Fate of Birth & Fertility in the Growth of a City-State: Singapore

Anantharajan Vivekbala
1004248

Dharmapuri Krishna Sathvik
1004286

Lim Jun Wei
1004379

Singapore University of Technology and Design

3 December 2021

1 Introduction

Since 1975, Singapore's birth rate has been declining below replacement levels. The Singapore government realised the importance of having a long-term policy to stabilise the population size. However, in the face of steep decline in birth rates during the last decade, especially for the population of Chinese ethnicity, the varied tuning of the policy toolkit that had in the past served the technocratic needs of demographic stability has been flailing at diagnosing and influencing the underlying factors that continue to drive these trends.

In this project, we investigate how local population growth and the birth rate of Singapore evolved as an economically developing country improves to a developed country with higher quality of life and advanced technological infrastructure. There are several metrics that serve to gauge the development of a country over the years. These would include changes in unit labour costs, gross domestic product (GDP), balance of payments, investments and taxes.

We want to introduce the idea of the circular flow of income in which major exchanges are represented as flows of money, goods and services within an economy. By using machine learning methods, we aim to determine the relationship between different economic agents behind the country's economic growth and how they affect the birth rate of a country.

We discern between families of more classical statistical models such as vector autoregression and vector autoregressive moving-average processes against the more modern neural network model architectures namely the Temporal Convolutional Network at forecasting the expected birth rates trained on our multivariate unit labour costs dataset across varied industry sectors.

We hope to present the data using meaningful time series graphs to show the correlations between the various features we have collected to help our government make informed choices to improve Singapore's birth rates based on the projected data, in hopes of preventing an increasing aging population and aging workforce of the future.

The code for this paper can be found at: https://github.com/KosmonautX/osiris.

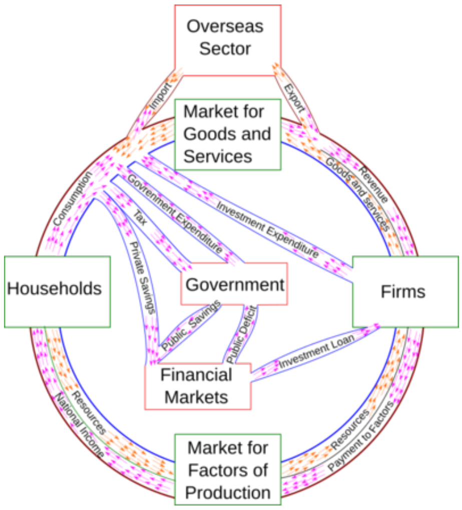

Figure 1: Circular flow of income

2 Dataset and Collection

We used datasets obtained from Data.gov.sg, the government's one-stop portal for publicly accessible datasets from various public agencies that contribute to the development of a country. The datasets obtained are:

- Quarterly unit labour cost indexes from 1980 to 2019 for various industries such as manufacturing, construction and business services
- Quarterly birth and fertility datasets from January 1986 to June 2021

Singapore Department of Statistics (https://www.singstat.gov.sg/) provided datasets on other economic indicators such as:

- Quarterly Singapore's balance of payments for exports and imports of goods from January 1986 to September 2021
- Quarterly Gross Domestic Product at current prices by Industry from January 1975 to September 2021
- Quarterly Taxes on Production by Industry from January 1980 to September 2021
- Quarterly Compensation Of Employees By Industry At Current Prices January 1980 to September 2021

We organised these aforementioned datasets into 4 major baskets during pre-processing. The baskets chosen reflect the constraints of the quarterly datasets that were publicly available while accounting for the relations mediated in the circular flow model of the economy. These break down primarily into broader categories of injections and withdrawals into the internal economy in the form of taxation and investment respectively and exchange between major sectors internally observed in factors of payments by firms to households and revenue of firms driven by consumption of households. Balance of Payment accounted for exchange between domestic and foreign markets critical in gauging the health of the internal economy of Singapore that is highly exposed and open to these foreign markets.

Baskets	Data Sets Compiled
Labour Factor Payments	Quarterly Compensation Of Employees By Industry At Current Prices Quarterly Unit Labour Cost Indexes for various industries
Consumption	Quarterly Gross Domestic Product at current prices by Industry
Taxation	Quarterly Taxes on Production by Industry
Balance of Payments (Export - Imports)	Quarterly Singapore's balance of payments for exports and imports of goods

Table 1: 4 major categories of the economy

3 Data Pre-processing

3.1 Data Cleaning

Standard data cleaning techniques were applied to the datasets obtained, such as transposing rows and columns and converting data types. The quarterly dates were also parsed to a real datetime type. Some examples of the datasets after cleaning are shown.

Unit Labour Cost Of Business Services	Unit Labour Cost Of Construction	Unit Labour Cost Of Manufacturing	year	quarter
1	19.3	44.3	1980	1
2	19.5	42.2	1980	2
3	20.2	42.9	1980	3

Table 2: Unit labour costs of various industries in Singapore

Total Live-Births By Birth Order	**1st Live-Birth**	**2nd Live-Birth**	**year**	**quarter**
9954	4414	3553	1986	1
9353	4186	3367	1986	2
9854	4398	3475	1986	3

Table 3: Singapore live-births by birth order

3.2 Data Visualisation

To visualise Singapore's birth rate and correlations between the feature sets, several time-series graphs were plotted.

