Kernel Regression Tree

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November 29, 2018

Writing a prediction function

```
tree_predict_kernel = function(tree_in, new_data, train, k, kernel){#train: with response in the last c
  #Cleaning the frame for the tree
  frame = tree_in$frame
  frame$var = as.character(frame$var)
  frame = cbind(frame,data.frame(split_l = frame$splits[,1]))
  frame = frame %>% select(-splits) %>% rownames_to_column('node_n') %>%
    mutate(split_l = str_remove(split_l,'<'),
           split_l = as.numeric(split_l), node_n = as.numeric(node_n)) #For numeric variables only
  response_index = ncol(train)

  #Function for finding the final leaf(number) number (recursive) for a new sample point x_s, starts wi
  f_node = function(x_s){
    x_s = x_s
    f_node_inner = function(node_num){
      node = frame %>% filter(node_n == node_num)
      if(is.na(node$split_l)){
        return(node_num)
      }else{
        if(x_s[,node$var] < node$split_l){
          node_num = node_num * 2
          f_node_inner(node_num)
        }else{
          node_num = node_num * 2 + 1
          f_node_inner(node_num)
        }
      }
    }
    f_node_inner(1)
  }


  #Function for spliting the training data into leaf space (return list of subsets of data for all leav
  f_train_leaves = function(train){
    leaf_n_list = frame %>% filter(is.na(split_l)) %>% pull(node_n)
    #function for dividing the data
    #input: one leaf number; output: the subset of the training data for the leaf
    f_divide = function(leaf_n){
      split_val = c()
      split_var = c()
      leaf_save = c(leaf_n)
      train_subset = train
      f_divide_inner = function(leaf_n){
        if(is.even(leaf_n)){
          leaf_n = leaf_n/2
```

```
        if(leaf_n!=1){
          leaf_save <<- c(leaf_n, leaf_save)
        }
        split_node = frame %>% filter(node_n == leaf_n)
        split_val <<- c(split_node %>% pull(split_l), split_val)
        split_var <<- c(split_node %>% pull(var), split_var)
        f_divide_inner(leaf_n)
      }else{
        if(leaf_n == 1){
          for(i in 1:length(split_var)){
            if(is.even(leaf_save[i])){
              train_subset <<- train_subset %>% filter((!!as.name(split_var[i])) < split_val[i])
            }else{
              train_subset <<- train_subset %>% filter((!!as.name(split_var[i])) >= split_val[i])
            }
          }
          train_subset
        }else{
          leaf_n = (leaf_n-1)/2
          if(leaf_n!=1){
            leaf_save <<- c(leaf_n, leaf_save)
          }
          split_node = frame %>% filter(node_n == leaf_n)
          split_val <<- c(split_node %>% pull(split_l), split_val)
          split_var <<- c(split_node %>% pull(var), split_var)
          f_divide_inner(leaf_n)
        }
      }
    }
    f_divide_inner(leaf_n)
  }
  lapply(leaf_n_list, f_divide)
}

#Function for running kernel for every sample point using the leaf-subset data (return predicted valu
f_kernel = function(x_s){
  node_num = f_node(x_s)
  leaf = leaves[[as.character(node_num)]]
  if(nrow(leaf) > k){
    kknn <- kknn(paste(names(leaf)[response_index],'~','.'), leaf, x_s, k = k, kernel = kernel, scale
    fitted(kknn)
  }else{
    kknn <- kknn(paste(names(leaf)[response_index],'~','.'), leaf, x_s, k = nrow(leaf)-1, kernel = ke
    fitted(kknn)
  }
}



#Starting regression process
leaves = f_train_leaves(train)#get the training leaves
names(leaves) = frame %>% filter(is.na(split_l)) %>% pull(node_n)#Put leaf number for each training l
as.numeric(by(new_data,1:nrow(new_data), f_kernel))
```

```
}
```

