

# On the Practical Security of White-Box Cryptography

IEEE Information Theory Society

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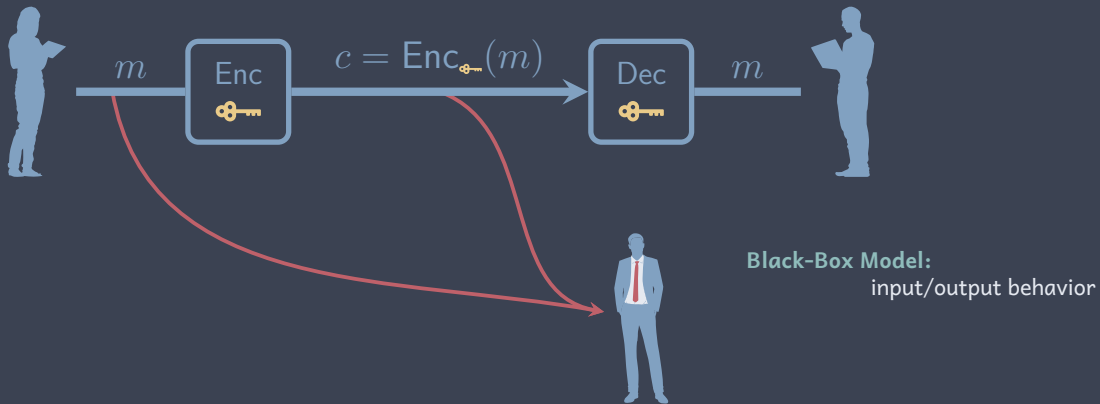
## White-Box Cryptography

- \* White-Box Security Model
- \* Historical White-Box Compilers
- \* Differential Computation Analysis Attack

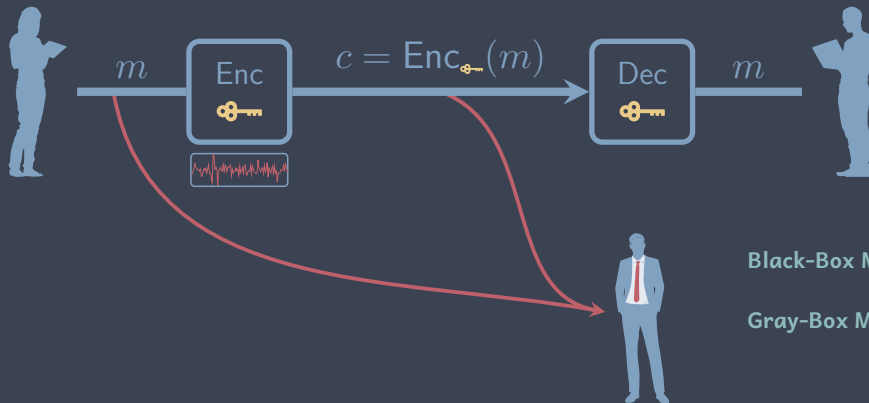
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## » Security Models: Shades of Gray



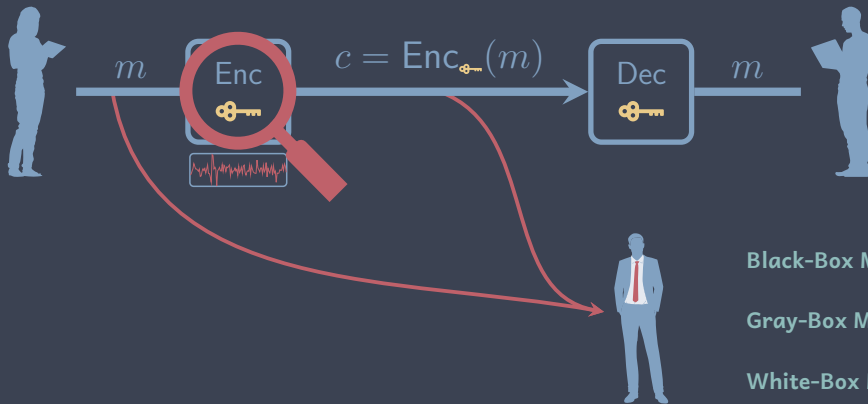
## » Security Models: Shades of Gray



**Black-Box Model:**  
input/output behavior

**Gray-Box Model:**  
side-channel leakage

## » Security Models: Shades of Gray



- Black-Box Model:**  
input/output behavior
- Gray-Box Model:**  
side-channel leakage
- White-Box Model:**  
“full” control

## » White-Box Threat Model

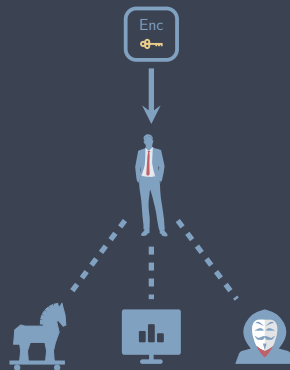
To extract a cryptographic key

**Where** from a software implementation of cipher

**Whom** by malwares, co-hosted applications, user themselves, ...

**How** by all kinds of means

- \* analyze the code
- \* spy on the memory
- \* interfere the execution
- \* ...



## » Motivation and Real-World Applications

- \* Why not using secure hardware ?
  - \* not always available
  - \* expensive (to produce, deploy, integrate, update)
  - \* usually has a long lifecycle
  - \* security breach is hard to mitigate
- \* Applications
  - \* Digital Content Distribution
  - \* Mobile Payment
  - \* Digital Contract Signing
  - \* Blockchains and cryptocurrencies



Credits to [Shamir, van Someren 99]



## White-Box Cryptography

- \* White-Box Security Model
- \* **Historical White-Box Compilers**
- \* Differential Computation Analysis Attack

## » White-Box Compiler

[Delerablée et al. 14]

A **white-box compiler** takes as input a *secret key* and generates a “white-box secure” program implementing some specific crypto. algo. with the specified secret key.

