On the Practical Security of White-Box Cryptography

Thesis Defense

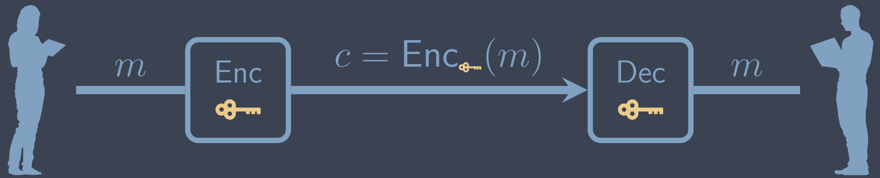
by Junwei Wang (王军委)

on June 24, 2020

Supervisors Jean-Sébastien Coron, Sihem Mesnager

Advisors Pascal Paillier, Matthieu Rivain

» Security Models: Shades of Gray



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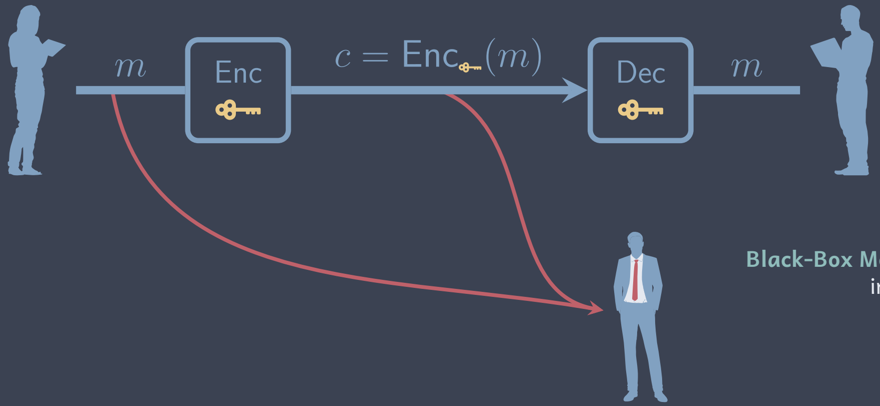

Black-Box Model:
input/output behavior

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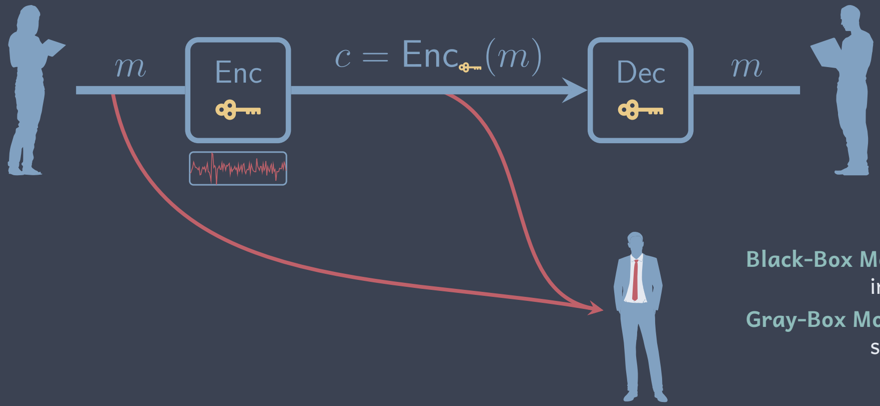

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input/output behavior

Gray-Box Model:
side-channel leakage

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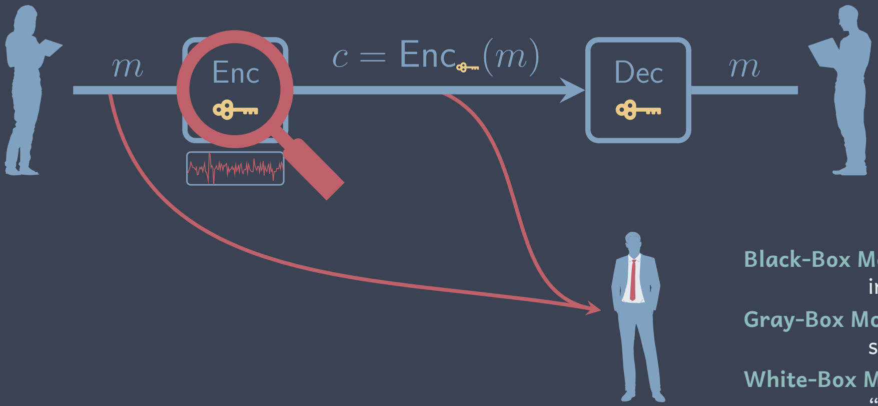

Black-Box Model:
input/output behavior

Gray-Box Model:
side-channel leakage

White-Box Model:
“full” control

» White-Box Threat Model

To extract a cryptographic key

Where from a software implementation of cipher

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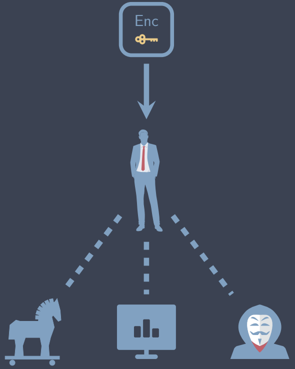
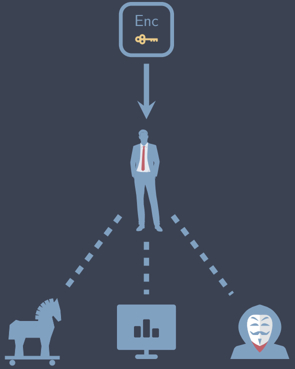
Where	from a software implementation of cipher
Whom	by malwares, co-hosted applications, user themselves, $\cdots$



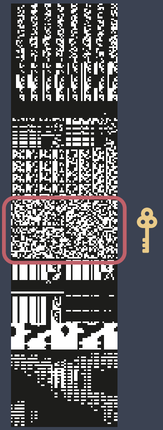
» White-Box Threat Model

	To extract a cryptographic key
Where	from a software implementation of cipher
Whom	by malwares, co-hosted applications, user themselves, $\cdots$
How	by all kinds of means

- analyze the code
- spy on the memory
- interfere the execution
- $\cdots$



» Motivation and Real-World Applications



Credits to [Shamir, van Someren 99]

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- Why not using secure hardware ?
  - not always available
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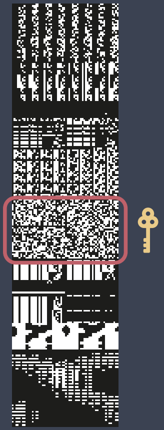

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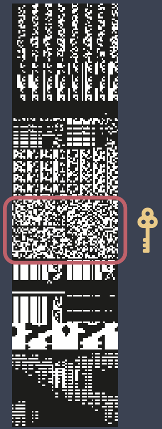
* Applications
  * Digital Content Distribution
  * Mobile Payment
  * Digital Contract Signing
  * Blockchains and cryptocurrencies



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» White-Box Compiler

[Delerablée et al. 14]

A **white-box compiler** takes as input *a secret key* and generates *a* "white-box secure" program implementing some specific crypto. algo. with the specified secret key.

