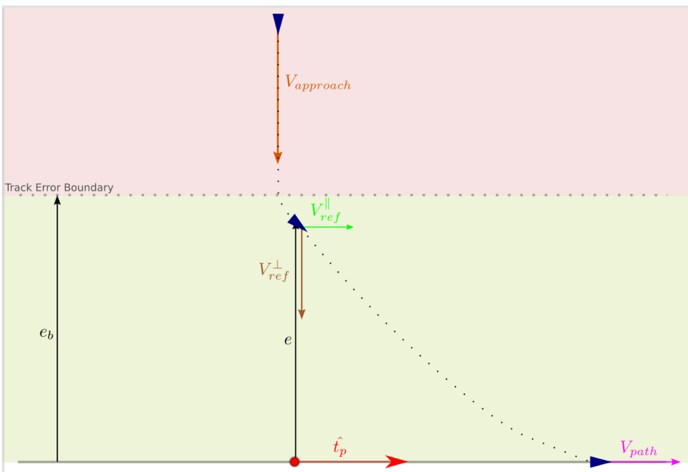
Week 9 Report - Jan 16, 2023 ~ Jan 22, 2023

💎 New Velocity curve formulation

After the last meeting, the new velocity curve formulation was drawn & evaluated.

Note, the nomenclature from this point on follows the diagram from the Week 8 Report:



Nomenclature

V_{max}	Maximum Speed vehicle can achieve
V_{min}	Minimum Speed Vehicle can achieve
V_{nom}	Nominal Speed Vehicle can achieve (cruise speed for Fixed Wing)
V_{path}	Desired speed on path
$V_{approach}$	Desired approaching speed orthogonal to path (outside track error boundary)
$V_{ref}^{\perp}$	Orthogonal component to path, of the reference (target) velocity
$V_{ref}^{\parallel}$	Parallel component to path, of the reference (target) velocity
e	Cross track error distance to the path
e_b	Track error boundary
$\hat{t_p}$	Unit tangent vector of the path segment

New Formulation in Equations

$With\ V_{approach-min}:\ Minimum\ desired\ approach\ speed\ (we\ can't\ approach\ slower\ than\ this\ to\ the\ path)$

1. $V_{approach} = max(V_{nom}, V_{path}, V_{approach-min})$
2. $e_b = V_{approach} * t_{const}$ $//\ Derive\ track\ error\ boundary$
3. $\theta_{la} = \frac{\pi}{2}(1 - \frac{e}{e_b})^2$ $//\ Look\ ahead\ angle\ (approaches\ \frac{\pi}{2},\ when\ on\ path)$
4. $if\ (V_{approach} > V_{path})$ $//\ If\ desired\ speed\ on\ path\ is\ below\ the\ nominal\ approach\ speed$
    a. $V^{||}_{approach} = V_{path} * sin(\theta_{la})$ $//\ Ramp\ in\ parallel\ velocity$
    b. $V^{\perp}_{approach} = V_{approach} * cos(\theta_{la})$ $//\ Ramp\ down\ approach\ velocity$
    c. $return\ [V^{||}_{approach}, V^{\perp}_{approach}]$
5. $else$ $//\ desired\ speed\ on\ path\ is\ high\ enough\ to\ draw\ unicyclic\ path\ like\ TJ's\ NPFG$
    a. $V_{gnd-min} = V_{tk} * (1 - \frac{e}{e_b})$ $//\ Calculate\ minimum\ ground\ speed\ from\ track\ keeping\ feature$
    b. $\widehat{V}_{bearing} = bearingVec(\widehat{t}_p, \theta_{la})$ $//\ Get\ the\ desired\ bearing\ vector\ (normalized)$
    c. $if\ (V_{gnd-min} > V_{max})$
        i. $return\ (V_{max} * \widehat{V}_{bearing})$ $//\ Return\ maximum\ airspeed\ ref\ vector\ in\ desired\ bearing$
    d. $else\ if\ (V_{gnd-min} > V_{nom})$ $//\ Desired\ minimum\ ground\ speed\ is\ between\ nom \sim max\ range$
        i. $return\ (V_{gnd-min} * \widehat{V}_{bearing})$
    e. $else$ $//\ minimum\ ground\ speed\ lower\ than\ nominal\ airspeed,\ definitely\ reachable$
        i. $return\ (V_{nom} * \widehat{V}_{bearing})$ $//\ Return\ nominal\ airspeed\ ref\ vector\ in\ desired\ bearing$