## \* White-box security notions

- \* **Unbreakability** (hardness of key-extraction)
- \* One-wayness
- \* Incompressibility
- \* Traceability



No provably secure unbreakable white-box compiler for standard block ciphers is known.

## » Theoretical Progress: Cryptographic Obfuscation

An **obfuscator** makes programs “unintelligible” while preserving their functionalities.

### \* Virtual Black-Box (VBB) Obfuscation

- \* Nothing is learned from the obfuscated programs except their I/Os.
- \* (**Impossibility**) VBB is impossible in general!
- \* VBB for point functions exist.
- \* *Can we VBB obfuscate a block cipher ?*

### \* Indistinguishability Obfuscation (iO)

- \* Literally, it hides the origin of an obfuscated program
- \* Has many implications
- \* Candidate constructions exist
- \* *Does not imply unbreakability directly !*

## » Historical White-Box Compilers

Transform a cipher into a series of randomized key-dependent lookup tables.

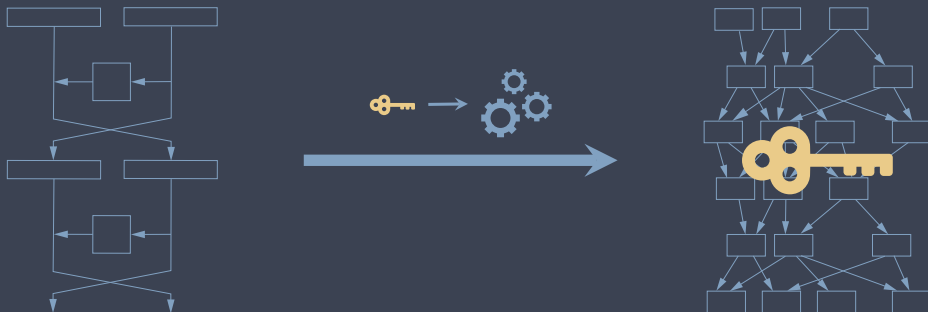
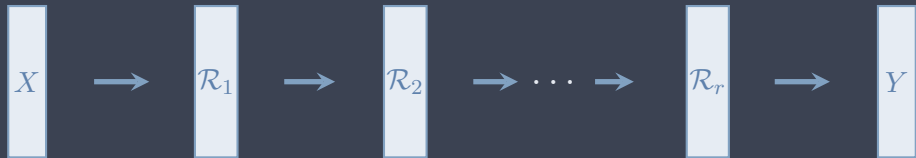
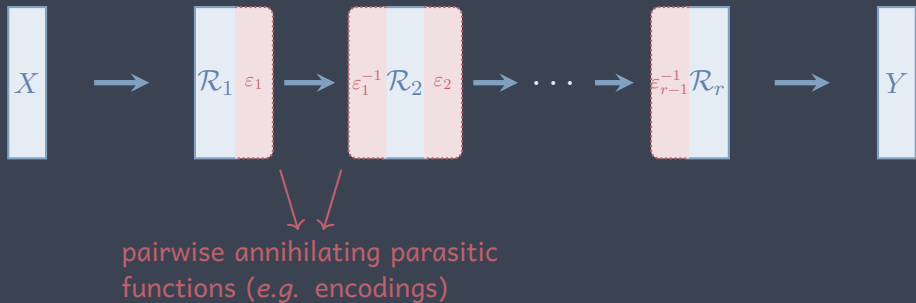


Illustration from [Wyseur12]

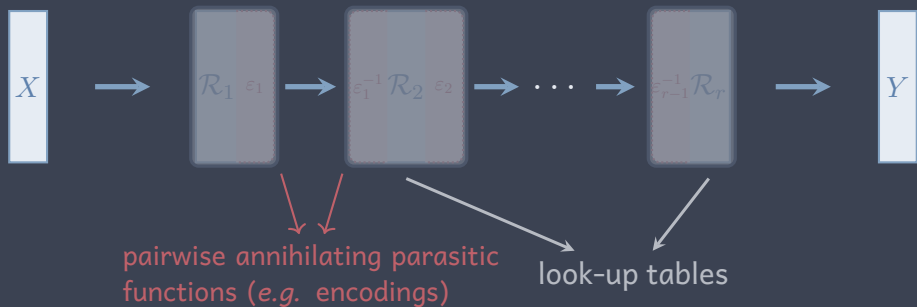
## » Historical White-Box Compilers: Internal Encodings



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## » Historical White-Box Compilers: Internal Encodings



## » Timeline: A Cat-And-Mouse Game



- \* 2002: seminal wb-DES [Chow et al. 02]
- \* 2003: seminal wb-AES [Chow et al. 03]
- \* 2005: variant wb-DES [Link, Neumann 05]
- \* 2006: variant wb-AES [Bringer et al. 06]
- \* 2009: variant wb-AES [Xiao, Lai 09]
- \* 2010: variant wb-AES [Karroumi 10]
- \* ...
- \* 2002: fault attack against wb-DES [Jacob et al. 02]
- \* 2004: BGE attack [Billet et al. 04]
- \* 2007: attack wb-DES [Goubin et al 07, Wyseur et al. 07]]
- \* 2009: attack wb-AES [Michiels et al. 09]
- \* 2010: attack Bringer et al. variant [De Mulder et al. 10]
- \* 2012: attack Xiao-Lai variant [De Mulder et al. 12]
- \* 2013: attack improvements and Karroumi variant [Lepoint et al .13]
- \* ...