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  - Unbreakability (hardness of key-extraction)
  - One-wayness
  - Incompressibility
  - Traceability

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No provably secure unbreakable white-box compiler for standard block ciphers is known.

» Historical White-Box Compilers

Transform a cipher into a series of randomized key-dependent lookup tables.

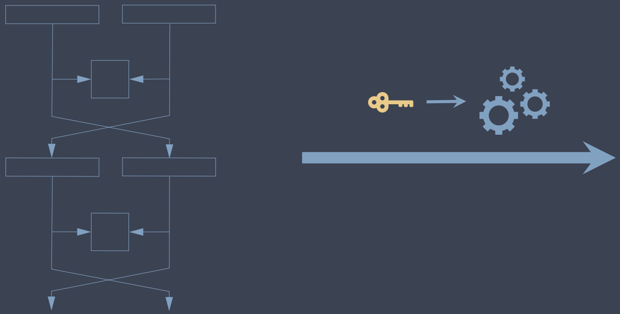

Illustration from [Wyseur12]

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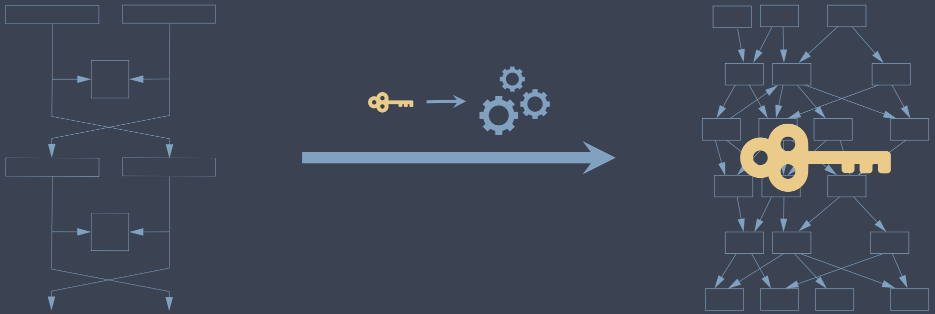

Illustration from [Wyseur12]

» Historical White-Box Compilers: Internal Encodings

$$\boxed{X} \longrightarrow \boxed{\mathcal{R}_1} \longrightarrow \boxed{\mathcal{R}_2} \longrightarrow \cdots \longrightarrow \boxed{\mathcal{R}_r} \longrightarrow \boxed{Y}$$

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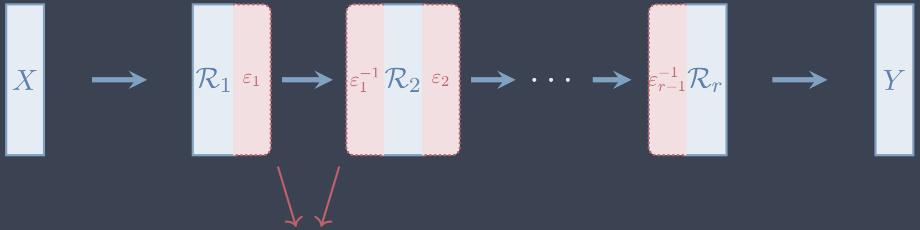

pairwise annihilating parasitic functions (*e.g.* encodings)

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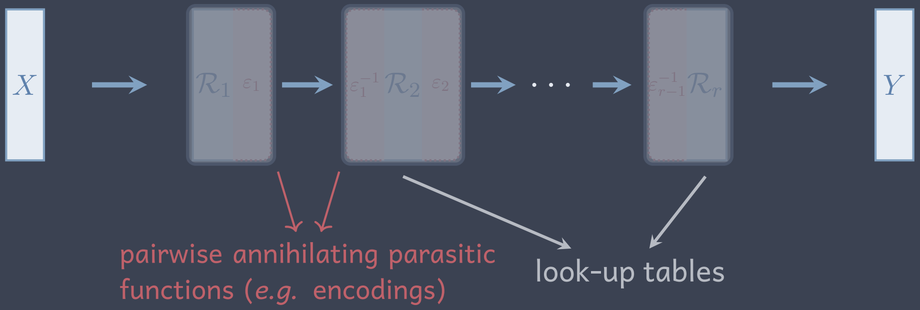
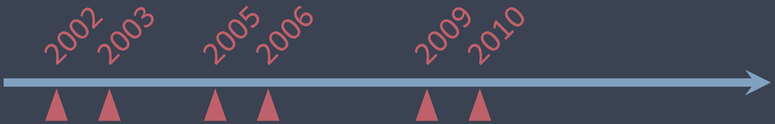


» Timeline: A Cat-And-Mouse Game



* 2002: seminal wb-DES [Chow et al. 02]
* 2003: seminal wb-AES [Chow et al. 03]
* 2005: variant wb-DES [Link, Neumann 05]
* 2006: variant wb-AES [Bringer et al. 06]
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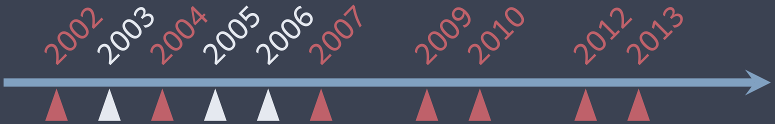

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* 2002: fault attack against wb-DES [Jacob et al. 02]
* 2004: BGE attack [Billet et al. 04]
* 2007: attack wb-DES [Goubin et al 07, Wyseur et al. 07]]
* 2009: attack wb-AES [Michiels et al. 09]
* 2010: attack Bringer et al. variant [De Mulder et al. 10]
* 2012: attack Xiao-Lai variant [De Mulder et al. 12]
* 2013: attack improvements and Karroumi variant [Lepoint et al .13]
* ...

» Obscurity as a Solution

* All public designs are broken
* No provably secure solution

* Growing demand in industry
* Huge application potential

$\Downarrow$

Security through obscurity: home-made design + obfuscation

Time consuming reverse engineering + structural analysis

» Differential Computation Analysis (DCA)

[Bos et al. 2016, Sanfelix et al. 2015]

Differential power analysis (DPA) techniques on computational leakages.

gray-box model

side-channel leakages (noisy)

e.g. power / EM / time / $\cdots$

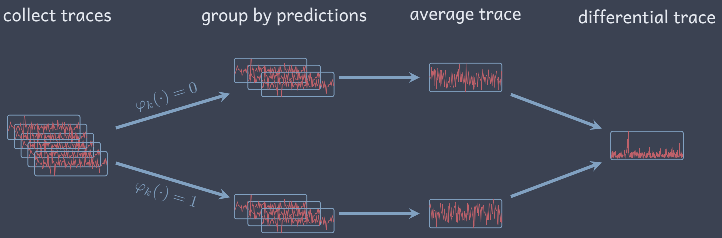
white-box model

computational leakages (noisy-free)

e.g. registers / accessed memory / $\cdots$

» Differential Computation Analysis (DCA) (cont.)

