Reveal Secrets in Adoring Poitras

A generic attack on white-box cryptography

Junwei Wang

CryptoExperts

University of Luxembourg

University of Paris 8

ECRYPT-NET School on
Correct and Secure Implementation

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Outline

1. White-Box Cryptography
   - What Is White-Box Cryptography (WBC)?
   - WhiBox Contest
2. Breaking Adoring Poitras
   - Cleaning the Code
   - De-Virtualization
   - Bitwise-Based Program to Boolean Circuits
   - Boolean Circuits Minimization
   - Data Dependency Analysis
   - Algebraic Analysis

What Is White-Box Cryptography (WBC)?

- WBC is resistant against **key extraction** in a **software** implementation of a cryptographic algorithm.
- The attacker **entirely** controls the running environment.
  - to record the computation trace (memory address/value, access type/time, etc)
  - to modify the control flow / intermediate value, etc
- No provably secure construction exists.
- All known practical constructions has been broken by **generic attacks** (DCA and DFA) before 2016.
- Applications:
  - digital rights management (DRM)
  - mobile payments

WhiBox Contest - CHES 2017 CTF

- Organized by ECRYPT CSA
- Two categories:
  - designers
  - breakers
- AES-128, physical limitation (<50M source code, <20M binary, <1s execution)
- 94 submitted challenges are all broken (most of them were alive $<$ 1 day)
- *Hardest* challenge: `Adoring Poitras`.
  - Surviving for 28 days (2.3 $\times$ the 2nd hardest one)
  - Submitted by `cryptolux` (Biryukov-Udovenko)
  - Only broken by `team_cryptoexperts` (Goubin-Paillier-Rivain-Wang)

Untidy Code

More than 1k functions

Readability Processing

- Duplicate / redundancy / unused codes elimination
- Functions / variables renaming
- Constants rewriting
- Code combination

Only 20 functions are remaining

```
void copy(uint KLedyCW, uint lRjmjY) {int_arr[(a+KLedyCW) & 0x3ffff] = int_arr[lRjmjY & 0x3ffff];}
void encode(uint owhjO, uint nlBqXn) {assign(owhjO, in_ptr[nlBqXn]);}
void decode(uint hFqeIO, uint jvXpt) {out_ptr[hFqeIO]=int_arr[(a+jvXpt)&0x3ffff];}
void rshift_xor(uint HcCOL, uint ISRfdIp, uint ufYFMX) { int_arr[HcCOL&0x3ffff]^=1&(int_arr[(a+ISRfdIp)&0x3ffff]>>ufYFMX); /*prir
void lshift_xor(uint NcJBw, uint OMQBXqa, uint AyoLFz) { uint dyfS=(int_arr[(a+AyoLFz)&0x3ffff])&1;int_arr[(a+NcJBw)&0x3ffff]^=dy

void expand_bit(uint SteQlD, uint ZubEP, uint ZiQz) {int_arr[(a+SteQlD)&0x3ffff]=-((int_arr[(a+ZubEP)&0x3ffff]>>ZiQz)&1);}

uint lookup1(uint AKBKig) {return int_arr[(a+AKBKig)&0x3ffff];}
uint lookup2(uint WAdV, uint ZcVdJ) {return int_arr[(WAdV+ZcVdJ)&0x3ffff];}

void assign(uint UbEJi, uint UmwjUhO) {int_arr[(a+UbEJi)&0x3ffff]=UmwjUhO;}
void assign_a(uint wEOkx) {a = wEOkx;}
void assign_b(uint fnmqxL) {b=int_arr[fnmqxL]&0x07fff;}
void update_a() {a=lookup2(1592,mix(b)); printf("%lu\n",a);}
void update_b() {b=0x7fff&lookup2(522,mix(b));}

void mistery(uint wJxea, uint QBGXUN) {uint t = (~int_arr[(a+QBGXUN)&0x3ffff])&0x7fff; assign(wJxea,lookup2(2979,mix(t)));}

// bitwise operation
void xor(uint oEHmwk, uint KCZu, uint MtCA) {int_arr[(a+oEHmwk)&0x3ffff]=int_arr[(a+KCZu)&0x3ffff]^int_arr[(a+MtCA)&0x3ffff];}
void and(uint bmmFp, uint UNFg, uint PqCtZYZ) {uint t = int_arr[(a+UNFg)&0x3ffff]&int_arr[(a+PqCtZYZ)&0x3ffff];}
void or(uint eTGI, uint udoXFs, uint mezPNN) {int_arr[(a+eTGI)&0x3ffff]=int_arr[(a+udoXFs)&0x3ffff]|int_arr[(a+mezPNN)&0x3ffff];}
void not(uint YfnT, uint JKTw) {int_arr[(a+YfnT)&0x3ffff]=~int_arr[(a+JKTw)&0x3ffff];}

// jump
void goto_f(uint LKhOC) {pc = bop + LKhOC;}
void jump_if(uint DbvJO, uint FleFNIf, uint LeHf) { if(lookup2(2979,mix(b))==lookup2(DbvJO, FleFNIf) || count >= 64) {printf("%d
```