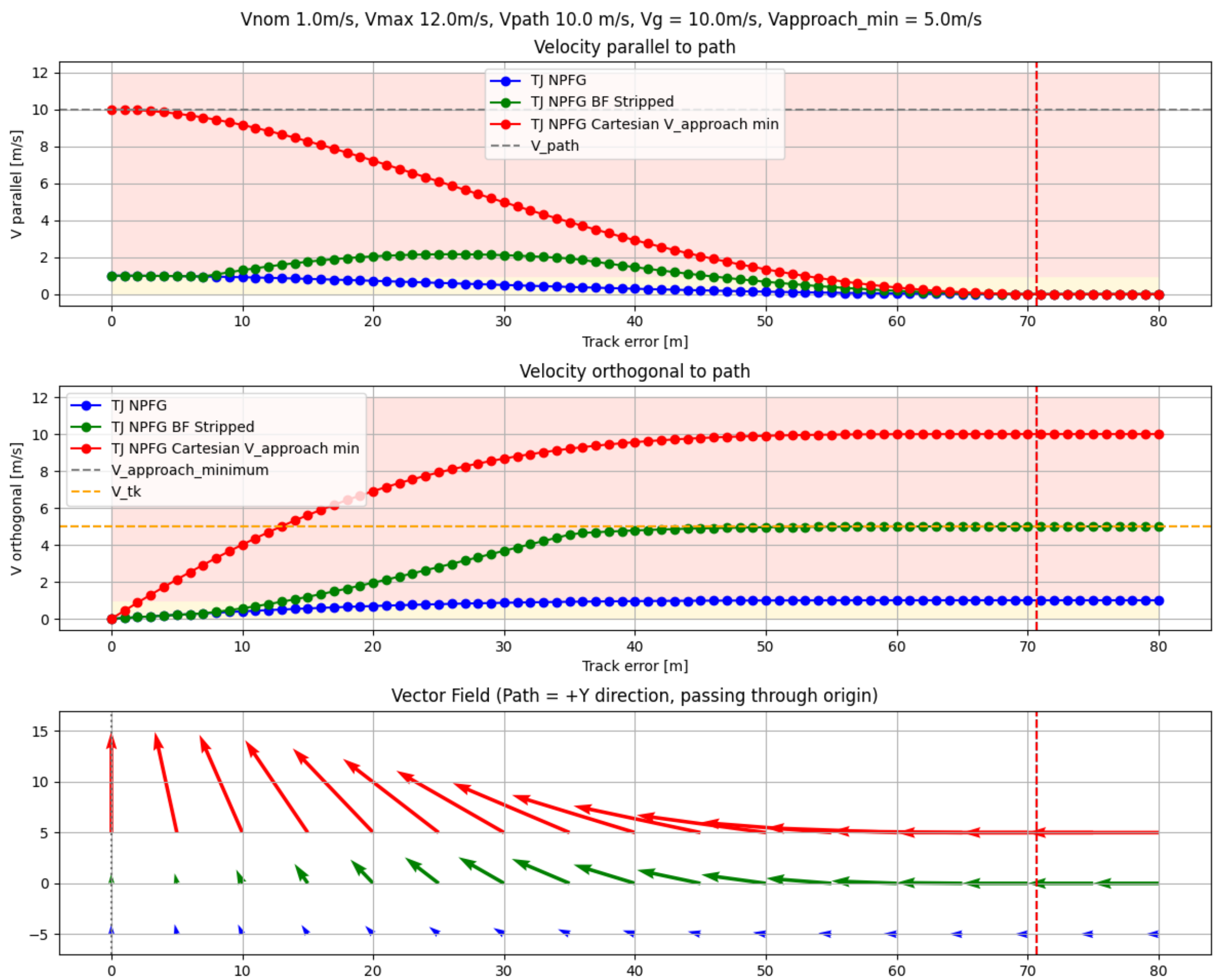
With this formulation, depending on the desired speed on the path, the algorithm is decoupled into using TJ's NPFG logic (assuming unicyclic motion), and ramping in using sine/cosine functions.

🧪 Testing the new formulation against different conditions

This is better summarized here: 12_VelocityCurveFormulation_230123.

