of the data computed from atomic models of differing accuracy shows that $F(K)$ values from Hartree–Fock wavefunctions (Tavard *et al* 1967) are sufficiently accurate; they are usually within 1 % of values including electron correlation effects.

We first tried to fit the data to equation (11). The attempt was unsuccessful chiefly because the fit to equation (11) requires $F(K)$ data for large K values beyond the range of the tabulation. Later we turned to a modified representation

$$\begin{gathered} F(K) = Z(1+x)^{-2}\left[1 + \exp(-\lambda y) \sum_{\nu=0}^{7} \kappa_\nu y^\nu + y^{30} \sum_{\nu=0}^{2} \mu_\nu y^{-\nu}\right] \\ x = \frac{K^2}{\alpha^2} \qquad y = (1+x)^{-1}x \end{gathered} \tag{12}$$

where $\alpha^2 = 4E_i$ and E_i is the ionization potential in rydbergs, and λ, κ_v, μ_v, are constants. The modified fitting turned out to be satisfactory, the details being discussed further in § 3.

At sufficiently large incident energy k^2, an asymptotic expansion for σ_{el} is useful. One may split the integral in equation (5) into two terms

$$\int_0^\infty - \int_{k_{max}}^\infty$$

and use equation (10) in the second term. Thus one obtains

$$\sigma_{el} = \pi k^2[A+Bk^{-2}+Ck^{-4}+Dk^{-6}+ \dots] \tag{13}$$

where

$$A = 8\int_0^\infty |Z-F(K)|^2K^{-3}\,dK \tag{14}$$

$$B = -Z^2 \tag{15}$$

$$C = 0 \tag{16}$$

$$D = -\pi Z\rho'(0)/3. \tag{17}$$

It may be noted that equation (13) represents a rigorous asymptotic form that applies when $F(K)$ satisfies equation (10). In our numerical work using equation (12), σ_{el} was evaluated at a series of k^2 values and the results were then fitted to the same form as equation (13) with three coefficients. The resulting coefficients (b and c in equation (20)) satisfy equations (15) and (16) only approximately, as they attempt to take into account D and higher terms, and to fit the Born cross section at all energies.

3. Results

The data used were the Hartree–Fock form factors of Tavard *et al* (1967), except for fluorine, where we also used the configuration-interaction (CI) results of Tanaka and Sasaki (1971), and for neon where we used the CI results of Peixoto *et al* (1971).

A first test of the reliability of our fits is the mean value of χ^2; for an m-parameter fit it is given as

$$\bar{\chi}^2 = (n-m-1)^{-1}\sum_{i=1}^{n}\left[\frac{F(K_i)_{data}-F(K_i)_{calc}}{F(K_i)_{data}}\right]^2. \tag{18}$$

As the $\bar{\chi}^2$ values in table 1 show, our fit deteriorates in accuracy with increasing Z, except for neon, where we achieve a remarkably tight fit. In general we fitted to 30 data

Table 1. $\bar{\chi}^2$ for $F(K)$.

$z =$	2	3	4	5	
$\bar{\chi}^2$	1.2×10^{-6}	3.0×10^{-4}	1.5×10^{-4}	2.3×10^{-3}	
$z =$	6	7	8	9	10
$\bar{\chi}^2$	2.7×10^{-3}	2.8×10^{-3}	3.7×10^{-3}	7.2×10^{-3}	2.5×10^{-4}

points over the range $0{\cdot}1 \leqslant K \leqslant 30$, and required our fits to go to the correct (Hartree–Fock) low-K limit (cf equation (9)), the Hartree–Fock values of $\langle x^2 \rangle$ being evaluated from the values of $\langle r^2 \rangle$ for each nl-orbital given by Fischer Froese (1972). In addition, we require that equation (10) is satisfied at large K. Neglecting the K^{-8} term in equation (10), we have

