Acoustic Horn

Nonlinear Sound Propagation using the Westervelt Model

DB# 54021

Background and Motivation

- This tutorial shows how to model the propagation of nonlinear acoustic waves generated by an exponential horn.
- The nonlinear behavior is due to a high amplitude input signal which results in high sound pressure levels in the horn throat.
- A harmonic input driven at the frequency $f_0 = 130$ Hz generates an acoustic wave with the frequency spectrum that contains the harmonics $2f_0$, $3f_0$, etc.
- This makes the frequency domain analysis irrelevant and requires a full nonlinear transient analysis of the system.

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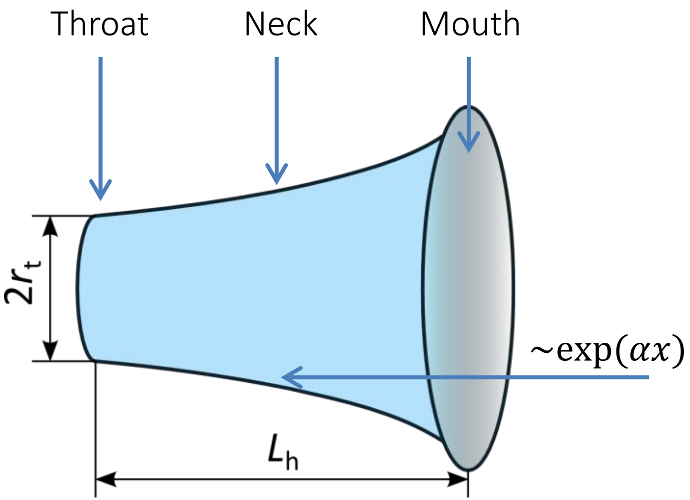
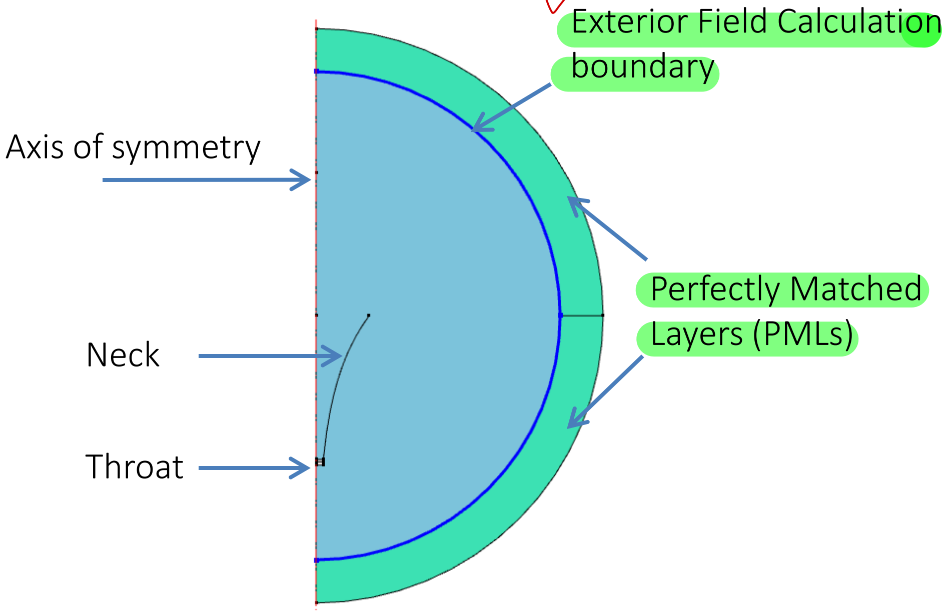
Model Geometry



Exponential horn sketch



2D axisymmetric model setup

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Model Setup: Physics Interfaces

- The *Pressure Acoustics, Transient* interface is used for transient computation of the acoustic pressure
- The dissipative—Thermally conducting and viscous—material model and the *Nonlinear Acoustics (Westervelt)* features describe the nonlinear propagation of acoustics in the physical domain
- PMLs are used together with the lossless Transient Pressure Acoustics Model to model the nonreflecting condition towards infinity
- The *Exterior Field Calculation* feature provides the radiation pattern of the acoustic field

Equation

Show equation assuming:

