

# **Bankruptcy and Liquidation Prediction Model**

Math Capstone PBL (Data Analysis) – Project #2

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Jaeseon Lee<sup>1</sup> Junwoo Yang<sup>2</sup>

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<sup>1</sup>Department of Economics & Finance  
Hanyang University

<sup>2</sup>Department of Finance  
Hanyang University

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# **Introduction**

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### **Bankruptcy and Liquidation Prediction by Logistic Regression**

- What are the variables associated with bankruptcy and liquidation?
- Which company does model suggest will go bankrupt or liquidate in the next three years?

## WRDS Compustat – Capital IQ <sup>1</sup>

- 226,866 observations × 981 variables
- Fundamentals annual of companies that are actively trading on the NYSE, AMEX, NASDAQ, TSX, or NYSE/Arca exchanges from 2000 to 2020

## United States Cities Database <sup>2</sup>

- 29,488 observations × 17 variables
- This data include city name, state abbr., state name, county fips, county name, longitude and latitude of city, etc.

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<sup>1</sup><http://wrds-web.wharton.upenn.edu.ssl.access.hanyang.ac.kr/wrds/ds/compd/funda/index.cfm?navId=83>

<sup>2</sup><https://simplemaps.com/data/us-cities>

## Variable groups

- Identifying Information
- Identifying Information, cont.
- Company Descriptor
- Balance Sheet Items
- Income Statement Items
- Cash Flow Items
- Miscellaneous Items
- Supplemental Data Items
- Map Items

# Data description

acctstd	auop	costat	dlc	ebit	gind	ivch	naics4	pidom	reajo	tfvce	txtubbegin
acdo	auopic	county_fips	dlclch	ebitda	glea	ivncf	naics5	pifo	recch	tfvl	txtubend
aco	bkvlp5	county_name	dldte	ein	glced	ivst	naics6	pnca	recco	tic	txtubposdec
acodo	BL	chshd	dlsrn	emp	glceps	ivstch	naicsh	pncad	recd	tlcf	txtubposinc
acominc	busdesc	cshi	dtis	epsfi	glcep	lat	ni	pncaebs	rect	tstk	txtubpspdec
acox	caps	csho	dlto	epsfx	gp	lco	niadj	pncwia	recta	tstkc	txtubospinc
act	capx	cshpri	dltp	epspi	gsector	lcax	nopi	pncwip	rectr	tstkn	txtubsettle
add1	capxv	csrh	dltr	epspx	gsubind	lcodxr	nopio	pnrsho	reuna	tstkp	txtubooflimit
addzip	census_region	cshtr_c	dtlt	esoptc	gvkey	lct	np	ppget	revt	txach	txtubatr
adjex_c	ceoso	cshtr_f	dm	esopdlt	ib	lifr	oancf	ppent	sale	txbco	txtubintbs
adjex_f	ceq	cstk	dn	esopnr	ibadj	lifrp	oiadp	ppeveb	scf	txbcf	txtubintis
ajex	ceql	cstkv	do	esopt	ibc	lno	oibdp	prca	seq	txc	txw
ajp	ceqt	cstke	donr	esub	ibcom	lo	opeps	prcad	seqo	txdb	upd
aldo	cfoso	curcd	dp	esubc	ibmii	lol2	oprexpx	pcaeps	sic	txdba	wcap
am	ch	curncd	dpact	exchg	icapt	long	optca	prcc_c	sich	txdbc	weburl
ano	che	currtr	dpc	exe	idbflag	loxdr	optdr	prcc_f	siv	txdbcl	xacc
ao	chech	cusip	dpvieb	fatb	idit	lpql1	optex	prch_c	spce	txdc	xad
acocidergl	ci	datadata	drc	fatc	incorp	lse	optextd	prch_f	spced	txfed	xi
aociother	cibegni	dc	drlt	fate	intan	lt	optfvgr	prcl_c	spceeps	txdfo	xido
aocipen	cicurr	dclo	ds	fatl	intano	lu3	optgr	prcl_f	spcindcd	txdi	xidoc
acocsecgl	cidergl	dcom	dt	fatn	intc	mib	optlife	priusa	spcseccd	txditc	xint
aodo	cik	dcpstk	dudd	fato	intpn	mibn	optosby	prsho	spcsrc	txds	xintopt
aol2	cimii	dcs	dv	fatp	invch	mibt	optosey	prstkc	spi	txfed	xopr
aodoch	ciother	dcvrs	dvc	fax	invfg	mii	optprcby	pstk	sppe	txfo	xpp
aox	cipen	dcvsub	dvp	fca	invo	mkvalt	optprcca	pstk	sppiv	txndb	xpr
ap	cisecgl	dcvt	dvpva	fdate	invrn	mrc1	optprcex	pstk	src	txndba	xrd
apalch	citolat	dd	dvpssp_c	fiao	invt	mrc2	optprcye	pstkn	sstk	txndbl	xrdp
apdedate	city	dd1	dvpssp_f	fic	invwip	mrc3	optprcgr	pstk	stalt	txndbr	xrent
aqc	cld2	dd2	dvpssx_c	fincf	ipodate	mrc4	optprcw	pstk	state	txo	xsga
aqi	cld3	dd3	dvpssx_f	folo	ismod	mrc5	optfr	rdip	state.name	txp	
aqpl1	cld4	dd4	dvt	folopo	itcb	mrct	optvol	rdipa	stkco	txpd	
aqs	cld5	dd5	dxd2	fyr	itci	mrcta	pdate	rdipd	stkcpa	txr	
at	cogs	dfs	dxd3	fyc	ivaco	msa	pddur	rdipeps	stko	txs	
au	comm	diladj	dxd4	gdwl	ivaeq	naics2	phone	re	teq	txt	
aul3	comm1	dilavx	dxd5	ggroup	ivao	naics3	pi	rea	tfa	txtubadjust	

**Table 1:** Variable names

## Response variable

The response variable BL is defined as binary as follows:

$$BL = \begin{cases} 1 & \text{if it went bankrupt or liquidated in 2011–13} \\ 0 & \text{otherwise (solvent company).} \end{cases}$$

year	All deletion	Bankruptcy	Liquidation	BL
2011	241	1	16	17
2012	363	6	29	35
2013	348	8	38	46
2014	369	3	47	50
2015	348	8	36	44
2016	356	10	31	41
2017	273	6	1	7
2018	243	8	1	9
2019	266	16	0	16
2020	99	4	0	4

**Table 2:** Number of deleted companies

## Explanatory variables: fundamentals of 2010

aco: current assets that are not included in cash, cash equivalents, receivables or inventory on the Balance Sheet.

aql1: assets measured at fair value using observable inputs based on unadjusted quoted prices for identical instruments in active markets.

caps: a group of capital accounts other than capital stock or retained earnings.

csho: net number of all common shares outstanding at year-end, excluding treasury shares and scrip.

cstk: total par, carrying, or stated value of all common/ordinary capital.

