a)
<K.P. Kaypee and T.N. Tian. Nathan’s scheme>

$$\int_0^1 f(t)\,dt \approx \frac{f(0)+f(1)+4f(\frac{1}{4})+4f(\frac{3}{4})+2f(\frac{1}{2})}{12},$$

Let [t0,t1,t2] = [0,½,1]
h=1/2
The first point, t0 = 0, has h/6 weights
(t0+t1)/2 has 4h/6 weights
t1 has 2h/6 weights
(t1+t2)/2 has 4h/6 weights
Lastly, t2=1 has h/6 weights
Therefore, this is a Composite Simpson's rule = ***ICS***
The error formula is
***ECS*(*f*)** = −1/2880*f””(c)*(1/2)^4= **-1/46080 * f””(c)**

<Trish’s scheme>

$$\int_0^1 f(t)\,dt \approx \frac{f(0)+f(1)+2f(\frac{1}{4})+2f(\frac{3}{4})+2f(\frac{1}{2})}{8}.$$

Let [t0,t1,t2,t3,t4] = [0,¼,½,¾,1]
h=1/4
t0 has h/2 weight
t1 has h weight
t2 has h weight
t3 has h weight
t4 has h/2 weight
Therefore, this is a Composite Trapezoid Rule = ***ICT***
***ECT*(*f*)** = −1/12*f”(c)*(1/4)^2 = **-1/192*f”(c)**

b)
<x^3 − 3x^2 + 2>
Since the 4th derivative of the function is 0 we can use Nathan’s scheme.
ECS(x^3 − 3x^2 + 2) = **0**

<arctan(3*x − 1)>
Since the 4th derivative’s max absolute value is much larger than the 2nd derivative’s we should use Trish’s scheme.
*ECT(*arctan(3*x − 1)*)*= -5.846*-1/12*h^2 (c=0.526)

<e^x>
Since f” and f”” is the same in this case and Nathan’s error formula has a bigger division, Nathan’s scheme has smaller errors than Trish’s.
ECS(*e^x*)=−1/2880 *e^1*h^4 (c=1)

<x^(5/2)>

Since the 4th derivative’s max absolute value is much larger than the 2nd derivative’s we should use Trish’s scheme.
ECT(x^(5/2))= (15*sqrt(1))/4*(-1/12)*h^2 (c=1)

<sqrt(|x − 1/2|)>
Both the 4th and 2nd derivatives have really big max absolute values but the 4th one is steeper so we should use Trish’s scheme.
ECT(sqrt(|x − 1/2 |))= -inf*(-1/12)*h^2 (c=lim(x→1/2) x)

c)
<x^3 − 3x^2 + 2>
I used Composite Simpson’s rule as the above error formula states, the error is zero regardless of how many points we have. So we can just use one subinterval

```
function res=Simpson_rule(f,a,b)
   res=(f(a)+4*f((a+b)/2)+f(b))*(b-a)/6;
end
f=@(x)x^3 - 3*x^2 + 2
Simpson_rule(f,0,1)
ans = 1.25
```

<arctan(3*x − 1)>
Error formula for Composite Trapezoid Rule is -1/12f’’*h^2*(1-0) <10^-8
5.846/12*h^2 <= 10^-8
h <= 0.000143272
N ~= 6980
So we need 6980 subintervals

```
function res=Trapezoid_rule(f,a,b)
   res=(f(a)+f(b))/2*(b-a);
end

t=linspace(0,1,6981);
f=@(x) atan(3*x-1);
for i=1:6980
      res=res+Trapezoid_rule(f,t(i),t(i+1));
end
res = 0.323584633878488
```

I used Composite Trapezoid Rule as higher derivatives have bigger values.

<e^x>
Since the function does not grow as the derivative goes higher, we can use the rule that has the highest degree of h, Composite Simpson’s rule. I think although there is one more evaluation of function in each subinterval, h^4 overcomes other rules.
The error formula for Composite Simpson’s rule is -1/2880*f’’’’*h^4*(1-0) <= 10^-8 →
2.718/2880*h^4 <= 10^-8
h <= 0.0570434
N~=18

```
f=@(x) exp(x);
t=linspace(0,1,19);
for i=1:18
      res=res+Simpson_rule(f,t(i),t(i+1));
```

```
end
```
res = **1.718281834141972**

<x^(5/2)>
The fourth derivative of the function can be really large depending on c. Since we don’t know c, it is safe to use Composite Trapezoid Rule.
-1/12*f’’*h^2*(1-0) <10^-8
h<0.00017888
N~=5591

```
f=@(x)x^(5/2);
t=linspace(0,1,5592);
for i=1:5591
        res=res+Trapezoid_rule(f,t(i),t(i+1));
end
```
res= 0.285714292378973

<sqrt(|x − 1/2|)>
The anti-derivative of the function is F(x)=(2*(x-1/2)^(5/2))/(3*abs(x-1/2)).
So the definite integral [0,1] is F(1)-F(0)= sqrt(2)/3~=0.4714045207910317

d)
<x^3 − 3x^2 + 2>
I-ICS=1.25-1.25=0

<arctan(3*x − 1)>
E = I-ICT=0.3235846354178851-0.323584633878488 = 1.5393971097843462*^-9

<e^x>
E = I-ICS=1.718281828459045-1.718281834141972 =-5.682927017858219*^-9

<x^(5/2)>
E = I-ICT=0.2857142857142857-0.285714292378973 = -6.664687302038175*^-9
<sqrt(|x − 1/2|)>
used anti-derivative E = 0

We can see that we all get errors less than 10^-8 and the error formula works.
Nathan’s scheme usually converges much faster than Trish’s but we have to be aware of the derivative’s value as we don’t know what the c value will be.