Study 1 - Frequency Domain, Frequency Domain

$$\frac{1}{\rho c^2}\frac{\partial^2 p_t}{\partial t^2} + \nabla\cdot\left(-\frac{1}{\rho}(\nabla p_t - \mathbf{q}_d)\right) = Q_m + \frac{\beta}{\rho^2 c^4}\frac{\partial^2 p_t^2}{\partial t^2}$$

Westervelt equation

$$\beta = 1 + \frac{B}{2A}$$

Model Input

Coefficient of Nonlinearity

From parameter of nonlinearity

Parameter of nonlinearity:

B/A From material

Material Contents

	Property	Variable	Value	Unit	Property group
✓	Bulk viscosity	muB	muB(T)	Pa·s	Basic
✓	Dynamic viscosity	mu	eta(T)	Pa·s	Basic
✓	Ratio of specific heats	gamma	1.4	1	Basic
✓	Heat capacity at constant pressure	Cp	Cp(T)	J/(kg·K)	Basic
✓	Density	rho	rho(pA,T)	kg/m³	Basic
✓	Thermal conductivity	k_iso ;...	k(T)	W/(m·K)	Basic
✓	Speed of sound	c	cs(T)	m/s	Basic
✓	Parameter of nonlinearity	BA	(def.gamma+1)/2	1	Nonlinear model
	Coefficient of thermal expansion	alpha_i...	alpha_p(pA,T)	1/K	Basic
	Mean molar mass	Mn	0.02897	kg/mol	Basic
	Relative permeability	mur_is...	1	1	Basic

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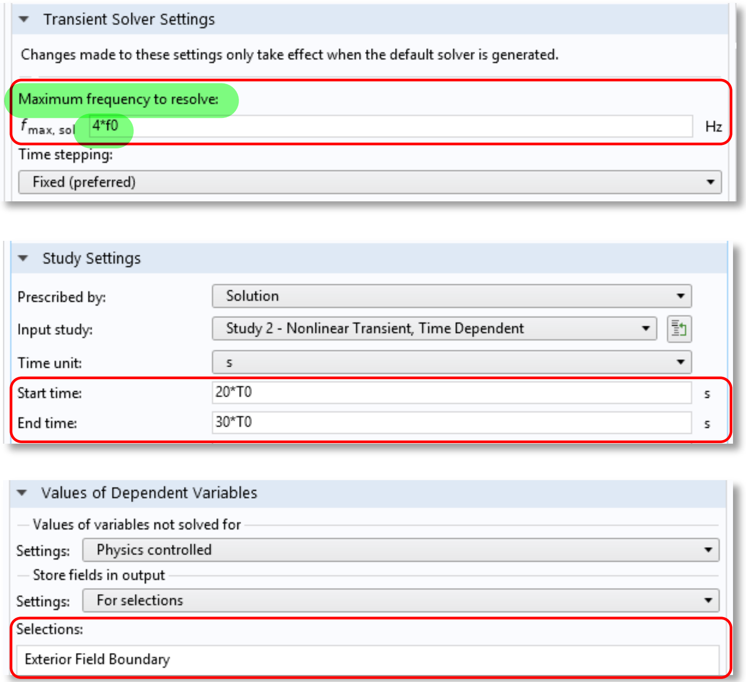
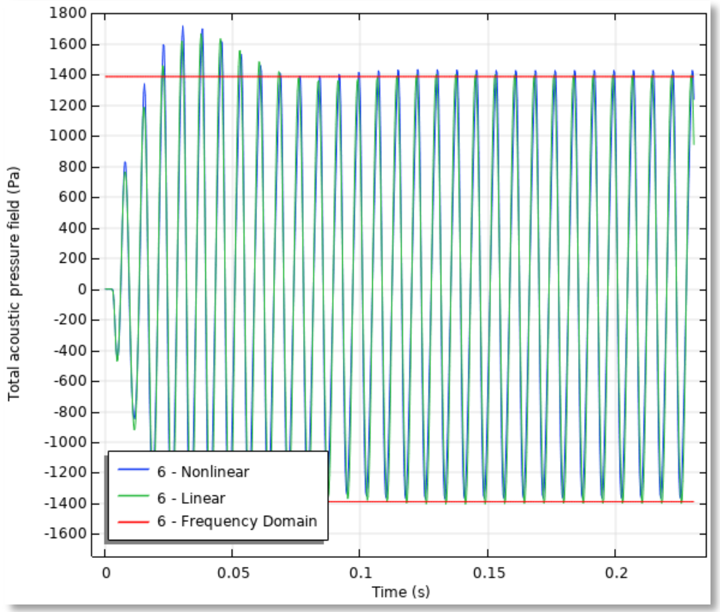
Model Setup: Study and Solver Settings