## Explanatory variables: fundamentals of 2010

glcea: after-tax gain or loss on a sale that is excluded from the Standard & Poor's Core Earnings calculation.

idbfalg: source of data for the company.

optfvgr: weighted average fair value of options granted during the year.

spced: Standard & Poor's Core Earnings EPS diluted value.

stalt: status alert as to whether the company is in bankruptcy or undergoing a leveraged buyout.

stkcpa: amount of stock-based compensation expensed on the Income Statement during the current period on an after-tax basis.

## Logistic regression model

$$y_i (= \text{BL}_i) \sim \text{Bernoulli}(p_i)$$

$$\text{logit}(p_i) = \log \frac{p_i}{1 - p_i} = \beta X_i, \quad p_i = \frac{1}{1 + e^{-\beta X_i}}$$

$$\text{where } \beta = \begin{bmatrix} \beta_0 & \beta_1 & \cdots & \beta_m \end{bmatrix}, \quad X_i = \begin{bmatrix} 1 & x_{1,i} & \cdots & x_{m,i} \end{bmatrix}^T$$

## Maximum Likelihood Estimation (MLE)

$$L(\beta | X_1, \dots, X_n) = \prod_{i=1}^n (p_i)^{y_i} (1 - p_i)^{1-y_i}$$

$$\log L(\beta | X_1, \dots, X_n) = \sum_{i=1}^n y_i \log p_i + \sum_{i=1}^n (1 - y_i) \log(1 - p_i)$$

# **EDA**

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## **North American Industrial Classification System** <sup>3</sup>

NAICS is a hierarchical structure and can consist of up to six digits/levels. It is a comprehensive system covering all economic activities. There are 20 sectors and 1,057 industries in 2017 NAICS United States.

## **NAICS vs. SIC**

The NAICS was developed to eliminate the inconsistent logic utilized in the SIC system and to increase specificity from the 4 digit SIC system by creating a 6 digit NAICS code. The last revision of the SIC was in 1987.

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<sup>3</sup><http://www.census.gov/epcd/www/naics.html>

# Handling missing values

Sector	#	Description
11	18	Agriculture, Forestry, Fishing and Hunting
21	426	Mining, Quarrying, and Oil and Gas Extraction
22	248	Utilities
23	78	Construction
31–33	2193	Manufacturing
42	169	Wholesale Trade
44–45	235	Retail Trade
48–49	148	Transportation and Warehousing
51	652	Information
52	2122	Finance and Insurance
53	341	Real Estate and Rental and Leasing
54	233	Professional, Scientific, and Technical Services
55	0	Management of Companies and Enterprises
56	111	Administrative and Support and Waste Management and Remediation Services
61	26	Educational Services
62	117	Health Care and Social Assistance
71	43	Arts, Entertainment, and Recreation
72	106	Accommodation and Food Services
81	17	Other Services (except Public Administration)
92	0	Public Administration
99	105	Nonclassifiable

**Table 3:** Structure of 2017 NAICS

# Handling missing values

Monster Beverage Corp		Kellogg Co	
31	Manufacturing	31	Manufacturing
312	Beverage and Tobacco Product Manufacturing	311	Food Manufacturing
3121	Beverage Manufacturing	3112	Grain and Oilseed Milling
31211	Soft Drink and Ice Manufacturing	31123	Breakfast Cereal Manufacturing
312111	Soft Drink Manufacturing	311230	Breakfast Cereal Manufacturing
Coca Cola Consolidated Inc		Nike Inc	
31	Manufacturing	31	Manufacturing
312	Beverage and Tobacco Product Manufacturing	316	Leather and Allied Product Manufacturing
3121	Beverage Manufacturing	3162	Footwear Manufacturing
31211	Soft Drink and Ice Manufacturing	31621	Footwear Manufacturing
312111	Soft Drink Manufacturing	316210	Footwear Manufacturing

**Table 4:** Replacing order is upward.

## F-test

Let  $X_{j,1}, \dots, X_{j,n_j}$  be i.i.d. random variables with normal density and  $\bar{X}_j$  be sample means for  $j = 1, 2$ . Then

$$F = \frac{s_1^2}{s_2^2} \sim F(n_1 - 1, n_2 - 1) \quad \text{where } s_j^2 = \frac{1}{n_j - 1} \sum_{i=1}^{n_j} (X_{j,i} - \bar{X}_j)^2.$$

$$H_{F,0}: \frac{\sigma_1^2}{\sigma_2^2} = 1 \quad \text{vs.} \quad H_{F,1}: \frac{\sigma_1^2}{\sigma_2^2} \neq 1.$$

## Variable selection

### Student's t-test (when $H_{F,0}$ is accepted)

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$$

$$\text{where } s_p = \sqrt{\frac{(n_1 - 1)s_{X_1}^2 + (n_2 - 1)s_{X_2}^2}{n_1 + n_2 - 2}}$$

### Welch's t-test (when $H_{F,1}$ is accepted)

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t(\nu) \quad \text{where } \nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

## Statistical test

$$H_{t,0}: \mu_1 - \mu_2 = 0 \quad \text{vs.} \quad H_{t,1}: \mu_1 - \mu_2 \neq 0.$$

