A Multiform Many-Objective Genetic Programming Method for Dynamic Flexible Job Shop Scheduling

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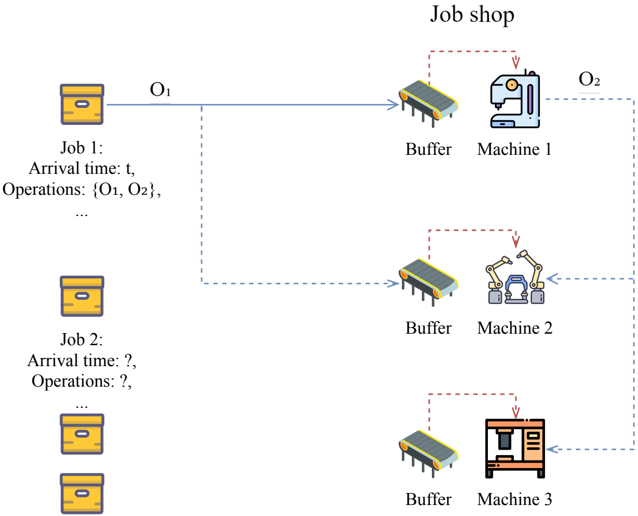
Introduction

Dynamic Flexible Job Shop Scheduling – **DFJSS**



Introduction: DFJSS

Dynamic event:

Uncertain weather conditions in the air traffic flow management[1]

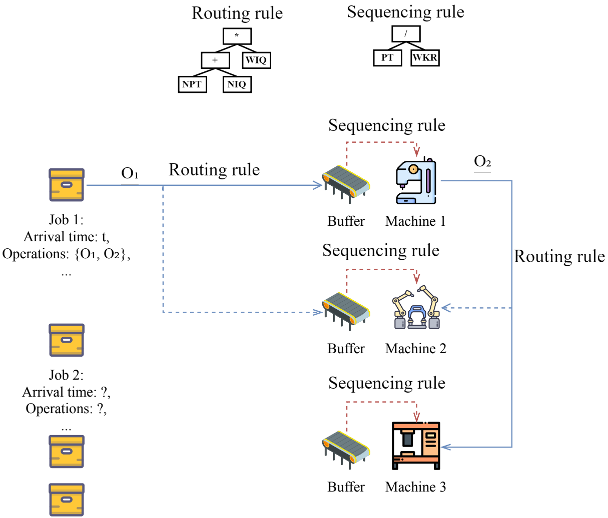
Flexible event:

Adaptation in type and quality of products for smart manufacturing[2]

[1] Shone R, Glazebrook K, Zografos K G. Applications of stochastic modeling in air traffic management: Methods, challenges and opportunities for solving air traffic problems under uncertainty[J]. European Journal of Operational Research, 2021, 292(1): 1-26.
[2] Y. Fang, C. Peng, P. Lou, Z. Zhou, J. Hu and J. Yan, "Digital-Twin-Based Job Shop Scheduling Toward Smart Manufacturing," in IEEE Transactions on Industrial Informatics, vol. 15, no. 12, pp. 6425-6435, Dec. 2019.

Introduction: GPHH for DFJSS

We use **genetic programming hyper-heuristic (GPHH)** to evolve scheduling heuristics for DFJSS.



Proposed problem and method

Proposed many-objective problem: Motivation

- Beyond effectiveness, diverse structural complexity measures promote the evolution of simpler models:
  - Model size[1]: bloat control
  - Number of unique features[2]: reduce redundant components
  - Number of non-arithmetic operators[3]: potentially improve interpretability
  - Dimension gap[4]: enhance physical meaning
  - ...

- However, no existing study has comprehensively considered a set of these measures.

[1] Luke S, Panait L. A comparison of bloat control methods for genetic programming[J]. Evolutionary computation, 2006, 14(3): 309-344.
[2] Zhang F, Mei Y, Nguyen S, et al. Evolving scheduling heuristics via genetic programming with feature selection in dynamic flexible job-shop scheduling[J]. ieee transactions on cybernetics, 2020, 51(4): 1797-1811.
[3] Virgolin M, De Lorenzo A, Medvet E, et al. Learning a formula of interpretability to learn interpretable formulas[C]//Parallel Problem Solving from Nature–PPSN XVI: 16th International Conference, PPSN 2020, Leiden, The Netherlands, September 5-9, 2020, Proceedings, Part II 16. Springer International Publishing, 2020: 79-93.
[4] Mei Y, Nguyen S, Zhang M. Constrained dimensionally aware genetic programming for evolving interpretable dispatching rules in dynamic job shop scheduling[C]//Simulated Evolution and Learning: 11th International Conference, SEAL 2017, Shenzhen, China, November 10–13, 2017, Proceedings 11. Springer International Publishing, 2017: 435-447.

