About

This document is Junwoo's effort in understanding "On Flying Backwards" paper [1]:

[1] T. Stastny and R. Siegwart, "TJ NPFG: On Flying Backwards: Preventing Run-away of Small, Low-speed, Fixed-wing UAVs in Strong Winds." arXiv, Aug. 04, 2019. doi: 10.48550/arXiv.1908.01381.

Also, the presentation & PX4's NPFG library is referenced: https://www.youtube.com/watch?v=LY6hYBCdy-0

Presentation Revisited - Jan 11, 2023

While writing down the new velocity formulation & pondering on how vehicle dynamics comes into play with path following guidance formulation, I wanted to revisit the formulation and find more information.

Adaptive Stability Bounds

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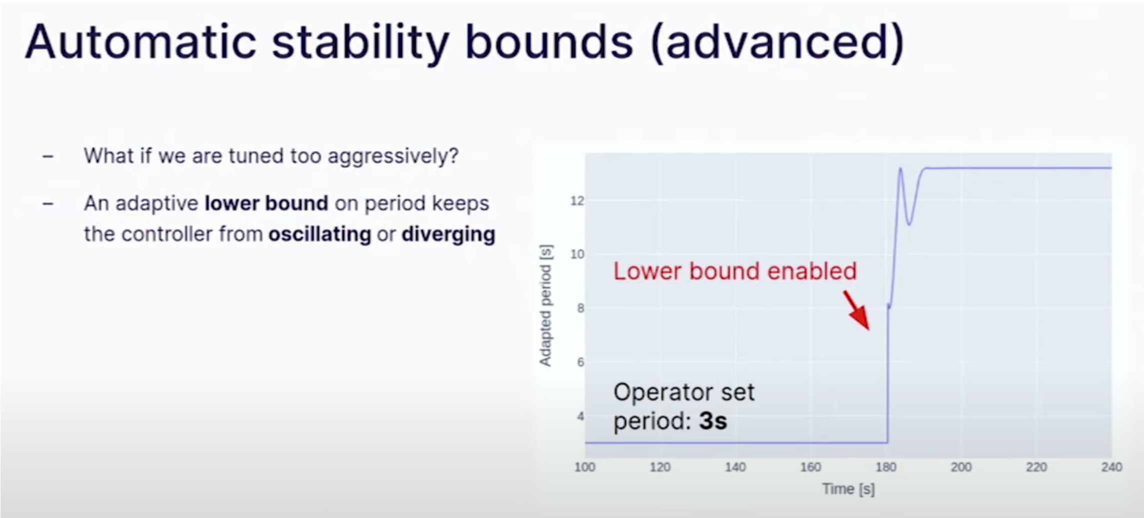
<td>
</td>
<td>NPFG_LB_PERIOD: Avoids limit cycling from a too aggressively tuned period/damping combination. If set to false, also disables the upper bound

NPFG_UB_PERIOD: Adapts period to maintain track keeping in variable winds and path curvature.</td>
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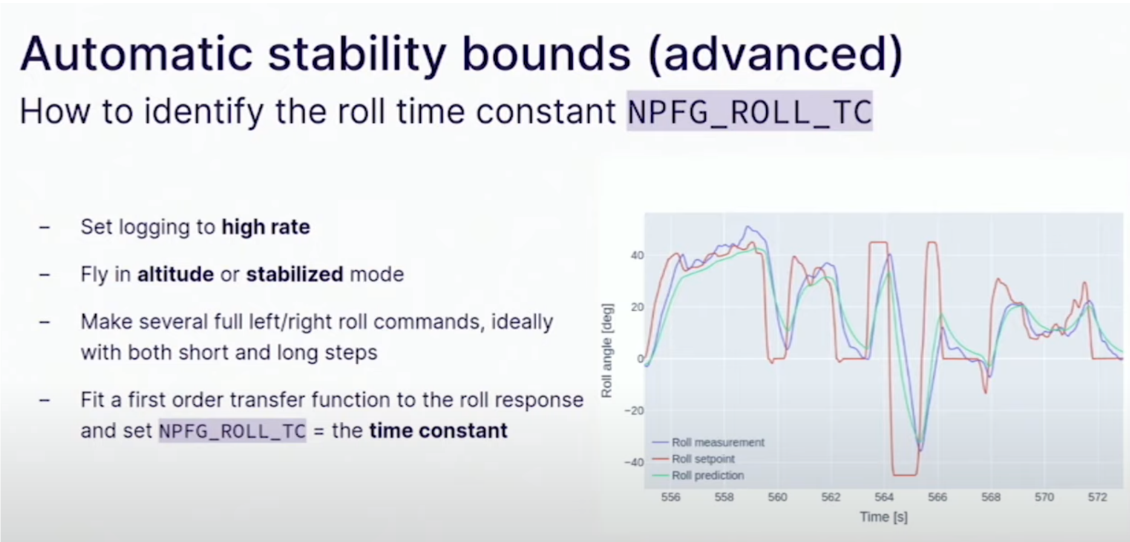
<td>
</td>
<td>NPFG_ROLL_TC: Depends on autopilot's behavior on roll attitude control</td>
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What can be improved