## » Obscurity as a Solution

- \* All public designs are broken
- \* No provably secure solution

- \* Growing demand in industry
- \* Huge application potential



Security through obscurity: home-made design + obfuscation



Time consuming reverse engineering + structural analysis

## White-Box Cryptography

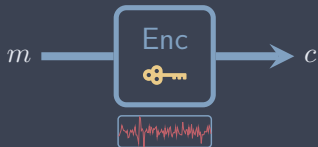
- \* White-Box Security Model
- \* Historical White-Box Compilers
- \* **Differential Computation Analysis Attack**

## » Differential Computation Analysis (DCA)

[Bos et al. 2016, Sanfelix et al. 2015]

Differential power analysis (DPA) techniques on computational leakages.

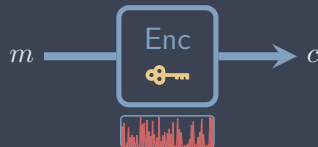
### gray-box model



side-channel leakages (noisy)

*e.g.* power / EM / time / ...

### white-box model

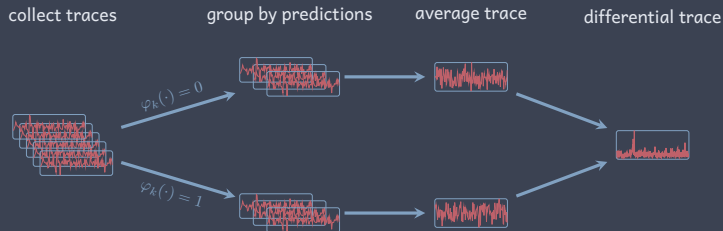


computational leakages (noisy-free)

*e.g.* registers / accessed memory / ...

## » Differential Computation Analysis (DCA) (cont.)

[Bos et al. 2016, Sanfelix et al. 2015]



Implying strong *linear correlation* between the sensitive variables  $\varphi_k$  and the leaked samples in the computational traces.

Many publicly available implementations are broken by DCA.

## » WhibOx Competitions

- \* Organized as CHES CTF events

*The competition gives an opportunity for researchers and practitioners to confront their (secretly designed) white-box implementations to state-of-the-art attackers*

– WhibOx 2017

- \* Designer: to submit the C source codes of AES-128 with secret key
- \* Attacker: to reveal the hidden key
- \* No need to disclose identity or underlying techniques

## » WhibOx Competitions (cont.)

### \* WhibOx 2017

- \* 94 submissions were **all broken** by 877 individual breaks
- \* most (86%) of them were alive for  $< 1$  day
- \* mostly broken by DCA [Bock and Treff 20]

### \* WhibOx 2019

- \* New rules encourage designers to submit “smaller” and “faster” implementations
- \* 27 submissions with 124 individual breaks
- \* 3 implementations survived, but broken after the competition

## » This Talk

- \* Analyze in-depth why DCA can break internal encodings and propose new efficient attacks against internal encodings
- \* Propose new different advanced gray-box attack paths
- \* Analyze advanced gray-box countermeasures against new attack paths
- \* Propose data-dependency analysis with substantially improved complexity over the existing attacks
- \* Break the winning challenges from two editions of WhibOx competitions

## » Publications

- [RW19] Rivain and Wang. “Analysis and Improvement of Differential Computation Attacks against Internally-Encoded White-Box Implementations”. In: **TCHES 2019 Issue 2**.
- [BRVW19] Bogdanov, Rivain, Vejre, and Wang. “Higher-Order DCA against Standard Side-Channel Countermeasures”. In: **COSADE 2019**.
- [GPRW20] Goubin, Paillier, Rivain, and Wang. “How to reveal the secrets of an obscure white-box implementation”. In: **Journal of Cryptographic Engineering Volume 10 Issue 1**.
- [GRW20] Goubin, Rivain, and Wang. “Defeating State-of-the-Art White-Box Countermeasures with Advanced Gray-Box Attacks”. In: **TCHES 2020 Issue 3**.



» **Passive Gray-Box Adversary Model**

[GRW20]

- \* For each of  $N$  chosen  $(x^{(i)})_{1 \leq i \leq N}$ , collect a computational trace of  $t$  samples

$$\mathbf{v} = (v_1, v_2, \dots, v_t)$$

- \* Build a distinguisher  $D$ :

$$(\gamma_k)_{k \in \mathcal{K}} = D \left( (x^{(i)})_i, (\mathbf{v}^{(i)})_i \right)$$

- \* Choose key candidate:  $\operatorname{argmax}_{k \in \mathcal{K}} \gamma_k$

- \* Number samples in attacked trace (window):  $t$

- \* Required number traces:  $N$

## » Outline

# White-Box Cryptography

## DCA Analysis and Improvements against Internal Encodings

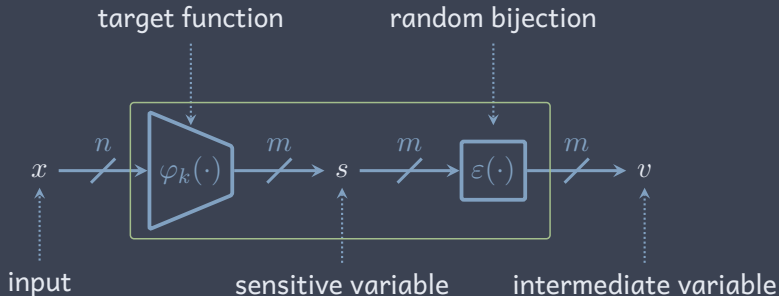
## Advanced Gray-Box Countermeasures and Attacks