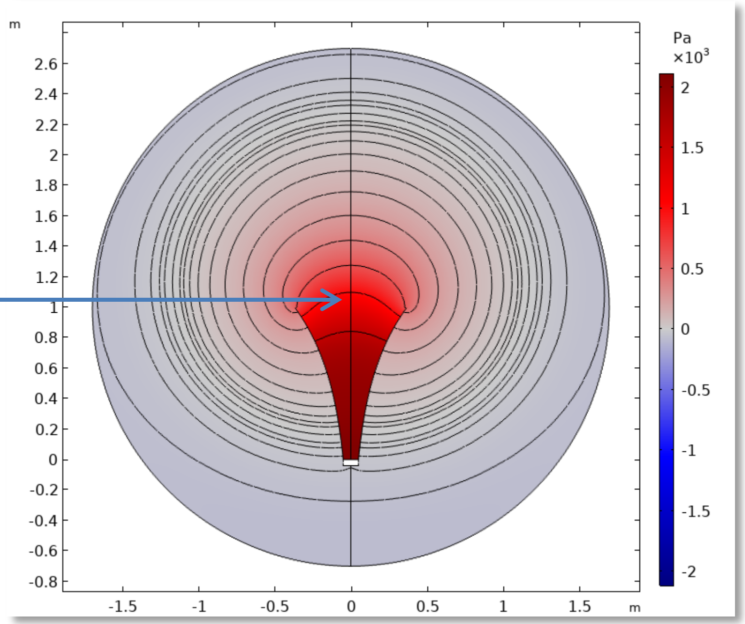
- This model resolves the frequency components up to $4f_0$
- The nonlinear transient study contains two steps: *Time Dependent* and *Time to Frequency FFT*
- The *Nonlinear Acoustics (Westervelt)* feature automatically tunes the Time Dependent Solver, which makes it effective for the underlying nonlinear problem
- The *Time to Frequency FFT* step is applied on the time interval where the solution has reached steady state. The result is only stored on the exterior field boundary that is used for the exterior field calculation.



Results: Acoustic Pressure



Acoustic pressure at point: linear vs nonlinear approach



Total nonlinear acoustic pressure profile
with contours

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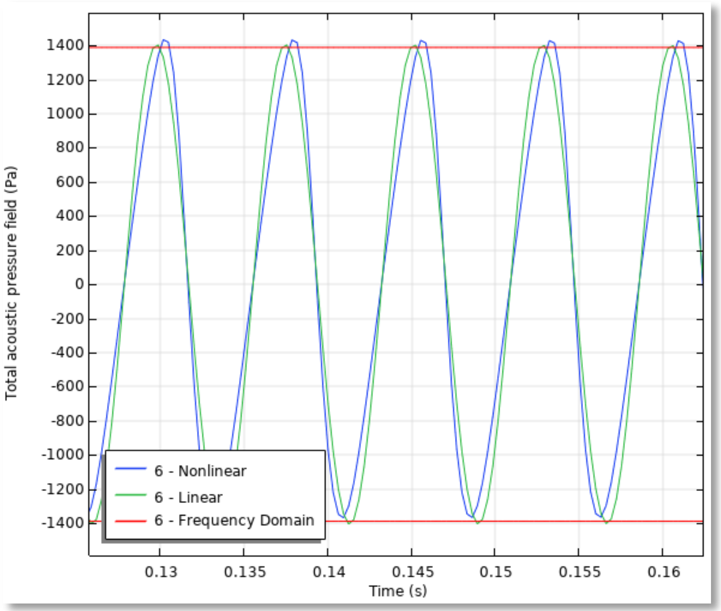
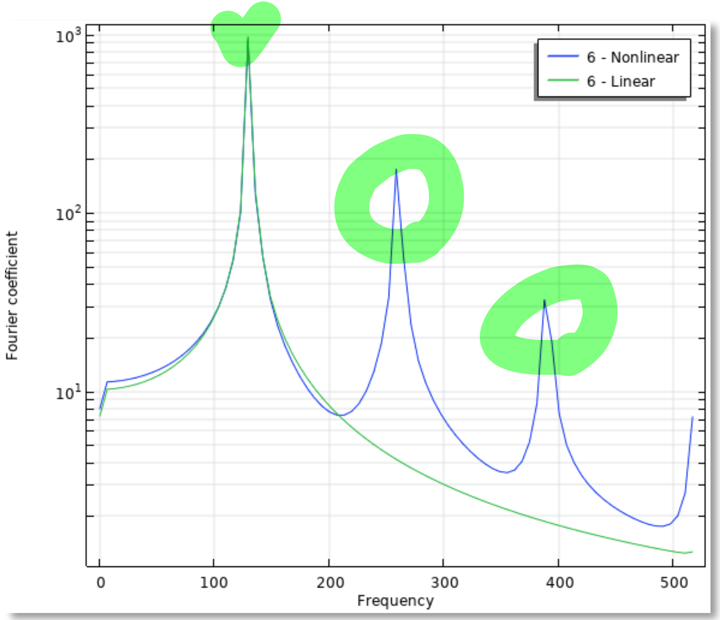
Results: Exterior Pressure