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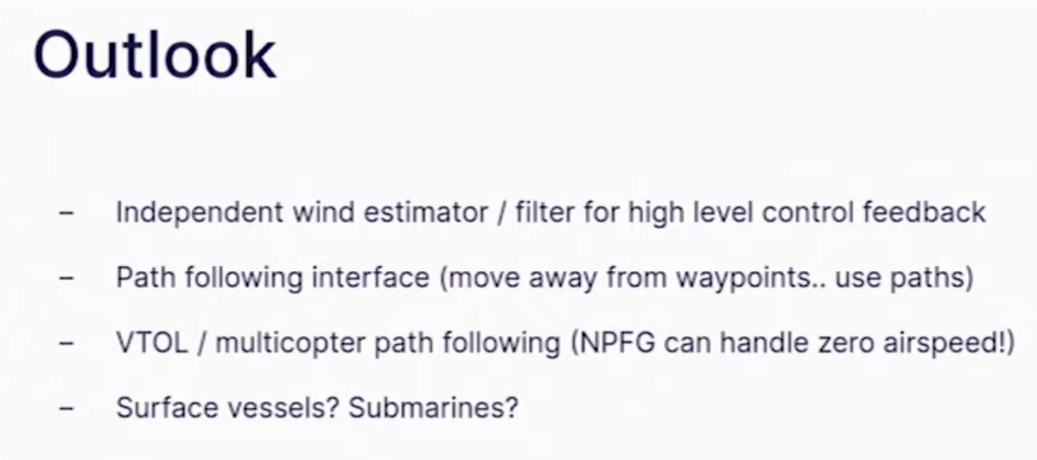

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<td>
1. Wind estimate is crucial for a reliable behavior of NPFG (PX4 Improvement)
2. Path following interface (PX4 Improvement)
3. VTOL / Multicopter application
4. Marine Vehicles application (especially in high currents)

During QnA
1. For Multicopter: Using the ‘jerk limited trajectory’ library to construct a track-error boundary is possible (makes sense)
    a. Tuning is still question mark
2.
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NPFG Parameters

https://docs.px4.io/main/en/advanced_config/parameter_reference.html#fw-npfg-control

- **NPFG_DAMPING**:
- **NPFG_PERIOD** [s]
    - Damping & Period defines:
        - P-gain for lateral acceleration control setpoint
        - Time-constant for track-error-boundary calculation
- **NPFG_PERIOD_SF**: Safety Factor. Multiplied by period for conservative minimum period bounding (when period lower bounding is enabled). 1.0 bounds at marginal stability.
- **NPFG_ROLL_TC** [s]: Time constant of roll controller command/response, modeled as first order delay. Used to determine lower period bound. Setting zero disables automatic period bounding.
    - Only used in ‘lower period bound’ function:
        - Less Wind: PI * ROLL_TC / Damping
        - Strong Wind: 4 * PI * ROLL_TC * Damping
        - The lower bound varies during runtime by feasibility & wind factor & air-turn-rate of the path (desired)
- **NPFG_SW_DST_MLT**: Multiplied by the track error boundary to determine when the aircraft switches to the next waypoint and/or path segment. Should be less than 1. 1/pi (0.32) sets the switch distance equivalent to that of the L1 controller.