## Data-Dependency Analysis

## DCA Analysis and Improvements against Internal Encodings

- \* DCA against Internal Encodings
- \* Collision Attack

## » Internal Encoding Leakage



- \*  $\varepsilon \circ \varphi_k$ , as a result of some **table look-ups**, is **leaked in the memory**
- \* To exploit the leakage of  $\varepsilon \circ \varphi_k$ , it is necessary that  $n > m$

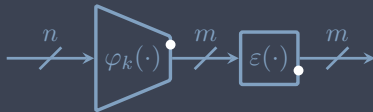
## DCA Analysis and Improvements against Internal Encodings

- \* DCA against Internal Encodings
- \* Collision Attack

## » DCA Analysis

Based on well-established theory – *Boolean correlation*, instead of *difference of means*: for any key guess  $k$

$$\rho_k = \text{Cor}\left( \varphi_k(\cdot)[i] \ , \ \varepsilon \circ \varphi_{k^*}(\cdot)[j] \right)$$



DCA success (roughly) requires:

$$|\rho_{k^*}| > \max_{k \neq k^*} |\rho_k|$$

» Distributions of  $\rho_{k^*}$  and  $\rho_{k^\times}$ 

- \* Ideal assumption:  $(\varphi_k)_k$  are mutually independent random  $(n, m)$  functions

Correct key guess  $k^*$ ,

$$\rho_{k^*} = 2^{2-m} N^* - 1$$

where

$$N^* \sim \mathcal{HG}(2^m, 2^{m-1}, 2^{m-1}).$$

Only depends on  $m$ .

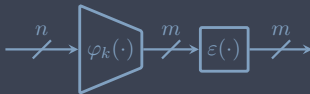
Incorrect key guess  $k^\times$ ,

$$\rho_{k^\times} = 2^{2-n} N^\times - 1$$

where

$$N^\times \sim \mathcal{HG}(2^n, 2^{n-1}, 2^{n-1}).$$

Only depends on  $n$ .



## » Lemma

## Lemma

Let  $\mathcal{B}(n)$  be the set of balanced  $n$ -bit Boolean functions. If  $f \in \mathcal{B}(n)$  and  $g \overset{\$}{\leftarrow} \mathcal{B}(n)$  independent of  $f$ , then the balanceness of  $f + g$  is  $B(f + g) = 4 \cdot N - 2^n$  where  $N \sim \mathcal{HG}(2^n, 2^{n-1}, 2^{n-1})$  denotes the size of  $\{x : f(x) = g(x) = 0\}$ .

With

$$\text{Cor}(f + g) = \frac{1}{2^n} B(f + g)$$

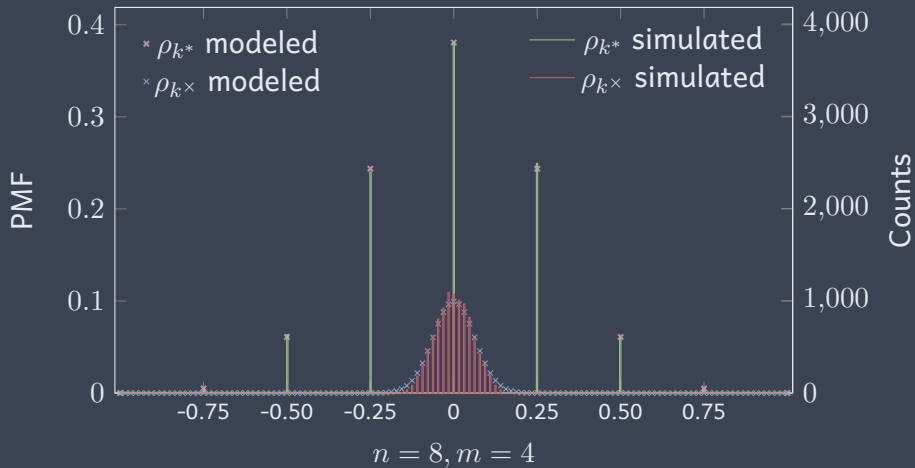
$\Rightarrow$

$$\rho_{k^*} = 2^{2-m} N^* - 1 \quad \text{and} \quad \rho_{k^\times} = 2^{2-n} N^\times - 1$$

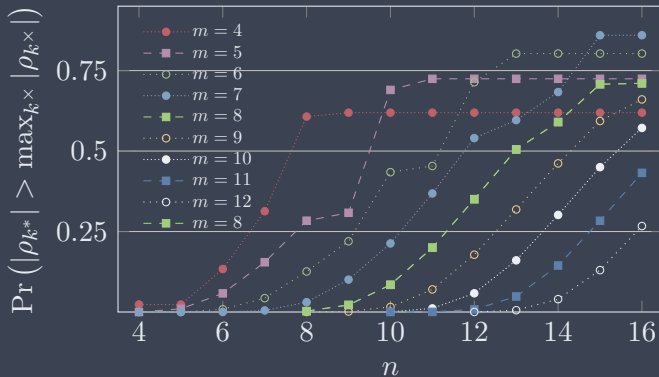
where  $N^* \sim \mathcal{HG}(2^m, 2^{m-1}, 2^{m-1})$  and  $N^\times \sim \mathcal{HG}(2^n, 2^{n-1}, 2^{n-1})$ .



## » Distributions of $\rho_{k*}$ and $\rho_{k\times}$



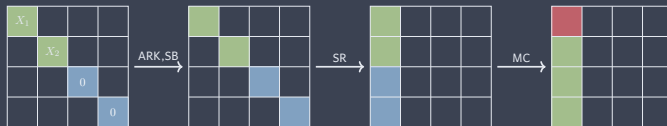
» **DCA Success Rate:**  $|\rho_{k^*}| > \max_{k \neq k^*} |\rho_{k^*}|$



DCA success probability converges towards  $\approx 1 - \Pr_{N^*}(2^{m-2})$  for  $n \geq 2m + 2$ .