Proposed many-objective problem

$$\min \mathsf{F}_1(\mathsf{x}) = (\mathit{eff}(\mathsf{x}), s(\mathsf{x}), \mathit{uf}(\mathsf{x}), \mathit{nao}(\mathsf{x}), \mathit{dg}(\mathsf{x})) \quad (1)$$

- *eff*: effectiveness measure
- *s*: model size
- *uf*: the number of unique features
- *nao*: the number of non-arithmetic operators
- *dg*: the dimension gap

Main Challenge:

- Search bias to simple models.

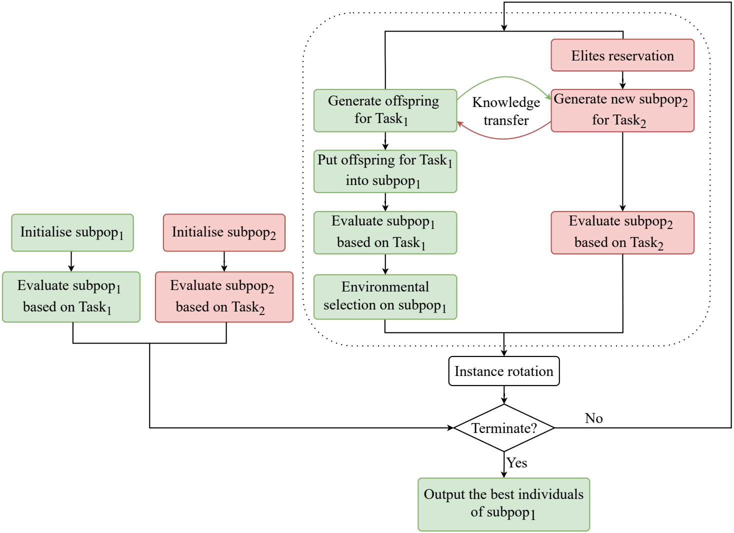
Multiform optimisation:

- An extension of multi-task optimisation.

Form definition:

- $\text{Task}_1: \ \min F_1(x) = (\mathit{eff}(x), s(x), \mathit{uf}(x), \mathit{nao}(x), \mathit{dg}(x))$
- $\text{Task}_2: \ \min F_2(x) = \mathit{eff}(x)$

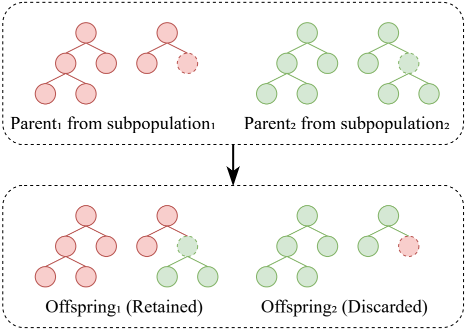
Proposed method: MFGPHH



Proposed method: MFGPHH

We use the origin-based offspring reservation strategy[1], designed for multitask GP, as the knowledge transfer method.

If applied crossover operator and $r < tmp$[2], **Then**



[1] F. Zhang, Y. Mei, S. Nguyen, K. C. Tan and M. Zhang, "Multitask Genetic Programming-Based Generative Hyperheuristics: A Case Study in Dynamic Scheduling," in IEEE Transactions on Cybernetics, vol. 52, no. 10, pp. 10515-10528, Oct. 2022.
[2] Where $r \leftarrow rand(0, 1)$, and tmp is the predefined random mating probability (rmp).

Experimental settings

Experimental setting: Simulator configuration

Simulator configuration

- 10 machines
- The job is released following a Poisson process
- Ops/job $\sim \mathcal{U}(1, 10)$
- Machines/op $\sim \mathcal{U}(1, 10)$
- Processing time $\sim \mathcal{U}(1, 99)$
- 20%, 60%, and 20% of jobs have weights of 1, 2, and 4
- Due date = 1.5 * processing time + arrival time
- Utilisation level: 0.85 / 0.95
- Simulator will stop after finishing 6,000 jobs (including 1,000 warm-up jobs).