With this new formulation, the behavior of the velocity curves under different parameters was tested. The especially interesting part was the following:

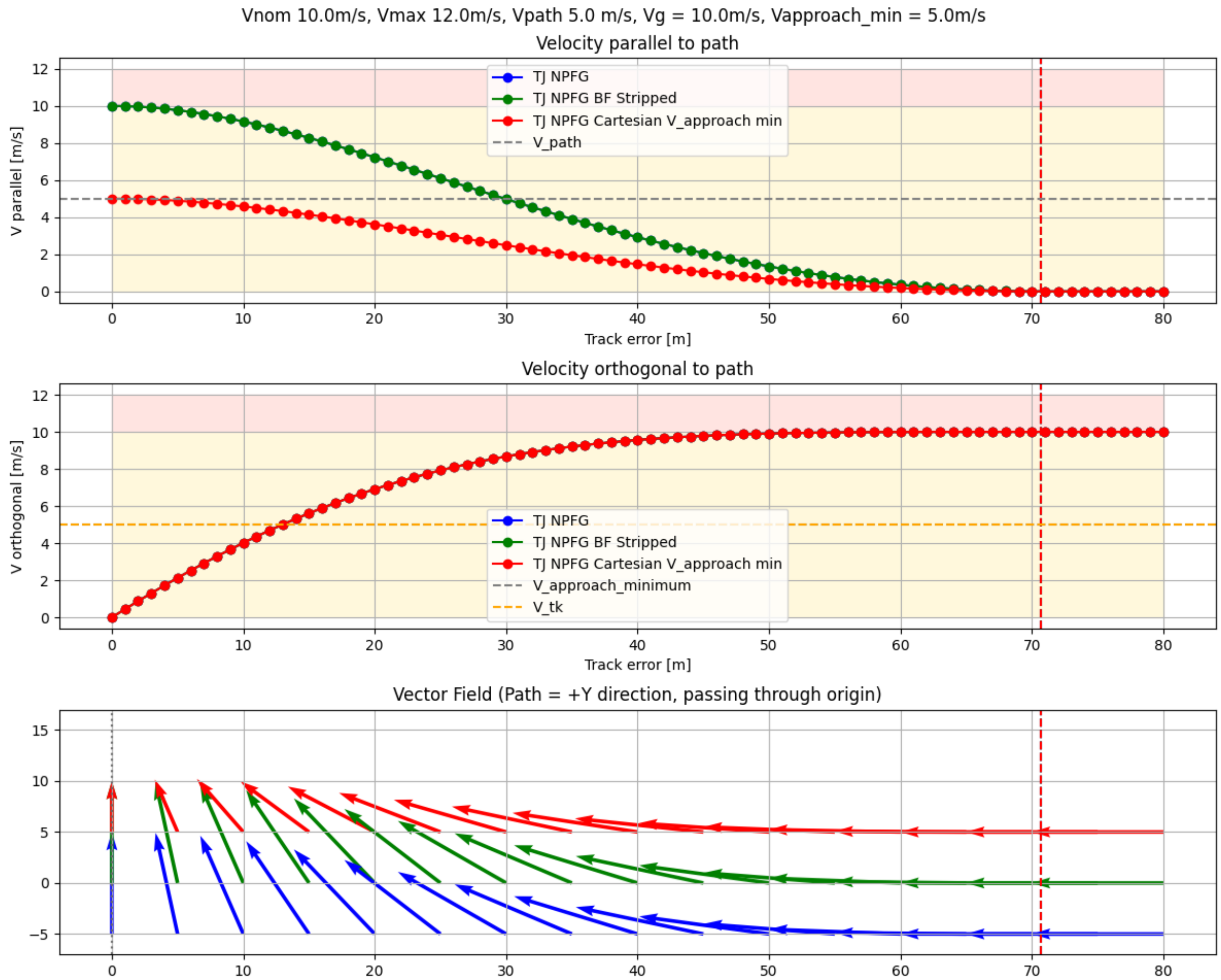
$V_{nom} < max(V_{path}, V_{approach-min})$: Low nominal vehicle speed



Due to the interference with the bearing feasibility function, the BF (Bearing Feasibility) Stripped [Green] formulation diverges from original TJ NPFG [Blue].

While the new formulation [Red] respects the actual desired V_{path}. Whereas TJ NPFG derivations always have a velocity magnitude of V_{nom}, which is very small.