Simulation study

function for getting the error and sd for one simulate dataset from different methods

```
#The last col of the sim_data data is the outcome we want to predict.
apply_methods_result = function(sim_data){
  #Spilit dataset into training and testing: 50:50
  set.seed(1)
  train_index = sample (1: nrow(sim_data), nrow(sim_data )/2)
  train = sim_data[train_index,]
  #getting the formular for other regressions
  formu = as.formula(paste(names(train)[ncol(train)],'~','.'))
  #building a tree
  tree_in=tree(formu,train)
  summary (tree_in )
  y = sim_data[-train_index,ncol(sim_data)]
  new_data = sim_data[-train_index,-ncol(sim_data)]#for testing
  #Kernel tree
  y_kernel = tree_predict_kernel(tree_in, new_data, train, k = 6, kernel =  "cos" )
  error_kernel = mean((y - y_kernel)^2)
  SD_kernel = sd((y - y_kernel)^2)
  #Normal tree
  y_tree = predict(tree_in, new_data)
  error_tree = mean((y - y_tree)^2)
  SD_tree = sd((y - y_tree)^2)


  #Bagging
  set.seed(1)
  bag.boston =randomForest(formu,data = train ,mtry=ncol(new_data), importance = TRUE)
  y_bagging = predict (bag.boston ,newdata = new_data)
  error_bag = mean((y - y_bagging)^2)
  SD_bag = sd((y - y_bagging)^2)
  #Random forest
  set.seed(1)
  rf.boston =randomForest(formu,data=train, mtry=6, importance =TRUE)
  y_rf = predict (rf.boston ,newdata = new_data)
  error_rf = mean((y - y_rf)^2)
  SD_rf = sd((y - y_rf)^2)
  #Boosting
  set.seed(1)
  boost.boston =gbm(formu,data=train, distribution= "gaussian",n.trees =5000 , interaction.depth =4)
  y_boosting =predict (boost.boston ,newdata = new_data, n.trees=5000)
  error_boosting = mean((y - y_boosting)^2)
  SD_boosting = sd((y - y_boosting)^2)
  #kernel knn global
  kknn <- kknn(formu, train, new_data, k = 6, kernel = 'cos', scale = F)
  y_knn_global = fitted(kknn)
```

```
  error_knn_global = mean((y - y_knn_global)^2)
  SD_knn_global = sd((y - y_knn_global)^2)
  #Result
  result = tibble(method = c('kernel_tree', 'tree', 'bagging', 'random forest', 'boosting', 'knn_global
                         error = c(error_kernel, error_tree, error_bag, error_rf, error_boosting, error_k
                         SD = c(SD_kernel,SD_tree,SD_bag,SD_rf,SD_boosting,SD_knn_global))
  return(result)
}
```

Linear model generated dataset

Outcome generated with all linear predictors

```
#Simulation for the linear model
simfun <- function(b_0=3,b_1=10,b_2=8,b_3=6,b_4=4,b_5=2,b_6=-10,b_7=-8,b_8=-6,b_9=-4,b_10=-2,
                     n=200,x1.sd=1,x2.sd=1,e.sd=1) {
   x1 <- rnorm(n, mean=0, sd=x1.sd)
   x2 <- rnorm(n, mean=0, sd=x2.sd)
   x3 <- rnorm(n, mean=0, sd=x1.sd)
   x4 <- rnorm(n, mean=0, sd=x2.sd)
   x5 <- rnorm(n, mean=0, sd=x1.sd)
   x6 <- rnorm(n, mean=0, sd=x2.sd)
   x7 <- rnorm(n, mean=0, sd=x1.sd)
   x8 <- rnorm(n, mean=0, sd=x2.sd)
   x9 <- rnorm(n, mean=0, sd=x1.sd)
   x10 <- rnorm(n, mean=0, sd=x2.sd)
   e <-  rnorm(n, mean=0, sd=e.sd)
   y1 <- b_0+b_1*x1+b_2*x2+b_3*x3+b_4*x4+b_5*x5+b_6*x6+b_7*x7+b_8*x8+b_9*x9+b_10*x10+e
   data.frame(x1,x2,x3,x4,x5,x6,x7,x8,x9,x10,y1)
}

#Simulate data
n = 10
sim_datas = vector("list", n)
for(i in 1:n){
  sim_data = simfun(n = 400)
  sim_datas[[i]] = sim_data
  i = i + 1
}

#applying all the results to the simluated datas
results = lapply(sim_datas, apply_methods_result)
```

```
## Warning: package 'bindrcpp' was built under R version 3.4.4
```

```
#bind them into one dataframe
result_linear_sim = do.call(rbind,results) %>% group_by(method) %>%
  summarise(mean_error = mean(error), mean_SD = mean(SD)) %>% mutate(method = forcats::fct_reorder(meth
```