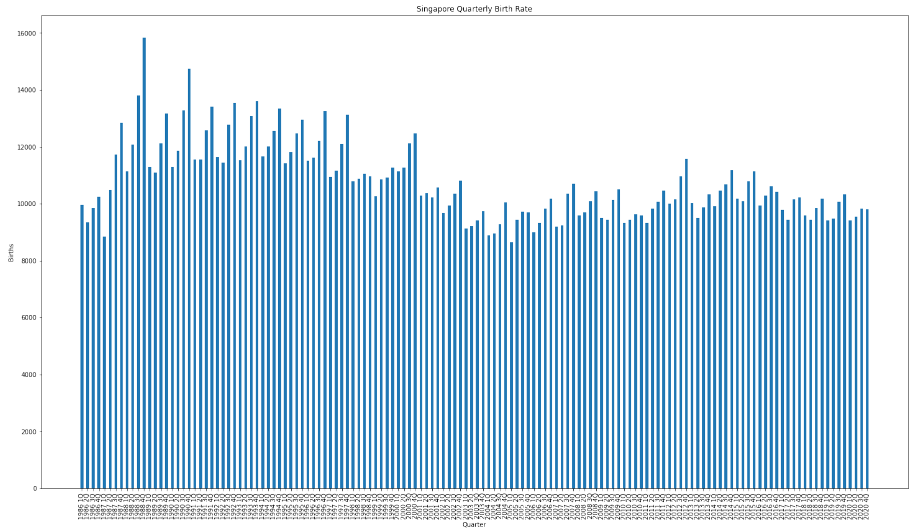

Figure 2: Singapore Quarterly Birth Rate

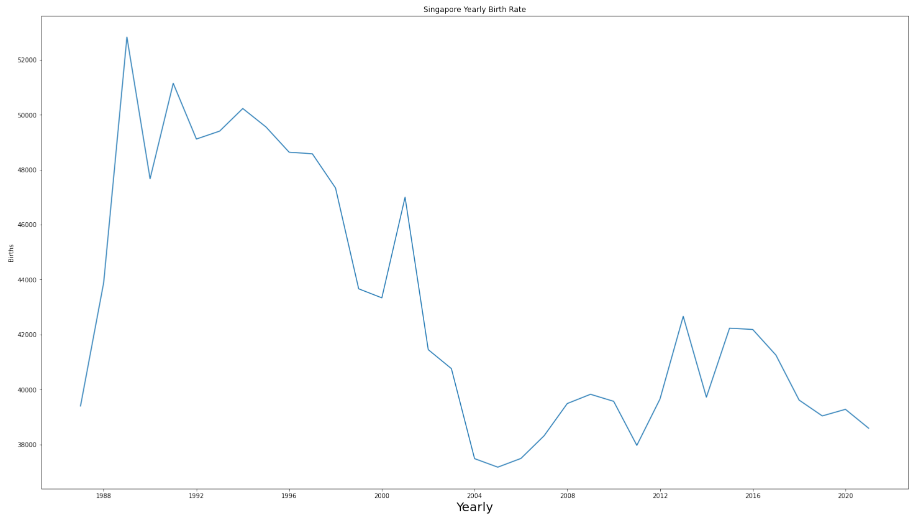

Figure 3: Singapore Yearly Birth Rate

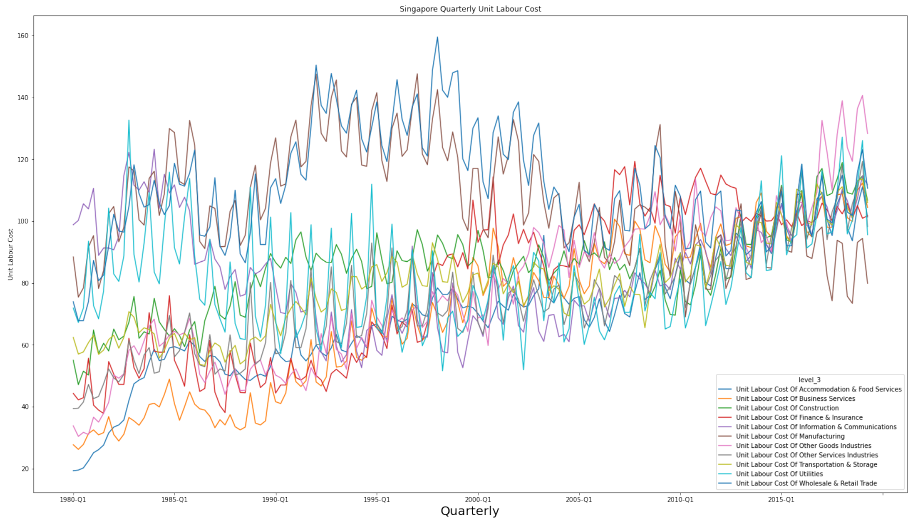

Figure 4: Singapore Quarterly Unit Labour Cost

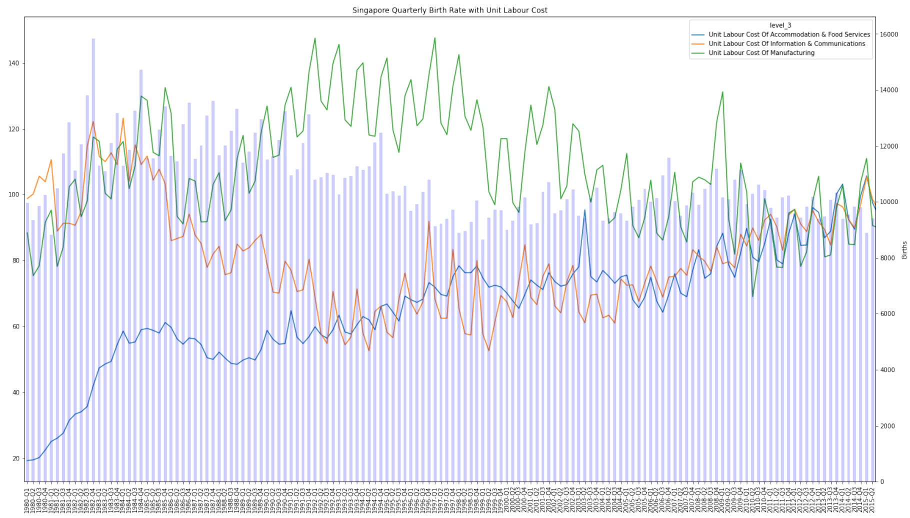

Figure 5: Singapore Unit Labour Cost with Birth Rates

The labour cost and birth rates were plotted together to view their correlation over the years. Cost of accommodation and food services in Singapore has been on a rise since 1980, which could be an interesting feature to look into. As seen from Figure 3, Singapore's birth rate has been on the decline.

