Mathematical Statistics I – Midterm Exam

April 25, 2019

1. A test indicates the presence of a particular disease 90% of the time when the disease is present and the presence of the disease 2% of the time when the disease is not present. If 1% of the population has the disease, calculate the probability that a person selected at random has the disease if the test indicates the presence of the disease.

2. Let X be the number of total Bernoulli trials until the rth success occurs, where the probability of success is p.

   (1) Provide the pmf of X.

   (2) Compute the variance of X.

3. Let X have a Poisson distribution with $P(X = 0) = 0.5$, $X \sim \text{Pois}(\lambda)$. A random variable Y has a pmf

$$f_Y(y) = \begin{cases} p + (1-p)f_X(y), & y = 0 \\ (1-p)f_X(y), & y = 1, 2, 3, \ldots \end{cases}$$

   where $p \in (0, 1)$ is a parameter and is called as zero-inflated Poisson distribution. Also, a random variable Z has a pmf

$$f_Z(z) = \frac{P(X = z)}{P(X > 0)}, \quad z = 1, 2, \ldots$$

   and it is called as zero-truncated Poisson distribution.

   (1) Derive the moment generating function (mgf) of a Poisson random variable, X after obtaining the value of λ.

   (2) Using the mgf of X, compute the variance of X.

   (3) Derive the moment generating function (mgf) of Y.

   (4) Using the mgf of Y, compute the mean and variance of Y.

   (5) Find the pmf of Z.

   (6) Compute the mean and variance of Z.

4. Let X be a random variable following the cumulative distribution

$$F_X(x) = \begin{cases} ce^{x}, & x \leq 0 \\ 1 - ce^{-x}, & x > 0 \end{cases}$$

   where c is a constant.

   (1) Find the value of c and the pdf of X.

   (2) Obtain the mgf of X.

   (3) Using the mgf of X, compute the mean and variance of X.