$$K^4 F(K) = a_1 + a_2 K^{-2}. \tag{19}$$

According to equation (10), $a_1 = -8\pi\rho'(0)$. We have calculated $\rho'(0)$ for neon using analytic Hartree–Fock wavefunctions (Clementi 1965) and find $a_1(\text{HF}) = 3{\cdot}12 \times 10^{-4}$. Our fit using the CI data of Tanaka and Sasaki (1971) to $K = 50$ extrapolates to give $2{\cdot}91 \times 10^4$, which is sufficiently accurate for our purpose. Our fits to $F(K)$ are least satisfactory for fluorine, where a graphical extrapolation of the Hartree–Fock and CI data (figure 1) yields $a_1 = 1{\cdot}88 \times 10^5$, but our best fit gives $a_1 = 1{\cdot}35 \times 10^5$. However

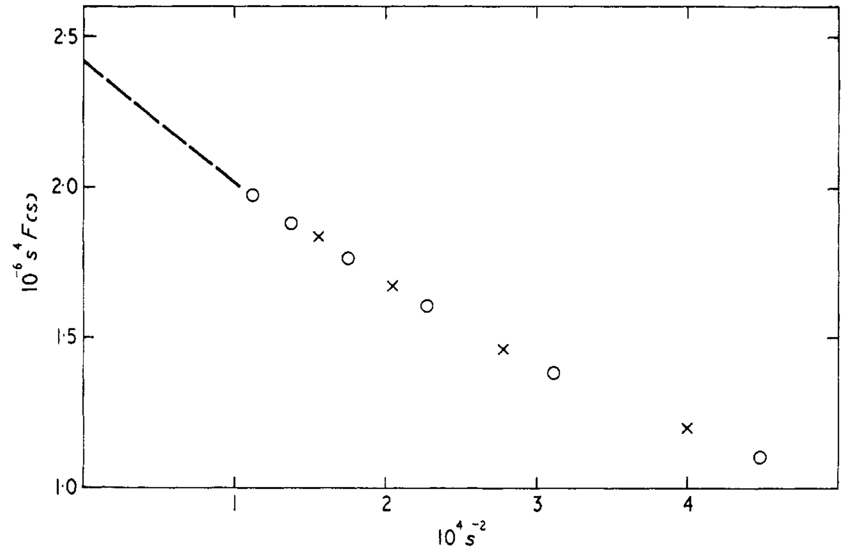

Figure 1. Form factor data for fluorine expressed in terms of equation (19). The variable q of Tavard *et al* (1967) stands for wavevector change expressed in Å^{-1}, and is related to K in au by $q = K/0{\cdot}52918$. The crosses are the Hartree–Fock values of Tavard *et al* (1967), the circles the CI values of Tanaka and Sasaki (1971). The dashed extrapolation gives $1{\cdot}88 \pm 0{\cdot}04 \times 10^5$.

our fits represent the tabulated values of $F(K)$ to better than 1% for $K \leqslant 30$ for $2 \leqslant Z \leqslant 10$, and we evaluate the Born total cross section in the energy range $5 \leqslant k^2 \leqslant 250$ au from our fits to $F(K)$ using a twenty-point Gaussian integration. The results were then fitted to a three-parameter form

$$\sigma_{\text{el}}(k^2) = (ak^{-2} + bk^{-4} + ck^{-6})\pi \tag{20}$$

the values of the parameters being given in table 2. They are compared with the asymptotic values (equations (14) and (15)), the value of A being estimated as

$$8 \int_0^{30{\cdot}0} K^{-3} |Z - F(K)|^2 \, dK.$$

Table 2. Coefficients of equation (20) compared with our best values for the asymptotic coefficients, equations (14) and (15). The bracketed values for helium are those of Kennedy (1968).