[Bos et al. 2016, Sanfelix et al. 2015]



Implying strong *linear correlation* between the sensitive variables φ_k and the leaked samples in the computational traces.

Many publicly available implementations are broken by DCA.

» WhibOx Competitions

* Organized as CHES CTF events

> *The competition gives an opportunity for researchers and practitioners to confront their (secretly designed) white-box implementations to state-of-the-art attackers*
>
> – WhibOx 2017

* Designer: to submit the C source codes of AES-128 with secret key
* Attacker: to reveal the hidden key
* No need to disclose identity or underlying techniques

» WhibOx Competitions (cont.)

* WhibOx 2017
  * 94 submissions were **all broken** by 877 individual breaks
  * most (86%) of them were alive for < 1 day
  * mostly broken by DCA [Bock and Treff 20]

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* Both winning implementations due to Biryukov and Udovenko, and broken in this thesis with Goubin, Paillier and Rivain

» Thesis Contribution

* Analyze in-depth why DCA can break internal encodings and propose new efficient attacks against internal encodings
* Propose new different advanced gray-box attack paths
* Analyze advanced gray-box countermeasures against new attack paths
* Propose data-dependency analysis with substantially improved complexity over the existing attacks
* Break the winning challenges from two editions of WhibOx competitions

» Publications

[RW19] Rivain and Wang. "Analysis and Improvement of Differential Computation Attacks against Internally-Encoded White-Box Implementations". In: **TCHES 2019 Issue 2**.

[BRVW19] Bogdanov, Rivain, Vejre, and Wang. "Higher-Order DCA against Standard Side-Channel Countermeasures". In: **COSADE 2019**.

[GPRW20] Goubin, Paillier, Rivain, and Wang. "How to reveal the secrets of an obscure white-box implementation". In: **Journal of Cryptographic Engineering Volume 10 Issue 1**.

[GRW20] Goubin, Rivain, and Wang. "Defeating State-of-the-Art White-Box Countermeasures with Advanced Gray-Box Attacks". In: **TCHES 2020 Issue 3**.

» Passive Gray-Box Adversary Model

[GRW20]

* For each of N chosen $(x^{(i)})_{1\leq i\leq N}$, collect a computational trace of t samples

$$\boldsymbol{v} = (v_1, v_2, \cdots, v_t)$$

* Build a distinguisher D:

$$(\gamma_k)_{k\in\mathcal{K}} = \mathsf{D}\left((x^{(i)})_i \;,\; (\boldsymbol{v}^{(i)})_i\right)$$

* Choose key candidate: $\mathrm{argmax}_{k\in\mathcal{K}}\gamma_k$

» Passive Gray-Box Adversary Model [GRW20]

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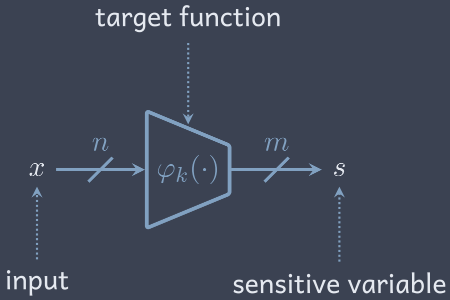
> - Number samples in attacked trace (window): t
> - Required number traces: N

» Outline

DCA Analysis and Improvements against Internal Encodings

* DCA against Internal Encodings
* Collision Attack

» Internal Encoding Leakage



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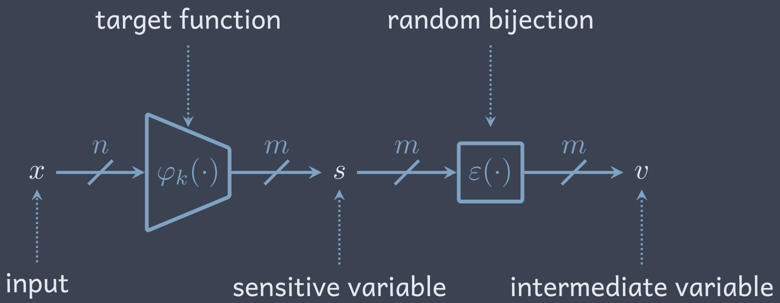

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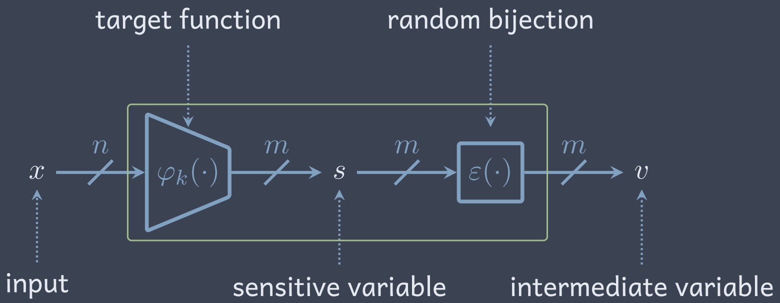

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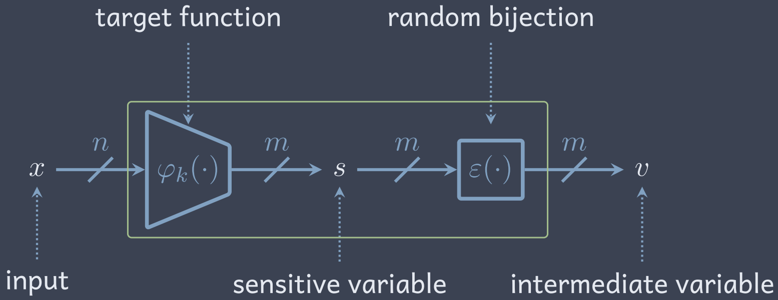

* $\varepsilon \circ \varphi_k$, as a result of some **table look-ups**, is **leaked in the memory**
* To exploit the leakage of $\varepsilon \circ \varphi_k$, it is necessary that $n > m$

DCA Analysis and Improvements against Internal Encodings

* DCA against Internal Encodings
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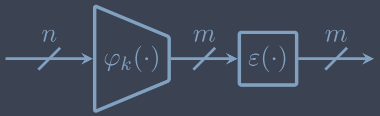
» DCA Analysis

Based on well-established theory – *Boolean correlation*, instead of *difference of means*: for any key guess k

$$\rho_k = \mathrm{Cor}\Big(\qquad\qquad , \qquad\qquad \Big)$$



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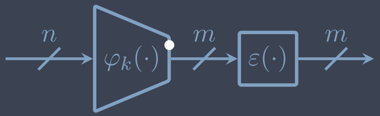
$$\rho_k = \mathrm{Cor}\Big(\varphi_k(\cdot)[i] \ , \qquad\qquad \Big)$$



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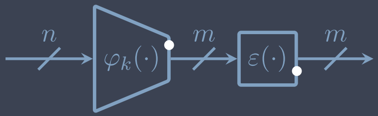

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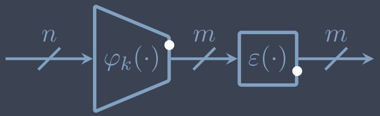

DCA success (roughly) requires:

$$\left|\rho_{k^*}\right| > \max_{k^\times} \left|\rho_{k^\times}\right|$$

» Distributions of ρ_{k^*} and $\rho_{k^\times}$

* **Ideal** assumption: $(\varphi_k)_k$ are mutually independent random (n, m) functions

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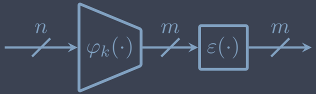
Correct key guess k^*,

$$\rho_{k^*} = 2^{2-m} N^* - 1$$

where

$$N^* \sim \mathcal{HG}(2^m, 2^{m-1}, 2^{m-1}) \ .$$

Only depends on m.