Outcome generated with all non-linear predictors

```
#Simulation for the non-linear model
simfun <- function(b_0=3,b_1=10,b_2=8,b_3=6,b_4=4,b_5=2,b_6=-10,b_7=-8,b_8=-6,b_9=-4,b_10=-2,
                     n=200,x1.sd=1,x2.sd=1,e.sd=1) {
   x1 <- rnorm(n, mean=0, sd=x1.sd)
   x2 <- rnorm(n, mean=0, sd=x2.sd)
   x3 <- rnorm(n, mean=0, sd=x1.sd)
   x4 <- rnorm(n, mean=0, sd=x2.sd)
   x5 <- rnorm(n, mean=0, sd=x1.sd)
   x6 <- rnorm(n, mean=0, sd=x2.sd)
   x7 <- rnorm(n, mean=0, sd=x1.sd)
   x8 <- rnorm(n, mean=0, sd=x2.sd)
   x9 <- rnorm(n, mean=0, sd=x1.sd)
   x10 <- rnorm(n, mean=0, sd=x2.sd)
   e <-  rnorm(n, mean=0, sd=e.sd)
   y1 <- b_0+b_1*(x1^2)+b_2*(x2^2)+b_3*(x3^2)+b_4*(x4^2)+b_5*(x5^2)+b_6*(x6^2)+b_7*(x7^2)+b_8*(x8^2)+b_
   data.frame(x1,x2,x3,x4,x5,x6,x7,x8,x9,x10,y1)
}

#Simulate data
n = 10
sim_datas = vector("list", n)
for(i in 1:n){
  sim_data = simfun(n = 400)
  sim_datas[[i]] = sim_data
  i = i + 1
}

#applying all the results to the simluated datas
results = lapply(sim_datas, apply_methods_result)

#bind them into one dataframe
result_nonlinear_sim = do.call(rbind,results) %>% group_by(method) %>%
  summarise(mean_error = mean(error), mean_SD = mean(SD)) %>% mutate(method = forcats::fct_reorder(metho
```

Outcome generated with selective linear predictors

```
#Simulation for the selective linear model
simfun <- function(b_0=3,b_1=10,b_2=8,b_3=6,b_4=4,b_5=2,b_6=-10,b_7=-8,b_8=-6,b_9=-4,b_10=-2,
                     n=200,x1.sd=1,x2.sd=1,e.sd=1) {
   x1 <- rnorm(n, mean=0, sd=x1.sd)
   x2 <- rnorm(n, mean=0, sd=x2.sd)
   x3 <- rnorm(n, mean=0, sd=x1.sd)
   x4 <- rnorm(n, mean=0, sd=x2.sd)
   x5 <- rnorm(n, mean=0, sd=x1.sd)
   x6 <- rnorm(n, mean=0, sd=x2.sd)
   x7 <- rnorm(n, mean=0, sd=x1.sd)
   x8 <- rnorm(n, mean=0, sd=x2.sd)
   x9 <- rnorm(n, mean=0, sd=x1.sd)
   x10 <- rnorm(n, mean=0, sd=x2.sd)
   e <-  rnorm(n, mean=0, sd=e.sd)
```

```
   y1 <- b_0+b_1*x1+b_3*x3+b_4*x4+b_6*x6+b_7*x7+b_8*x8+b_10*x10+e
   data.frame(x1,x2,x3,x4,x5,x6,x7,x8,x9,x10,y1)
}

#Simulate data
n = 10
sim_datas = vector("list", n)
for(i in 1:n){
  sim_data = simfun(n = 400)
  sim_datas[[i]] = sim_data
  i = i + 1
}

#applying all the results to the simluated datas
results = lapply(sim_datas, apply_methods_result)

#bind them into one dataframe
result_linear_select_sim = do.call(rbind,results) %>% group_by(method) %>%
  summarise(mean_error = mean(error), mean_SD = mean(SD)) %>%  mutate(method = forcats::fct_reorder(meth
```