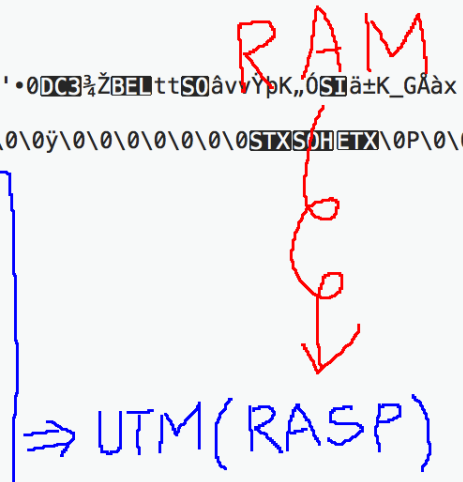
Universal Turing Machine

```
2262 uint kIKfgI, VDpRwo;
2263 uchar *JqcadL;
2264 uchar *xkpRp, *Puix;
2265 uchar *sutlnu;
2266 uchar *unYAQ;
2267 const char pDeoW[] = "äSTXUS}BS&ýÖ:eK±Ï¿üì¡÷¿žåPê‹˜Ùþç•Àâ]'•0DC3¾ŽBELttSOâvvŸþK„ÓSIä±K_GÅàx    ETB¿íf
2268 uint ooGoRv[sizeof(pDeoW)/sizeof(uint)];
2269 char JGNNvi[] = "SOH\0\n\0\0\0\0\0\0STXSOHxSUBSOH\0\0\0\0\0ÿ\0\0\0\0\0\0\0STXSOHETX\0P\0\0\0\0\0\0
2270 uchar *klspCVy = (uchar*)JGNNvi;
2271 void aLLl() {
2272 unYAQ = klspCVy;
2273 sutlnu = klspCVy + sizeof(JGNNvi)/sizeof(uchar);
2274 JqcadL = klspCVy;
2275 while (JqcadL < sutlnu) {
2276 uchar eMmr = *JqcadL++;
2277 if (eMmr == 0) {
2278 void (*QiEb)();
2279 QiEb = (void*)funcptrs[*JqcadL++];
2280 uint *AnezsV = (uint*)JqcadL;
2281 JqcadL += eMmr*8;
2282 QiEb();
2283 }
2284 else if (eMmr == 1) {
2285 void (*QiEb)(uint);
2286 QiEb = (void*)funcptrs[*JqcadL++];
2287 uint *AnezsV = (uint*)JqcadL;
2288 JqcadL += eMmr*8;
2289 QiEb(AnezsV[0]);
2290 }
2291 ...
2327 }
2328 void AES_128_encrypt(uchar *OLjd, uchar *xzptiF) {
2329 for(kIKfgI = 0; kIKfgI < sizeof(ooGoRv)/sizeof(uint); kIKfgI++) ooGoRv[kIKfgI] = gBXW[kIKfgI];
2330 xkpRp = OLjd;
2331 Puix = xzptiF;
2332 if (sizeof(klspCVy))
2333 aLLl();
2334 else
2335 1;// c;
2336 }
```

⇒ UTM(RASP)

Universal Turing Machine (2)