4 Models

Our team used three models to predict Singapore's birth rates with our multivariate time series data, Vector Autoregression, Vector Autoregressive Moving Average and Temporal Convolutional Networks, as they were found to be useful for time series forecasting.

A research conducted by Li, H. and Lu, Y. [HY17] used a sparse vector autoregression model to project US and UK mortality rates. The spatial-temporal autoregressive approach performed well through various benchmarks such as Lee-Carter model [DR92] and Li-Lee model [NR05], producing reasonable forecasts in the long-run.

However, VAR models may require sufficient lag length in order to express the series reasonably. This would translate to a less precise model since many parameters have to be estimated. Since Singapore is relatively young compared to the rest of the countries in the world, the dataset available regarding our birth rates and economic activities are small. The problem could be avoided by using a Vector Autoregressive Moving Average (VARMA) model, a combination of vector autoregression and vector moving average models.

On the other hand, Gan et al. concluded that temporal convolutional networks have consistently outperformed baseline recurrent architectures such as LSTM for sequence modeling tasks [Z+21].

4.1 Vector Autoregression

We first used Vector Autoregression (VAR) to model our time series. VAR was chosen over other Autoregressive models like AR, ARMA or ARIMA model because those models are uni-directional, where the predictors influence the live births and not vice-versa whereas VAR is bi-directional. We were unsure of how the different data influences the other, so we decided to do a couple of tests before fitting the model to figure out certain correlation and causality statistics.

A typical autoregression model ($AR(p)$) for univariate time series can be represented by

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + ... + \beta_p Y_{t-p} + \epsilon_t$$

where

Y_{t-i} indicates the variable value at periods earlier.

β_1, β_2 till β_p are the coefficients of the lags of Y till order p.

ϵ_t is an error term.

α is an intercept of the model.

Here the order p means up to p-lags of y is used. In the VAR model, each variable is modeled as a linear combination of past values of itself and the past values of other variables in the system. Hence, the more the variables, the longer the equation.

Before building the model, we conducted a few tests to check on the relationship between all the variables in order to narrow down the features being fed onto the model such as the Granger's causality test and Johansen's cointegration test.

4.1.1 Granger's Causality Test

Granger's causality tests the null hypothesis if the coefficients of past values in the regression equation is zero. In simpler terms, it checks whether the past values of all the time series data (X) will cause any changes on the live births (Y). So, if the p-value obtained from the test is lesser than the significance level of 0.05, we can safely reject the null hypothesis. The causality test code implements the Granger's Causality test for all possible combinations of the time series of the data being fed and stores the p-values of each combination in an output matrix.

	Unit Labour Cost Of Accommodation & Food Services_x	Unit Labour Cost Of Business Services_x	Unit Labour Cost Of Construction_x	Unit Labour Cost Of Finance & Insurance_x	Unit Labour Cost Of Information & Communications_x	Unit Labour Cost Of Manufacturing_x	Unit Labour Cost Of Other Goods Industries_x	Unit Labour Cost Of Other Services Industries_x	Unit Labour Cost Of Overall Economy_x
Unit Labour Cost Of Accommodation & Food Services_y	1.0000	0.0000	0.0043	0.0149	0.0066	0.0000	0.0003	0.0000	0.0001
Unit Labour Cost Of Business Services_y	0.0000	1.0000	0.0000	0.0000	0.0001	0.0000	0.0008	0.0000	0.0026
Unit Labour Cost Of Construction_y	0.0000	0.0000	1.0000	0.0001	0.0000	0.0000	0.0002	0.0000	0.0799
Unit Labour Cost Of Finance & Insurance_y	0.0023	0.0003	0.0000	1.0000	0.0049	0.0000	0.0004	0.0000	0.0002
Unit Labour Cost Of Information & Communications_y	0.0031	0.0072	0.0008	0.0015	1.0000	0.0000	0.0247	0.0000	0.1890
Unit Labour Cost Of Manufacturing_y	0.0004	0.0000	0.0000	0.0376	0.0001	1.0000	0.0013	0.0000	0.0000
Unit Labour Cost Of Other Goods Industries_y	0.0000	0.0001	0.0000	0.0269	0.0052	0.0000	1.0000	0.0004	0.4872
Unit Labour Cost Of Other Services Industries_y	0.0000	0.0000	0.0000	0.0014	0.0000	0.0000	0.0003	1.0000	0.0000
Unit Labour Cost Of Overall Economy_y	0.0316	0.0207	0.0776	0.1669	0.3703	0.0060	0.0061	0.0054	1.0000
Unit Labour Cost Of Transportation & Storage_y	0.0000	0.0007	0.0000	0.0276	0.0805	0.0000	0.0669	0.0002	0.0000
Unit Labour Cost Of Utilities_y	0.0000	0.0003	0.0003	0.0220	0.0000	0.0000	0.0083	0.0000	0.0005
Unit Labour Cost Of Wholesale & Retail Trade_y	0.0015	0.0000	0.0000	0.0000	0.0001	0.0000	0.0797	0.0000	0.0000

Figure 6: p-value results

If a given p-value is ¡ significance level (0.05), we can reject the null hypothesis and conclude that the corresponding data series (column) causes the change in live births. If not, we drop the respective columns from the dataframe. From our data we could see that most of the variables (time series) in the system are interchangeably related to one another. Hence our data is a good candidate for using VAR to forecast.