Scenario = ⟨effectiveness objective, utilisation level⟩

- $F_{\mathbf{max}} = \mathbf{max}_{j \in J}\{C_i - r_i\}$
- $F_{\mathrm{mean}} = \frac{\sum_{j \in J}(C_i - r_i)}{|J|}$
- $WF_{\mathrm{mean}} = \frac{\sum_{j \in J} w_i(C_i - r_i)}{|J|}$
- $T_{\mathbf{max}} = \mathbf{max}_{j \in J}\{T_i\}$
- $T_{\mathrm{mean}} = \frac{\sum_{j \in J} T_i}{|J|}$
- $WT_{\mathrm{mean}} = \frac{\sum_{j \in J} w_i T_i}{|J|}$

Experimental setting: Comparison configuration

Compared algorithms:

NSGP-III[1], αMOGP-a[2], VMT_αNSGP[3]

Shared parameter settings

- Generation: 51
- Initialisation: ramped half-and-half method with tree depth between 2 and 6
- Maximum tree depth: 8
- Terminals/non-terminals selection probabilities: 0.1/0.9
- Independent runs: 50
- Training set: 51 instances
- Test set: 50 unseen instances with the same distribution

Terminal and functions Sets

Terminal	Description
NIQ	the number of operations in the machine buffer
WIQ	the total processing time in the machine buffer
MWT	the machine waiting time.
PT	the processing time of the operation
NPT	the processing time of the next operation
OWT	the waiting time of the operation
WKR	the total processing time of remaining operations
NOR	the number of remaining operations of the job
W	the job weight
TIS	the job duration in the system
Function	$\{+, -, *, /, max, min\}$

Performance Indicator

- HV
- IGD

[1] K. Deb and H. Jain, "An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point-Based Nondominated Sorting Approach, Part I: Solving Problems With Box Constraints," in IEEE Transactions on Evolutionary Computation, vol. 18, no. 4, pp. 577-601, Aug. 2014.
[2] S. Wang, Y. Mei and M. Zhang, "A Multi-Objective Genetic Programming Algorithm With α Dominance and Archive for Uncertain Capacitated Arc Routing Problem," in IEEE Transactions on Evolutionary Computation, vol. 27, no. 6, pp. 1633-1647, Dec. 2023.
[3] F. Zhang, G. Shi, Y. Mei and M. Zhang, "Multiobjective Dynamic Flexible Job Shop Scheduling With Biased Objectives via Multitask Genetic Programming," in IEEE Transactions on Artificial Intelligence, vol. 6, no. 1, pp. 169-183, Jan. 2025.

Experimental setting: Parameter configuration

Non-shared parameter settings

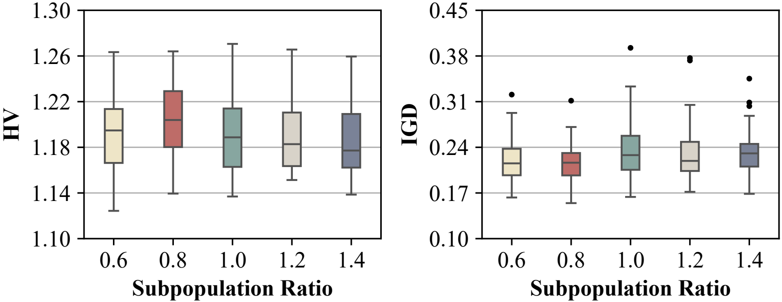
Parameter	NSGP-III	αMOGP-a	VMT_αNSGP	MFGPHH
Crossover operator rate	80%	80%	80%	80%
Mutation rate	20%	20%	15%	20% for $\mathbb{S}_1$, 15% for $\mathbb{S}_2$
Reproduction operator rate	N/A	N/A	5%	N/A for $\mathbb{S}_1$, 5% for $\mathbb{S}_2$
Elitism	N/A	N/A	10	N/A for $\mathbb{S}_1$, 10 for $\mathbb{S}_2$
Tournament selection	2	2	7	2 for $\mathbb{S}_2$, 7 for for $\mathbb{S}_1$
Population size	500	500	900 for $\mathbb{S}_1$ 100 for $\mathbb{S}_2$	$250 * \theta$ for $\mathbb{S}_1$ $1000 - 2 * 250 * \theta$ for $\mathbb{S}_2$
transfer ratio (*rmp*)	N/A	N/A	0.1	0.6