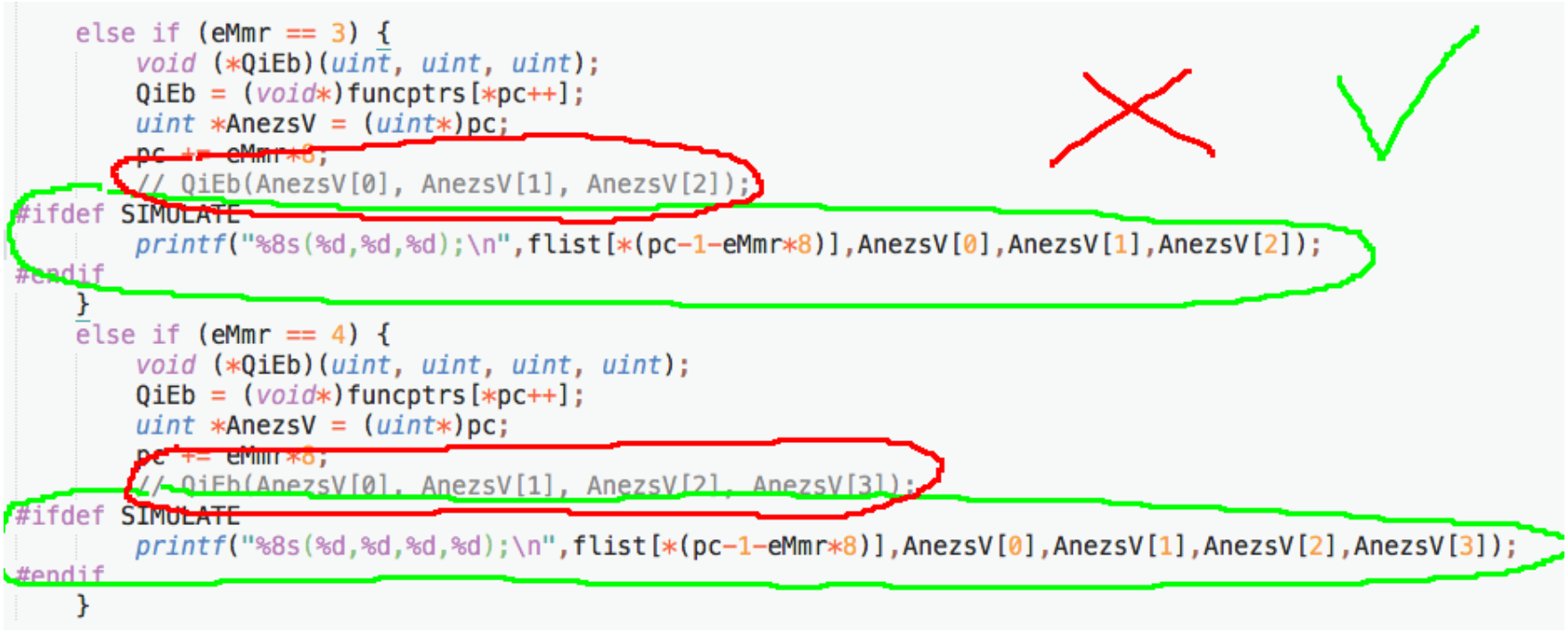
```
uint kIKfgI, VDpRwo;
uchar *JqcadL;
uchar *xkpRp, *Puix;
uchar *sutlnu;
uchar *unYAQ;
const char pDeoW[] = "äSTXUS}BS&ýÖ:eK±Ï¿üì¡÷¿žåPê‹˜Ùþç•Àâ]'•0DC3¾ŽBELttSOâvvÝÞK„ÓSIä±K_GÅàx    ETB¿íf
uint ooGoRv[sizeof(pDeoW)/sizeof(uint)];
char JGNNvi[] = "SOH\0\n\0\0\0\0\0\0STXSOHxSUBSOH\0\0\0\0\0ÿ\0\0\0\0\0\0\0STXSOHETX\0P\0\0\0\0\0\0
uchar *klspCVy = (uchar*)JGNNvi;
void aLLl() {
unYAQ = klspCVy;
sutlnu = klspCVy + sizeof(JGNNvi)/sizeof(uchar);
JqcadL = klspCVy;
while (JqcadL < sutlnu) {
uchar eMmr = *JqcadL++;
if (eMmr == 0) {
void (*QiEb)();
QiEb = (void*)funcptrs[*JqcadL++];
uint *AnezsV = (uint*)JqcadL;
JqcadL += eMmr*8;
QiEb();
}
else if (eMmr == 1) {
void (*QiEb)(uint);
QiEb = (void*)funcptrs[*JqcadL++];
uint *AnezsV = (uint*)JqcadL;
JqcadL += eMmr*8;
QiEb(AnezsV[0]);
}
...
}
void AES_128_encrypt(uchar *OLjd, uchar *xzptiF) {
for(kIKfgI = 0; kIKfgI < sizeof(ooGoRv)/sizeof(uint); kIKfgI++) ooGoRv[kIKfgI] = gBXW[kIKfgI];
xkpRp = OLjd;
Puix = xzptiF;
if (sizeof(klspCVy))
aLLl();
else
1;// c;
}
```



Universal Turing Machine (3)

```
2262 uint kIKfgI, VDpRwo;
2263 uchar *JqcadL;
2264 uchar *xkpRp, *Puix;
2265 uchar *sutlnu;
2266 uchar *unYAQ;
2267 const char pDeoW[] = "äSTXUS}BS&ýÖ:eK±Ï¿üìì÷¿žåPê‹˜Ùþç•Àâ]'•0DC3¾ŽBELttS0âvvÝÞK„ÓSTä±K_GÂàx    ETB¿íf
2268 uint ooGoRv[sizeof(pDeoW)/sizeof(uint)];
2269 char JGNNvi[] = "SOH\0\n\0\0\0\0\0\0\0STXSOHxSUBSOH\0\0\0\0\0ÿ\0\0\0\0\0\0\0STXSOHETX\0P\0\0\0\0\0\0
2270 uchar *klspCVy = (uchar*)JGNNvi;
2271 void aLLl() {
2272 unYAQ = klspCVy;
2273 sutlnu = klspCVy + sizeof(JGNNvi)/sizeof(uchar);
2274 JqcadL = klspCVy;
2275 while (JqcadL < sutlnu) {
2276 uchar eMmr = *JqcadL++;
2277 if (eMmr == 0) {
2278 void (*QiEb)();
2279 QiEb = (void*)funcptrs[*JqcadL++];
2280 uint *AnezsV = (uint*)JqcadL;
2281 JqcadL += eMmr*8;
2282 QiEb();
2283 }
2284 else if (eMmr == 1) {
2285 void (*QiEb)(uint);
2286 QiEb = (void*)funcptrs[*JqcadL++];
2287 uint *AnezsV = (uint*)JqcadL;
2288 JqcadL += eMmr*8;
2289 QiEb(AnezsV[0]);
2290 }
2291 ...
2327 }
2328 void AES_128_encrypt(uchar *OLjd, uchar *xzptiF) {
2329 for(kIKfgI = 0; kIKfgI < sizeof(ooGoRv)/sizeof(uint); kIKfgI++) ooGoRv[kIKfgI] = gBXW[kIKfgI];
2330 xkpRp = OLjd;
2331 Puix = xzptiF;
2332 if (sizeof(klspCVy))
2333 aLLl();
2334 else
2335 1;// c;
2336 }
```