4.1.2 Cointegration Test

Cointegration test helps to establish the presence of a statistically significant connection between two or more time series. When two or more time series are cointegrated, they have a long-run statistically significant relationship.

```
Name   ::  Test Stat > C(95%)    =>   Signif
 ----------------------------------------
Unit Labour Cost Of Accommodation & Food Services ::  476.69    > 311.1288  =>   True
Unit Labour Cost Of Business Services ::  377.61    > 263.2603  =>   True
Unit Labour Cost Of Construction ::  293.91    > 219.4051  =>   True
Unit Labour Cost Of Finance & Insurance ::  236.28    > 179.5199  =>   True
Unit Labour Cost Of Information & Communications ::  187.31    > 143.6691  =>   True
Unit Labour Cost Of Manufacturing ::  140.74    > 111.7797  =>   True
Unit Labour Cost Of Other Goods Industries ::  100.59    > 83.9383   =>   True
Unit Labour Cost Of Other Services Industries ::  73.11     > 60.0627   =>   True
Unit Labour Cost Of Overall Economy ::  47.76     > 40.1749   =>   True
Unit Labour Cost Of Transportation & Storage ::  29.03     > 24.2761   =>   True
Unit Labour Cost Of Utilities ::  14.16     > 12.3212   =>   True
Unit Labour Cost Of Wholesale & Retail Trade ::  2.23      > 4.1296    =>   False
```

Figure 7: Cointegration test results

4.1.3 Augmented Dickey-Fuller Test (ADF Test)

Since the VAR model requires the time series we want to forecast to be stationary, it is customary to check all the time series in our data for stationarity. A stationary time series is one whose characteristics like mean and variance does not change over time. We used an ADF test to check if each time series is stationary. If a series is found to be non-stationary, we make it stationary by taking the differential of the series once and repeating the test again until it becomes stationary. In our data, we had to take the differential at least once in order to make the dataset stationary.

```
   Augmented Dickey-Fuller Test on "Unit Labour Cost Of Overall Economy"
   -----------------------------------------------
Null Hypothesis: Data has unit root. Non-Stationary.
Significance Level    = 0.05
Test Statistic        = -3.1449
No. Lags Chosen       = 10
Critical value 1%     = -3.488
Critical value 5%     = -2.887
Critical value 10%    = -2.58
=> P-Value = 0.0234. Rejecting Null Hypothesis.
=> Series is Stationary.

   Augmented Dickey-Fuller Test on "Unit Labour Cost Of Transportation & Storage"
   -----------------------------------------------
Null Hypothesis: Data has unit root. Non-Stationary.
Significance Level    = 0.05
Test Statistic        = -2.1607
No. Lags Chosen       = 12
Critical value 1%     = -3.489
Critical value 5%     = -2.887
Critical value 10%    = -2.58
=> P-Value = 0.2208. Weak evidence to reject the Null Hypothesis.
=> Series is Non-Stationary.

   Augmented Dickey-Fuller Test on "Unit Labour Cost Of Utilities"
   -----------------------------------------------
Null Hypothesis: Data has unit root. Non-Stationary.
Significance Level    = 0.05
Test Statistic        = -5.5292
No. Lags Chosen       = 6
Critical value 1%     = -3.486
Critical value 5%     = -2.886
Critical value 10%    = -2.58
=> P-Value = 0.0. Rejecting Null Hypothesis.
=> Series is Stationary.
```

Figure 8: ADF test results

To select the right order of the VAR model, we iteratively fit increasing orders of the VAR model and pick the order that gives a model with least Akaike information criterion (AIC). Though the usual practice is to look at the AIC, we could also check other best fit comparison estimates of Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC).

	AIC	BIC	FPE	HQIC
0	189.5	189.9*	1.947e+82*	189.6*
1	192.2	198.5	3.035e+83	194.8
2	192.0	204.3	2.993e+83	197.0
3	193.6	211.9	2.800e+84	201.1
4	190.1	214.4	3.151e+83	200.0
5	190.6	220.9	7.244e+84	202.9
6	183.7*	220.0	2.197e+84	198.5

Figure 9: VAR order selection

Based on the estimators, a p-value of 6 would be the most optimal for our model as it has the lowest AIC. We then fit the model based on the lag and move towards forecasting our values.

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Fri, 03, Dec, 2021
Time:                     11:18:05
--------------------------------------------------------------------
No. of Equations:         16.0000    BIC:                    219.986
Nobs:                     119.000    HQIC:                   198.459
Log likelihood:          -12082.2    FPE:                2.19702e+84
AIC:                      183.741    Det(Omega_mle):     1.58244e+80
--------------------------------------------------------------------
Results for equation Unit Labour Cost Of Accommodation & Food Services
========================================================================================================================
                                                      coefficient       std. error           t-stat            prob
------------------------------------------------------------------------------------------------------------------------
const                                                    0.480306         3.663966            0.131           0.896
L1.Unit Labour Cost Of Accommodation & Food Services     0.247911         0.211699            1.171           0.242
L1.Unit Labour Cost Of Business Services                 1.012484         0.951795            1.064           0.287
L1.Unit Labour Cost Of Construction                      0.298074         0.239900            1.242           0.214
L1.Unit Labour Cost Of Finance & Insurance               0.338993         0.183743            1.845           0.065
L1.Unit Labour Cost Of Information & Communications      0.000000         0.000000            0.263           0.792
L1.Unit Labour Cost Of Manufacturing                     0.081795         0.270718            0.302           0.763
L1.Unit Labour Cost Of Other Goods Industries           -0.289678         0.207106           -1.399           0.162
L1.Unit Labour Cost Of Other Services Industries        -0.061481         0.160159           -0.384           0.701
L1.Unit Labour Cost Of Transportation & Storage          0.322798         0.315461            1.023           0.306
L1.Unit Labour Cost Of Utilities                        -0.202110         0.089620           -2.255           0.024
L1.Unit Labour Cost Of Wholesale & Retail Trade         -0.000000         0.000000           -0.218           0.827
L1.Total Live-Births By Birth Order                      0.089447         0.101608            0.880           0.379
```

Figure 10: Summary of regression results

4.2 VARMAX

The results from VAR were not promising, so we decided to use the VARMAX model. The vector autoregression moving-average with exogenous regressors (VARMAX) is an extension of the VARMA model that also includes the modeling of exogenous variables. The VARMA model is a combination of VAR and VMA models. The Vector Moving Average (VMA) model is a generalization of the Moving Average Model for multivariate time series where the time series is stationary and we consider only the order of moving average 'q' in the model. Thus VARMA helps with multivariate time series modelling by considering both lag order and order of moving average (p and q respectively) in the model. Similar to the VAR model, we run the same tests (Causality, ADF & Cointegration tests) on the dataset and choose the p and q values based on the lowest AIC estimator before fitting the model for forecasting.