# Variable selection

acdo	capvx	cshtr_c	dm	emp	ibcom	lifrp	oiadp	pncaeps	spced	txditc	txtubxintbs
aco	ceq	cshtr_f	dn	esopct	ibmii	lo	oibdp	ppegt	spceeps	txds	txw
acodo	ceql	cstk	dp	esopt	icapt	loxdr	optdr	ppent	sppe	txfed	wcap
acominc	ceqt	cstke	dpact	esub	idit	lse	optex	ppeveb	sppiv	txfo	xacc
acox	ch	dc	dpc	esubc	intan	lt	optexd	prsho	stkco	txndb	xad
act	chech	dclo	dpvieb	fatb	intano	lul3	optfvgr	prstkc	stkpca	txndba	xidoc
aldo	ci	dcs	drc	fatc	intc	mibn	optgr	pstk	teq	txndbl	xint
am	cibegni	dcvsub	drlt	fate	intpn	mibt	optosby	pstkn	tfvce	txp	xopr
ao	cimii	dd	ds	fatn	invfg	mii	optosey	rdipd	tlcf	txpd	xpp
aocipen	cipen	dd1	dt	fato	invo	mkvalt	optprcby	rdipeps	tstk	txr	xpr
aodo	cisecgl	dd2	dudd	fatp	invrn	mrc1	optprcca	re	tstk	txs	xrd
aox	citotal	dd3	dv	fincf	inwwip	mrc2	optprcex	reajo	tstkn	txt	xrdp
ap	cl2	dd4	dvc	fopo	itcb	mrc3	optprcye	rech	txbco	txtubbegin	xrent
aqc	cl3	dd5	dvpv	popox	itci	mrc4	optprcgr	recd	txbcf	txtubend	xsga
aqlp1	cl4	dfs	dvt	gdwl	ivaeq	mrc5	optprcw	rect	txc	txtubposdec	
aqs	cl5	dilavx	dxd2	glica	ivstch	mrct	optvnl	recta	txdb	txtubposinc	
at	cogs	dlcch	dxd3	glcep	lco	mrcta	pi	rectr	txdba	txtubpospdec	
aul3	cshfd	dldte	dxd4	gp	lcox	ni	pidom	reuna	txdbc	txtubpospinc	
bkvlp5	cshi	dlto	dxd5	ib	lcodxr	niadj	pifo	revt	txdbcl	txtubsettle	
caps	csho	dltp	ebit	ibadj	lct	nopio	pnca	sale	txdc	txtubsoflimit	
capx	cshpri	dltt	ebitda	ibc	lifr	oancf	pncad	seq	txdfo	txtubtxtr	
adjex_c	aoloch	currtr	do	epspi	glceeps	lno	oprepsex	prcaeaps	pstkr	spi	txo
adjex_f	apalch	dcom	donr	epspx	invch	lol2	optca	prcc_c	pstkrv	sstk	txtubadjust
ajex	aqi	dcpstk	dvp	esopdlt	invt	long	optlife	prcc_f	rdip	tfva	txtubxintis
ajp	che	dcvsr	dvpssp_c	esopnr	ivaco	lp1	oprfr	prch_c	rdipa	tvf	xi
ano	cicurr	dcvt	dvpssp_f	exe	ivao	mib	pncwia	prch_f	rea	tstkp	xido
acidergl	cidergl	diladj	dvpssx_c	fatl	ivch	msa	pncwip	prcl_c	recco	txach	xintopt
aciother	ciother	dlc	dvpssx_f	fca	ivncf	nopi	pnrsht	prcl_f	seqo	txdfed	
acisecgl	cshr	dltis	epsfi	fiao	ivst	np	prca	pstkc	siv	txdi	
aol2	cstkcv	dltr	epsfx	glced	lat	opeps	prcad	pstkl	spce	txnbr	

**Table 5:** Selected and removed variables by t-test (245/350)

How to solve multicollinearity?

## Variance Inflation Factor (VIF)

$$\text{VIF}_i = \frac{1}{1 - R_i^2}$$

where  $R_i^2$  is the coefficient of determination of the regression equation

$$X_i = \beta_0 + \beta_1 X_1 + \cdots + \beta_{i-1} X_{i-1} + \beta_{i+1} X_{i+1} + \cdots + \beta_n X_n + \varepsilon.$$

We eliminate  $X_j$  with the highest VIF. We recalculate the VIF with the rest of the variables except  $X_j$ , and eliminate a variable with the highest VIF again. Repeat this process until  $\max \text{VIF} < 10$ .

# Variable selection

acdo	caps	dclo	dvc	fate	intc	ivstch	optgr	rdipeps	txbco	txr	xidoc
aco	chech	dcs	dvpd	fatn	invfg	mii	optprcw	recch	txbcf	txs	xpp
aldo	cipen	dcvsub	emp	fato	invo	mrcta	optvol	recta	txdbca	txtubposdec	
aocipen	cld3	dltp	esopct	fatp	invrn	nopio	pidom	spced	txdbcl	txtubsettle	
aqc	csho	dm	esopt	fincf	invwip	optdr	pnca	sppe	txdc	txtubsoflimit	
aqpl1	cshtr_c	drc	esubc	glcea	itcb	optex	pncad	stkcpa	txdfo	txw	
aqs	cstk	drlt	fatb	idit	itci	optextd	prsho	tfvce	txfed	wcap	
bkvips	dc	dudd	fatc	intano	ivaeq	optfvgr	prstkc	tstkn	txp	xad	
acodo	ceq	cshfd	dlcch	dxd2	ibadj	lo	mrct	pi	rectr	txdb	txtubpospdec
acominc	ceql	cshi	dldte	dxd3	ibc	loxdr	ni	pifo	reuna	txdba	txtubpospinc
acox	ceqt	cshpri	dlto	dxd4	ibcom	lse	niadj	pncaeps	revt	txditc	txtubtxtr
act	ch	cshtr_f	dltt	dxd5	ibmii	lt	oancf	ppegt	sale	txds	txtubxitbs
am	ci	cstke	dn	ebit	icapt	lul3	oiadp	ppent	seq	txfo	xacc
ao	cibegni	dd	dp	ebitda	intan	mibn	oibdp	ppeveb	spceeps	txndb	xint
aodo	cimii	dd1	dpact	esub	intpn	mibt	optosby	pstk	sppiv	txndba	xopr
aox	cisecgl	dd2	dpc	fopo	lco	mkvalt	optosey	pstk	stkco	txndbl	xpr
ap	citotal	dd3	dpvieb	fopox	lc ox	mrc1	optprcby	rdipd	teq	txpd	xrd
at	cld2	dd4	ds	gdwl	lc oxdr	mrc2	optprcca	re	tlcf	txt	xrdp
aul3	cld4	dd5	dt	glcep	lct	mrc3	optprcex	reajo	tstk	txtubbegin	xrent
capx	cld5	dfs	dv	gp	lifr	mrc4	optprc ey	recd	tstk	txtubend	xsga
capxv	cogs	dilavx	dvt	ib	lifrp	mrc5	optprcgr	rect	txc	txtubposinc	

