Modern Algebra 1 – Final Exam

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1. Prove or disprove.
   (1) $\mathbb{Z}_4 \oplus \mathbb{Z}_{18} \oplus \mathbb{Z}_{15} \approx \mathbb{Z}_3 \oplus \mathbb{Z}_{36} \oplus \mathbb{Z}_{10}$.
   (2) $\mathbb{Z}_8 \oplus \mathbb{Z}_{10} \oplus \mathbb{Z}_{24} \approx \mathbb{Z}_4 \oplus \mathbb{Z}_{12} \oplus \mathbb{Z}_{40}$.
2. Determine the number of element of order 5 in $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$.
3. Find all Abelian groups (up to isomorphism) of order 360.
4. Show that if H and N are subgroups of a group G, and N is normal subgroup in G, then $H \cap N$ is normal subgroup in H.
5. Let G and $\overline{G}$ be groups and let ϕ be a homomorphism from G to $\overline{G}$.
   (1) Prove that $G/\ker(\phi) \approx \phi(G)$.
   (2) Let $SL(2,\mathbb{R}) = \{A \in GL(2,\mathbb{R}) \mid \det(A) = 1\}$. Show that
   $$GL(2,\mathbb{R})/SL(2,\mathbb{R}) \approx \mathbb{R}^*.$$
6. What is the characteristic of $\mathbb{Z}_6 \oplus \mathbb{Z}_{15}$?
7. Prove that the only ideals of a field F are $\{0\}$ and F itself.
8. Let R be a finite commutative ring with unity. Show that I is prime ideal if and only if I is maximal ideal.