수리통계학 2 – HW3

파이낸스경영학과
2017004093
양 정 우

7.1-6. $n=37$, $\bar{x}=11.95$, $s=11.80$

$$\left[11.95 - z_{0.025}\frac{11.8}{\sqrt{37}},\ 11.95 + z_{0.025}\frac{11.8}{\sqrt{37}}\right] = [8.15,\ 15.75]$$

$$\approx \left[11.95 - t_{0.025}(36)\cdot\frac{11.8}{\sqrt{37}},\ 11.95 + t_{0.025}(36)\frac{11.8}{\sqrt{37}}\right] = [8.016,\ 15.884]$$

7.1-16

$X \sim N(\mu, \sigma^2)$, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$$P\left[\chi^2_{1-\frac{\alpha}{2}}(n-1) \le \frac{(n-1)S^2}{\sigma^2} \le \chi^2_{\frac{\alpha}{2}}(n-1)\right] = 1-\alpha$$

$$P\left[(n-1)S^2/\chi^2_{\frac{\alpha}{2}}(n-1) \le \sigma^2 \le (n-1)S^2/\chi^2_{1-\frac{\alpha}{2}}(n-1)\right] = 1-\alpha$$

$n=13$, $12S^2 = \sum_{i=1}^{13}(x_i-\bar{x})^2 = 128.41$

$$P\left[12S^2/\chi^2_{0.05}(12) \le \sigma^2 \le 12S^2/\chi^2_{0.95}(12)\right] = 0.9$$

$$\Rightarrow P\left[128.41/21.02 \le \sigma^2 \le 128.41/5.226\right] = 0.9$$

$$\Rightarrow [6.10718,\ 24.57124]$$ ▨

7.2-3

i) $X \sim (\mu_X, \sigma_X^2)$, $Y \sim (\mu_Y, \sigma_Y^2)$, $\sigma_X^2 = \sigma_Y^2 = \sigma^2$

$\bar{X} \sim (\mu_X, \frac{\sigma^2}{12})$, $\bar{Y} \sim (\mu_Y, \frac{\sigma^2}{15})$

$\bar{X}-\bar{Y} \sim (\mu_X-\mu_Y, \frac{\sigma^2}{12}+\frac{\sigma^2}{15})$

$$Z = \frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{S_X^2}{12}+\frac{S_Y^2}{15}}} \longrightarrow N(0,1) \text{ by CLT}$$

$$\left[(65.1-68.2) - z_{0.01}\cdot\sqrt{\frac{16}{12}+\frac{9}{15}},\ (65.1-68.2) + z_{0.01}\cdot\sqrt{\frac{16}{12}+\frac{9}{15}}\right]$$

$\Rightarrow$ 98% C.I for $\mu_X-\mu_Y$ is $[-5.134,\ 0.134]$

ii) $\dfrac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}} \sim t(n+m-2)$

$\Rightarrow \left[(\bar{X}-\bar{Y}) - t_{0.01}\cdot S_p\sqrt{\frac{1}{n}+\frac{1}{m}},\ (\bar{X}-\bar{Y}) + t_{0.01}\cdot S_p\sqrt{\frac{1}{n}+\frac{1}{m}}\right]$

$\Rightarrow \left[(65.1-68.2) - 2.48\cdot 3.41\sqrt{\frac{1}{12}+\frac{1}{15}}\right.$

$\left.(65.1-68.2) + 2.48\cdot 3.41\sqrt{\frac{1}{12}+\frac{1}{15}}\right]$

$\Rightarrow [-5.845,\ 0.845]$

7.2-14

$X \sim N(\mu_X, \sigma_X^2)$ $\bar{X} \sim N(\mu_X, \frac{\sigma_X^2}{n})$

$Y \sim N(\mu_Y, \sigma_Y^2)$ $\bar{Y} \sim N(\mu_Y, \frac{\sigma_Y^2}{m})$

$\dfrac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \sim F(m-1, n-1)$

$P\left[F_{1-\frac{\alpha}{2}}(m-1,n-1) \le \dfrac{S_Y^2/\sigma_Y^2}{S_X^2/\sigma_X^2} \le F_{\frac{\alpha}{2}}(m-1,n-1)\right] = 1-\alpha$

$P\left[F_{1-\frac{\alpha}{2}}(m-1,n-1)\cdot\dfrac{S_X^2}{S_Y^2} \le \dfrac{\sigma_X^2}{\sigma_Y^2} \le F_{\frac{\alpha}{2}}(m-1,n-1)\cdot\dfrac{S_X^2}{S_Y^2}\right] = 1-\alpha$

If $n=13$, $m=9$, $12S_X^2 = 128.41$, $8S_Y^2 = 36.12$, $\alpha = 0.02$

$\Rightarrow \left[0.1764\cdot\dfrac{128.41/12}{36.12/8},\ 4.4993\cdot\dfrac{128.41/12}{36.12/8}\right]$

$\Rightarrow$ 98% C.I for $\dfrac{\sigma_X^2}{\sigma_Y^2}$ is $[0.4114,\ 10.4895]$

$\dfrac{\sigma_X}{\sigma_Y}$ is $[0.6414,\ 3.2381]$

7.3-6

$X \sim \text{Ber}(p)$. $\sum_{i=1}^{n} X_i \sim \text{Bin}(n,p)$.

$\dfrac{\sum X_i/n - p}{\sqrt{p(1-p)/n}} \to N(0,1)$ by CLT

$\Leftrightarrow P\left(\dfrac{\sum X_i}{n} - z_{\frac{\alpha}{2}}\sqrt{\dfrac{p(1-p)}{n}} \le p \le \dfrac{\sum X_i}{n} + z_{\frac{\alpha}{2}}\sqrt{\dfrac{p(1-p)}{n}}\right) = 1-\alpha$

$\Rightarrow P\left(\dfrac{\sum X_i}{n} - z_{\frac{\alpha}{2}}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \le p \le \dfrac{\sum X_i}{n} + z_{\frac{\alpha}{2}}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right) = 1-\alpha$

$\Rightarrow P\left(\hat{p} - z_{0.01}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{5757}} \le p \le \hat{p} + z_{0.01}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{5757}}\right) = 0.98$

$$\hat{p} = \frac{1491}{5751} = 0.26$$

$$\Rightarrow P\left(0.26 - 2.32\sqrt{\frac{0.26 \cdot 0.74}{5751}},\ 0.26 + 2.32 \cdot \sqrt{\frac{0.26 \cdot 0.74}{5751}}\right) = 0.98.$$

$\Rightarrow$ 98% C.I for p is $(0.2465,\ 0.2734)$.

7.3 - 11

C.I for $p_1 - p_2$

$$\therefore \frac{y_1}{n_1} - \frac{y_2}{n_2} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{(y_1/n_1)(1-y_1/n_1)}{n_1} + \frac{(y_2/n_2)(1-y_2/n_2)}{n_2}}$$

$n_1 = 1300,\ n_2 = 1100,\ y_1 = 520,\ y_2 = 385$

$$\frac{y_1}{n_1} = 0.4,\ \frac{y_2}{n_2} = 0.35$$

$$\Rightarrow [0.4 - 0.35 - 1.96 \cdot 0.019,\ 0.4 - 0.35 + 1.96 \cdot 0.019]$$

$$\Rightarrow [0.011223,\ 0.088777]$$