**Table 6:** Selected and removed variables by VIF (90/245)

## Standardization

$$X' = \frac{X - \mu}{\sigma}$$

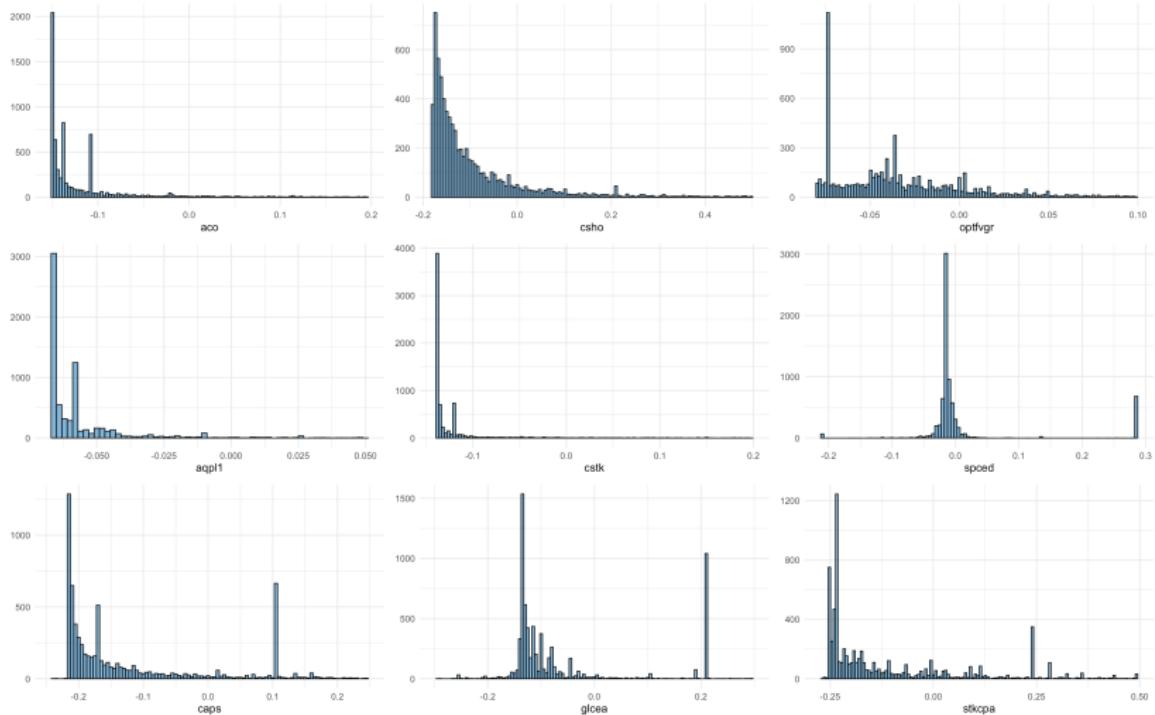
Standardized regression coefficients can be used to directly compare the effects of independent variables because standardized variables have the effect of eliminating the measurement unit or variation of the original variable.

# Summary statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
aco	7,380	0.000	1.000	-0.150	-0.149	-0.107	42.624
aqpl1	7,380	0.000	1.000	-0.067	-0.067	-0.056	55.435
caps	7,380	0.000	1.000	-0.389	-0.209	-0.029	34.987
csho	7,380	0.000	1.000	-0.179	-0.163	-0.053	44.172
cstk	7,380	0.000	1.000	-0.138	-0.138	-0.119	35.533
glcea	7,380	0.000	1.000	-6.029	-0.134	-0.043	71.202
optfvgr	7,380	0.000	1.000	-0.080	-0.065	-0.008	66.953
spced	7,380	0.000	1.000	-83.980	-0.016	-0.006	5.143
stkcpa	7,380	0.000	1.000	-1.930	-0.235	-0.005	42.503

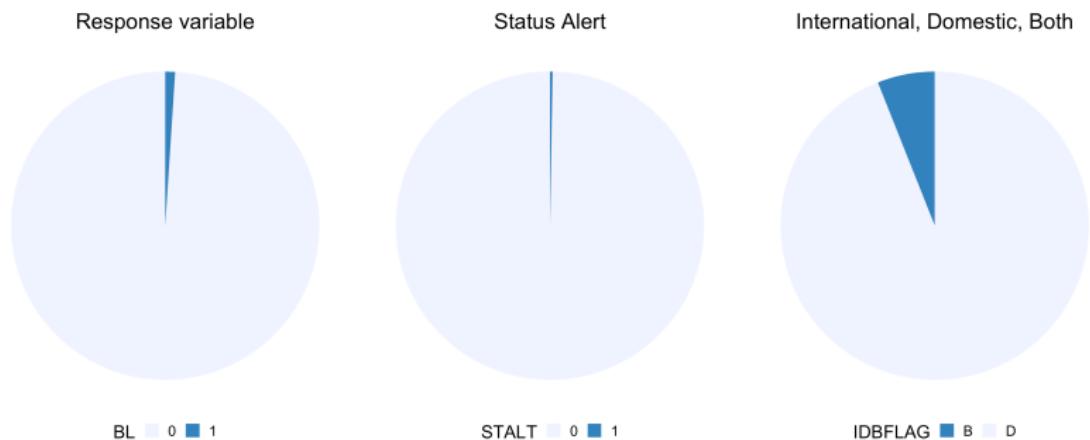
**Table 7:** Summary statistics of continuous variables

# Visualization



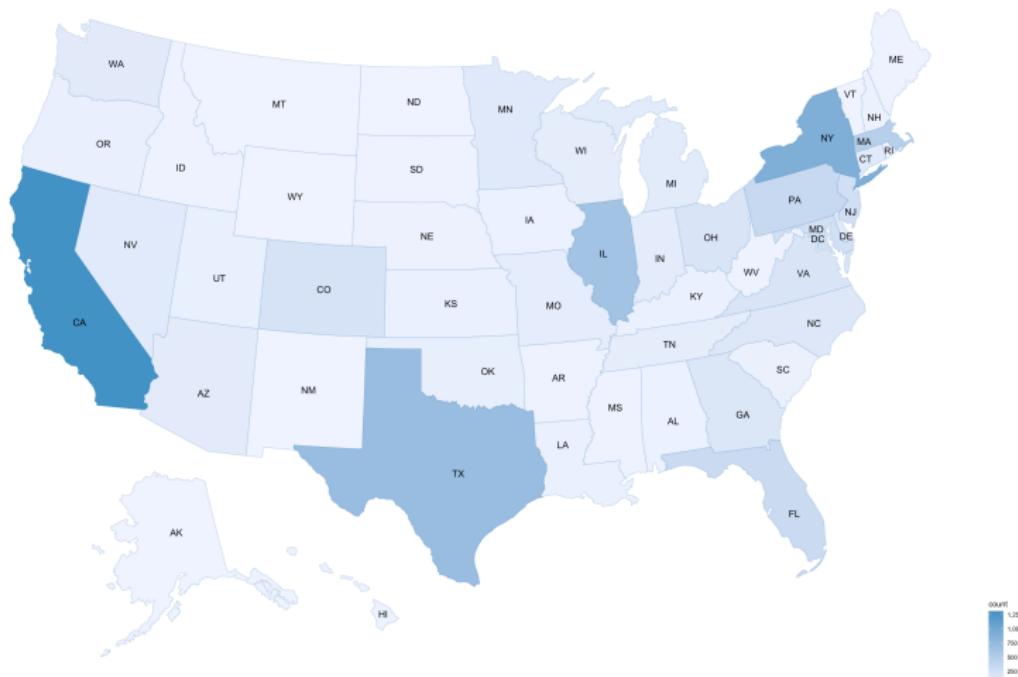
**Figure 1:** Histogram of continuous variables (truncated)