Statistical test

- Mean rank: rank provided by the Friedman test
- The Wilcoxon rank sum test with Bonferroni correction
- A significance level of 0.05

Experimental results

Parameter sensitive analysis



- <WF_{mean}, 0.95>
- Subpopulation ratio θ: 0.8
- Subpopulation size: 200 for $\mathbb{S}_1$, 600 for $\mathbb{S}_2$

Test performance: HV

Table: Mean (Standard Deviation) of HV Values Across 12 Scenarios.

Scenario	NSGP-III	αMOGP-a	VMT_αNSGP	MFGPHH
$<T_{max}, 0.85>$	0.2505 (0.164) $\downarrow$	0.4586 (0.126) $\downarrow$	0.4833 (0.194) $\approx$	0.5389 (0.156)
$<T_{max}, 0.95>$	0.1471 (0.024) $\downarrow$	0.3029 (0.114) $\approx$	0.3373 (0.153) $\approx$	0.3609 (0.160)
$<T_{mean}, 0.85>$	1.2201 (0.011) $\downarrow$	1.0023 (0.134) $\downarrow$	1.1328 (0.022) $\downarrow$	1.2740 (0.030)
$<T_{mean}, 0.95>$	1.1686 (0.018) $\downarrow$	1.0375 (0.095) $\downarrow$	1.1073 (0.087) $\downarrow$	1.2171 (0.027)
$<WT_{mean}, 0.85>$	1.1760 (0.023) $\downarrow$	0.9534 (0.129) $\downarrow$	1.0444 (0.040) $\downarrow$	1.2256 (0.025)
$<WT_{mean}, 0.95>$	1.1342 (0.031) $\downarrow$	1.0117 (0.109) $\downarrow$	1.0080 (0.061) $\downarrow$	1.1849 (0.025)
$<F_{max}, 0.85>$	0.3398 (0.230) $\downarrow$	0.4818 (0.125) $\downarrow$	0.4843 (0.185) $\downarrow$	0.6640 (0.135)
$<F_{max}, 0.95>$	0.4059 (0.259) $\downarrow$	0.4739 (0.159) $\downarrow$	0.5336 (0.173) $\downarrow$	0.7004 (0.150)
$<F_{mean}, 0.85>$	1.2530 (0.007) $\downarrow$	1.0699 (0.097) $\downarrow$	1.1599 (0.034) $\downarrow$	1.3050 (0.025)
$<F_{mean}, 0.95>$	1.1283 (0.017) $\downarrow$	0.9912 (0.096) $\downarrow$	1.0993 (0.081) $\downarrow$	1.2028 (0.030)
$<WF_{mean}, 0.85>$	1.2188 (0.026) $\downarrow$	1.0151 (0.098) $\downarrow$	1.0461 (0.063) $\downarrow$	1.2609 (0.022)
$<WF_{mean}, 0.95>$	1.0385 (0.026) $\downarrow$	0.8939 (0.075) $\downarrow$	0.8938 (0.067) $\downarrow$	1.0830 (0.024)
win-draw-lose	0-0-12	0-1-12	0-2-10	N/A
mean rank	2.6667	3.5000	2.8333	1.000

Test performance: IGD

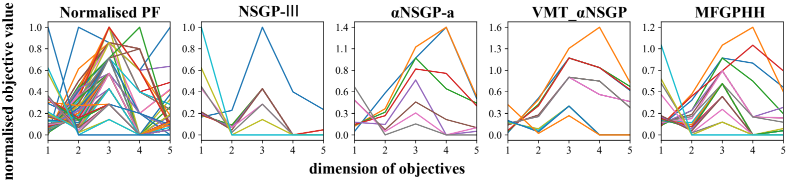
Table: Mean (Standard Deviation) of IGD Values Across 12 Scenarios.