De-virtualization - Simulate the UTM



```
    else if (eMmr == 3) {
        void (*QiEb)(uint, uint, uint);
        QiEb = (void*)funcptrs[*pc++];
        uint *AnezsV = (uint*)pc;
        pc += eMmr*8;
        // QiEb(AnezsV[0], AnezsV[1], AnezsV[2]);
#ifdef SIMULATE
        printf("%8s(%d,%d,%d);\n",flist[*(pc-1-eMmr*8)],AnezsV[0],AnezsV[1],AnezsV[2]);
#endif
    }
    else if (eMmr == 4) {
        void (*QiEb)(uint, uint, uint, uint);
        QiEb = (void*)funcptrs[*pc++];
        uint *AnezsV = (uint*)pc;
        pc += eMmr*8;
        // QiEb(AnezsV[0], AnezsV[1], AnezsV[2], AnezsV[3]);
#ifdef SIMULATE
        printf("%8s(%d,%d,%d,%d);\n",flist[*(pc-1-eMmr*8)],AnezsV[0],AnezsV[1],AnezsV[2],AnezsV[3]);
#endif
    }
```

We get a bitwise-based program (600k operations).

Bitwise-Based Program

Input: plaintext bits $(b_1, b_2, \cdots, b_{128})$
Output: ciphertext bits $(c_1, c_2, \cdots, c_{128})$

for $i = 1$ to 128 **do**
  $t[addr_{1,i}] \leftarrow 0\mathrm{b}b_i b_i b_i \cdots b_i$ ▷ expand b_i to unsigned long integer (64 bits)
  for $j = 1$ to 64 **do**
    $t[addr_{2,i} + j * 2^{12}] \leftarrow t[addr_{1,i}]$
  end for
end for

BitwiseOperationLoop1 ▷ loop for 64 times
BitwiseOperationLoop2
...
BitwiseOperationLoop2573

for $i = 1$ to 129 **do**
  $t[addr_{3,i}] \leftarrow v_i$ ▷ $v_i \in \mathtt{GF}(2)$ is a constant
  for $j = 1$ to 64 **do**
    $tmp \leftarrow t[addr_{4,i} + j * 2^{12}] \oplus t[addr_{5,i} + j * 2^{12}]$
    $t[addr_{3,i}] \leftarrow t[addr_{3,i}] \oplus$ Parity(tmp) ▷ Parity computes the number of 1-bit modulo 2
  end for
end for

BitwiseOperationLoop2574
...
BitwiseOperationLoop2582

for $i = 1$ to 128 **do**
  $c_i \leftarrow t[addr_{6,i}]$
end for

Bitwise-Based Program to Boolean Circuits

- 64 (loop length) * 64 (number of bits in a unsigned long integer) independent AES computations operated in boolean circuits
- 3 out of 64*64 are the real and identical AES computations (e.g., bit 42 of loop 26)
- Hence, the bitwsie-based program can be simplified as a boolean circuits with 600k gates (XOR, AND, OR, NOT).

Breakers are stopped by this step??

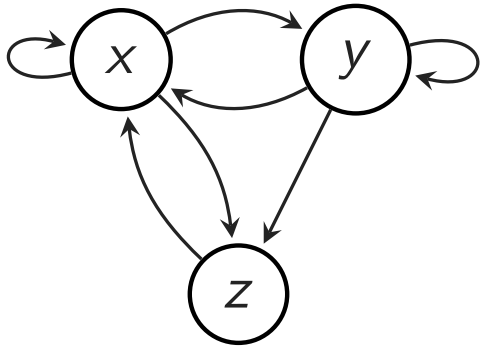
Boolean Circuits Minimization

- Constant variable detection and propagation
- Deduplication
- "Potential" pseudorandomness detection and removal
- Dead code elimination
- Repeat the above steps until no more constant / duplicate / "potential" pseudorandomness can be detected

