

## Exercise 5

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## 1 sequential updating

1. Denote  $\theta|x^{(n)} \sim \pi$ . Since  $x_i|\theta$  is i.i.d, then  $x^{(n)}|\theta \sim \nu_\theta^n$ . Suppose  $\mu$  is the common dominating measure of  $\nu$  and  $\rho$ , i.e.  $p(x_i|\theta) = \frac{d\nu}{d\mu}(x_i)$  and  $p(\theta) = \frac{d\rho}{d\mu}(\theta)$ , then by Bayes' Theorem, we have:

$$\frac{d\pi}{d\mu}(\theta) = p(\theta|x^{(n)}) = \frac{p(x^{(n)}|\theta) \cdot p(\theta)}{\int_{\Theta} p(x^{(n)}|\theta) \cdot p(\theta) d\mu}$$

Hence

$$\pi(d\theta) = \frac{p(x^{(n)}|\theta) \cdot \rho(d\theta)}{\int_{\Theta} p(x^{(n)}|\theta) \cdot \rho(d\theta)} = \frac{\nu_\theta^n(dx) \cdot \rho(d\theta)}{\int_{\Theta} \nu_\theta^n(dx) \cdot \rho(d\theta)}$$

2. Using  $\pi$  as the new prior, then by Bayes's Theorem, the posterior distribution of  $\theta|x_{n+1}$  is given by:

$$\begin{aligned} & \frac{p(x_{n+1}|\theta) \cdot \pi(d\theta)}{\int_{\Theta} p(x_{n+1}|\theta) \cdot \pi(d\theta)} \\ &= \frac{\nu_\theta(dx_{n+1}) \cdot \nu_\theta^n(dx) \cdot \rho(d\theta)}{\int_{\Theta} \nu_\theta(dx_{n+1}) \cdot \nu_\theta^n(dx) \cdot \rho(d\theta)} \\ &= \frac{\nu_\theta^{n+1}(dx) \cdot \rho(d\theta)}{\int_{\Theta} \nu_\theta^{n+1}(dx) \cdot \rho(d\theta)} \end{aligned}$$

We can see that is equivalent of using  $\rho$  as prior with  $(x_1, \dots, x_{n+1})$  observations.

## 2 Bayesian inference in the limit of increasing data

This time, the model index is our parameter, and it has prior on  $\text{Unif}(\{0, 1\})$ . Given the model index, we will use the corresponding model. we want to infer under the observation, the probability of the model index equals to 1, or equivalently, the posterior mean of model index, so that we could determine whether model 1 has higher chance or not. So we will implement a new ppl program and then use posterior function to compute the posterior mean.

```

1 unified_model = function() {
2   # model ~ unif({0,1}) and it represents if we choose the model with slope(=1), or without
   slope(=0)
3   model = simulate(Bern(1 / 2))
4   if (model == 1) {
5     logistic_regression()
6   }
7   else {
8     logistic_regression_2()
9   }
10  return(model)
11 }
12 posterior(unified_model, 1000)
13 # 0.533340959655548

```

From above, we can see that under the observation, model index being 1 has higher probability, thus we should choose the model with slope.