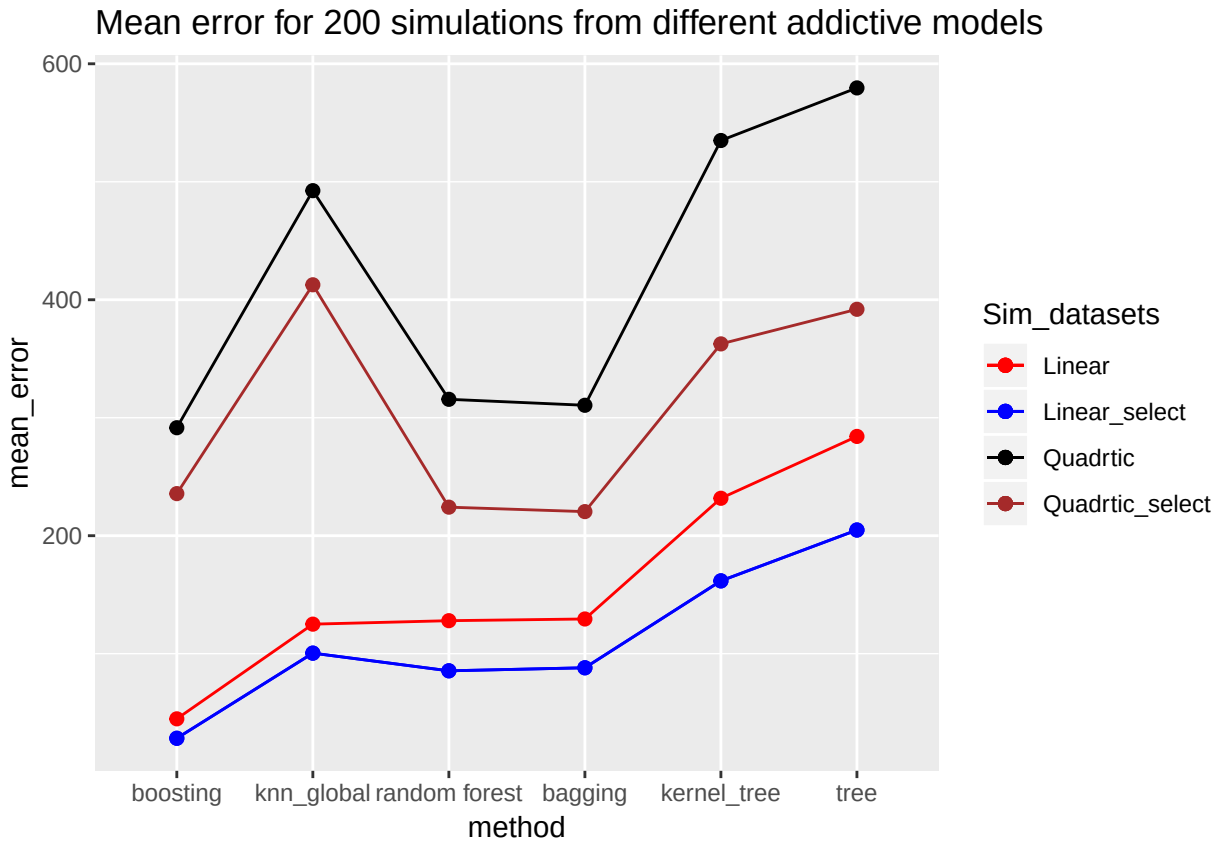
Outcome generated with selective non-linear predictors

```
#Simulation for the selective linear model
simfun <- function(b_0=3,b_1=10,b_2=8,b_3=6,b_4=4,b_5=2,b_6=-10,b_7=-8,b_8=-6,b_9=-4,b_10=-2,
                     n=200,x1.sd=1,x2.sd=1,e.sd=1) {
   x1 <- rnorm(n, mean=0, sd=x1.sd)
   x2 <- rnorm(n, mean=0, sd=x2.sd)
   x3 <- rnorm(n, mean=0, sd=x1.sd)
   x4 <- rnorm(n, mean=0, sd=x2.sd)
   x5 <- rnorm(n, mean=0, sd=x1.sd)
   x6 <- rnorm(n, mean=0, sd=x2.sd)
   x7 <- rnorm(n, mean=0, sd=x1.sd)
   x8 <- rnorm(n, mean=0, sd=x2.sd)
   x9 <- rnorm(n, mean=0, sd=x1.sd)
   x10 <- rnorm(n, mean=0, sd=x2.sd)
   e <-  rnorm(n, mean=0, sd=e.sd)
   y1 <- b_0+b_1*(x1^2)+b_3*(x3^2)+b_4*(x4^2)+b_6*(x6^2)+b_7*(x7^2)+b_8*(x8^2)+b_10*(x10^2)+e
   data.frame(x1,x2,x3,x4,x5,x6,x7,x8,x9,x10,y1)
}

#Simulate data
n = 10
sim_datas = vector("list", n)
for(i in 1:n){
  sim_data = simfun(n = 400)
  sim_datas[[i]] = sim_data
  i = i + 1
}

#applying all the results to the simluated datas
results = lapply(sim_datas, apply_methods_result)
```

```r
#bind them into one dataframe
result_nonlinear_select_sim = do.call(rbind,results) %>% group_by(method) %>%
  summarise(mean_error = mean(error), mean_SD = mean(SD)) %>% mutate(method = forcats::fct_reorder(metho
```