## » Attack a NSC Variant: a White-Box AES

- \* Byte encoding protected AES
- \* DCA has failed to break it *before this work*
- \* Our approach: target an output byte of MixColumn in the first round



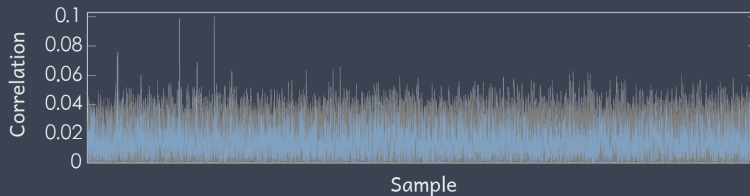
$$\varphi_{k_1||k_2}(x_1||x_2) = 2 \cdot \mathbf{Sbox}(x_1 \oplus k_1) \oplus 3 \cdot \mathbf{Sbox}(x_2 \oplus k_2) \oplus \mathbf{Sbox}(k_3) \oplus \mathbf{Sbox}(k_4) \oplus c$$

$$\varepsilon' = \varepsilon \circ \oplus_c ,$$

$$n = 16, m = 8 , |\mathcal{K}| = 2^{16}.$$

## » Attack a NSC Variant: a White-Box AES

\* Attack results:  $\sim 1800$  traces



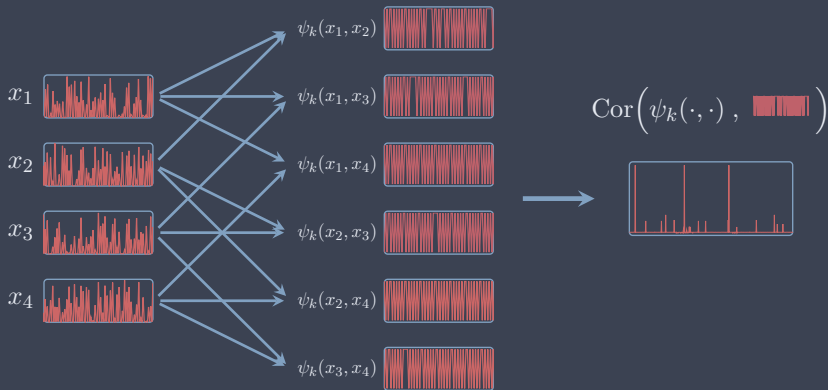
\* Byte encoding can be efficiently broken

## DCA Analysis and Improvements against Internal Encodings

- \* DCA against Internal Encodings
- \* Collision Attack

## » Collision Attack

$N$  inputs & raw traces       $\binom{N}{2}$  collision predictions & traces



$$\psi_k(x_1, x_2) := \left( \varphi_k(x_1) = \varphi_k(x_2) \right)$$

## » Collision Attack Explanation

Based on the principle:

$$\varphi_k(x_1) = \varphi_k(x_2) \Leftrightarrow \varepsilon \circ \varphi_k(x_1) = \varepsilon \circ \varphi_k(x_2)$$

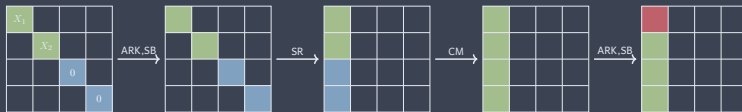
Trace Complexity:

$$N = \mathcal{O}\left(2^{\frac{m}{2}}\right)$$

		Predictions					
		1	2	3	4	5	6
key guesses	$k_1$	○○○	●○○	●●○	○○○	●●○	○○○
	$k_2$	●●○	○○○	○○○	●○○	○○○	●●○
	$k_3$	●○○	●○○	●○○	●○○	○○○	●○○
	$k_4$	○○○	●●○	●●○	○○○	●○○	○○○

## » Attack the NSC Variant

- \* Same to DCA: targeting at one 1-st round MixColumn output byte



$$\varphi_{k_1||k_2}(x_1||x_2) = 2 \cdot \mathbf{Sbox}(x_1 \oplus k_1) \oplus 3 \cdot \mathbf{Sbox}(x_2 \oplus k_2)$$

$$\varepsilon' = \varepsilon \circ \oplus_c \quad \text{or} \quad \varepsilon'' = \varepsilon \circ \mathbf{Sbox} \circ \oplus_{c \oplus k'_1}$$

- \* Attack results: 60 traces



## » Contribution Summary

- \* DCA against internal encodings has been analysed in depth
  - \* Allows to attack wider encodings
- \* Propose new class of collision attacks with very low trace complexity
- \* Mutual information analysis with similar trace complexity but higher computation complexity
- \* Hence, protecting AES with internal encodings in the beginning rounds is insufficient

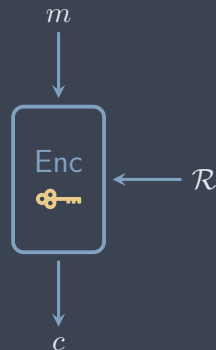
## Advanced Gray-Box Countermeasures and Attacks

- \* Linear Masking, Higher-Order DCA, and Linear Decoding Analysis
- \* Algebraic Security and Non-Linear Masking
- \* Shuffling

## » Random Source

[BRVW19]

- \* Countermeasures need randomness.