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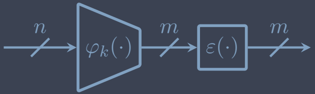

» Lemma

Lemma

Let $\mathcal{B}(n)$ be the set of balanced n-bit Boolean functions. If $f \in \mathcal{B}(n)$ and $g \xleftarrow{\$} \mathcal{B}(n)$ independent of f, then the balanceness of $f+g$ is $\mathrm{B}(f+g) = 4 \cdot N - 2^n$ where $N \sim \mathcal{HG}(2^n, 2^{n-1}, 2^{n-1})$ denotes the size of $\{x : f(x) = g(x) = 0\}$.

With

$$\mathrm{Cor}(f+g) = \frac{1}{2^n}\mathrm{B}(f+g)$$

$\Rightarrow$

$$\rho_{k^*} = 2^{2-m} N^* - 1 \quad \text{and} \quad \rho_{k^\times} = 2^{2-n} N^\times - 1$$

where $N^* \sim \mathcal{HG}(2^m, 2^{m-1}, 2^{m-1})$ and $N^\times \sim \mathcal{HG}(2^n, 2^{n-1}, 2^{n-1})$.

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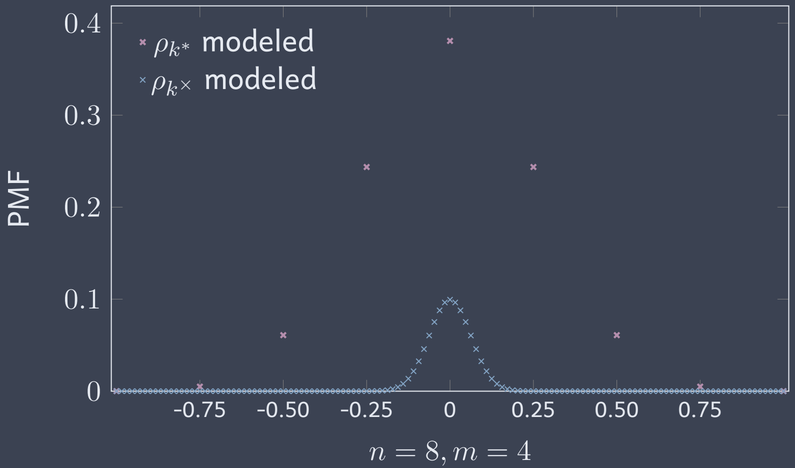

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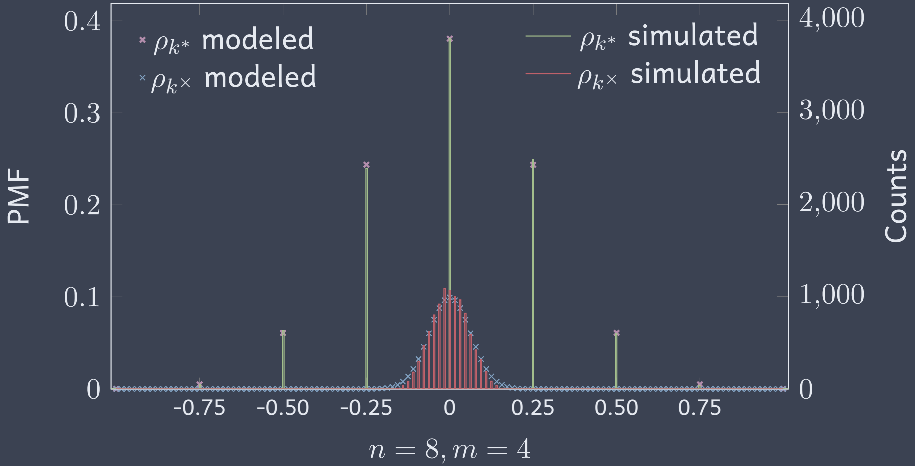
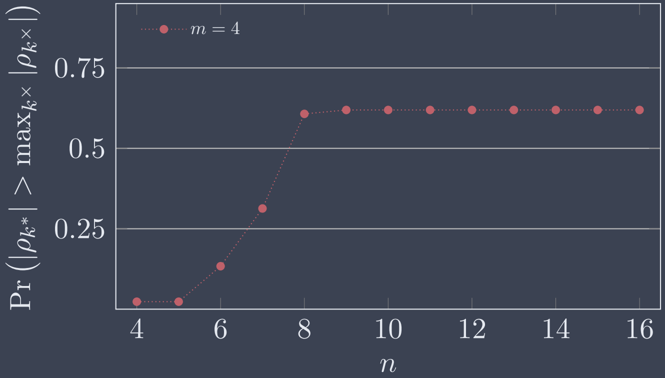


» DCA Success Rate: $\left|\rho_{k^*}\right| > \max_{k^\times} \left|\rho_{k^\times}\right|$



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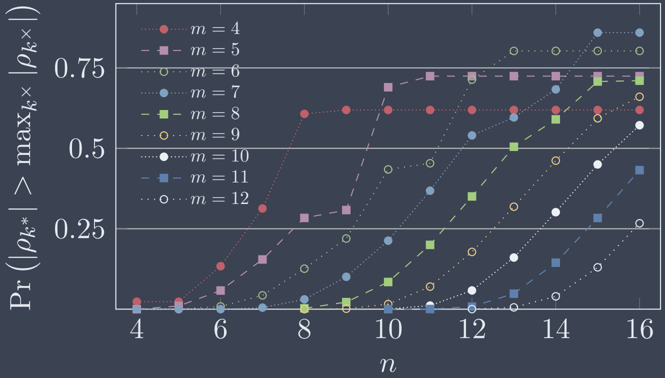