Z	a	A	$-b$	Z^2	c
1	2·333	—	1·0	1·0	—
2	3·526	3·509	4·06	4·0	2·33
	(3·540)		(4·14)		(2·85)
3	23·8	23·8	9·14	9·0	10·4
4	35·4	35·3	16·0	16·0	25·0
5	47·1	47·1	25	25	50
6	50·5	50·4	35	36	64
7	53·6	52·5	46	49	80
8	58	61	59	64	97
9	60	63	72	81	101
10	64·0	63·6	90·3	100	131

It is clear that the asymptotic expression for $\sigma_{el}(k^2)$, equation (13), is adequate at all energies for which the first Born approximation is likely to be reliable for $Z \leqslant 7$. The discrepancies for $Z = 8$ and 9 are probably due to the diminishing number of significant figures of the tabulated form-factor data for large K.

The variation of the leading coefficient a is shown in figure 2. In general, it varies smoothly along an nl shell, the small departures from smooth behaviour again being due to the quality of the original data, the maximum uncertainty being 2% in O($Z = 8$).

Our σ_{el} for helium agree with those of Kennedy (1968) to one part in 10^4.

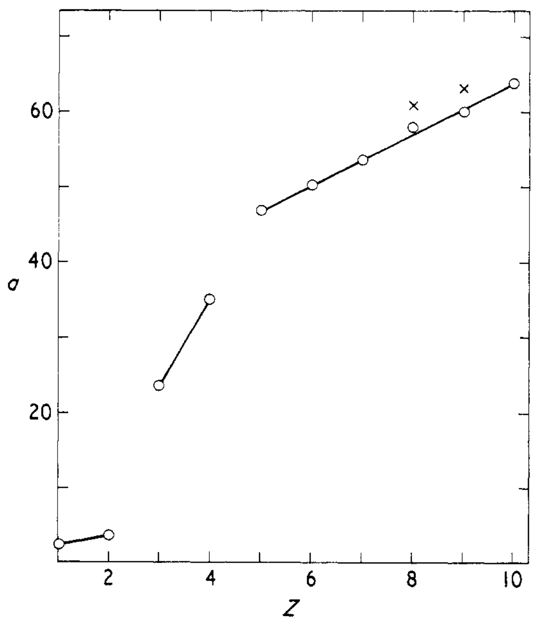

Figure 2. Variation of the leading Born coefficient a equation (20) with Z. The crosses show the estimated values of the leading asymptotic coefficient A equation (14), using a cut-off at $K = 30$, for $Z = 8, 9$.

Bethe and Jackiw (1968) give a general discussion of the reliability of the first Born approximation. A variety of theoretical and experimental evidence now permits us to estimate the reliability in more detail. We first compare our Born results for helium with the polarized-orbital calculations of LaBahn and Callaway (1969) and of Purcell *et al* (1970). This comparison is given in table 3. Note that the cross sections in this table are given in units of a_0^2. The differential cross sections obtained by Purcell *et al* (1970) are in essence fits to the measurements of Bromberg (1968) and other workers, and the absolute value given for the total cross section depends on the normalization of the experimental data. This is uncertain by perhaps $\pm 6\%$ at 500 eV, (cf Crooks and Rudd 1972) so that Born values appear to be consistent with experiment to better than $\pm 20\%$ at 500 eV, in agreement with an earlier conclusion of Vriens *et al* (1968). This is sufficient for our needs in dispersion-relation analysis.

Table 3. Total elastic-scattering cross sections in a_0^2 for helium.

E (eV)	100	150	200	300	400	500
Purcell *et al*	2·23	1·38	0·95	0·61	0·44	(0·35)
LaBahn and Callaway	2·51	1·47	1·00	0·61	0·43	0·35
Born, present work	1·27	1·18	0·70	0·48	0·36	0·29

The Glauber approximation has been applied to elastic scattering of electrons by atomic hydrogen (Franco 1968) and by helium (Franco 1970). For both atoms and for $k^2 > 10$, the *integrated* cross section σ_{el} from the Glauber approximation is virtually the same as that from the Born approximation, although angular distributions given by the two approximations differ appreciably for extremely small and large scattering angles.