```
Searching order of p and q for : Total Live-Births
Performing stepwise search to minimize aic
 ARIMA(1,1,1)(0,0,0)[0] intercept   : AIC=1767.045, Time=0.12 sec
 ARIMA(0,1,0)(0,0,0)[0] intercept   : AIC=1784.857, Time=0.01 sec
 ARIMA(1,1,0)(0,0,0)[0] intercept   : AIC=1782.765, Time=0.01 sec
 ARIMA(0,1,1)(0,0,0)[0] intercept   : AIC=1768.493, Time=0.02 sec
 ARIMA(0,1,0)(0,0,0)[0]             : AIC=1782.870, Time=0.00 sec
 ARIMA(2,1,1)(0,0,0)[0] intercept   : AIC=1751.434, Time=0.11 sec
 ARIMA(2,1,0)(0,0,0)[0] intercept   : AIC=1760.511, Time=0.03 sec
 ARIMA(3,1,1)(0,0,0)[0] intercept   : AIC=1714.675, Time=0.15 sec
 ARIMA(3,1,0)(0,0,0)[0] intercept   : AIC=1729.710, Time=0.03 sec
 ARIMA(4,1,1)(0,0,0)[0] intercept   : AIC=1712.808, Time=0.08 sec
 ARIMA(4,1,0)(0,0,0)[0] intercept   : AIC=1710.268, Time=0.06 sec
 ARIMA(5,1,0)(0,0,0)[0] intercept   : AIC=1712.164, Time=0.16 sec
 ARIMA(5,1,1)(0,0,0)[0] intercept   : AIC=1713.841, Time=0.28 sec
 ARIMA(4,1,0)(0,0,0)[0]             : AIC=1708.374, Time=0.04 sec
 ARIMA(3,1,0)(0,0,0)[0]             : AIC=1727.894, Time=0.02 sec
 ARIMA(5,1,0)(0,0,0)[0]             : AIC=1710.262, Time=0.11 sec
 ARIMA(4,1,1)(0,0,0)[0]             : AIC=1710.932, Time=0.06 sec
 ARIMA(3,1,1)(0,0,0)[0]             : AIC=1712.830, Time=0.12 sec
 ARIMA(5,1,1)(0,0,0)[0]             : AIC=1711.916, Time=0.25 sec