# Visualization



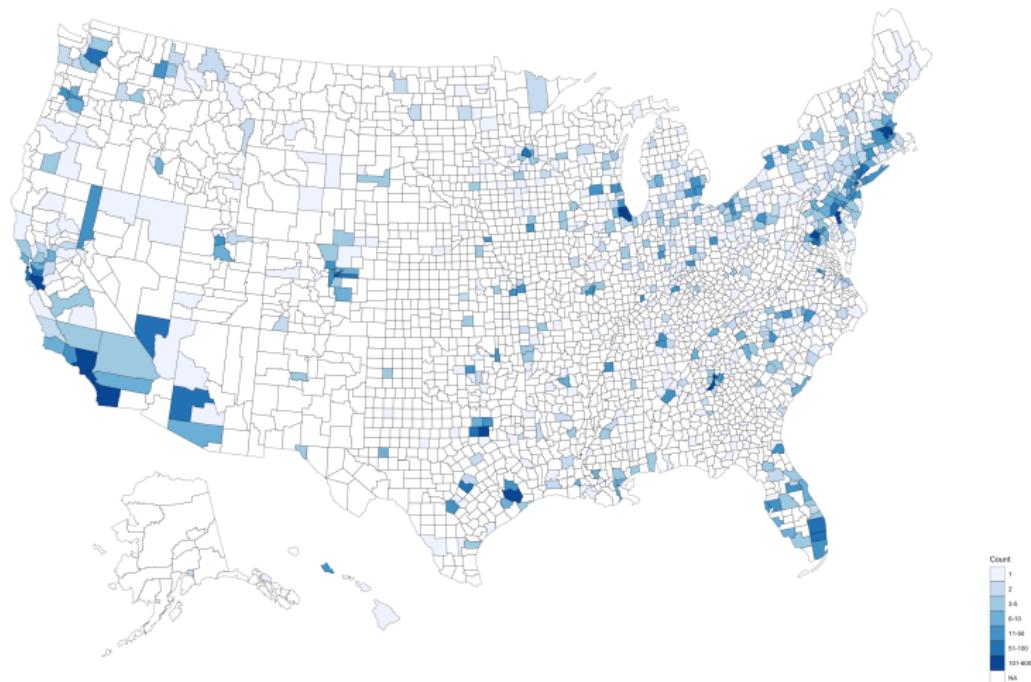
**Figure 2:** Pie chart of categorical variables

# Visualization



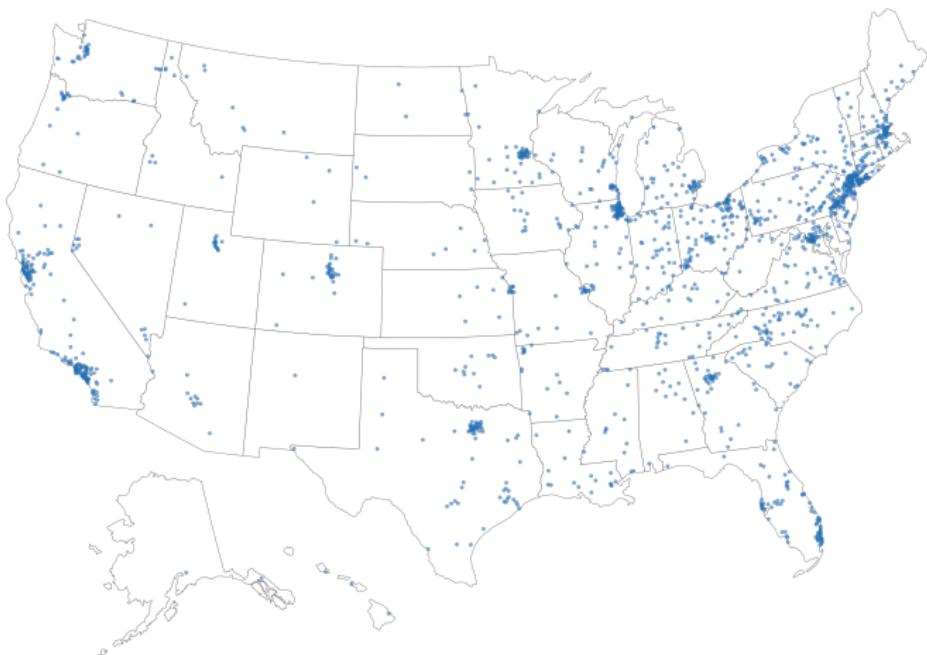
**Figure 3:** Number of companies by state