## » Random Source

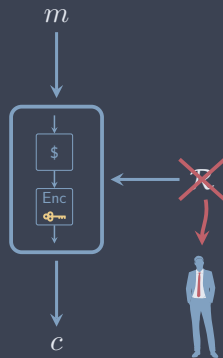
[BRVW19]

- \* Countermeasures need randomness.
- \* Plaintext is the only source of randomness
- \* Security criteria:

**Pseudorandomness** no statistical flaw

**Obscurity** the design should be kept secret

**Obfuscation** hard to distinguish from other intermediate variables



## Advanced Gray-Box Countermeasures and Attacks

- \* Linear Masking, Higher-Order DCA, and Linear Decoding Analysis
- \* Algebraic Security and Non-Linear Masking
- \* Shuffling

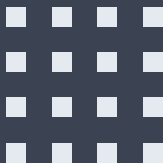
## » Linear Masking

[Ishai et al. 03]

- \* Intermediate value  $x$  is split into  $n$  shares

$$x = x_1 \oplus x_2 \cdots \oplus x_n$$

original states

Masking  


masked states

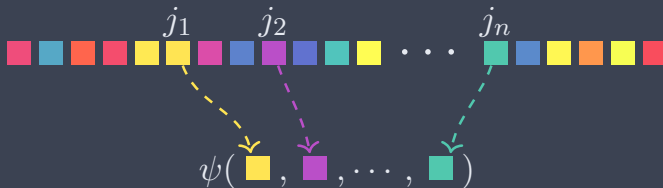


- \* Shares are manipulated separately such that any subset of at most  $n - 1$  shares is independent of  $x$
- \* Resistant against  $(n - 1)$ -th order DCA attacks

## » Higher-Order DCA (HO-DCA)

[BVRW19]

- \* Trace **pre-processing**: an  $n$ -th order trace contains  $q = \binom{t}{n}$  points:



- \* The natural combination function  $\psi$  is XOR sum
- \* Perform DCA attacks on the higher-order traces
- \* Linear masking can be broken
  - \*  $\exists$  fixed  $n$  positions in which the shares are

$$\binom{1000}{5} \approx 2^{43}$$

## » Linear Decoding Analysis (LDA)

[GPRW20]

- \* Assumption: there exists a linear decoding function

$$D(v_1, v_2, \dots, v_t) = a_0 \oplus \left( \bigoplus_{1 \leq i \leq t} a_i \cdot v_i \right) = \varphi_k(x)$$

for some sensitive variable  $\varphi_k$  and some fixed coefficients  $a_0, a_1, \dots, a_t$ .

- \* Record the  $v_i$ 's over  $N$  executions:

$$\begin{bmatrix} 1 & v_1^{(1)} & \cdots & v_t^{(1)} \\ 1 & v_1^{(2)} & \cdots & v_t^{(2)} \\ 1 & \vdots & \ddots & \vdots \\ 1 & v_1^{(N)} & \cdots & v_t^{(N)} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \varphi_k(x^{(1)}) \\ \varphi_k(x^{(2)}) \\ \vdots \\ \varphi_k(x^{(N)}) \end{bmatrix}$$



## » Linear Decoding Analysis (LDA) (cont.)

[GPRW20]

- \* Record the  $v_i$ 's over  $N$  executions:

$$\begin{bmatrix} 1 & v_1^{(1)} & \cdots & v_t^{(1)} \\ 1 & v_1^{(2)} & \cdots & v_t^{(2)} \\ 1 & \vdots & \ddots & \vdots \\ 1 & v_1^{(N)} & \cdots & v_t^{(N)} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \varphi_k(x^{(1)}) \\ \varphi_k(x^{(2)}) \\ \vdots \\ \varphi_k(x^{(N)}) \end{bmatrix}$$

- \* Linear masking is vulnerable to LDA

- \* system solvable for  $k^*$
- \* but not for incorrect key guess  $k^\times$

- \* Trace Complexity  $t + \mathcal{O}(1)$

- \* Computation complexity  $\mathcal{O}(t^{2.8} \cdot |\mathcal{K}|)$

$$1000^{2.8} \approx 2^{28}$$

[GPRW20]

- \* Small windows located for target variables
- \* The 3rd s-box  $t = 35, \min(n) = 2 \implies$  HO-DCA:  $2^{18}$ , and LDA:  $2^{22}$

Bit	Encoding coefficients
1	000000101011100010101000000000000000
2	000000100110111111100000000000000000
3	000000001010001110111000000000000000
4	000000000011000111011100000000000000
5	000000001100100000011100000000000000
6	000000000000100000000000001000000000
7	000000010001001010101000000000000000
8	000000001000001001100000000000000000

- \* The 14th s-box:  $t = 45, \min(n) = 9 \implies$  HO-DCA:  $2^{49}$ , and LDA:  $2^{23}$

## Advanced Gray-Box Countermeasures and Attacks

- \* Linear Masking, Higher-Order DCA, and Linear Decoding Analysis
- \* Algebraic Security and Non-Linear Masking
- \* Shuffling

## » Algebraic Security and Non-Linear Masking

[Biryukov and Udovenko 18]

- \* Introduced by Biryukov and Udovenko at Asiacrypt 2018
- \* To capture LDA like algebraic attack

A  $d$ -th degree algebraically-secure non-linear masking ensures that any function of up to  $d$  degree to the intermediate variables should not compute a “predictable” variable.

## » First-Degree Secure Non-Linear Masking

[Biryukov and Udovenko 18]

- \* Quadratic decoding function

$$(a, b, c) \mapsto ab \oplus c$$

- \* Secure gadgets for bit XOR, bit AND, and refresh
- \* Provably secure composition
- \* But vulnerable to DCA attack

$$\text{Cor}(ab \oplus c, c) = \frac{1}{2}$$

- \* Empirically, suggest using a combination of linear masking and non-linear masking to thwart both DCA (probing security) and LDA (algebraic security).