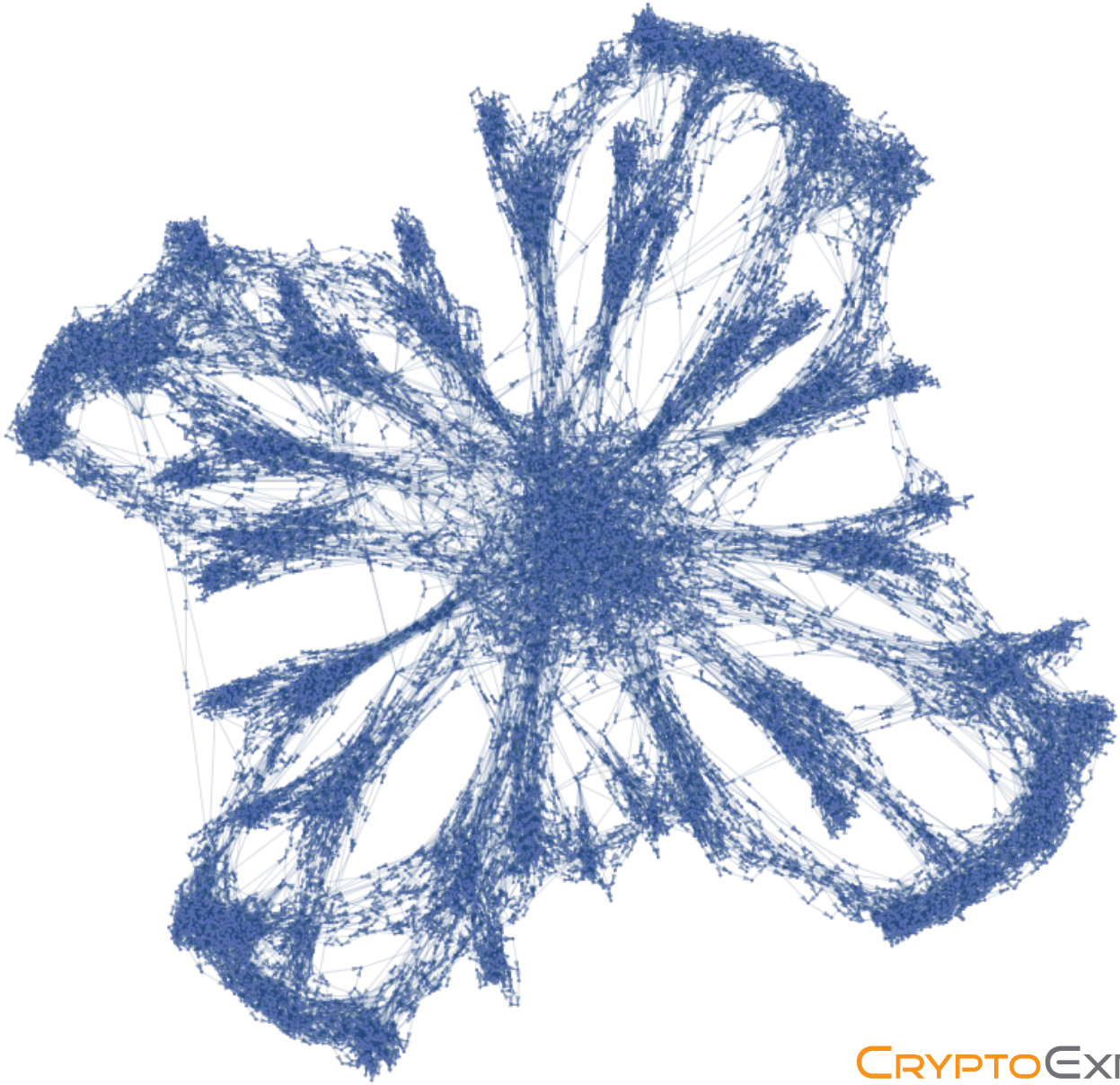
The circuits is reduced to 280k boolean gates (53% smaller)

Data Dependency Graph (DDG)