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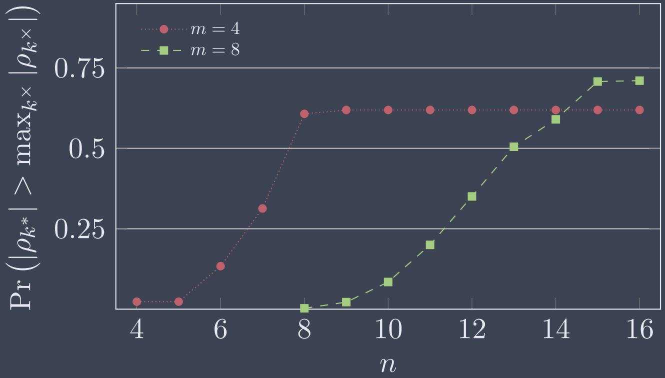

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* *Byte* encoding protected AES
* DCA has failed to break it *before this work*

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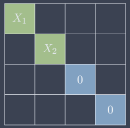

x_1 x_2

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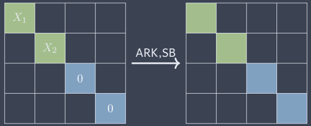

$\mathbf{Sbox}(x_1 \oplus k_1)$ $\mathbf{Sbox}(x_2 \oplus k_2)$ $\mathbf{Sbox}(k_3)$ $\mathbf{Sbox}(k_4)$

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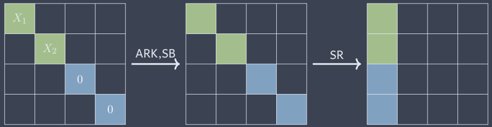

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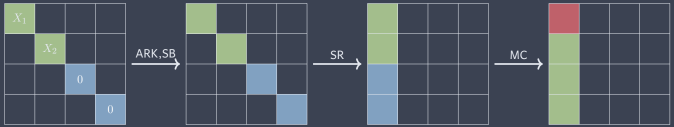
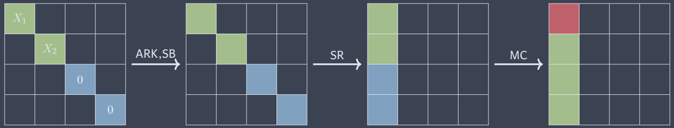


$$2 \cdot \mathbf{Sbox}(x_1 \oplus k_1) \oplus 3 \cdot \mathbf{Sbox}(x_2 \oplus k_2) \oplus \mathbf{Sbox}(k_3) \oplus \mathbf{Sbox}(k_4)$$

» Attack a NSC Variant: a White-Box AES

- *Byte* encoding protected AES
- DCA has failed to break it *before this work*
- Our approach: target an output byte of MixColumn in the first round



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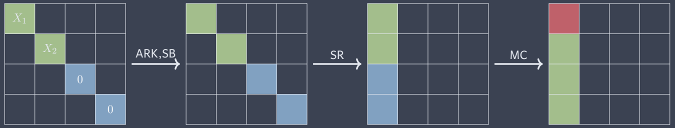


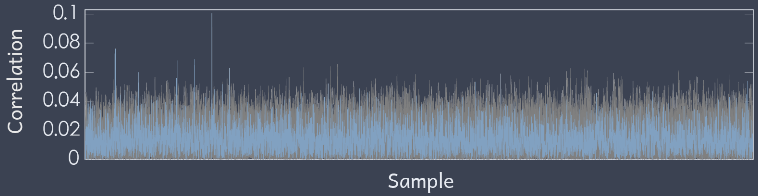
$$\varphi_{k_1||k_2}(x_1||x_2) = 2 \cdot \mathbf{Sbox}(x_1 \oplus k_1) \oplus 3 \cdot \mathbf{Sbox}(x_2 \oplus k_2)$$

$$\varepsilon' = \varepsilon \circ \oplus_c\ ,$$

$$n = 16, m = 8\ , |\mathcal{K}| = 2^{16}.$$

» Attack a NSC Variant: a White-Box AES

* Attack results: $\sim$ 1800 traces



* Byte encoding can be efficiently broken

DCA Analysis and Improvements against Internal Encodings

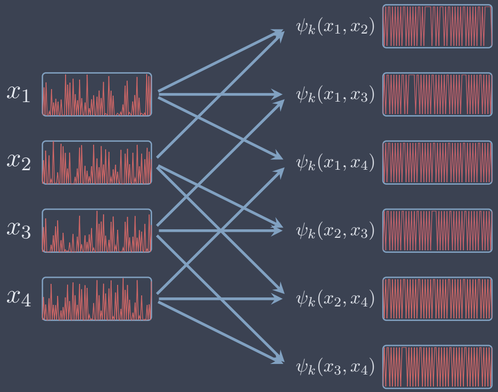
* DCA against Internal Encodings
* Collision Attack

» Collision Attack

N inputs & raw traces

» Collision Attack

N inputs & raw traces $\binom{N}{2}$ collision predictions & traces



$$\psi_k(x_1, x_2) := \Big(\varphi_k(x_1) = \varphi_k(x_2)\Big)$$

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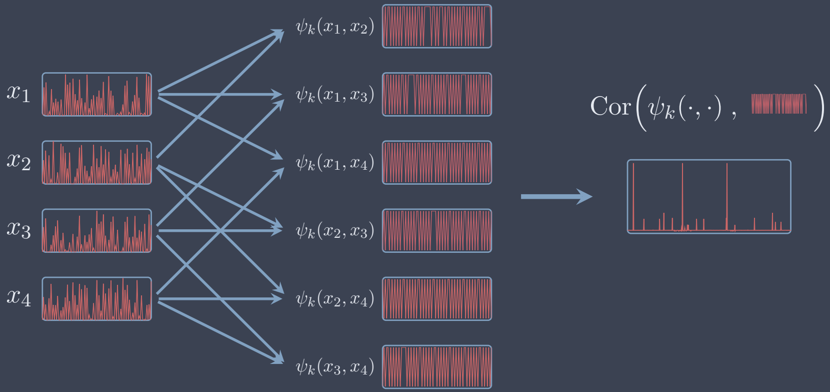

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» Collision Attack Explanation

Based on the principle:

$$\varphi_k(x_1) = \varphi_k(x_2) \quad \Leftrightarrow \quad \varepsilon \circ \varphi_k(x_1) = \varepsilon \circ \varphi_k(x_2)$$

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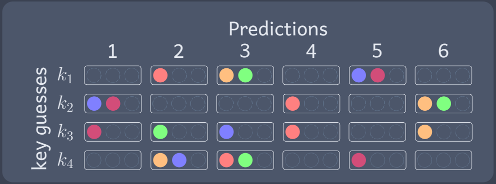
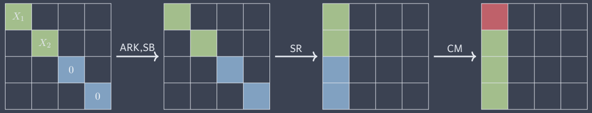
Trace Complexity:

$$N = \mathcal{O}\left(2^{\frac{m}{2}}\right)$$



» Attack the NSC Variant

* Same to DCA: targeting at one 1-st round MixColumn output byte



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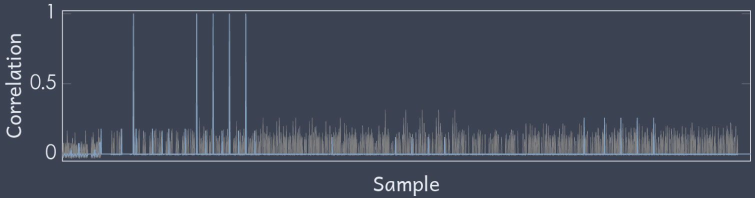
* Attack results: 60 traces