» **Combination of Linear Masking and Non-linear Masking**

[GRW20]

\* Three possible natural combinations:

1. apply linear masking on top of non-linear masking

$$x = (a_1 \oplus a_2 \oplus \cdots \oplus a_n)(b_1 \oplus b_2 \oplus \cdots \oplus b_n) \oplus (c_1 \oplus c_2 \oplus \cdots \oplus c_n)$$

2. apply non-linear masking on top of linear masking

$$x = (a_1 b_1 \oplus c_1) \oplus (a_2 b_2 \oplus c_2) \oplus \cdots \oplus (a_n b_n \oplus c_n) .$$

3. merge the two maskings into a new encoding

$$x = ab \oplus c_1 \oplus c_2 \oplus \cdots \oplus c_n .$$

\* For first two combinations, the combined masking gadgets can be simply derived from the original gadgets of both schemes.

## » Higher-Degree Decoding Analysis (HDDA)

[GPRW20]

- \* Assume the decoding function is of degree  $d$
- \* Trace **pre-processing**: a  $d$ -th degree trace contains all monomials of degree  $\leq d$



- \* Perform LDA attacks on the higher-degree traces
- \* Higher-degree trace samples:  $\sum_{i=0}^d \binom{t}{d} = \binom{t+d}{d} \ll t^d$
- \* Complexity:  $\mathcal{O}(t^{2.8d} \cdot |\mathcal{K}|)$ , practical when  $t, d$  are small.

$$t^{2.8d} < 2^{50}$$



$$\begin{aligned} d = 2 &\Rightarrow t < 487 \\ t = 100 &\Rightarrow d \leq 5 \end{aligned}$$

## Advanced Gray-Box Countermeasures and Attacks

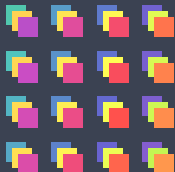
- \* Linear Masking, Higher-Order DCA, and Linear Decoding Analysis
- \* Algebraic Security and Non-Linear Masking
- \* **Shuffling**



## » Shuffling

- \* The order of execution is randomly chosen for each run of the implementation.
- \* To increase noise in the adversary's observation

masked states



iteration in *normal* order



iteration in *randomized* order



## » Shuffling (cont.)

[BRVW19]

- \* Not enough in white-box model: traces can be aligned by memory
- \* Thus, the memory location of shares has to be shuffled.

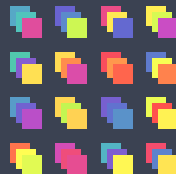
masked states



memory shuffling



memory shuffled states



## » HO-DCA and Integrated HO-DCA against Masking and Shuffling

[BRVW19]

- \*  $\nexists n$  fixed locations for all shares
- \* Shuffling degree is  $\lambda$ 
  - \* correlation score decreased by a factor of  $\lambda$
  - \* attack slow down by a factor of  $\lambda^2$
- \* Integrate values from all  $\lambda$  slots
  - \* correlation score decreased by a factor of  $\sqrt{\lambda}$
  - \* attack slow down by a factor of  $\lambda$

» **Multivariate HO-DCA**

[BRVW19]

- \* The multivariate HO-DCA optimizes the attack by exploiting joint information of the higher-order samples on the secrets
- \* Based on a maximum likelihood distinguisher

$$\gamma_k = \Pr \left( K = k \mid (V^{(i)})_i = (v^{(i)})_i \wedge (X^{(i)})_i = (x^{(i)})_i \right)$$

- \* We show that

$$\gamma_k \propto \prod_{i=1}^N C_k(v^{(i)}, x^{(i)})$$

where *the counter*

$C_k(v, x) :=$  the number of  $n$ -tuples  $s.t.$   $v_{j_1} \oplus \dots \oplus v_{j_n} = \varphi_k(x)$  in one trace.

## » Attack Comparison

	linear masking		linear + NL masking	
	trace	computation	trace	computation
<i>without shuffling</i>				
<b>LDA / HDDA</b>	$t + \mathcal{O}(1)$	$\mathcal{O}( \mathcal{K}  \cdot t^{2.8})$	$\mathcal{O}(t^2)$	$\mathcal{O}( \mathcal{K}  \cdot t^{5.6})$
<b>HODCA</b>	$c$	$\mathcal{O}( \mathcal{K}  \cdot t^n)$	$4c$	$\mathcal{O}( \mathcal{K}  \cdot t^n)$
<i>with shuffling of degree <math>\lambda</math></i>				
<b>HO-DCA</b>	$c\lambda^2$	$\mathcal{O}( \mathcal{K}  \cdot t^n \cdot \lambda^2)$	$4c\lambda^2$	$\mathcal{O}( \mathcal{K}  \cdot t^n \cdot \lambda^2)$
<b>Intg. HO-DCA</b>	$c\lambda$	$\mathcal{O}( \mathcal{K}  \cdot t^n \cdot \lambda)$	$4c\lambda$	$\mathcal{O}( \mathcal{K}  \cdot t^n \cdot \lambda)$
<b>MV HO-DCA</b>	$\mathcal{O}(t^n)$	$\mathcal{O}( \mathcal{K}  \cdot t^{2n})$		

## Data-Dependency Analysis

- \* Data-Dependency Graph
- \* Data-Dependency Analysis against Masking Combinations

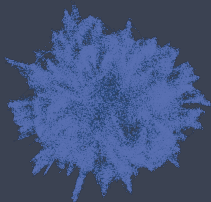
## Data-Dependency Analysis

- \* Data-Dependency Graph
- \* Data-Dependency Analysis against Masking Combinations

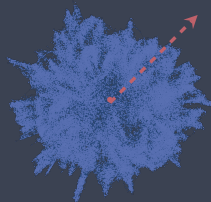
## » Data Dependency Graph

[GPRW20]

