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# Table of Contents

.....	1
set directories .....	1
Problem 1 .....	1
part 1a - set up state space, parameters .....	1
part 1b - set up action space .....	1
part 1c - define utility of extraction .....	2
part 1d - define flow utility matrix .....	2
part 1e - identify state in next period .....	2
part 1f - state transition matrix .....	2
part 1g - value function iteration .....	3
part 1h - find optimal transition matrix .....	3
part 1i - simulate for t periods .....	4
part 1j - plots .....	4

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Computational Problem Set, Enviro I, Problem 1
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%
% Last Edit Date: Nov 7, 2022
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

## set directories

```
clear; clc;

cd '/Users/zachkuloszewski/Dropbox/My Mac (Zachs-MBP.lan)/Documents/';
cd 'GitHub/phd_psets/year2/environmental';
addpath(genpath('figures'));
addpath(genpath('functions'));
```

## Problem 1

Solving the DDP by value function iteration; discrete state space.

### part 1a - set up state space, parameters

```
N      = 501;
S_tot  = 1000;
r      = 0.05;

delta  = 1/(1+r);
S      = linspace(0,S_tot,N);
```

### part 1b - set up action space

```
A  = S;
```

---

```
nA = numel(A);
```

## part 1c - define utility of extraction

```
% pick utility function of interest
u_fun_flag = 1; % choose 1 or 2

if u_fun_flag == 1
    u        = @(x) 2*x.^0.5;
    u_prime  = @(x) x.^(-0.5);
elseif u_fun_flag == 2
    u        = @(x) 5*x-0.05*x.^2;
    u_prime  = @(x) 5-0.1*x;
end
```

## part 1d - define flow utility matrix

```
U = u(repmat(A,nA,1));

% def -inf upper triangular matrix
infs = repmat(-Inf, N);
flow_UT = triu(infs,1);

% sum to get final flow utility matrix
U = U + flow_UT;
```

## part 1e - identify state in next period

init matrix w same dimension as flow utility

```
transition = nan(N,nA);

for i=1:N % iter through rows
    for j=1:nA % iter through columns
        if j > i % if extracting more than full stock
            transition(i,j) = 1;
        else
            transition(i,j) = i-j+1;
        end
    end
end
```

## part 1f - state transition matrix

```
T = nan(N,N*nA);
rows = 1:501;

for k=1:nA
    inds = rows - A(k)/2;
    inds(inds<=0) = 1;
```

---

```

        T(:,1+(k-1)*N:k*N) =
        sparse(rows(inds>0),inds(inds>0),ones(numel(inds(inds>0)),1),N,nA);
    end

T = sparse(T);

```

## part 1g - value funciton iteration

```

% init parameters
error      = 1e12;
error_tol  = 1e-8;

% init value and optim choice
V_hist = nan(N,1000);
C_hist = nan(N,1000);

% iteration counter
n_iter = 0;

V      = repelem(0,N)';

Vnext = nan(N,nA);

while error > error_tol

    % count number iterations
    n_iter = n_iter + 1;
    Vold   = V;

    Vnext = zeros(N,nA);
    for k=1:nA %looping thru action space
        Vnext(:,k) = T(:,1+(k-1)*N:k*N)*V;
    end

    % grab optimized value and action column
    [V, C] = max(U + delta .* Vnext,[],2);

    % store values and choices
    V_hist(:,n_iter) = V;
    C_hist(:,n_iter) = C;

    error = max(abs(V-Vold));

end

```

## part 1h - find optimal transition matrix

```

% init matrix
Topt = zeros(N,nA);

for i=1:N
    Topt(i, C_hist(i,n_iter)) = 1;
end

```

---

## part 1i - simulate for t periods

```
t      = 80;
st     = S_tot; %init stock = 1000

% init storage for output
V_hist = nan(t,1); % value function
C_hist = nan(t,1); % extraction
S_hist = nan(t,1); % state
P_hist = nan(t,1); % price

for i=1:80

    st_ind = find(S == st); % find state index
    action = find(Topt(st_ind,:) == 1);

    st = st - A(action); % update stock

    S_hist(i) = st;
    C_hist(i) = A(action);
    V_hist(i) = u(A(action));

    if action > 1
        P_hist(i) = u_prime(A(action));
    else
        P_hist(i) = nan;
    end
end
```

## part 1j - plots

```
figure;
plot([P_hist, C_hist]);
legend("Price", "Extraction", Location="northeast");
legend box off
xlabel("Period")

if u_fun_flag == 1
    u_txt = "2\sqrt{y}$";
elseif u_fun_flag == 2
    u_txt = "5y - 0.05y^2$";
end

title(strcat("Linear State Space Simulations, $u=", u_txt), ...
      'Interpreter','latex');

saveas(gcf, ['figures/part1j_u' num2str(u_fun_flag) '.png']);
```

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