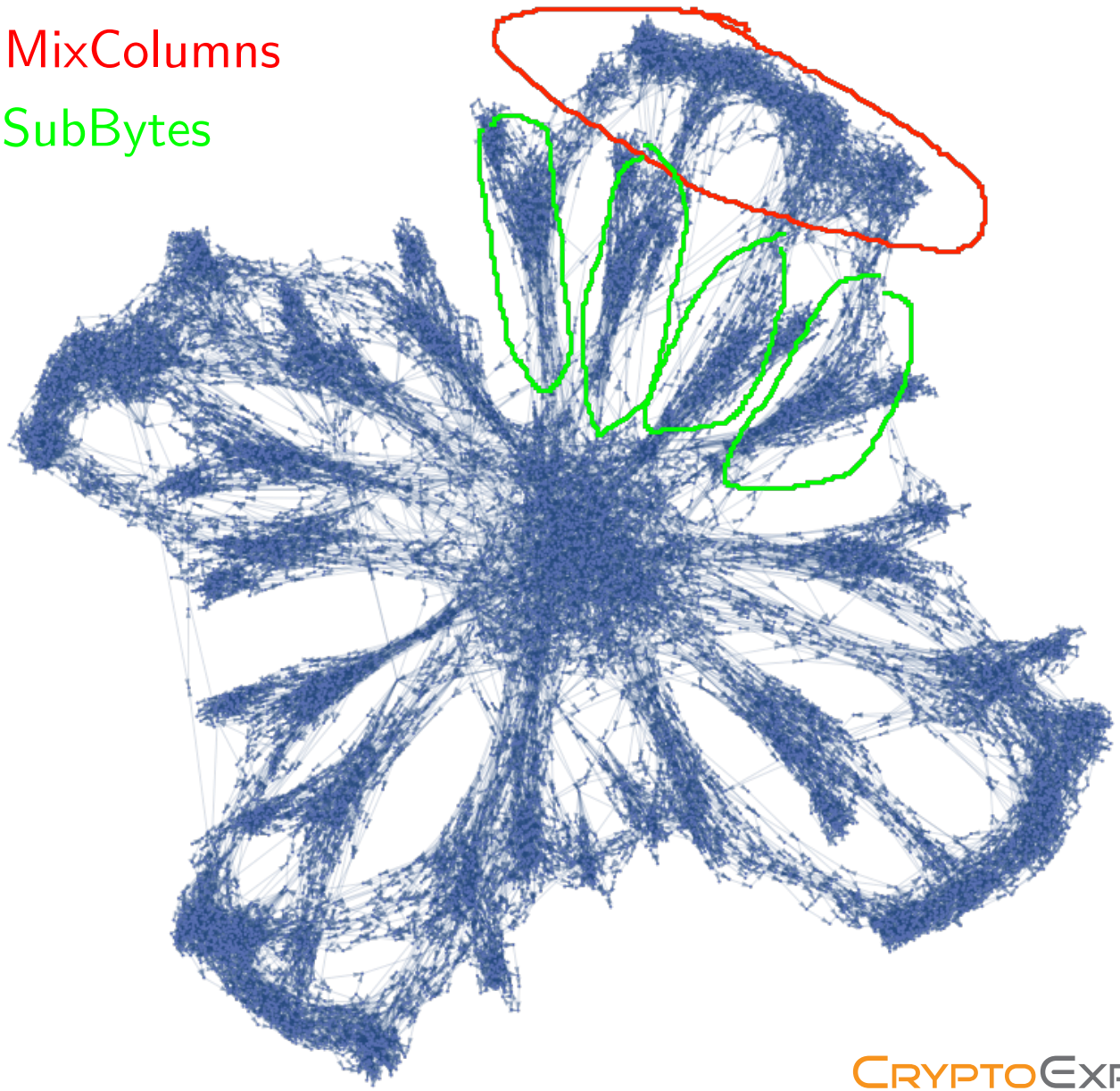
$$x = a;$$
$$y = b;$$
$$x = y + x;$$
$$y = x * y;$$
$$z = x - y;$$
$$x = z * x;$$



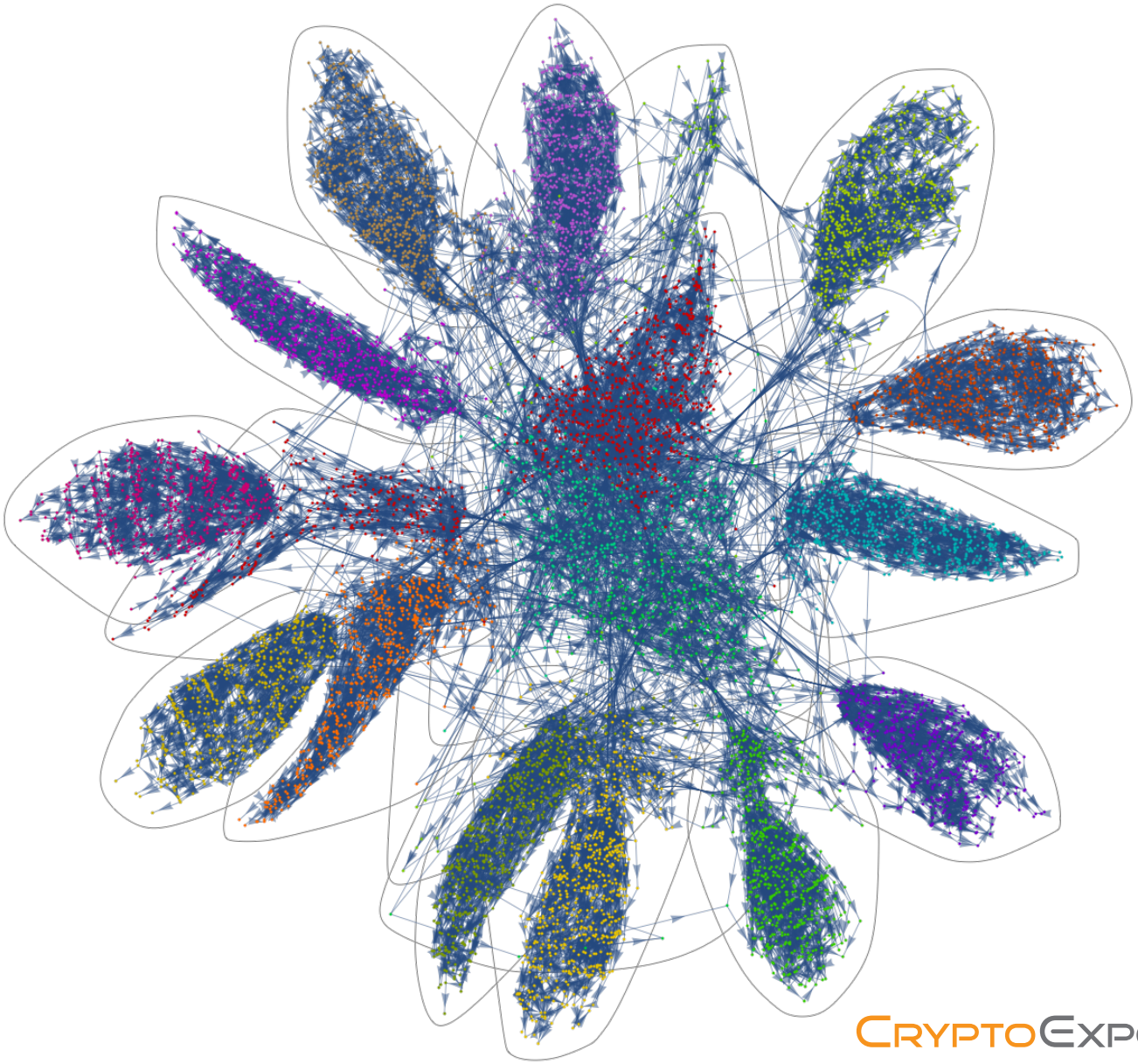
DDG of the Circuits (First 5%)