Our Born results for lithium agree with the values reported by Walters (1973). Walters' Glauber fixed-core model gives cross sections appreciably lower than the Born below 500 eV. However both Born and Glauber cross sections are in complete disagreement with experiment below 50 eV. This disagreement is discussed further in § 4. Walters argues that in this case also the Born values cannot be reliable below 500 eV, and we would agree that our cross sections may be in error by at least 10% below this energy in all cases.

4. Comparison with experiment

Our Born results for lithium agree with those obtained by Walters (1973) and by Sarkar *et al* (1973). When extrapolated back to 50 eV they are smaller than the experimental results (Kasdan *et al* 1973) by a factor of 2 even when allowance is made for inelastic scattering by using the result of Inokuti *et al* (1974). It is clear from Walter's work that the fixed-core Glauber approximation provides no substantial improvement at low energies.

Sarkar *et al* (1973) have included a polarization potential in the interaction in both Born and fixed-core Glauber approximations. Their polarized-Born result is a gross overestimate, but as may be seen from figure 4, their polarized-Glauber calculation gives

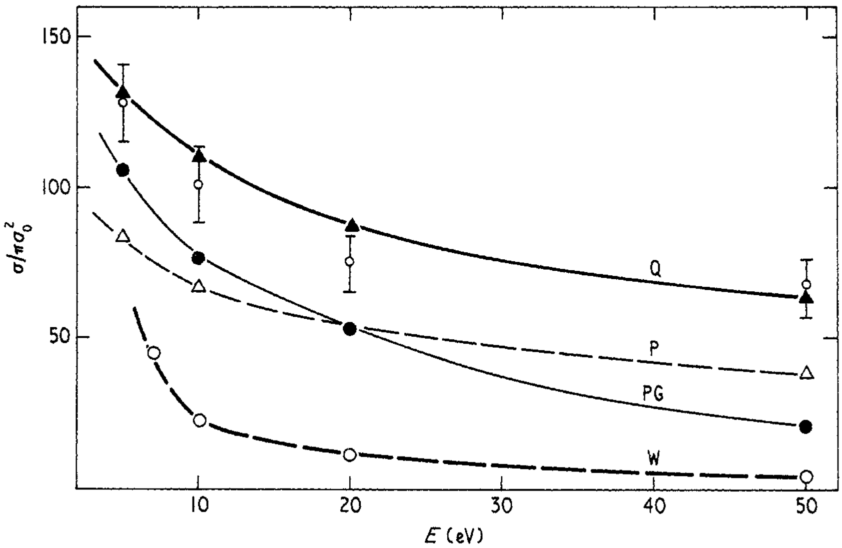

Figure 3. Total cross section for electrons on lithium. The experimental data of Kasdan *et al* (1971) is shown by circles with error bars ($\pm 12\%$). The Glauber approximation of Walters (1973) is the curve W, the polarized Glauber calculation of Sarker *et al* (1973) is the curve PG. Our elastic cross section equation (25) is the curve P, while curve Q is the total cross section obtained by allowing for inelastic contributions (see text).

a substantial improvement. However such polarized-Glauber models are of doubtful validity as they appear to include the effects of polarization inconsistently.

Alternatively we may adopt an impact-parameter treatment. Relating the impact parameter ξ with angular momentum l as $\xi = (l+\frac{1}{2})/k$ and replacing the standard partial-wave sum by an integral over ξ, we have

$$\sigma_{\mathrm{el}}(k^2) = 8\pi \int_0^\infty \xi \sin^2\eta(\xi)\,\mathrm{d}\xi \tag{21}$$

where $\eta(\xi)$ is the phase shift. Adopting the random-phase approximation for small ξ (and neglecting spin), we may write

$$\sigma_{\mathrm{el}}(k^2) = \pi\left[2\xi_1^2 + 8\int_{\xi_1}^\infty \eta^2(\xi)\xi\,\mathrm{d}\xi\right] \tag{22}$$