Plotting for simulation data

```r
ggplot() +
  geom_point(data = result_linear_sim, aes(x = method, y = mean_error, color = 'Linear'), size = 2) +
  geom_line(data = result_linear_sim, aes(x = method, y = mean_error, group = 1, color = 'Linear')) +
  geom_point(data = result_linear_select_sim, aes(x = method, y = mean_error, color = 'Linear_select'),
  geom_line(data = result_linear_select_sim, aes(x = method, y = mean_error, group = 2,color = 'Linear_s
  geom_point(data = result_linear_select_sim, aes(x = method, y = mean_error, color = 'Linear_select'),
  geom_line(data = result_linear_select_sim, aes(x = method, y = mean_error, group = 2,color = 'Linear_s
  geom_point(data = result_nonlinear_sim, aes(x = method, y = mean_error, color = 'Quadrtic'), size = 2)
  geom_line(data = result_nonlinear_sim, aes(x = method, y = mean_error, group = 2,color = 'Quadrtic'))
  geom_point(data = result_nonlinear_select_sim, aes(x = method, y = mean_error, color = 'Quadrtic_selec
  geom_line(data = result_nonlinear_select_sim, aes(x = method, y = mean_error, group = 2,color = 'Quadr
  scale_colour_manual("Sim_datasets",
                      values = c("Linear"="red", "Linear_select"="blue",
                                 "Quadrtic"="black", "Quadrtic_select" = "brown")) +
  labs(title = "Mean error for 200 simulations from different addictive models")
```