» Contribution Summary

* DCA against internal encodings has been analysed in depth
  * Allows to attack wider encodings
* Propose new class of collision attacks with very low trace complexity
* Mutual information analysis with similar trace complexity but higher computation complexity
* Hence, protecting AES with internal encodings in the beginning rounds is insufficient

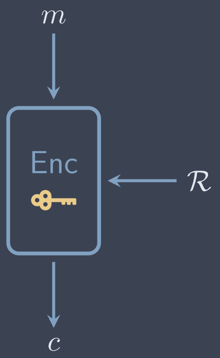
Advanced Gray-Box Countermeasures and Attacks

* Linear Masking, Higher-Order DCA, and Linear Decoding Analysis
* Algebraic Security and Non-Linear Masking
* Shuffling

» Random Source

[BRVW19]

* Countermeasures need randomness.



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* Countermeasures need randomness.
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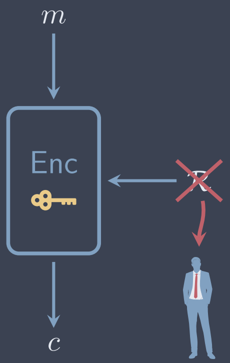
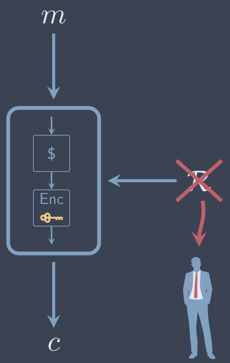


» Random Source

[BRVW19]

- Countermeasures need randomness.
- Plaintext is the only source of randomness
- Security criteria:

Pseudorandomness no statistical flaw
Obscurity the design should be kept secret
Obfuscation hard to distinguish from other intermediate variables



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» Linear Masking

[Ishai et al. 03]

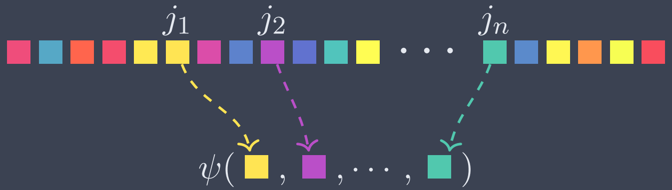
- Intermediate value x is split into n shares

$$x = x_1 \oplus x_2 \cdots \oplus x_n$$

- Shares are manipulated separately such that any subset of at most $n-1$ shares is independent of x
- Resistant against $(n-1)$-th order DCA attacks

» Higher-Order DCA (HO-DCA) [BVRW19]

- Trace **pre-processing**: an n-th order trace contains $q = \binom{t}{n}$ points:



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- Perform DCA attacks on the higher-order traces
- Linear masking can be broken
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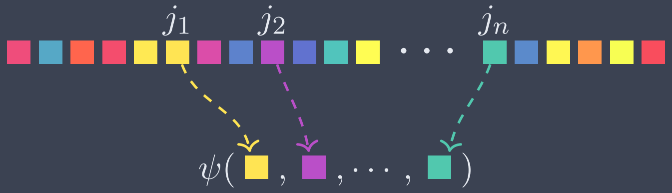

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$$\binom{1000}{5} \approx 2^{43}$$

» Linear Decoding Analysis (LDA) [GPRW20]

* Assumption: there exists a linear decoding function

$$D(v_1, v_2, \cdots, v_t) = a_0 \oplus \left(\bigoplus_{1 \le i \le t} a_i \cdot v_i \right) = \varphi_k(x)$$

for some sensitive variable φ_k and some fixed coefficients $a_0, a_1, \cdots, a_t$.

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* Record the v_i's over N executions:

$$v_1^{(1)} \quad \cdots \quad v_t^{(1)} \qquad\qquad x^{(1)}$$

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» Linear Decoding Analysis (LDA) (cont.) [GPRW20]

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* Linear masking is vulnerable to LDA
  * system solvable for k^*
  * but not for incorrect key guess $k^\times$
* Trace Complexity $t + \mathcal{O}(1)$
* Computation complexity $\mathcal{O}\left(t^{2.8} \cdot |\mathcal{K}|\right)$

$$1000^{2.8} \approx 2^{28}$$

» Breaking WhibOx 2017 Winning Challenge with LDA [GPRW20]

- Small windows located for target variables
- The 3rd s-box $t = 35, \min(n) = 2 \quad \Longrightarrow \quad$ HO-DCA: 2^{18}, and LDA: 2^{22}

Bit	Encoding coefficients
1	0 0 0 0 0 0 1 0 1 0 1 1 1 0 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
2	0 0 0 0 0 0 1 0 0 1 1 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
3	0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
4	0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
5	0 0 0 0 0 0 0 1 1 0 0 1 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
6	0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
7	0 0 0 0 0 0 1 0 0 0 1 0 0 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
8	0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

The solution of the system of equations for each bit in the 3rd byte.

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8	0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

The solution of the system of equations for each bit in the 3rd byte.

* The 14th s-box: $t = 45, \min(n) = 9$ $\implies$ HO-DCA: 2^{49}, and LDA: 2^{23}

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* Shuffling

» Algebraic Security and Non-Linear Masking

[Biryukov and Udovenko 18]

* Introduced by Biryukov and Udovenko at Asiacrypt 2018
* To capture LDA like algebraic attack

> A d-th degree algebraically-secure non-linear masking ensures that any function of up to d degree to the intermediate variables should not compute a "predictable" variable.

» First-Degree Secure Non-Linear Masking

[Biryukov and Udovenko 18]

* Quadratic decoding function

$$(a, b, c) \mapsto ab \oplus c$$

* Secure gadgets for bit XOR, bit AND, and refresh
* Provably secure composition
* But vulnerable to DCA attack

$$\mathsf{Cor}(ab \oplus c,\ c) = \frac{1}{2}$$

* Empirically, suggest using a combination of linear masking and non-linear masking to thwart both DCA (probing security) and LDA (algebraic security).