Best model:  ARIMA(4,1,0)(0,0,0)[0]
Total fit time: 1.668 seconds
optimal order for:Total Live-Births is: (4, 1, 0)
```

Figure 11: Summary of results for VARMAX

Based on the most optimal order through the auto-ARIMA test which gives us the optimal values (lag order, moving average order, differential order), p value is 4 and q value is 1.

4.3 Temporal Convolutional Networks

Temporal Convolutional Networks (TCN) are a variant upon the more extensively studied Convolutional Neural Networks that have demonstrated potent ability to learn and extract high level feature representations from input in varied contexts. CNNs primarily are built across three major layers:

- Convolutional Layer
- Pooling Layer
- Fully-Connected Layer

The distinguishing constraints imposed by TCN are as follows. Firstly, it can take a sequence of any length and output it as a sequence of the same length with the input, just like using an RNN. Second, convolution is a causal convolution, which means that there is no information "leakage" from future to past.[Kol+18] To reach the first goal, the TCN uses a one-dimensional, fully convolutional network (1D FCN) architecture. That is, each hidden layer will be padded zero to maintain the same length with the input layer. To achieve the second point, the causal convolution, where an output at time t is convolved only with elements from time t and earlier in the previous layer, is adopted. In short, TCN is the sum of 1D FCN and causal convolutions. This allows TCN to perform well on temporal data since the model at a particular timestep cannot depend on data from future timesteps, allowing the information to only flow to the future.

Models	Mean Squared Error	Mean Absolute Error
Vector Autoregression	164.071240888	12.803315186
Vector Autoregressive Moving Average	0.157515394	0.326737601
Temporal Convolutional Networks	0.050030038	0.18635967

Table 4: Results of models that were trained using unit labour cost dataset

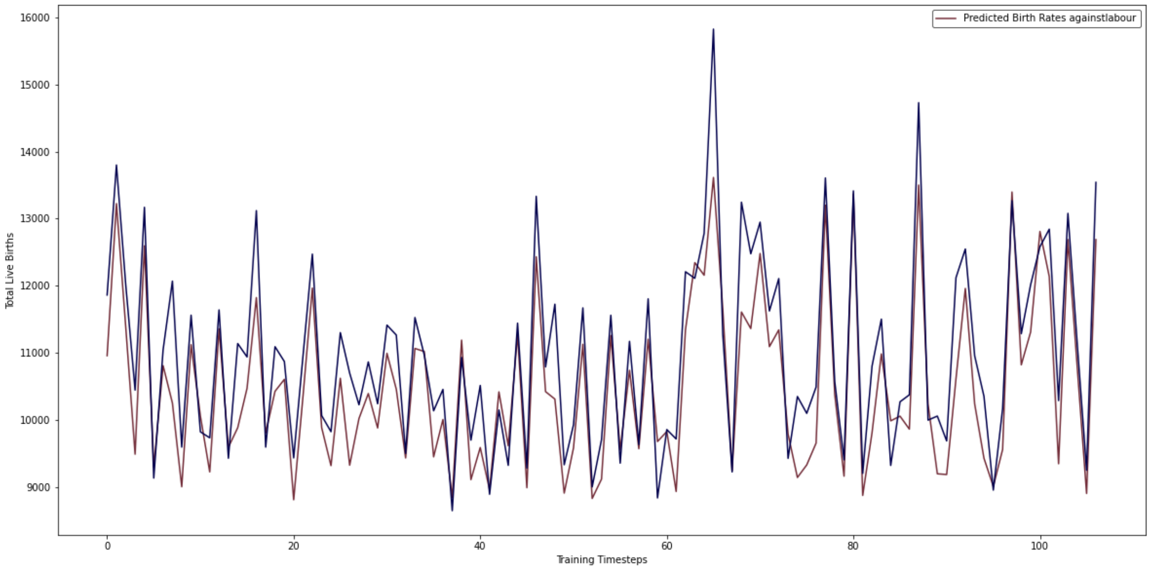

Figure 12: Training loss of TCN model after a few iterations

5 Evaluation of Models and Results

We used a common methodology to compare all three models, by using the mean square error and mean absolute error of each model, to visualise the accuracy of our forecasts.

5.1 Vector Autoregression

For the VAR model, we chose to forecast for the next 6 quarters. We also split our train and test data prior to fitting the model whereby the test data are the last 6 quarters of our data and training data is the entire data-series excluding the last 6. Test data is depicted in Figure 13 which ends at Q2 2019.

	Unit Labour Cost Of Accommodation & Food Services_forecast	Unit Labour Cost Of Business Services_forecast	Unit Labour Cost Of Construction_forecast	Unit Labour Cost Of Finance & Insurance_forecast	Unit Labour Cost Of Information & Communications_forecast	Unit Labour Cost Of Manufacturing_forecast	Unit Labour Cost Of Other Goods Industries_forecast
2018-03-31	105.242599	-21.916789	195.154874	-173.986393	-1.947441e+15	16.871614	143.750296
2018-06-30	40.845601	-21.465012	-46.851204	-187.713248	-1.095650e+15	88.783369	89.591045
2018-09-30	-44.468572	29.681568	-212.910453	-247.616893	2.174706e+15	158.528236	169.851032
2018-12-31	77.244334	58.768092	-217.934836	-359.365089	3.185285e+15	189.077528	219.710532
2019-03-31	164.581782	65.354272	-129.345411	-180.557699	1.647926e+15	119.501591	105.146638
2019-06-30	175.413995	74.799323	-117.634403	116.197608	1.039228e+15	138.099233	40.977251

Figure 13: VAR results

Before forecasting, we had to revert back our differential values back to its original scale, since we took the differential of our values once, we performed the inverse transformation taking the entire dataset as a matrix. After inverting, we then plot the last six quarters of the forecasted dataset and compare with the test dataset.

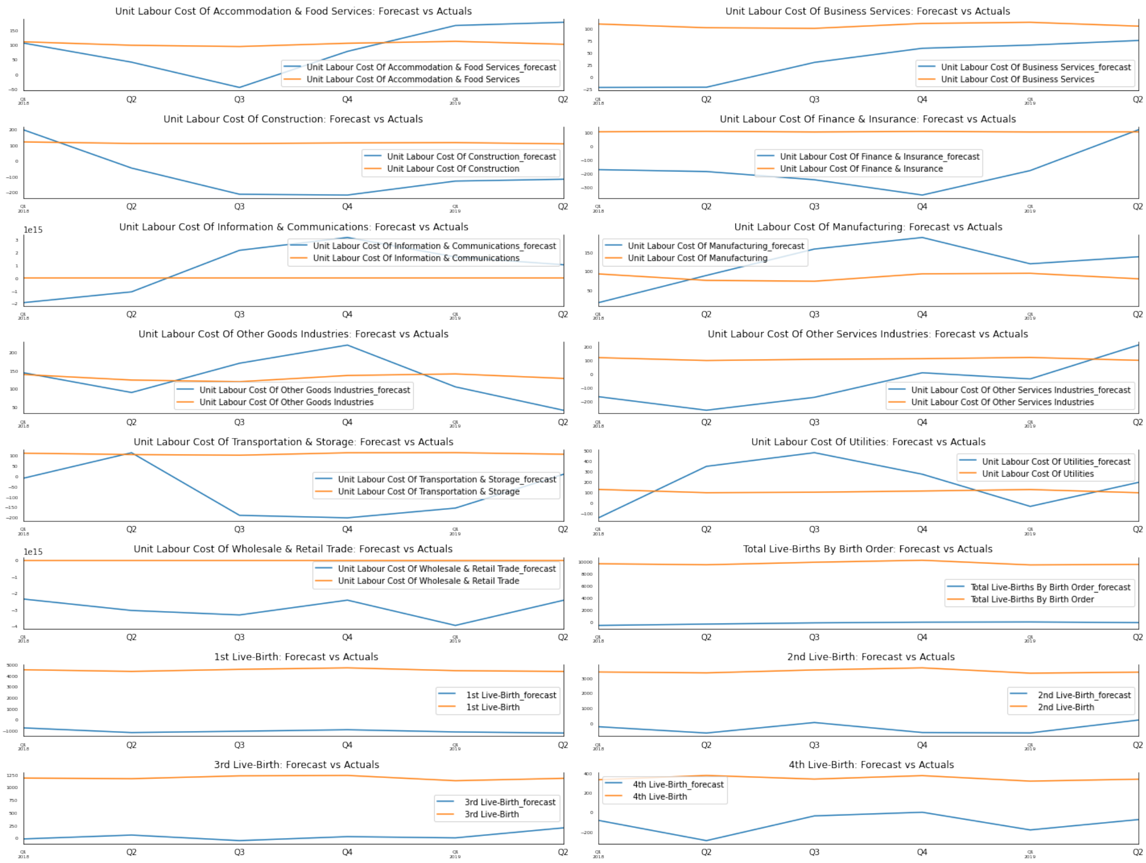

Figure 14: VAR results

Based on the results in Figure 14, VAR may not be appropriate to fit the model as the forecasted data is not able to follow the trends and values of the actual data. We could infer this before fitting the model as we were choosing the lag order of the model. Although AIC is the best estimator of choosing the lag order, which is the lowest at p=6 , most of the other estimators point towards choosing p=0. If we were to choose p=0 as our lag order, the VAR model would just be a simple regression and may mean that the past values have very little contribution to the next value. Thus it may be the case that VAR is not the best model for our dataset.

5.2 VARMAX