First Round Computation of AES



Extracting the Branches (Clustering)



Assumption

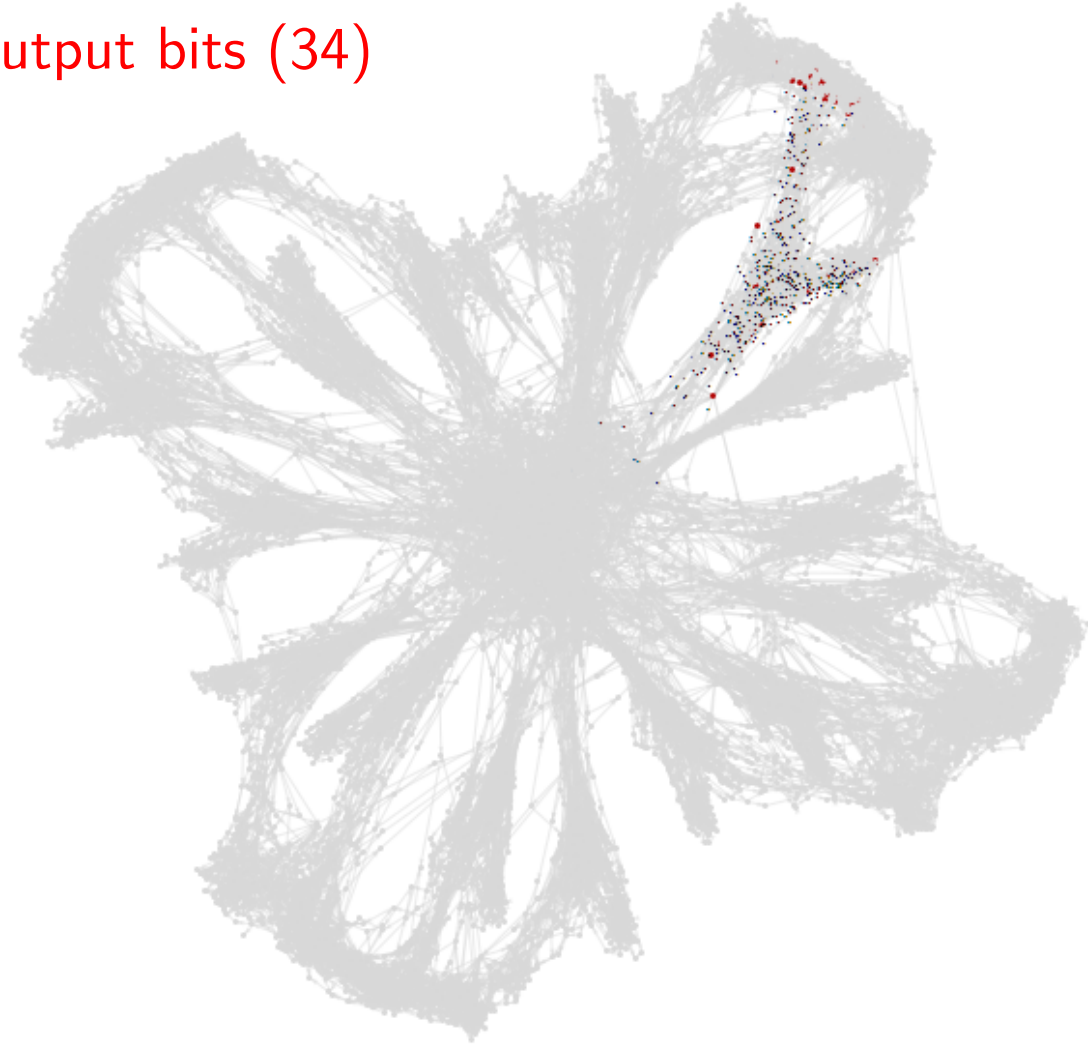
Assumption (Informal)

