STAT 447C: Bayesian Statistics **Fall 2024**

Exercise 2

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1 define a Bayesian model

1. The unknown quantity is the value of p, and the data is the observed value of Y_i's.
2.

$$X \sim \rho, \rho = (\rho_k)_{k=1}^{K}$$
$$Y|X \sim \text{Bernoulli}(X)$$

2 posterior and point estimates

1.
```
prior_probabilities = NULL
realizations = NULL
for (k in (0 : K)) {
    prior_probabilities = c(prior_probabilities, (k/K)*(1-k/K))
    realizations = c(realizations, k/K)
}
prior_probabilities
realizations
plot(realizations, prior_probabilities, type="h", main = "prior pmf")

```

2.
```
posterior_probabilities = NULL
Z = 0
for (k in (0 : K)) {
  # likelihood = P(X = k/K) * P(Y = (1,1,1) | X = k/K)
  likelihood = realizations[k+1]**3 * prior_probabilities[k+1]
  Z = Z + likelihood
  posterior_probabilities = c(posterior_probabilities, likelihood)
}
posterior_probabilities = posterior_probabilities / Z
posterior_probabilities
plot(realizations, posterior_probabilities, type = "h", main = "posterior pmf")


```

3. From the plot, we can see the posterior mode is at $X = 0.8$
4.
```
# posterior mean
m = 0
for (k in (0 : K)) {
  x = realizations[k+1]
  pi_k = posterior_probabilities[k+1]
  m = m + x * pi_k
}
m #0.712497759455099


```

3 Bayes action

1. xxx

2. xxx
3. xxx