where ξ_1 is the largest value of ξ for which $\eta(\xi)$ exceeds $\pi/4$. For $\eta(\xi) < \pi/4$ it suffices to use the Born-approximation phase shift

$$\eta_{\mathrm{B}}(\xi) = \pi\alpha/(8k\xi^3) \tag{23}$$

for a pure polarization potential, where αa_0^3 is the dipole polarizability of the atom. Thus, we obtain

$$\sigma_{\mathrm{el}}(k^2) = 3{\cdot}234\pi\xi_1^2 \tag{24}$$

with

$$\xi_1^3 = (\alpha/2k). \tag{25}$$

The experimental value of the polarizability of Li is $160a_0^3$ (Molof *et al* 1974) and the resulting values of $\sigma_{\mathrm{el}}(k^2)$ are shown by the dashed curve marked P in figure 4.

We follow Walters (1973) in allowing for inelastic scattering by adding the resonance-excitation cross section $\sigma(2s \rightarrow 2p)$ measured by Hughes and Hendrickson (1966) for energies up to 30 eV, and estimating the total inelastic-scattering cross section at 50 eV as $25\pi a_0^2$ from the result of Inokuti *et al* (1974). The resultant total cross sections are in reasonable accord with experiment between 5 eV and 50 eV. Recent work by Leap and Gallagher (1974) for the $2s \rightarrow 2p$ resonance transition (plus cascade) is in close agreement with the results of Hughes and Hendrickson (1966), and is consistent with our adopted value at 50 eV.

Total cross sections for electrons on rare gases have been measured by Normand (1930). Plotting his results for helium and neon (times the energy) against energy in figures 4 and 5, we may compare with the total cross sections obtained by adding our elastic cross sections to the inelastic-scattering cross sections of Inokuti *et al* (1974). For helium, our results lie 50 % higher than Normand's at the highest energy at which he measured (400 eV). Although we have argued that the Born approximation is unreliable below 500 eV in all cases, the polarization cross sections obtained by LaBahn and Callaway (1964) and Purcell *et al* (1970), and given in table 2, indicate that errors of 50 % are unlikely. Bransden *et al* (1974) have shown that for positron scattering by helium, replacement of the high-energy experimental results by our values leads to an accurate prediction of the scattering length via a sum rule. Similarly, if we adopt our values and join them smoothly to the low-energy experimental data of Golden and Bandel (1966) we satisfy the dispersion relation at low energies (Naccache and McDowell 1974), but fail to do so at any energy, if we accept Normand's results, which therefore appear to be 50 % too low at 400 eV†.

