

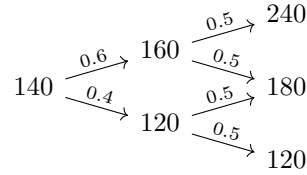
# Financial Mathematics I (SNU) – Exam 1

May 7, 2020

1. Consider a two-period binomial model with the time index  $t = 0, 1, 2$ . This model has two underlying assets; a bank account process  $(G_t)_{t=0,1,2}$  given by

$$G_0 = G_1 = 1, \quad G_2 = 1.2$$

and stock with price process given by the following tree.



- (a) Construct a sample space  $\Omega$ , a probability  $\mathbb{P}$ , a filtration  $(\mathcal{F}_t)_{t=0,1,2}$ , and stock price process  $(S_t)_{t=0,1,2}$  representing this market.
- (b) Consider an option with payoff

$$X_2 = \begin{cases} 30 & \text{if } S_2 = 240 \\ 15 & \text{if } S_2 = 180 \\ 0 & \text{if } S_2 = 120 \end{cases}$$

and maturity  $t = 2$ .

- i. Find the price process  $(X_t)_{t=0,1,2}$  and the hedging portfolio of this option.
- ii. Find the risk-neutral measure  $\mathbb{Q}$ .
2. Let  $C_E$  and  $C_A$  be the prices of European call and American call options, respectively, for the same maturity and strike. Show that  $C_E = C_A$  when the short rate  $r \geq 0$ .
3. Let  $B$  be a Brownian motion. Calculate  $\mathbb{E}(B_1|B_3)$ .
4. Solve the following problems.
- (a) Let  $(X_t)_{t \geq 0}$  be a RCLL Gaussian process. Show that

$$\left( \int_0^t X_s ds \right)_{t \geq 0}$$

is a continuous Gaussian process. You may use, without proof, the fact that the limit (in the sense of convergence in distribution) of a sequence of normal random variables is normal.

- (b) Calculate  $\mathbb{E}(e^{\int_0^T B_u du})$ .