Since VARMAX takes into account the moving averages unlike VAR, our group decided to use Bollinger Bands to visualise and evaluate our results. Bollinger Bands is a technical indicator normally used to measure the stock market's volatility, and is used to evaluate the statistical significance of our predicted birth rate results. It calculates the moving averages of a given dataset, while having upper and lower bands to evaluate the "stability" of the data. The upper and lower Bollinger Bands are calculated with the formulas below.

$$BOLU = MA(Birth, 16) + 2\sigma[Birth, 16]$$

$$BOLD = MA(Birth, 16) - 2\sigma[Birth, 16]$$

where

$BOLU$ = Upper Bollinger Band.

$BOLD$ = Lower Bollinger Band

MA = Moving average or mean

$Birth$ = Birth rate of current quarter

$\sigma[Birth, 16]$ = Standard deviation over last 16 quarters (4 years)

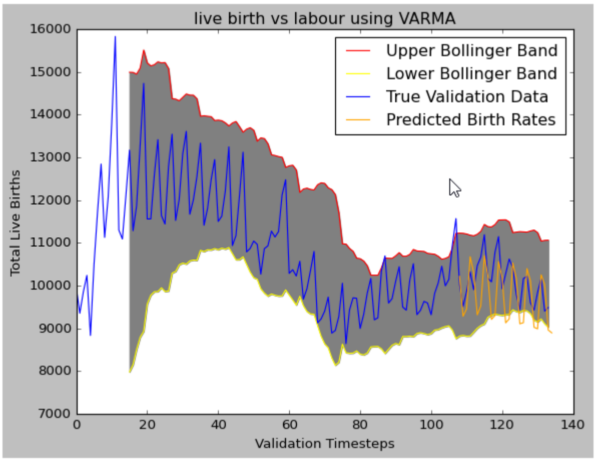

Figure 15: Bollinger Bands evaluation of VARMA

The predicted data trained with VARMAX is shown to be somewhat within the bands. Our group concluded that the model seems like a good fit, performing better than VAR. However, the MSE of VARMAX trained using labour data and birth rates was not as low as the MSE of the TCN model. The VARMA model was also unequipped to capture the underlying patterns in our multivariate time series data that expressed non-linearity. VARMA and its broader family of associated statistical models as aforementioned rested upon the stationary assumption or stability after taking the differential, otherwise the pattern will not be captured. These limitations led us to explore the ability conferred by Temporal Convolutional Networks in capturing these non-linear and non-stationary dynamics found in demographic and population trends in cities.

5.3 Temporal Convolutional Networks

We used the same Bollinger Bands to evaluate the statistical significance of our predicted birth rate results.

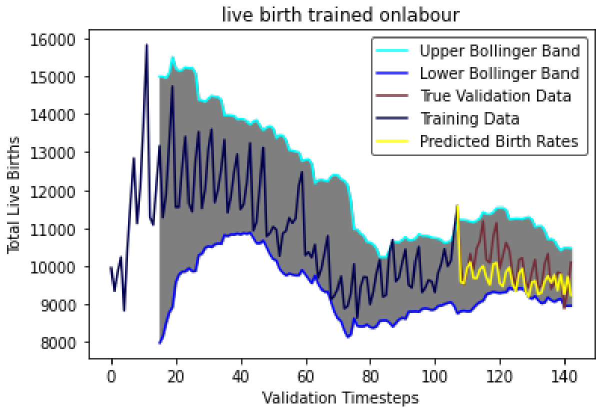

Figure 16: Bollinger Bands evaluation of TCN

The predicted values are well within the Bollinger Bands as shown in Figure 16, suggesting that the birth rates are stable. It was also able to capture the underlying patterns in our multivariate time series data to a small extent.

Hence, together with unit labour costs and birth rates, we decided to train our TCN model with the rest of our statistical data. The training is stopped after the validation loss converges for 300 times, allowing the mean squared error and mean absolute error to be calculated.

Training	Iterations	Mean Squared Error	Mean Absolute Error
Labour	420	0.050030038	0.18635967
Gross Domestic Product	584	0.058415506	0.21119718
Investment	321	0.11384143	0.26864198
Taxes on Industry	302	0.12587605	0.12587605
Balance of Payments	319	0.10194871	0.26412103

Table 5: Results of TCN training

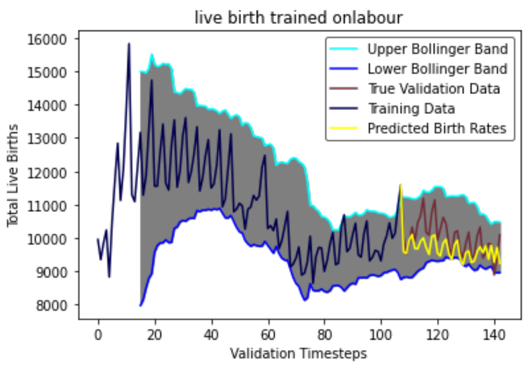

Figure 17: Predicted results for unit labour cost

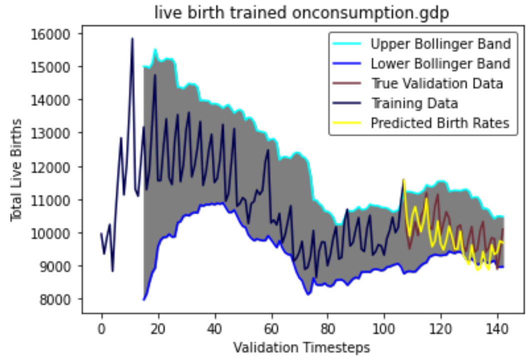

Figure 18: Predicted results for GDP

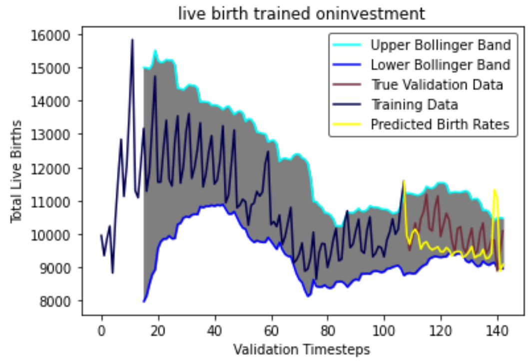

Figure 19: Predicted results for investments

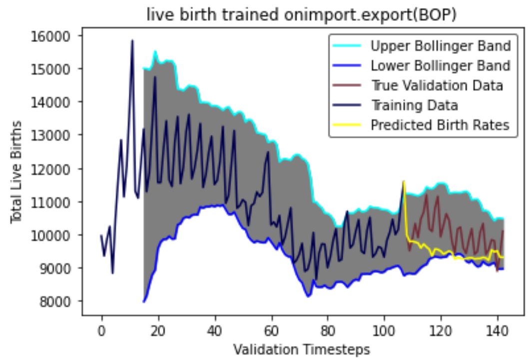

Figure 20: Predicted results for balance of payments

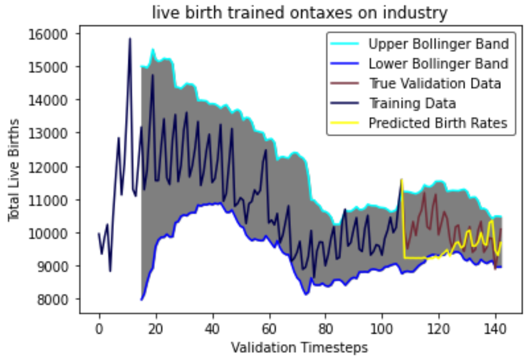

Figure 21: Predicted results for taxes on industries

6 Interpretation

Across the live-birth rates trained against the 4 major baskets, those trained against Factor Payments for Labour across industries and GDP for consumption across industries were most performant during evaluation against the validation dataset even in the face of major shocks in the final 4 time steps between 2020 Q1 and 2021 Q3. The shifts in prominence across industries have only accentuated underlying features observed before this decade. The difference in labour market costs over 6 quarters though momentous drops of over 25% in manufacturing, food/beverage and retail, signify the higher level features have already discounted for such a possibility.

Other than the unit labour cost data, the model had good results with GDP data as well. Considering that unit labour cost is how much a business pays its workers in the industry, there are deep interdependencies between labour costs and GDP. As a worker's salary increases, he will be able to spend and consume more goods and services in the country, raising the GDP. In this regard, we can make an assumption that since they are so closely related, they have a similar effect on the model.

The connection between birth rate and economic growth has been studied by many researchers and economists before. Li et al concluded that birth rate has a negative impact on economic growth, and that it was highly possible that the one-child policy played a part in the improvement of Chinese economy after it was implemented [HJ07]. Our experiment went further by comparing various economic indicators to evaluate which performed as better predictors on our TCN model.