# Visualization



**Figure 4:** Number of companies by county

# Visualization

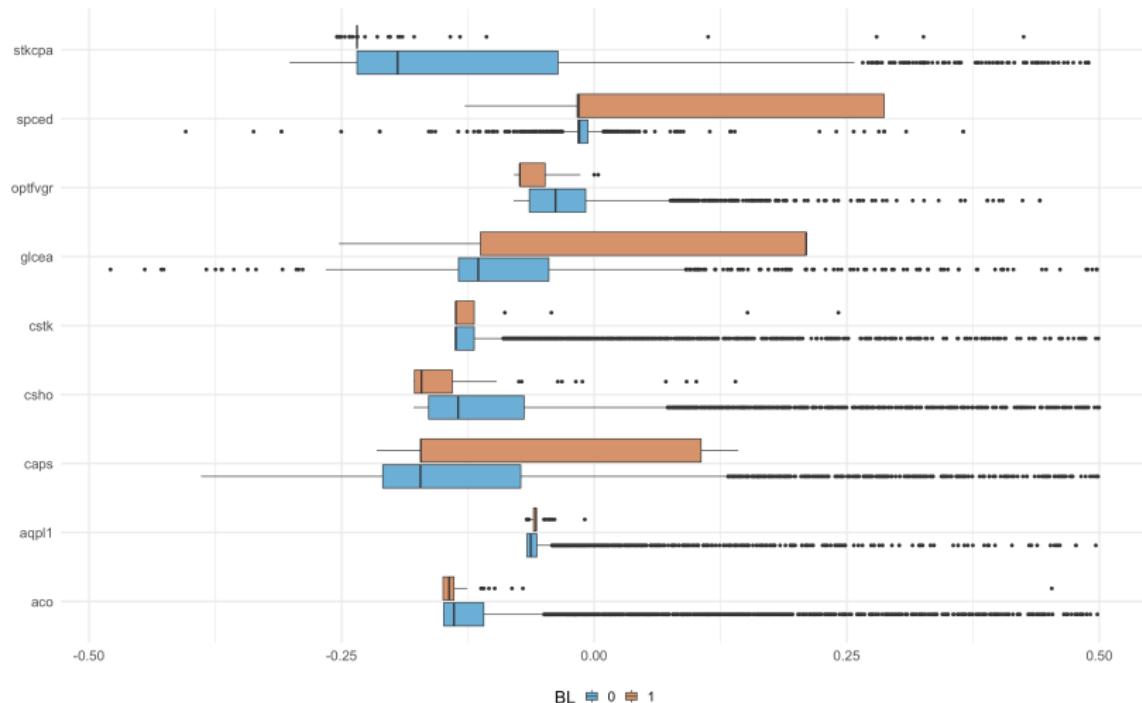


**Figure 5:** Cities that companies are located

# **Correlation Analysis**

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## BL — continuous variable



**Figure 6:** Box plots by BL (truncated)

BL — categorical variable

		stalt		idbflog	
		1	0	B	D
BL	1	4	72	1	0
	0	16	7288	0	445
20	7360	7380	76	76	7304
				445	6935
					7380

**Table 8:** Contingency tables

BL — categorical variable

## Pearson's chi-squared test

$$\chi^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(n_{ij} - e_{ij})^2}{e_{ij}} \sim \chi^2((r-1)(c-1)) \quad \text{where } e_{ij} = \frac{n_{i\cdot} n_{\cdot j}}{n}$$

$$H_0: p_{ij} = p_{i\cdot} p_{\cdot j} \quad \forall i, j \quad \text{vs.} \quad H_1: \exists i, j \text{ s.t. } p_{ij} \neq p_{i\cdot} p_{\cdot j}.$$

## Fisher's exact test

$$p = \frac{\binom{a+b}{a} \binom{c+d}{c}}{\binom{n}{a+c}} \sim \text{Hypergeometric}(n, a+b, a+c)$$

$$H_0: OR = 1 \quad \text{vs.} \quad H_1: OR \neq 1$$

where  $OR$  is true odds ratio.

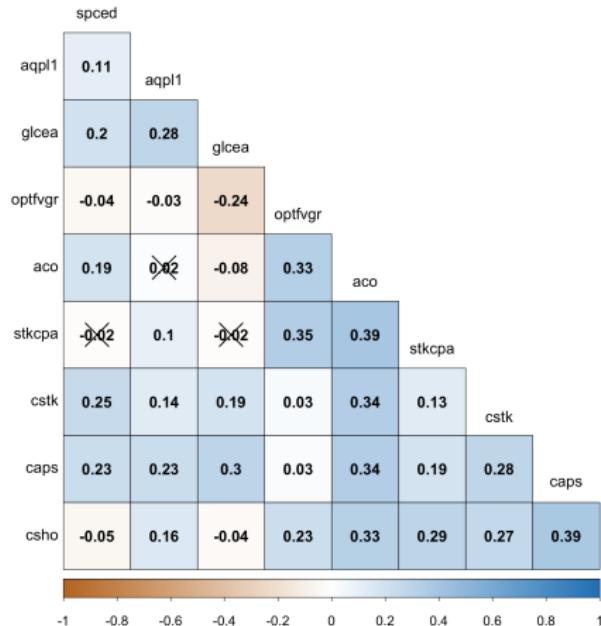
## BL — categorical variable

	Pearson's chi-squared test			Fisher's exact test			
	$\chi^2$	df	p-value	odds ratio	95% CI	p-value	
stalt	53.376	1	$2.755 \times 10^{-13}$	25.238	5.994	80.874	$4.441 \times 10^{-5}$
idbfalg	3.911	1	0.048	$\infty$	1.299	$\infty$	0.014

**Table 9:** Result of tests

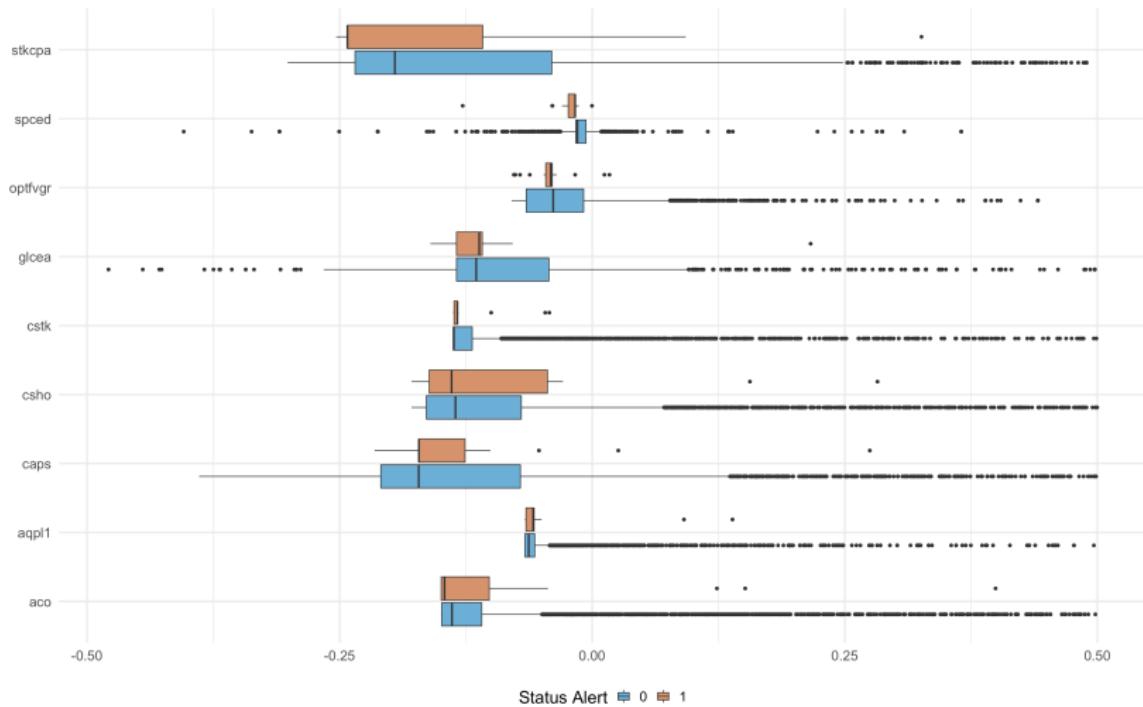
Neither `stalt` nor `idbfalg` are independent of BL. That is, the two categorical variables are associated with BL.