Period and Damping Ratios of the NPFG (and L1)

```/** * L1 period * * Used to determine the L1 gain and controller time constant. This parameter is * proportional to the L1 distance (which points ahead of the aircraft on the path * it is following). A value of 18-25 seconds works for most aircraft. Shorten * slowly during tuning until response is sharp without oscillation. * * @unit s * @min 7.0 * @max 50.0 * @decimal 1 * @increment 0.5 * @group FW L1 Control */ PARAM_DEFINE_FLOAT(FW_L1_PERIOD, 20.0f); /** * L1 damping * * Damping factor for L1 control. * * @min 0.6 * @max 0.9 * @decimal 2 * @increment 0.05 * @group FW L1 Control */ PARAM_DEFINE_FLOAT(FW_L1_DAMPING, 0.75f);```	Even in the original paper, the period & damping is only referenced for the case of straight path following. How is the Period & Damping actually defined for NPFG & L1? How come can we actually use them unanimously between two controllers? What are the theoretical backgrounds behind the calculation of lower period bound & p-gain (for lateral acceleration) and time constant for track-error boundary calculation?

END

Questions - from Dec 2, 2022

Question 1: Reasoning behind the Bearing Feasibility buffer zone formulation

Originally, the feasibility function is proposed as in equation 8:

(see Section III for reference command generation). In [8], the following continuous feasibility function was proposed:

$$\boxed{\operatorname{feas}(\lambda, \beta) = \frac{\sqrt{1-(\beta \sin \bar{\lambda})^2}}{\cos \bar{\lambda}}} \tag{3}$$

where $\operatorname{feas}(\lambda, \beta) \in [0,1]$ transitions from a value of 1 at "fully" feasible conditions ($\beta < 1$) to 0 in infeasible conditions (definition in (1)), see Fig. 3 (left). Input $\bar{\lambda} = \operatorname{sat}\left(|\lambda|, 0, \frac{\pi}{2}\right)$, where operator $\operatorname{sat}(\cdot, \min, \max)$ saturates the input at the bounds min and max.

However, a concept of 'buffer zone' is introduced for dealing with multiple factors, but primarily binary jump on feasibility around the boundary: $|\lambda| \geq \frac{\pi}{2} \cap \beta = 1$.

$$\operatorname{feas}(\lambda, \beta) = \begin{cases} 0 & \beta > \beta_+ \\ \cos^2\left(\frac{\pi}{2} \operatorname{sat}\left(\frac{\beta-\beta_-}{\beta_+-\beta_-}, 0, 1\right)\right) & \beta > \beta_- \\ 1 & \text{else} \end{cases} \tag{4}$$

where the upper limit of the transitioning region β_+ is approximated as a piecewise-continuous function with a linear finite cut-off to avoid singularities, the cut-off angle λ_{co} chosen small such that the regular operational envelope is not affected:

$$\beta_+ = \begin{cases} \beta_{+_{co}} + m_{co}\left(\lambda_{co} - \bar{\lambda}\right) & \bar{\lambda} < \lambda_{co} \\ 1/\sin \bar{\lambda} & \text{else} \end{cases} \tag{5}$$

with $\beta_{+_{co}} = 1/\sin \lambda_{co}$ and $m_{co} = \cos \lambda_{co}/\sin \lambda_{co}^2$. The lower limit of the transitioning region β_- is similarly made piecewise-continuous to correspond with β_+:

$$\beta_- = \begin{cases} \beta_{-_{co}} + m_{co}\left(\lambda_{co} - \bar{\lambda}\right)\beta_{buf} & \bar{\lambda} < \lambda_{co} \\ \left(1/\sin \bar{\lambda} - 2\right)\beta_{buf} + 1 & \text{else} \end{cases} \tag{6}$$

where $\beta_{-_{co}} = \left(1/\sin \lambda_{co} - 2\right)\beta_{buf} + 1$.

However, the formulation of β_+ and β_- isn't so straightforward, because the distance between the two values doesn't equal β_{buf}, which I expected to be the case.

- ~~What are the mathematical significances defining the~~ ~~β_+ and β_-~~ ~~in that specified way (Equation 5 & 6)?~~
  - ~~It does make the function piecewise-continuous, but why wasn't~~ ~~$\beta_{-\infty}$~~ ~~defined to something like:~~ ~~$\beta_{+\infty} - \beta_{buf}$~~ ~~instead?~~
- ~~Which value should~~ ~~λ_{co}~~ ~~take? It is mentioned that it is chosen 'small' to not affect regular operational envelope. Does that imply that~~ ~~λ_{co}~~ ~~takes a value above~~ ~~$\frac{\pi}{2}$~~~~?~~
  - ~~If that is the case, since~~ ~~$\bar{\lambda} = sat(|\lambda|, 0, \pi/2)$~~~~, the~~ ~~$\bar{\lambda}$~~ ~~can't take a value over~~ ~~$\frac{\pi}{2}$~~~~. So the~~ ~~$\bar{\lambda} < \lambda_{co}$~~ ~~will always be true. This wouldn't make sense.~~
- ~~Should the formulation (Equation 5 & 6) rather compare the~~ ~~λ_{co}~~ ~~to the actual raw bearing to wind vector value~~ ~~λ~~~~, instead of~~ ~~$\bar{\lambda}$~~~~, due to the reason mentioned above? (I.e, does the paper have a typo?)~~

>> The PX4 code gives the latest reference, TJ said I shouldn’t need to deal with this old logic

Note: an effort to code-ify this function is being done in: https://colab.research.google.com/drive/1jhuTI5EvrFLxM09NVj4rUQbBFtf0lNid?usp=sharing.

Question 2: