Naive Bayes with Correlation Factor for Text Classification Problem

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    - Neural networks v.s. Naive Bayes

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- For each document d, define its label vector

$$y(d) = (y_1(d), y_2(d), \cdots, y_k(d))$$

If document d is in class C_i, we have $y_i(d) = 1$, else are 0.

Basics of Naive Bayes

- Each class C_i $(1 \leq i \leq k)$ with words probability vector $\theta_i = (\theta_{i_1}, \theta_{i_2}, ..., \theta_{i_v})$. Each $P(word_j \in d | d \in C_i) = \theta_{i_j}$ and they satisfy: $\sum_{j=1}^{v} \theta_{i_j} = 1$.

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How to find θ_{i_j}?

Estimating: maximum likelihood

- For a given class C_i we estimate $\theta_i = (\theta_{i_1}, \theta_{i_2}, ..., \theta_{i_v})$

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How good is the estimator?

Error analysis

Theorem

Assume we have normalized length of each document, that is: $\sum_{j=1}^{V} x_j = m$ *for all documents* $d \in S$, *the estimator* (1) *satisfies following properties:*

1. $\hat{\theta}_{i_j}$ *is unbiased.*
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More words ($m \nearrow$), more documents ($|C_i| \nearrow$) $\Rightarrow$ less error.

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Insights from Neural net

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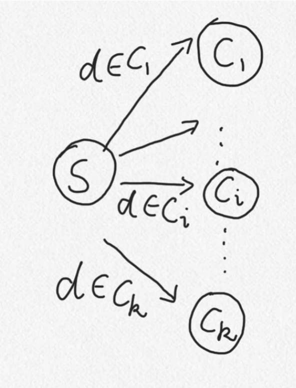

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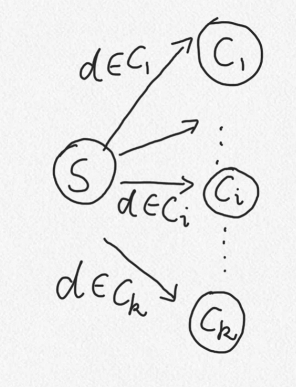

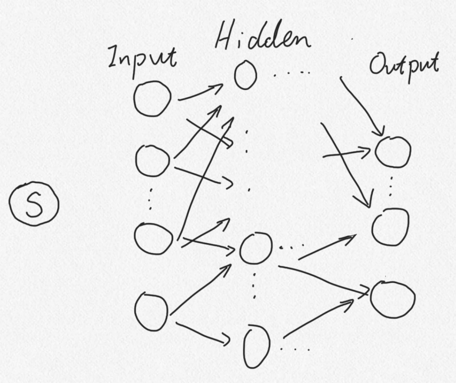

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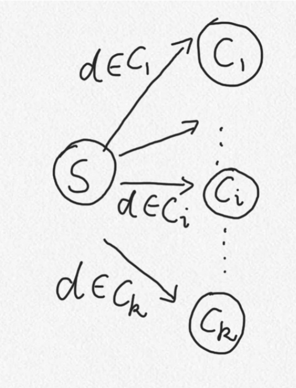

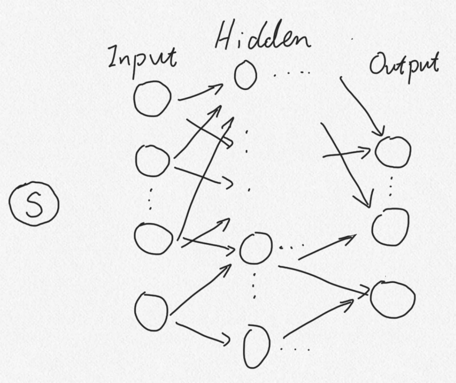

Can we use all data for different classes in Naive Bayes?

Correlation factor

Modify loss function: $\log L(C_i, \theta) = \sum_{d \in C_i} \sum_{j=1}^{v} x_j \log \theta_{i_j} \rightarrow$

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$$\hat{\theta}_{i_j}^{L_1} = \frac{\sum_{d \in S} (y_i(d) + t) x_j}{\sum_{j=1}^{v} \sum_{d \in S} (y_i(d) + t) x_j}$$

v.s. original naive bayes $\hat{\theta}_{i_j} = \frac{\sum_{d \in C_i} x_j}{\sum_{d \in C_i} \sum_{l=1}^{v} x_l}$

Error analysis

Theorem

Assume $|C_i|/|S|(1 \leq i \leq k)$ *are of same order, and* $k << v$*. we have normalized length for each document, that is:* $\sum_{j=1}^{v} x_j = m$*. Then*

1. $\hat{\theta}_{i_j}^{L_1}$ *is biased, with:* $|E[\hat{\theta}_{i_j}^{L_1}] - \theta_{i_j}| = O(t)$
2. $E[|\hat{\theta}_{i_j}^{L_1} - E[\hat{\theta}_{i_j}^{L_1}]|^2] = O(\frac{1}{m|S|})$ *when* $t \approx \frac{1}{k}$.

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Note: The order could have a large constant and lead to larger error than the original Naive bayes MSE, $\frac{\theta_{i_j}(1-\theta_{i_j})}{m|C_i|}$

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We take two different datasets: 10 largest groups in Reuter-21578 dataset and 20 news group dataset.

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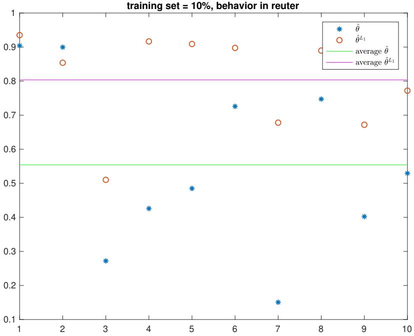
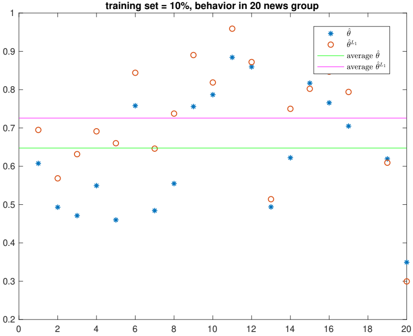
Small training: take 10% of the data as training set.

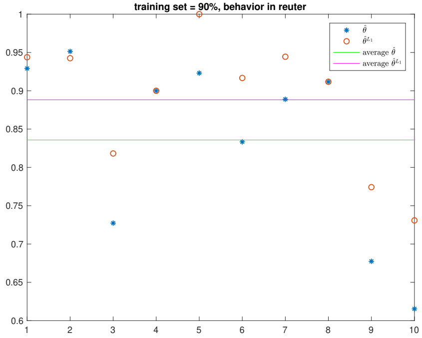




The y-axis is the accuracy, and the x-axis is the class index.

Simulation: large training set

Now take 90% of the data as training set:



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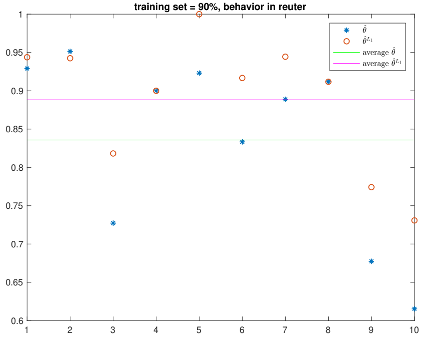

The bias term is dominant in 20 newsgroup.

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- Can we modify t that it adapts to different classes?

References

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Thank you!