# Between continuous variables



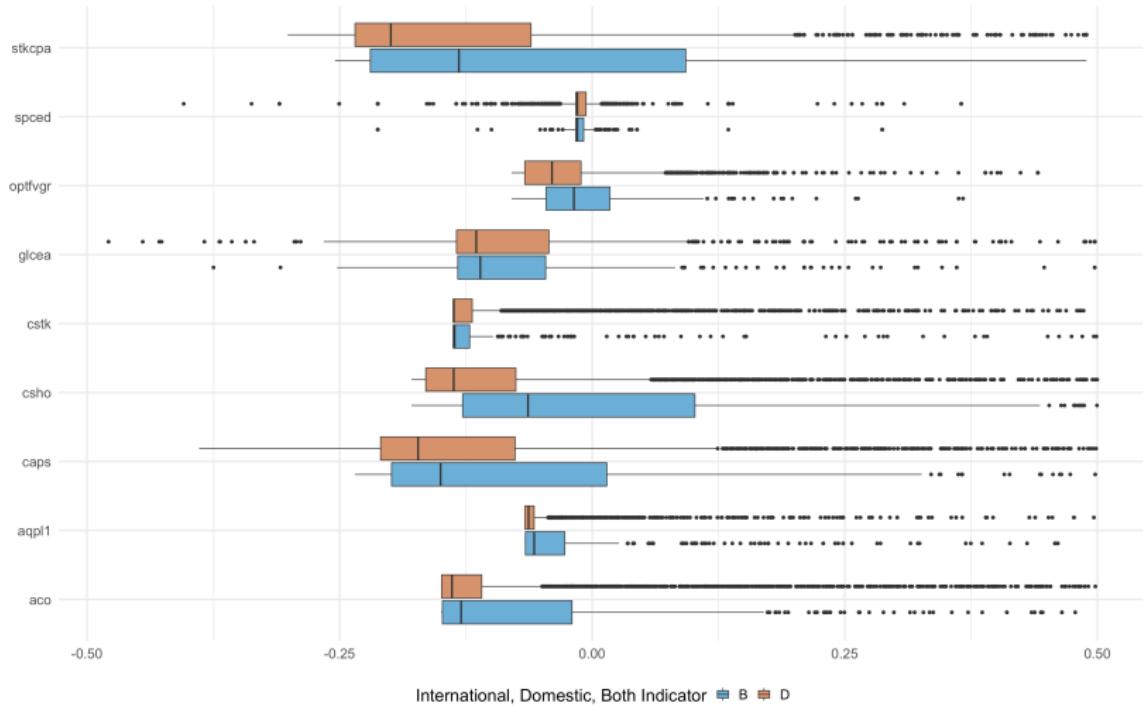
**Figure 7:** Spearman correlogram with significance test

## Between continuous and categorical variables



**Figure 8:** Box plots by stalt (truncated)

## Between continuous and categorical variables



**Figure 9:** Box plots by idbflag (truncated)

## Between categorical variables

		idbflag		
		B	D	
stalt	1	1	19	20
	0	444	6916	7360
		445	6935	7380

**Table 10:** Contingency table between stalt and idbflag

Pearson's chi-squared test			Fisher's exact test			
$\chi^2$	df	p-value	odds ratio	95% CI	p-value	
$2.648 \times 10^{-29}$	1	1	1.219	0.193 - 50.795	1	

**Table 11:** Result of tests

Therefore, stalt and idbflag are independent!

# **Modeling**

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## Method 1: subsampling training data

The first method is to subsample the negative set to reduce it to be the same size as the positive set, then fit the logistic regression model with the reduced data set.

## Method 2: weighted logistic regression

For a data set containing 5% positives and 95% negatives, we can assign each positive observation a weight of 0.95, and each negative observation a weight of 0.05. The weighted likelihood can be written as

$$L(\beta) = \prod_{i=1}^n (p_i)^{(1-w)y_i} (1 - p_i)^{w(1-y_i)}$$

where  $w$  represents proportion of events in the population.

## Type I error vs. Type II error

Both of them predict a fair amount of true positives as positives and true negatives as positives. This means that Type II error decreases, but Type I error increases. However, it is more dangerous for a company that is actually going to go bankrupt to be predicted not to go bankrupt!

## Subsampling – Training set vs. Test set

**Training set:** 60 bankrupt companies and 600 not bankrupt companies

**Test set:** 16 bankrupt companies and 160 not bankrupt companies

## Akaike Information Criterion (AIC)

Let  $k$  be the number of estimated parameters in the model and  $\hat{L}$  be the maximum value of the likelihood function for the model.

$$\text{AIC} = 2k - 2 \log \hat{L}$$

factor		numeric									
BL	idbflag	aco	chech	dm	fincf	mrcta	optfvgr	prstkc	stkcpa	txfed	
census_region	naics2	aqpl1	csho	dvc	glcea	nopio	optgr	recch	tstkn	txs	
exchg	stalt	bkvlp5	cshter_c	emp	idit	optex	optprcwa	recta	txdbca	wcap	
fic	state	caps	cstk	fate	intano	optexd	optvol	spced	txdc	xad	

**Table 12:** 44 variables before stepwise selection

## Final model

### Final model

Finally, our logistic regression model is

$$\begin{aligned}\log \frac{p_i}{1 - p_i} = & -203.6 - 7.19x_{aco,i} + 8.9x_{aqppl1,i} + 2.12x_{caps,i} \\ & - 7.58x_{csho,i} - 13.51x_{cstk,i} + 1.56x_{glcea,i} + 197.01x_{idbflag_D,i} \\ & - 19.96x_{optfvgr,i} - 2.42x_{spced,i} + 2.66x_{stalt_1,i} - 3.31x_{stkcpa,i}\end{aligned}$$

solving for  $p_i$ ,

$$p_i = (1 + \exp(203.6 + 7.19x_{aco,i} - 8.9x_{aqppl1,i} + \dots + 3.31x_{stkcpa,i}))^{-1}.$$