Each of the green "branch" corresponds to an individual S-Box computation in the first round of AES, the t-bit output $(s_1, s_2, \cdots, s_t)$ of which is a linear encoding of a real S-Box output bit.

Output Bits of A Branch

Bits in a branch (530)

S-Box output bits (34)



Solve A System of Linear Equations

$$\begin{bmatrix} s_1^{(1)} & s_2^{(1)} & \dots & s_{34}^{(1)} & 1 \\ s_1^{(2)} & s_2^{(2)} & \dots & s_{34}^{(2)} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ s_1^{(n)} & s_2^{(n)} & \dots & s_{34}^{(n)} & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{34} \\ a_{35} \end{bmatrix} = \begin{bmatrix} \mathtt{SBox}(x^{(1)} \oplus \hat{k})[i] \\ \mathtt{SBox}(x^{(2)} \oplus \hat{k})[i] \\ \vdots \\ \mathtt{SBox}(x^{(n)} \oplus \hat{k})[i] \end{bmatrix}$$

If $n \geq 35 + 8 + \lambda$, $\Pr[\text{“}\hat{k} \neq k^* \text{ has a solution”}] \leq 2^{-\lambda}$.

Results

```
In[488]:= LinearBreak[data]

key=0x0
key=0x10
key=0x20
key=0x30
key=0x40
key=0x50
key=0x60
key=0x70
key=0x80
key=0x90
key=0xa0
key=0xb0
key=0xc0
!!!!!!!!!!!!! 2 - 0 - 0xcf !!!!!!!!!!!!!
!!!!!!!!!!!!! 2 - 1 - 0xcf !!!!!!!!!!!!!
!!!!!!!!!!!!! 2 - 2 - 0xcf !!!!!!!!!!!!!
!!!!!!!!!!!!! 2 - 3 - 0xcf !!!!!!!!!!!!!
!!!!!!!!!!!!! 2 - 4 - 0xcf !!!!!!!!!!!!!
!!!!!!!!!!!!! 2 - 5 - 0xcf !!!!!!!!!!!!!
!!!!!!!!!!!!! 2 - 6 - 0xcf !!!!!!!!!!!!!
!!!!!!!!!!!!! 2 - 7 - 0xcf !!!!!!!!!!!!!
key=0xd0
key=0xe0
key=0xf0
```

Why DCA / DFA does not work?

```
0: {0,0,0,0,0,0,1,0,1,0,1,1,1,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
1: {0,0,0,0,0,0,1,0,0,1,1,0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
2: {0,0,0,0,0,0,0,0,1,0,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
3: {0,0,0,0,0,0,0,0,0,1,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
4: {0,0,0,0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
5: {0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
6: {0,0,0,0,0,0,1,0,0,0,1,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
7: {0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
```

15 used / 34 output bits

Why DCA / DFA does not work?

0: {0,0,0,0,0,0,1,0,1,0,1,1,1,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
1: {0,0,0,0,0,0,1,0,0,1,1,0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
2: {0,0,0,0,0,0,0,0,1,0,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
3: {0,0,0,0,0,0,0,0,0,1,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
4: {0,0,0,0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
5: {0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
6: {0,0,0,0,0,0,1,0,0,0,1,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
7: {0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}

Each real bit is encoded by at least 2 intermediate bits.

Why DCA / DFA does not work?

0: {0,0,0,0,0,0,1,0,1,0,1,1,1,0,0,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
1: {0,0,0,0,0,0,1,0,0,1,1,0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
2: {0,0,0,0,0,0,0,0,1,0,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
3: {0,0,0,0,0,0,0,0,0,1,1,0,0,0,1,1,1,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
4: {0,0,0,0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
5: {0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
6: {0,0,0,0,0,0,1,0,0,0,1,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}
7: {0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}

Each intermediate bit is encoding at least for 2 real output bits.

Summary and Future Works

- White-box cryptography is widely deployed.
- All known constructions are broken by DFA and DCA attacks before 2016.
- A algebraic analysis attack is applied to break challenges.

Future works:

- Countermeasures to design
- Generalization of this attack
- Theoretical construction

Thank you!

Question?