Quiz 1. 50 minutes.

Calculus 4 Class 6. May 2nd, 2024

ID#:_____ **Name:**_____

This quiz contains four questions. Except for Question 1, partial credit will only be given if you do not show your computation process.

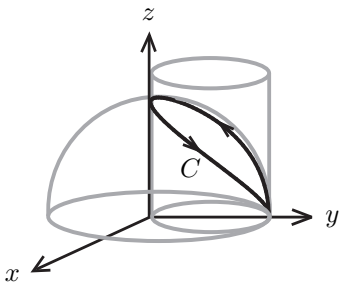
1. (3 pts) Determine whether the statement is true or false. In the following, the functions f, P, and Q are functions from $\mathbb{R}^2$ to $\mathbb{R}$.
   (a) $\int_{-C} f(x,y)ds = -\int_C f(x,y)ds$.
   (b) $\int_{-C} P(x,y)dx + Q(x,y)dy = -\int_C P(x,y)dx + Q(x,y)dy$.
   (c) The work done by a conservative force field in moving a particle around a closed path is zero.

2. (4 pts) Show that $F(x,y) = (1+xy)e^{xy}\mathbf{i} + (e^y + x^2e^{xy})\mathbf{j}$ is a conservative vector field, and find a function f such that $F = \nabla(f)$.

3. (6 pts) Compute the line integral

$$\int_C z^2 dx - x^2 dy + 2yz dz,$$

where C is the curve of the intersection of the upper half sphere $z = \sqrt{4 - x^2 - y^2}$ and the circular cylinder $x^2 + y^2 = 2y$, orientated counterclockwise viewed from the above, as Figure.



4. (7 pts) Let the vector field $F(x,y) = \frac{x^2y}{(x^2+y^2)^2}\mathbf{i} - \frac{x^3}{(x^2+y^2)^2}\mathbf{j}$, C_1 be the curve $|x| + |y| = 1$ and C_2 be the curve $x^2 + (y-2)^2 = 1$. Find $\int_{C_1} F \cdot d\gamma$ and $\int_{C_2} F \cdot d\gamma$