Scenario	NSGP-III	αMOGP-a	VMT_αNSGP	MFGPHH
<T_{max}, 0.85>	0.8748 (0.147) $\downarrow$	0.4051 (0.072) $\uparrow$	0.4192 (0.107) $\uparrow$	0.5143 (0.107)
<T_{max}, 0.95>	0.8628 (0.090) $\downarrow$	0.5236 (0.169) $\approx$	0.4653 (0.110) $\approx$	0.5074 (0.149)
<T_{mean}, 0.85>	0.5519 (0.062) $\downarrow$	0.3183 (0.048) $\approx$	0.3277 (0.066) $\approx$	0.3014 (0.053)
<T_{mean}, 0.95>	0.4138 (0.065) $\downarrow$	0.3064 (0.055) $\downarrow$	0.2994 (0.051) $\downarrow$	0.2506 (0.041)
<WT_{mean}, 0.85>	0.5040 (0.069) $\downarrow$	0.3011 (0.035) $\downarrow$	0.3081 (0.042) $\downarrow$	0.2858 (0.052)
<WT_{mean}, 0.95>	0.4777 (0.040) $\downarrow$	0.3167 (0.065) $\downarrow$	0.3102 (0.062) $\downarrow$	0.2595 (0.058)
<F_{max}, 0.85>	0.8179 (0.204) $\downarrow$	0.3741 (0.055) $\uparrow$	0.3685 (0.077) $\uparrow$	0.4391 (0.119)
<F_{max}, 0.95>	0.8044 (0.223) $\downarrow$	0.3474 (0.078) $\uparrow$	0.3232 (0.065) $\uparrow$	0.4117 (0.102)
<F_{mean}, 0.85>	0.4116 (0.040) $\downarrow$	0.2711 (0.038) $\downarrow$	0.2664 (0.037) $\downarrow$	0.2447 (0.052)
<F_{mean}, 0.95>	0.4225 (0.069) $\downarrow$	0.2781 (0.038) $\downarrow$	0.2610 (0.045) $\downarrow$	0.2141 (0.031)
<WF_{mean}, 0.85>	0.5300 (0.060) $\downarrow$	0.3114 (0.043) $\approx$	0.2966 (0.036) $\approx$	0.2987 (0.064)
<WF_{mean}, 0.95>	0.3585 (0.052) $\downarrow$	0.2769 (0.042) $\downarrow$	0.2721 (0.035) $\downarrow$	0.2108 (0.042)
win-draw-lose	0-0-12	3-3-6	3-3-6	N/A
mean rank	4.000	2.6250	1.7083	1.6667

Approximated Pareto front distribution

In <WF_{mean}, 0.95>, for each method, select the non-dominated solutions from one run with median HV.

- Diverse individuals found by MFGPHH
- A dense region along the approximated Pareto front



Performance on optimising two objectives

Two optimisation Objectives

- Effectiveness
- Model size

Table: Mean (Standard Deviation) of HV and IGD Values Across 6 Scenarios.

Scenario	αMOGP-a	VMT_αNSGP	MFGPHH$_2$	αMOGP-a	VMT_αNSGP	MFGPHH$_2$
$<T_{max}, 0.85>$	0.7598 (0.086) ↑	0.6784 (0.144) ↑	0.5974 (0.175)	0.2122 (0.069) ↑	0.2369 (0.079) ↑	0.3798 (0.133)
$<T_{max}, 0.95>$	0.4918 (0.210) ↑	0.5547 (0.137) ↑	0.4221 (0.164)	0.3317 (0.178) ≈	0.2515 (0.096) ↑	0.3689 (0.133)
$<T_{mean}, 0.85>$	1.0162 (0.046) ↓	1.0610 (0.027) ≈	1.0721 (0.037)	0.1116 (0.028) ≈	0.1225 (0.039) ≈	0.1166 (0.041)
$<T_{mean}, 0.95>$	0.9310 (0.080) ↓	0.9954 (0.056) ≈	1.0073 (0.066)	0.1784 (0.054) ↓	0.1558 (0.044) ↓	0.1372 (0.049)
$<WT_{mean}, 0.85>$	1.0258 (0.045) ↓	1.0746 (0.025) ≈	1.0769 (0.036)	0.1279 (0.040) ≈	0.1373 (0.028) ↓	,0.1157 (0.047)
$<WT_{mean}, 0.95>$	0.9268 (0.062) ↓	0.9760 (0.051) ↓	0.9990 (0.047)	0.1428 (0.034) ↓	0.1405 (0.028) ≈	0.1276 (0.033)
win-draw-lose	2-0-4	2-3-1	N/A	1-3-2	2-2-2	N/A
mean rank	2.5000	1.8333	1.6667	2.000	2.1667	1.8333

Semantic analyses

- Evolved scheduling heuristics by MFGPHH can be explained
- Using more structural complexity measures can bring benefits to explanation.