» Combination of Linear Masking and Non-linear Masking [GRW20]

* Three possible natural combinations:
  1. apply linear masking on top of non-linear masking

$$x = \left(a_1 \oplus a_2 \oplus \cdots \oplus a_n\right)\left(b_1 \oplus b_2 \oplus \cdots \oplus b_n\right) \oplus \left(c_1 \oplus c_2 \oplus \cdots \oplus c_n\right)$$

  2. apply non-linear masking on top of linear masking

$$x = \left(a_1 b_1 \oplus c_1\right) \oplus \left(a_2 b_2 \oplus c_2\right) \oplus \cdots \oplus \left(a_n b_n \oplus c_n\right) .$$

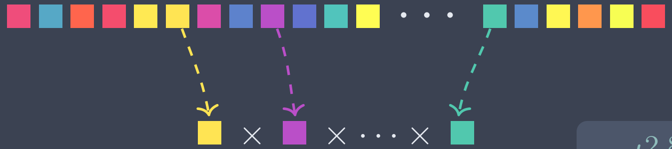
  3. merge the two maskings into a new encoding

$$x = ab \oplus c_1 \oplus c_2 \oplus \cdots \oplus c_n .$$

* For first two combinations, the combined masking gadgets can be simply derived from the original gadgets of both schemes.

» Higher-Degree Decoding Analysis (HDDA) [GPRW20]

- Assume the decoding function is of degree d
- Trace **pre-processing**: a d-th degree trace contains all monomials of degree $\leq d$



- Perform LDA attacks on the higher-degree traces
- Higher-degree trace samples: $\sum_{i=0}^{d} \binom{t}{d} = \binom{t+d}{d} \ll t^d$
- Complexity: $\mathcal{O}\left(t^{2.8d} \cdot |\mathcal{K}|\right)$, practical when t, d are small.

$$t^{2.8d} < 2^{50}$$

$$\Downarrow$$

$$d = 2 \quad \Rightarrow \quad t < 487$$
$$t = 100 \quad \Rightarrow \quad d \leq 5$$

Advanced Gray-Box Countermeasures and Attacks

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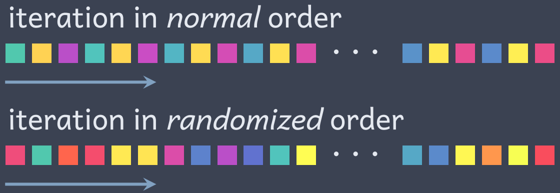
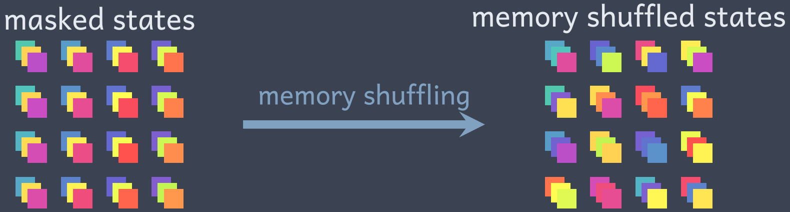
» Shuffling

- The order of execution is randomly chosen for each run of the implementation.
- To increase noise in the adversary's observation



» Shuffling (cont.) [BRVW19]

- Not enough in white-box model: traces can be aligned by memory
- Thus, the memory location of shares has to be shuffled.



» HO-DCA and Integrated HO-DCA against Masking and Shuffling

[BRVW19]

- $\nexists\, n$ fixed locations for all shares
- Shuffling degree is λ
  - correlation score decreased by a factor of λ
  - attack slow down by a factor of λ^2
- Integrate values from all λ slots
  - correlation score decreased by a factor of $\sqrt{\lambda}$
  - attack slow down by a factor of λ

» Multivariate HO-DCA [BRVW19]

* The multivariate HO-DCA optimizes the attack by exploiting joint information of the higher-order samples on the secrets
* Based on a maximum likelihood distinguisher

$$\gamma_k = \Pr\left(K = k | (\boldsymbol{V}^{(i)})_i = (\boldsymbol{v}^{(i)})_i \wedge (X^{(i)})_i = (x^{(i)})_i\right)$$

* We show that

$$\gamma_k \propto \prod_{i=1}^{N} \mathsf{C}_k(\boldsymbol{v}^{(i)}, x^{(i)})$$

where *the counter*

$\mathsf{C}_k(\boldsymbol{v}, x) :=$ the number of n-tuples *s.t.* $v_{j_1} \oplus \cdots \oplus v_{j_n} = \varphi_k(x)$ in one trace.

» Attack Comparison

	linear masking		linear + NL masking	
	trace	computation	trace	computation
without shuffling				
LDA / HDDA	$t + \mathcal{O}(1)$	$\mathcal{O}\left(\lvert\mathcal{K}\rvert \cdot t^{2.8}\right)$	$\mathcal{O}\left(t^2\right)$	$\mathcal{O}\left(\lvert\mathcal{K}\rvert \cdot t^{5.6}\right)$

Note that c is some small empirical factor

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Intg. HO-DCA	$c\,\lambda$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t^n \cdot \lambda)$	$4\,c\,\lambda$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t^n \cdot \lambda)$
MV HO-DCA	$\mathcal{O}(t^n)$	$\mathcal{O}\left(\lvert\mathcal{K}\rvert \cdot t^{2n}\right)$		

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Data-Dependency Analysis

* Data-Dependency Graph
* Data-Dependency Analysis against Masking Combinations

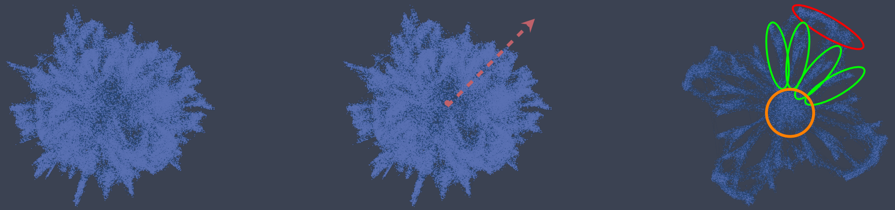
Data-Dependency Analysis

* Data-Dependency Graph
* Data-Dependency Analysis against Masking Combinations

» Data Dependency Graph

[GPRW20]

- White-box adversary also observes data-flow.
- Data-dependency graph (DDG) can visually reveal the structure of the implementation.