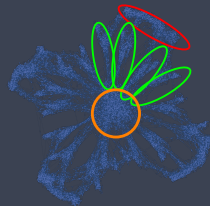
- \* White-box adversary also observes data-flow.
- \* Data-dependency graph (DDG) can visually reveal the structure of the implementation.



first 20%



first 10%



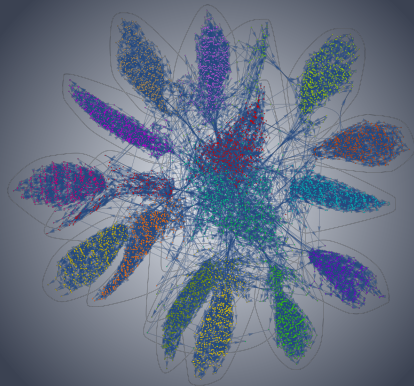
first 5%



## » Data Dependency Clusters

[GPRW20]

- \* Data-dependency can extract computation clusters and determines the location of sensitive computation
  - \* Round:  $\sim 28$  k nodes
  - \* S-box cluster:  $\sim 500$  nodes
  - \* Trace windows containing targets:  $\sim 50$  nodes



## Data-Dependency Analysis

- \* Data-Dependency Graph
- \* **Data-Dependency Analysis against Masking Combinations**

» **AND Gadget for Linear Masking**

[Ishai et al. 03]

$$(x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n) \mapsto (z_1, z_2, \dots, z_n) \quad \text{s.t.} \quad \bigoplus_i x_i \cdot \bigoplus_i y_i = \bigoplus_i z_i.$$

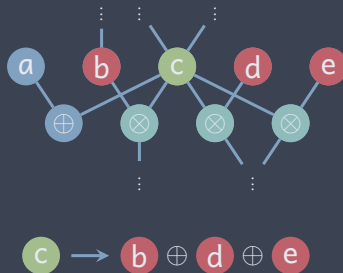
$$\begin{bmatrix} x_1 y_1 & 0 & 0 \\ x_1 y_2 & x_2 y_2 & 0 \\ x_1 y_3 & x_2 y_3 & x_3 y_3 \end{bmatrix} \oplus \begin{bmatrix} 0 & x_2 y_1 & x_3 y_1 \\ 0 & 0 & x_3 y_2 \\ 0 & 0 & 0 \end{bmatrix}^T \oplus \begin{bmatrix} 0 & r_{1,2} & r_{1,3} \\ r_{1,2} & 0 & r_{2,3} \\ r_{1,3} & r_{2,3} & 0 \end{bmatrix} \xrightarrow{\text{sum rows}} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Each  $x_i$  is multiplied with all shares of  $y$ :  $(y_j)_j$ , vice versa.

## » Data-Dependency Analysis against Masking Combinations

[GRW20]

- \* Find co-operands of each node for  $\otimes$
- \* Collecting data-dependency (DD) traces
  - \* Sum co-operands values
- \* Launch HO-DCA attacks on DD traces
  - \* Biased variables can be covered in DD trace
- \* Computation complexity substantially improved
- \* Successfully applied to break WhibOx 2019 winning implementations



## » Attack Comparison

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<i>without shuffling</i>				
<b>LDA/HDDA</b>	$t + \mathcal{O}(1)$	$\mathcal{O}( \mathcal{K}  \cdot t^{2.8})$	$\mathcal{O}(t^2)$	$\mathcal{O}( \mathcal{K}  \cdot t^{5.6})$
<b>HODCA</b>	$c$	$\mathcal{O}( \mathcal{K}  \cdot t^n)$	$4c$	$\mathcal{O}( \mathcal{K}  \cdot t^n)$
<b>DD-DCA</b>	$c$	$\mathcal{O}( \mathcal{K}  \cdot t)$	$4c$	$\mathcal{O}( \mathcal{K}  \cdot t)$
<i>with shuffling of degree <math>\lambda</math></i>				
<b>HO-DCA</b>	$c\lambda^2$	$\mathcal{O}( \mathcal{K}  \cdot t^n \cdot \lambda^2)$	$4c\lambda^2$	$\mathcal{O}( \mathcal{K}  \cdot t^n \cdot \lambda^2)$
<b>Intg. HO-DCA</b>	$c\lambda$	$\mathcal{O}( \mathcal{K}  \cdot t^n \cdot \lambda)$	$4c\lambda$	$\mathcal{O}( \mathcal{K}  \cdot t^n \cdot \lambda)$
<b>MV HO-DCA</b>	$\mathcal{O}(t^n)$	$\mathcal{O}( \mathcal{K}  \cdot t^{2n})$		
<b>DD-DCA</b>	$c\lambda^2$	$\mathcal{O}( \mathcal{K}  \cdot t \cdot \lambda^2)$	$4c\lambda^2$	$\mathcal{O}( \mathcal{K}  \cdot t \cdot \lambda^2)$
<b>Intg. DD-DCA</b>	$c\lambda$	$\mathcal{O}( \mathcal{K}  \cdot t \cdot \lambda)$	$4\lambda$	$\mathcal{O}( \mathcal{K}  \cdot t \cdot \lambda)$

Note that  $c$  is some small empirical factor

## » Conclusion

- \* This talks covers practical security of white-box cryptography
  - \* Demonstrated the capabilities of gray-box adversary in white-box model
  - \* Understood why gray-box attacks work against internal encodings
  - \* Proposed new gray-box attacks
  - \* Quantified different gray-box attack performances against different countermeasures
- \* A good level of practical resistance against these attacks can be achieved
  - \* under an assumption of adversary's uncertainty on attacked window within a full computation trace
  - \* for some choice of the parameters for countermeasures
  - \* Stressed the importance to hide structural knowledge of implementation

# On the Practical Security of White-Box Cryptography

IEEE Information Theory Society

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on July 1, 2020

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