

I am going to only buy 10a as I have 3 grace days left.

A)

let  $w(t)=t^2+b*t+c$

integral of  $w(t)$  from 3 to 4 is  $(6*c+21*b+74)/6$

integral of  $t*w(t)$  from 3 to 4 is  $(42*c+148*b+525)/12$

since both results have to be 0

$$(6*c+21*b+74)/6=0;$$

$$(42*c+148*b+525)/12=0;$$

$b=-7$  and  $c=73/6$

therefore  $w(t)=t^2-7*t+73/6 = (t-(7/2+1/(2*\sqrt{3})))*(t-(7/2-1/(2*\sqrt{3})))$

and the root of this function is

$$t1= 7/2-1/(2*\sqrt{3})$$

$$t2= 7/2+1/(2*\sqrt{3})$$

B)

I'll use Lagrange polynomial interpolation to find  $w1$  and  $w2$ .

$$t1 \sim 3.2113$$

$$t2 \sim 3.7887$$

$$t3=7/2, w(t3)= -1/12$$

(using  $t1$  and  $t3$ )

$$p1 = (x-3.2113)/(3.5-3.2113)*(-1/12) = -0.290664*(-3.2113 + x)$$

(using  $t2$  and  $t3$ )

$$p2 = (x-3.7887)/(3.5-3.7887)*(-1/12) = 0.28865*(-3.7887 + x)$$

$$\text{integral}(p1)[3,4] = -0.0839146968 \sim 0 + w2*p1(t2) \rightarrow w2 = 0.5$$

$$\text{integral}(p2)[3,4] = -0.083333255 \sim w1*p2(t1) + 0 \rightarrow w1 = 0.5$$

so  $w1 = 0.5, w2 = 0.5, t1=3.2113, t2=3.7887$

C)

$$f(x)=x^3+1$$

$$\text{integral}[3,4](f(x))=((3.2113)^3+1+(3.7887)^3+1)*0.5=44.75$$

we got an exact answer using the above method

D)

Using above method

$$\text{integral}[3,4](e^x) \sim e^{(3.2113)*0.5}+e^{(3.7887)*0.5}=34.5051$$

$$\text{integral}[3,4](1/(x-2)) \sim (1/(3.2113-2)+1/(3.7887-2))*0.5=0.6923$$

$$\text{integral}[3,4](\cos(x)) \sim (\cos(3.2113)+\cos(3.7887))*0.5=-0.8977$$

Using composite Midpoint rule

```
function res=MidPointRule(f,a,b)
```

```
    res=f((a+b)/2)*(b-a);
```

```
end
```

```
f=@(x)exp(x);
```

```
t=linspace(3,4,3);
```

```
for i=1:2
```

```
    res=res+MidPointRule(f,t(i),t(i+1));
```

```
end  
res = 34.155710958627921
```

```
f=@(x)1/(x-2);  
res= 0.685714285714286  
f=@(x)cos(x);  
res= -0.907344516710054
```

Lastly using Simpson

```
function res=Simpson_rule(f,a,b)  
    res=(f(a)+4*f((a+b)/2)+f(b))*(b-a)/6;  
end  
f=@(x)exp(x);  
Simpson_rule(f,3,4) = 34.524249131850191  
f=@(x)1/(x-2);  
res = 0.694444444444444  
f=@(x)cos(x);  
res = -0.898243811104540
```

Using antiderivatives for integral(same order as above)=

34.51261310995657  
0.6931471805599453  
0.8979225033677955

For all three functions, the output from Gaußian rules is closest to the actual value therefore error is the smallest.