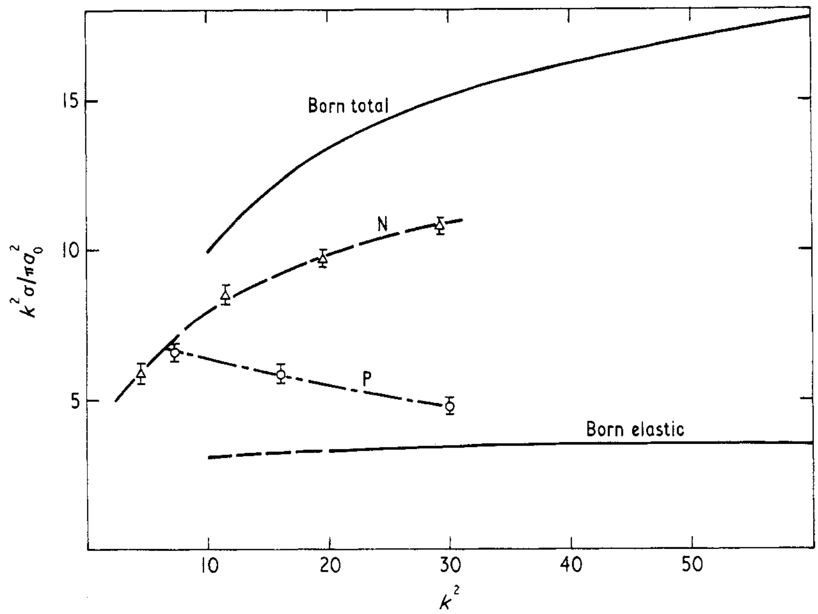

Figure 4. $k^2\sigma_{el}$ against k^2 for helium. The Born elastic and Bethe–Born total cross section results are shown together with the experimental data of Normand (1930), marked N, for electrons and Canter *et al* (1973) for positrons (marked P).

† *Note added in proof.* A reanalysis of the experimental data by de Heer *et al* (private communication) suggests that $\sigma = 1{\cdot}67\,\sigma$ (Normand) at 400 eV, A remeasurement of the position data (Canter *et al* 1974) bring them into agreement.

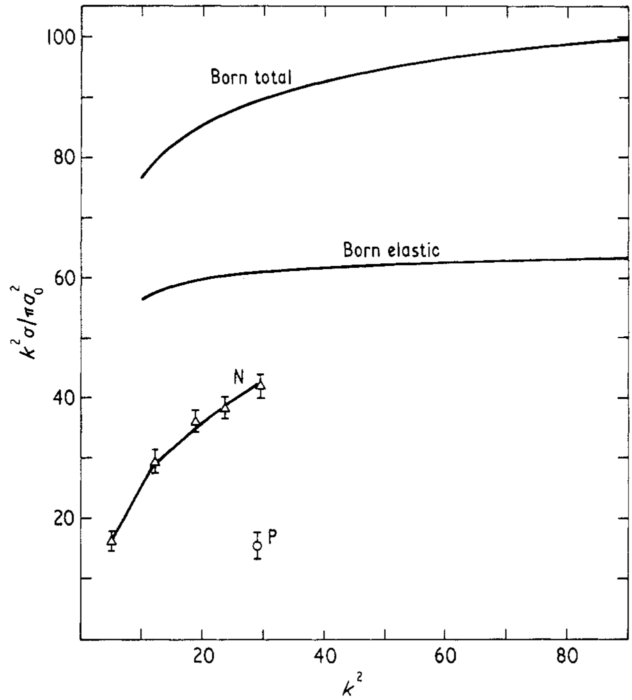

Figure 5. As figure 4, but for neon.

The situation in neon is not so clear. However, it should be noted that the total cross section for positron impact of $0{\cdot}53\pi a_0^2$ measured by Canter *et al* (1973) at 400 eV is a factor of three below Normand's result for electron impact at the same energy. Both results are strongly incompatible with our Born value which is a factor of two larger at 400 eV than Normand's results. The measurements are of course difficult, one difficulty being sharp forward peaking of the total differential cross section.

We have evaluated $I(\theta)\sin\theta$ in the Born approximation at $k^2 = 30{\cdot}0$ (408 eV) for the atoms from helium to neon and the results are shown in figures 6 and 7. Our results for helium (figure 6) show a maximum of $0{\cdot}09a_0^2$ at 16°, the experimental values of Vriens *et al* (1968) when renormalized at 5° to Chamberlain *et al* (1970) having their maximum of $0{\cdot}11a_0^2$ at 10°, but being in close accord with the Born values for $\theta > 20°$. The recent results of Crooks and Rudd (1972) show however a maximum value of $0{\cdot}13a_0^2$ at 10° and remain 20% larger than the Born value even at 30°. Other recent measurements in helium, at impact energies above 100 eV, by McConkey and Preston (1974) and by Bromberg (1969, 1974) are however consistent with our Born values of $I(\theta)\sin\theta$ at energies greater than 400 eV.