Table: Scheduling Heuristics Evolved for Solving <F_{mean}, 0.95>.

Algorithm	VMT_αNSGP	MFGPHH
Routing rule	$R_{\text{VMT}} = \texttt{PT} + \texttt{NOR} + R^a_{\text{VMT}} - R^b_{\text{VMT}}$ $R^a_{\text{VMT}} = (\texttt{PT} + \texttt{NOR} + \texttt{W/NPT}) * \texttt{NIQ}$ $R^b_{\text{VMT}} = \max(\texttt{MWT} - \texttt{OWT}, \texttt{MWT})$	$R_{\text{MF}} = \texttt{WIQ} - \texttt{MWT} - R^a_{\text{MF}}$ $R^a_{\text{MF}} = \texttt{MWT} - (\texttt{MWT} * \texttt{PT}/(\texttt{WIQ} - \texttt{MWT}))$
Sequencing rule	$S_{\text{VMT}} = \texttt{WKR} + \texttt{PT}$	$S_{\text{MF}} = \texttt{PT} * \texttt{WKR} * (\texttt{WIQ} + \texttt{WKR}) * \texttt{WKR}$
eff / *s* *uf* / *nao* / *dg*	605.06 / 22 8 / 1 / 10.5	599.15 / 22 4 / 0 / 0

eff: effectiveness   *s*: model size   *uf*: number of unique features
nao: number of non-arithmetic operators   *dg*: dimension gap

Semantic analyses

Table: Scheduling Heuristics Evolved for Solving <F_{mean}, 0.95>.

Algorithm	VMT_αNSGP	MFGPHH
Routing rule	$R_{\mathrm{VMT}} = \mathrm{PT} + \mathrm{NOR} + R^a_{\mathrm{VMT}} - R^b_{\mathrm{VMT}}$ $R^a_{\mathrm{VMT}} = (\mathrm{PT} + \mathrm{NOR} + \mathtt{W}/\mathtt{NPT}) * \mathtt{NIQ}$ $R^b_{\mathrm{VMT}} = \max(\mathtt{MWT} - \mathtt{OWT}, \mathtt{MWT})$	$R_{\mathrm{MF}} = \mathtt{WIQ} - \mathtt{MWT} - R^a_{\mathrm{MF}}$ $R^a_{\mathrm{MF}} = \mathtt{MWT} - (\mathtt{MWT} * \mathrm{PT}/(\mathtt{WIQ} - \mathtt{MWT}))$
Sequencing rule	$S_{\mathrm{VMT}} = \mathrm{WKR} + \mathrm{PT}$	$S_{\mathrm{MF}} = \mathrm{PT} * \mathrm{WKR} * (\mathtt{WIQ} + \mathrm{WKR}) * \mathrm{WKR}$

VMT_αNSGP:

- Difficult to explain based on R_{VMT}
  - $\mathtt{W}$
  - $\mathrm{PT} + \mathrm{NOR}$
  - $\max(\mathtt{MWT} - \mathtt{OWT}, \mathtt{MWT})$

MFGPHH:

$$R_{\mathrm{MF}} \approx \begin{cases} -2 * \mathtt{MWT} - \mathrm{PT} & \text{if } \mathtt{WIQ} = 0, \\ \mathtt{WIQ} & \text{if } \mathtt{MWT} = 0. \end{cases}$$

Conclusions

Conclusions

- **Proposed problem**:
  - Involve effectiveness and a wide range of model structural complexity measures.
- **Proposed method**:
  - Build an auxiliary task to optimise effectiveness only.
  - Solve the original task and its auxiliary task in a multitask manner.
- **Experiment validation**:
  - Verify the capability of multiform optimisation to improve search performance.
  - Highlight the benefits of utilising various structural complexity measures in explanation.

Thank you for your attention!

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