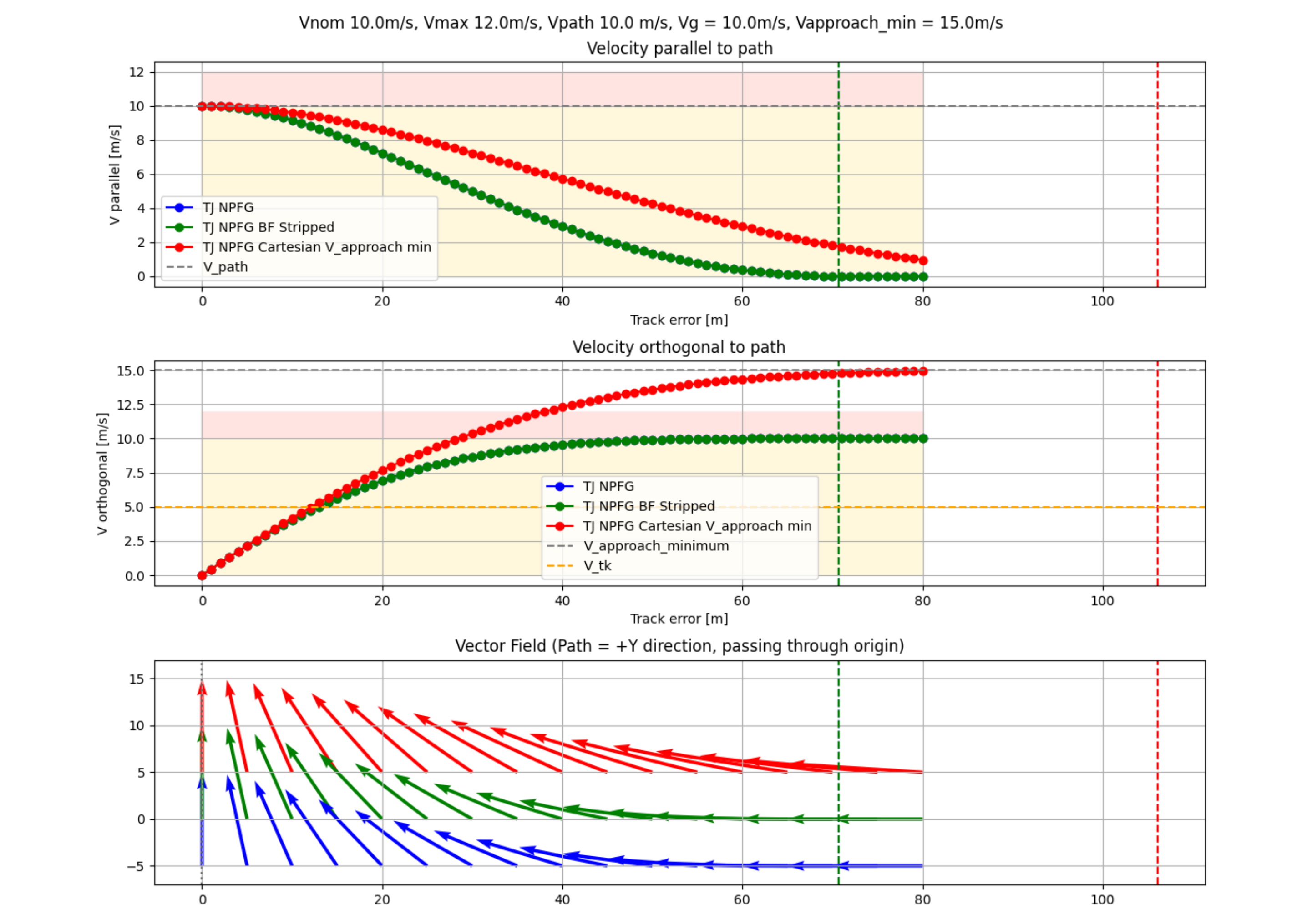
$V_{path} < max(V_{nom}, V_{approach-min})$: Low desired speed on path



With low desired speed on the path, the unicyclic motion assumption is broken, the cartesian ramp-in/out is applied.

New formulation [Red] is therefore respecting the V_{path} setting when on path (parallel velocity plot, when Track error = 0), whereas TJ's NPFG derivations maintain V_{nom} magnitude, ignoring V_{path}.

$V_{approach-min} > max(V_{nom}, V_{path})$: High desired approach speed



In this case, track error boundary expands as $V_{approach}$ is higher than cases before. And the cartesian velocity ramping works, and leads the velocity curve to desired V_{path} respectively.

☑️ Conclusions

As demonstrated by various cases shown above, **<u>the new formulation satisfies the most basic requirements that original TJ's NPFG couldn't satisfy</u>**, specific to quadcopter path following cases, namely:

- $V^{||}_{approach}(e = 0) == V_{path}$ $// When\ on\ path,\ follow\ desired\ speed\ on\ path\ (Quadcopter\ can\ have\ V_{path} \neq V_{nom})$
- $V_{approach} > V_{nom}$ $when\ V_{nom}\ is\ too\ low\ //\ Allows\ faster\ convergence\ to\ path\ (Quadcopter\ has\ low\ V_{nom})$

However, more quantitative analysis on how effective this algorithm is in terms of acceleration constraints has not been evaluated.

END