Again neon seems to be anomalous. Recent measurements at small angles by Bromberg (1974) at energies $E \geqslant 200$ eV are quite inconsistent with the values quoted by McConkey and Preston (1973 and private communication) for $E = 100$ eV and $\theta \geqslant 20°$. The results obtained by the several experimental groups are compared in table 4 (see also Naccache and McDowell 1974). It is clear that the measurements in helium and argon are reasonably consistent in absolute value (at least at energies $E \geqslant 200$ eV) though more detailed comparison (McConkey and Preston 1974, Williams and Willis 1974, Bromberg 1974) shows up some differences of a minor character in angular variation. This is not the case for neon; any reasonable extrapolation of

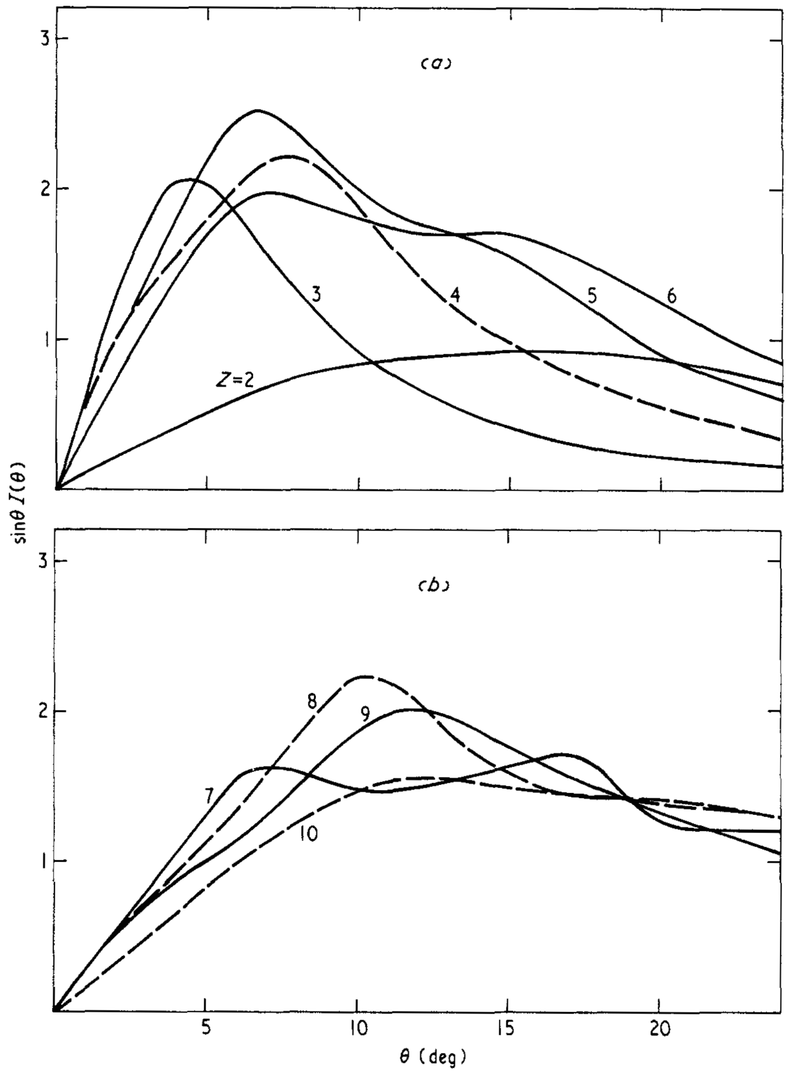

Figure 6. $I(\theta)\sin\theta$ for elastic scattering of electrons by neutral atoms with (*a*) $Z = 2, 3, 4, 5, 6$ at $k^2 = 30{\cdot}0$ (408 eV). The results shown for helium $Z = 2$ are ten times the Born values. (*b*) $Z = 7, 8, 9, 10$.

Bromberg's measurements down to 100 eV gives results a factor of three higher than those of McConkey and Preston.