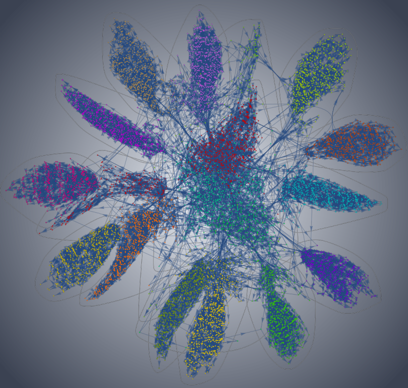


first 20%   first 10%   first 5%

» Data Dependency Clusters

[GPRW20]

- Data-dependency can extract computation clusters and determines the location of sensitive computation
  - Round: ∼ 28 k nodes
  - S-box cluster: ∼ 500 nodes
  - Trace windows containing targets: ∼ 50 nodes



Data-Dependency Analysis

* Data-Dependency Graph
* Data-Dependency Analysis against Masking Combinations

» AND Gadget for Linear Masking

[Ishai et al. 03]

$$(x_1,\ x_2, \cdots,\ x_n),\ (y_1,\ y_2, \cdots,\ y_n) \mapsto (z_1,\ z_2, \cdots,\ z_n) \quad \text{s.t.} \bigoplus_i x_i \cdot \bigoplus_i y_i = \bigoplus_i z_i\ .$$

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$$\begin{bmatrix} x_1y_1 & 0 & 0 \\ x_1y_2 & x_2y_2 & 0 \\ x_1y_3 & x_2y_3 & x_3y_3 \end{bmatrix} \oplus \begin{bmatrix} 0 & x_2y_1 & x_3y_1 \\ 0 & 0 & x_3y_2 \\ 0 & 0 & 0 \end{bmatrix}^T \oplus \begin{bmatrix} 0 & r_{1,2} & r_{1,3} \\ r_{1,2} & 0 & r_{2,3} \\ r_{1,3} & r_{2,3} & 0 \end{bmatrix} \xrightarrow{\text{sum rows}} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

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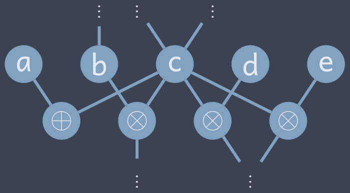
Each x_i is multiplied with all shares of y: $(y_j)_j$, vice versa.

» Data-Dependency Analysis against Masking Combinations

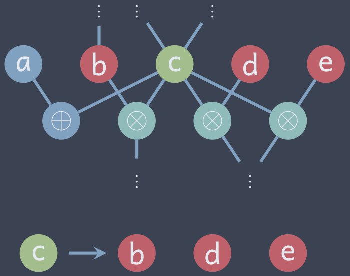
[GRW20]



» Data-Dependency Analysis against Masking Combinations

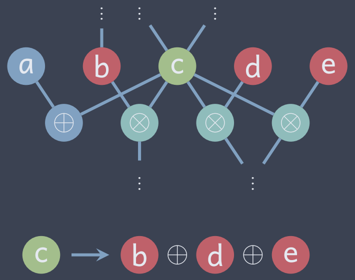
[GRW20]

* Find co-operands of each node for $\otimes$



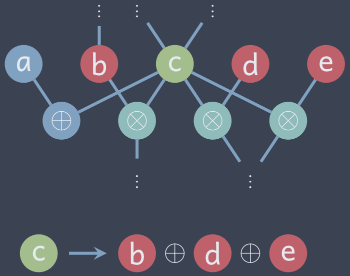
» Data-Dependency Analysis against Masking Combinations [GRW20]

* Find co-operands of each node for ⊗
* Collecting data-dependency (DD) traces
    * Sum co-operands values



» Data-Dependency Analysis against Masking Combinations [GRW20]

- Find co-operands of each node for $\otimes$
- Collecting data-dependency (DD) traces
  - Sum co-operands values
- Launch HO-DCA attacks on DD traces
  - Biased variables can be covered in DD trace



» Data-Dependency Analysis against Masking Combinations

[GRW20]

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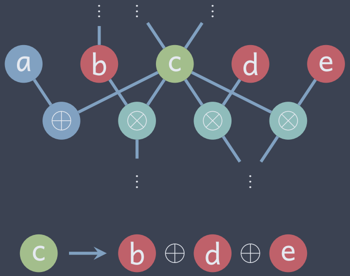
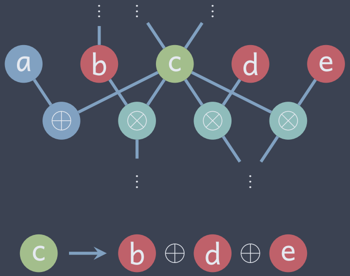
- Computation complexity substantially improved



» Data-Dependency Analysis against Masking Combinations [GRW20]

* Find co-operands of each node for $\otimes$
* Collecting data-dependency (DD) traces
  * Sum co-operands values
* Launch HO-DCA attacks on DD traces
  * Biased variables can be covered in DD trace
* Computation complexity substantially improved
* Successfully applied to break WhibOx 2019 winning implementations



» Attack Comparison

	linear masking		linear + NL masking	
	trace	computation	trace	computation
without shuffling				
LDA/HDDA	$t + \mathcal{O}(1)$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t^{2.8})$	$\mathcal{O}(t^2)$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t^{5.6})$
HODCA	c	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t^n)$	$4\,c$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t^n)$
DD-DCA	c	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t)$	$4\,c$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t)$

Note that c is some small empirical factor

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DD-DCA	c	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t)$	$4\,c$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t)$
with shuffling of degree λ				
HO-DCA	$c\,\lambda^2$	$\mathcal{O}\left(\lvert\mathcal{K}\rvert \cdot t^n \cdot \lambda^2\right)$	$4\,c\,\lambda^2$	$\mathcal{O}\left(\lvert\mathcal{K}\rvert \cdot t^n \cdot \lambda^2\right)$
Intg. HO-DCA	$c\,\lambda$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t^n \cdot \lambda)$	$4\,c\,\lambda$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t^n \cdot \lambda)$
MV HO-DCA	$\mathcal{O}(t^n)$	$\mathcal{O}\left(\lvert\mathcal{K}\rvert \cdot t^{2n}\right)$		
DD-DCA	$c\,\lambda^2$	$\mathcal{O}\left(\lvert\mathcal{K}\rvert \cdot t \cdot \lambda^2\right)$	$4\,c\,\lambda^2$	$\mathcal{O}\left(\lvert\mathcal{K}\rvert \cdot t \cdot \lambda^2\right)$

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Intg. DD-DCA	$c\,\lambda$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t \cdot \lambda)$	$4\;\lambda$	$\mathcal{O}(\lvert\mathcal{K}\rvert \cdot t \cdot \lambda)$

Note that c is some small empirical factor

» Thesis Summary

* This thesis concentrates on practical security of white-box cryptography
  * Demonstrated the capabilities of gray-box adversary in white-box model
  * Understood why gray-box attacks work against internal encodings
  * Proposed new gray-box attacks
  * Quantified different gray-box attack performances against different countermeasures
* A good level of practical resistance against these attacks can be achieved
  * under an assumption of adversary's uncertainty on attacked window within a full computation trace
  * for some choice of the parameters for countermeasures
  * Stressed the importance to hide structural knowledge of implementation

» Future Research Perspectives

- To ensure the uncertainty assumption on the attacked trace window
  - to counter data-dependency leakage
  - circuit obfuscation
- To build formal security arguments in passive gray-box attack model, *e.g.* to show that
  - the proposed attacks are somehow optimal
  - and the best the adversary can do can be made arbitrarily hard
- To construct higher-degree algebraically-secure gadgets

On the Practical Security of White-Box Cryptography

Thesis Defense

by Junwei Wang (王军委)

on June 24, 2020

Supervisors Jean-Sébastien Coron, Sihem Mesnager

Advisors Pascal Paillier, Matthieu Rivain