## Final model

	Estimate	Std. Error	z value	$P(> z )$
(Intercept)	-203.596	970.743	-0.210	0.834
aco	-7.190	6.036	-1.191	0.234
aqpl1	8.901	3.520	2.529	0.011
caps	2.117	0.961	2.204	0.028
csho	-7.575	3.088	-2.453	0.014
cstk	-13.510	11.405	-1.185	0.236
glcea	1.561	0.675	2.312	0.021
idbflag <sub>D</sub>	197.005	970.673	0.203	0.839
optfvgr	-19.957	6.620	-3.015	0.003
spced	-2.420	1.488	-1.627	0.104
stalt <sub>1</sub>	2.663	1.242	2.144	0.032
stkcpa	-3.308	1.449	-2.284	0.022

**Table 13:** Coefficients of final model

### Likelihood ratio test

$LR = 2(ULF - RLF) \sim \chi^2_{df=q}$  where  $q$  is # of restrictions.

$$H_0: \beta_i = 0 \quad \forall i \quad \text{vs.} \quad H_1: \exists i \text{ s.t. } \beta_i \neq 0.$$

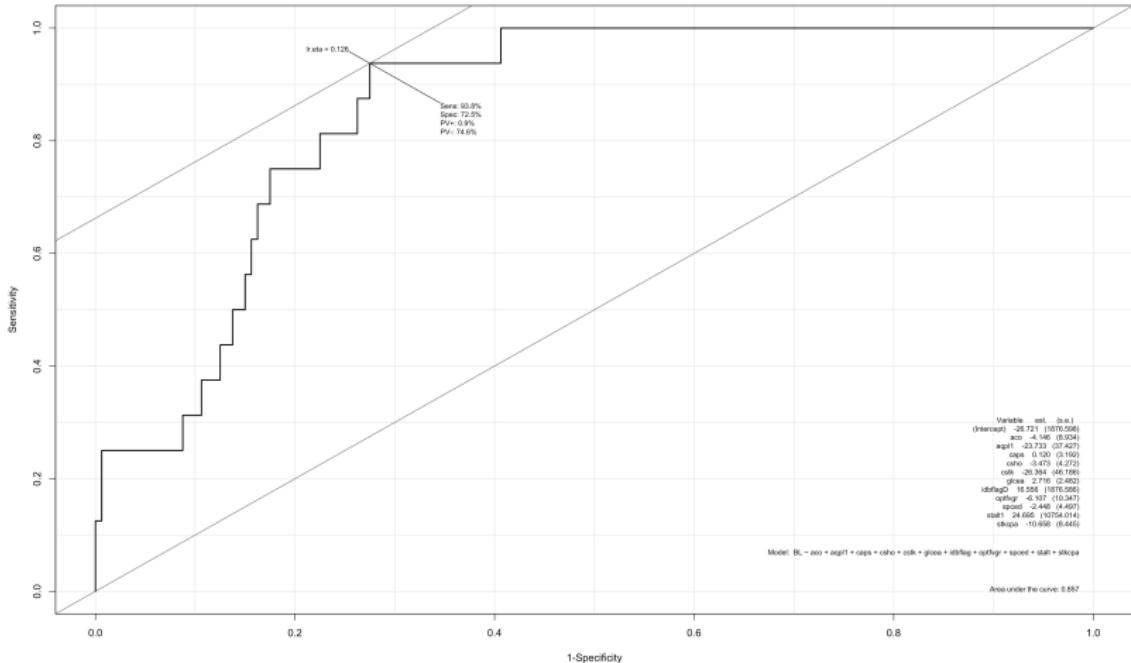
$$ULF - RLF = 402.12 - 311.41 = 90.71$$

$$q = 659 - 648 = 11$$

$$p\text{-value} = 1.210039 \times 10^{-14}$$

Therefore, at least one explanatory variable is significant in predicting the response variable.

# Measuring performance



**Figure 10:** ROC curve

# Measuring performance

		Predicted		
		Positive	Negative	
Actual	Positive	14	2	16
	Negative	43	117	160
		57	119	176

**Table 14:** Confusion matrix with cut-off of 0.1267

## Terminology

**TP:** True Positive. These are cases in which we predicted positive (they will go bankrupt or liquidate), and they actually went bankrupt or liquidated.

**TN:** True Negatives. We predicted negative, and they didn't actually go bankrupt or liquidate.

**FP:** False Positives. We predicted positive, but they didn't actually go bankrupt or liquidate. Also known as a Type I error.

**FN:** False Negatives. We predicted negative, but they actually went bankrupt or liquidated. We cared more about this error. Also known as a Type II error.

## Accuracy

Overall, how often is the classifier correct?

$$\frac{\text{TP} + \text{TN}}{\text{Total}} = \frac{14 + 117}{176} = 0.7443$$

## Misclassification Rate (Error Rate)

Overall, how often is it wrong?

$$\frac{\text{FP} + \text{FN}}{\text{Total}} = \frac{43 + 2}{176} = 0.2557$$

# Measuring performance

## True Positive Rate (Sensitivity, Recall)

When it's actually positive, how often does it predict positive?

$$\frac{\text{TP}}{\text{Actual Positive}} = \frac{14}{16} = 0.875$$

## False Positive Rate

When it's actually negative, how often does it predict positive?

$$\frac{\text{FP}}{\text{Actual Negative}} = \frac{43}{160} = 0.26875$$

## True Negative Rate (Specificity)

When it's actually negative, how often does it predict negative?

$$\frac{\text{TN}}{\text{Actual Negative}} = \frac{117}{160} = 0.73125$$

# Measuring performance

## Precision

When it predicts positive, how often is it correct?

$$\frac{\text{TP}}{\text{Predicted Positive}} = \frac{14}{57} = 0.2456$$

## Prevalence

How often does the positive condition actually occur in test set?

$$\frac{\text{Actual Positive}}{\text{Total}} = \frac{16}{176} = 0.0909$$

## AUC (Area Under an ROC Curve)

$$\text{AUC} = 0.857$$

## **Conclusion**

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# Conclusion

## 11 selected variables

aco, aqpl1, caps, csho, cstk, glcea, idbfalg, optfvgr, spced,  
stalt, stkcpa

## Performance

Accuracy: 74.43%, Sensitivity: 87.5%, Specificity: 73.13%

## Forecasting

Due to many missing values in above variables on 2020, we failed in forecasting. If variables are chosen in consideration of the 2020 missing values, the prediction will be successful because of good performance.

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