We compare the Born differential cross sections with Bromberg's (1974) results in table 5. The agreement is extremely poor at all energies up to 700 eV, and for angles out to 25°. However Bromberg's results would account for the large discrepancy between Normand's (1930) total cross section measurements, and our Born calculations. This again would suggest that at least in the case of neon, the Born approximation could not be considered reliable at energies below 2 keV. However the large discrepancy between Bromberg's neon data, and that of McConkey and Preston (1973) discussed above must first be resolved before any firm conclusions can be reached.

Table 4(*a*). Elastic differential cross sections in neon in units of $a_0^2\,\mathrm{sr}^{-1}$.

E (eV)	100	200	300	400	500	700
$\theta = 20°$	0·767	2·691	2·137	1·823	1·542	1·285
$\theta = 25°$	0·707	1·767	1·303	1·099	0·903	0·720

The results are those of Bromberg (1974) except for those at 100 eV, which are due to McConkey and Preston (1973), renormalized downwards by 13% (McConkey 1974, private communication).

Table 4(*b*). Elastic differential cross sections in the rare gases at 20° in $a_0^2\,\mathrm{sr}^{-1}$.

	E (eV)	100	200	300	400
He	(*a*)	0·251 ± 0·03	—	0·339 ± 0·03	—
	(*b*)	—	—	0·364 ± 0·009	—
Ne	(*a*)	0·767 ± 0·08	—	—	—
	(*b*)	—	2·691	2·132	—
Ar	(*a*)	6·375 ± 0·6	—	—	—
	(*b*)	—	5·087 ± 0·125	3·865 ± 0·090	3·196
	(*c*)	11·2	3·98	3·45	2·80

(*a*) McConkey and Preston (1973, 1974).
(*b*) Bromberg (1974).
(*c*) Williams and Willis (1974) and private communication.

Table 5. Elastic differential cross sections in neon in units of $a_0^2\,\mathrm{sr}^{-1}$, (*a*) Born, this paper, (*b*) experiment, Bromberg (1974).

E (eV)	400		500		700	
θ (deg)	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
3	8·74	10·67	8·72	9·92	8·68	9·62
5	8·61	8·62	8·56	7·98	8·45	7·70
8	8·73	6·34	8·02	5·79	7·52	5·51
10	7·72	5·19	7·30	4·69	6·48	4·38
14	6·17	3·42	5·53	3·03	4·58	2·72
20	4·19	1·82	3·61	1·54	2·71	1·29
25	3·03	1·10	2·43	0·90	1·62	0·72

Much less is known about total inelastic differential cross sections. There does not appear to be any experimental data on the systems of interest here, at any energy.

In the Born region and for $k^{-2} \ll \theta \ll 1$ radian we can make an estimate of the total inelastic differential cross section (Inokuti 1971, p 334) in terms of the incoherent scattering function $S_{\mathrm{inc}}(K)$:

$$I_{\mathrm{inel}}(\theta) \simeq 4K^{-2}ZS_{\mathrm{inc}}(K). \tag{26}$$

Using tabulated values of the incoherent-scattering function for helium and neon (Kim and Inokuti 1970, Tanaka and Sasaki 1971, Peixoto *et al* 1971) we find the values given in table 6, for $k^2 = 30$. Assuming the Born approximation has some validity at this

Table 6. Inelastic differential cross sections at 408 eV (a_0^2).

	θ (deg)	$I(\theta)$	$I(\theta)\sin\theta$
He	1·74	2·44	0·09
	3·47	2·93	0·18
	5·21	2·82	0·27
Ne	5·38	7·25	0·68
	11·06	5·77	1·11
	16·42	4·63	1·33
	32·84	2·41	1·39

energy for the systems considered and combining these results with the elastic data presented in figures 6(*a*) and 6(*b*), we can conclude that the large discrepancies between Born total cross sections in helium and neon and the measurements of Normand (1930) cannot be attributed solely to a failure to detect scattering through angles θ less than some θ_0, say 10°.

Further experimental work, particularly in total inelastic differential cross sections would be of considerable value.

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