Labour continues to be the central in determining underlying factors that raise age of childbearing years with the deepening of career paths of women. Reconciling labour participation with motherhood would be central beyond our reliance of outsourcing the roles of the family to services such as child care.

Policymakers through our basket approach would be able to sieve through and leverage predictiveness of said basket in approaching second-order effects of policy effects and design instruments that target beyond the policies such as birth bursaries that act upon immediate symptoms while deeper underlying levers where policy can intervene. Singapore system architects have been able to create policy levers that can be pulled on and adjusted through empiricism in sectors of public housing to education and crime. Focusing on parameters such as subsidies might miss deeper interdependencies in the system that feedback signals to households such as the labour market in the fertility context.

7 Conclusion

All in all, we find that predicting Singapore's birth rates using the industry sector's labour index as inputs was most accurate, probably due to the numerous indicators provided by the various sectors. These indicators include accommodation, food services, business services and finance industries which had a steady growth since the early days of Singapore. GDP was also a fairly good indicator that predicted birth rate with relatively good accuracy.

The multivariate analysis by both VAR models were able to do predictions of all variables in correlation to one another at the same time. We have also shown that VARMA models produced far more accurate forecasts on limited data as compared to VAR models, though the statistical models might have been quite difficult to apply in practice if the open-sourced tools such as statsmodels Python modules were not provided to conduct the analysis.

The TCN model performed well given the quarterly datasets that have been compiled in the economic statistics, adding to the fact that Singapore is a relatively young country. Comparing statistical models such as VAR and VARMAX with a neural network model expressed the shortfalls and strengths of our chosen approach compared to the traditional statistical approaches.

8 Future Work

There are other ways of exploring this work in the future, which includes using other advanced machine learning models such as random forest or deep neural networks such as LSTM. Given the broad datasets we collected for various economic indicators of Singapore's growth, we can explore using pre-processing techniques like Principal Components Analysis (PCA) on our TCN model to reduce dimensionality of the datasets, in hopes of getting more accurate predictions for birth rate trends. Building out a more interpretable and introspectable model in the future can enable feature importance weighing that might enable deeper insight to the data analysed

The availability of quarterly data to derive greater timestamps limited the nature of the baskets we could group to primarily economic factors. There is wealth of annual and decennial data collected by Singapore Statistics by engaging public sector data suppliers and through the Censuses that we thus had not considered to bundle into baskets. These range from societal factors such as crime, welfare and environment to demographic trends and distribution. These might shed a wider scope for policy makers to leverage.

References

[DR92] Lee R. D. and Carter L. R. "Modeling and forecasting U. S. Mortality". In: *Journal of the American Statistical Association* 87 (1992), p. 659. DOI: `10.2307/2290201`.

[NR05] Li N. and Lee R. "Coherent mortality forecasts for a group of populations: An extension of the lee-carter method". In: *Demography* 42 (2005), pp. 575–594. DOI: `https://doi.org/10.1353/dem.2005.0021`.

[HJ07] Li H. and Zhang J. "Do high birth rates hamper economic growth?" In: *Review of Economics and Statistics* 89 (2007), pp. 110–117. DOI: `https://doi.org/10.1162/rest.89.1.110`.

[HY17] Li H. and Lu Y. "Coherent forecasting of mortality rates: A sparse vector-autoregression approach". In: *ASTIN Bulletin* 47 (2017), pp. 563–600. DOI: 10.1017/asb.2016.37.

[Kol+18] Koltun et al. "An empirical evaluation of generic convolutional and recurrent networks for sequence modeling." In: (2018). DOI: 10.1088/1742-6596/1213/4/042050.

[Z+21] Gan Z. et al. "Temporal convolutional networks interval prediction model for wind speed forecasting". In: *Electric Power Systems Research* 191 (2021), p. 106865. DOI: 10.1016/j.epsr.2020.106865.