Acoustic pressure at point: Transient



Acoustic pressure at point: Frequency spectrum

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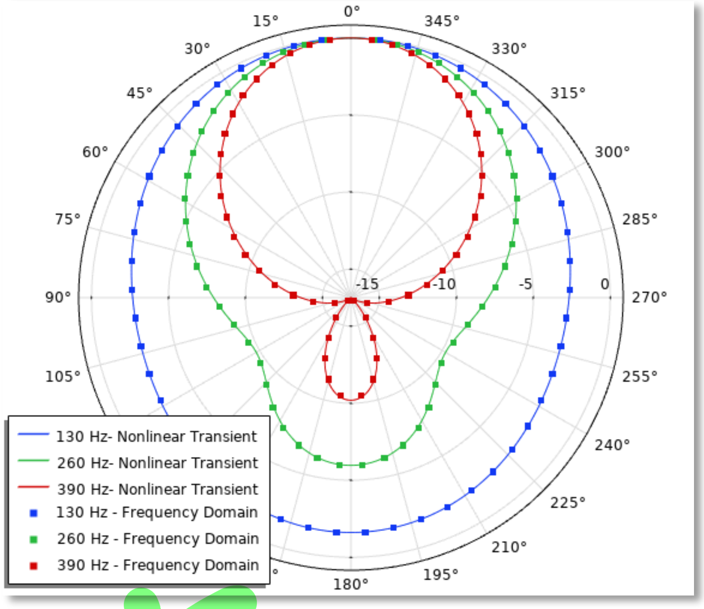
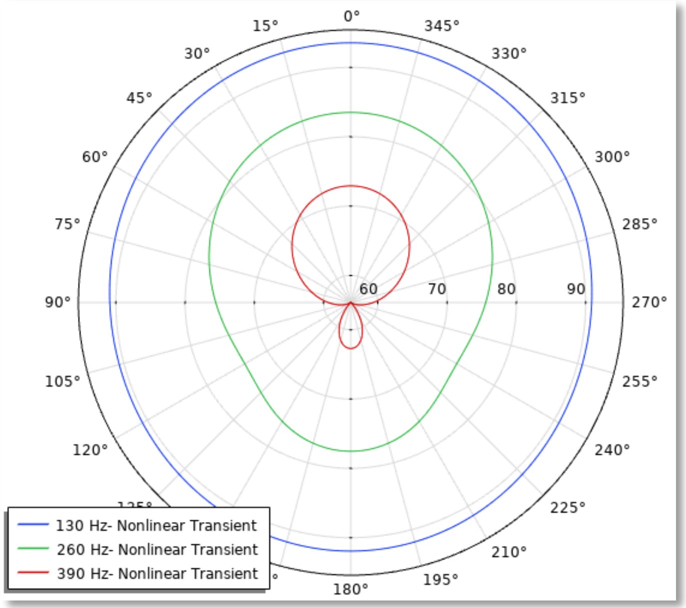
Results: Exterior Field SPL



Normalized exterior field SPL at 10 m:
nonlinear analysis vs single frequency domain



Exterior field SPL at 10 m:
Nonlinear transient analysis, first three harmonics