Real datasets

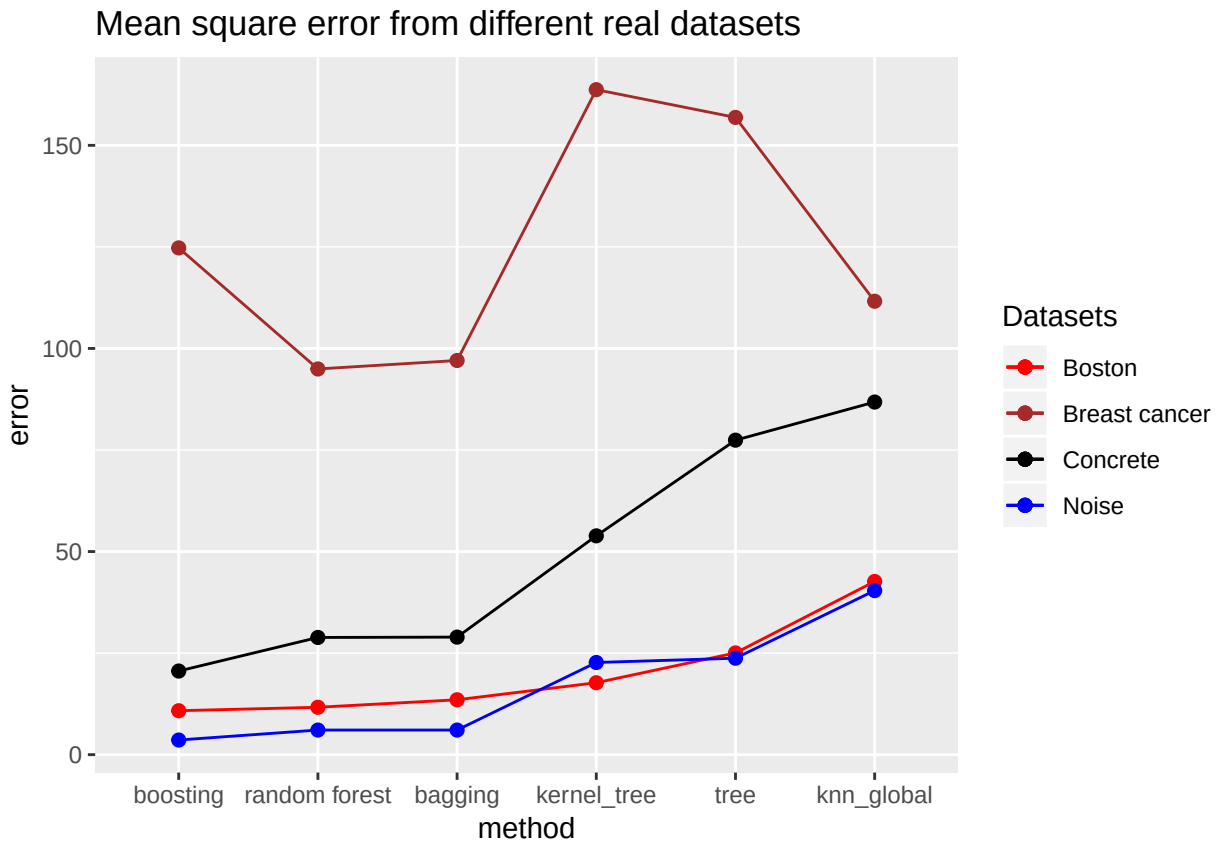
```
#Boston datase
result_boston = apply_methods_result(Boston) %>% mutate(method = forcats::fct_reorder(method, error))
#Airfoil Self-Noise Data Set
noise = read.table('http://archive.ics.uci.edu/ml/machine-learning-databases/00291/airfoil_self_noise.da
          header = F) # 5 predictors
result_noise = apply_methods_result(noise) %>% mutate(method = forcats::fct_reorder(method, error))
```

```
## Warning in randomForest.default(m, y, ...): invalid mtry: reset to within
## valid range
```

```
#Concrete Compressive Strength Data
concrete = read_excel("./data/Concrete_Data.xls", sheet = 1) %>% janitor::clean_names()
result_concrete = apply_methods_result(concrete) %>% mutate(method = forcats::fct_reorder(method, error
#Breast cancer recurrence
breast = read.table('http://archive.ics.uci.edu/ml/machine-learning-databases/breast-cancer-wisconsin/w
                    sep = ',') %>%
  select(-1,-2,-35, -V3, V3)
result_breast = apply_methods_result(breast) %>% mutate(method = forcats::fct_reorder(method, error)) %
```

ploting for the real datasets

```
ggplot() +
  geom_point(data = result_boston, aes(x = method, y = error, color = 'Boston'), size = 2) +
  geom_line(data = result_boston, aes(x = method, y = error, group = 1, color = 'Boston')) +
  geom_point(data = result_noise, aes(x = method, y = error, color = 'Noise'), size = 2) +
  geom_line(data = result_noise, aes(x = method, y = error, group = 2,color = 'Noise')) +
  geom_point(data = result_concrete, aes(x = method, y = error, color = 'Concrete'), size = 2) +
  geom_line(data = result_concrete, aes(x = method, y = error, group = 2,color = 'Concrete')) +
  geom_point(data = result_breast, aes(x = method, y = error, color = 'Breast cancer'), size = 2) +
  geom_line(data = result_breast, aes(x = method, y = error, group = 2,color = 'Breast cancer')) +
  scale_colour_manual("Datasets",
                      values = c("Boston"="red", "Noise"="blue",
                                 "Concrete"="black", "Breast cancer" = "brown")) +
  labs(title = "Mean square error from different real datasets")
```


Mean square error from different real datasets
150
100
50
0
error
boosting
random forest
bagging
kernel_tree
tree
knn_global
method
Datasets
Boston
Breast cancer
Concrete
Noise