미분방정식 HW3-1

수리과학부
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양 준 수

4.3-3

$y''' + y'' + y' + y = 2e^{-t} + 4t$

$r^3 + r^2 + r + 1 = 0.$

$(r+1)(r^2+1) = 0 \quad r = -1, \pm i \rightarrow y_c = c_1e^{-t} + c_2\cos t + c_3\sin t$

Guess $y_p = Ate^{-t} + Bt + C$

$y_p' = (A - At)e^{-t} + B$

$y_p'' = (At - 2A)e^{-t}$

$y_p''' = (3A - At)e^{-t}$

$y_p''' + y_p'' + y_p' + y_p = 2Ae^{-t} + Bt + B + C = 2e^{-t} + 4t \Rightarrow A = 1,\ B = 4,\ C = -4$

$\therefore y_p = te^{-t} + 4t - 4$

$\therefore y = te^{-t} + 4t - 4 + c_1e^{-t} + c_2\cos t + c_3\sin t$

4.3-5

$y^{(4)} + 2y'' + y = 4 + \cos 2t$

$r^4 + 2r^2 + 1 = (r^2+1)^2 = 0. \quad r^2 = \pm i$ 중근

$y_c = c_1\cos t + c_2\sin t + c_3t\cos t + c_4t\sin t$

(check that $t\cos t$, $t\sin t$: solutions of homo. DE.

$(t\cos t)^{(4)} + 2(t\cos t)'' + t\cos t = (4\sin t + t\cos t) + 2(-2\sin t - t\cos t) + t\cos t = 0$)

Guess $y_p = A\cos 2t + B\sin 2t + C$ 1

$y_p' = 2B\cos 2t - 2A\sin 2t$

$y_p'' = -4A\cos 2t - 4B\sin 2t$ 2

$y_p''' = -8B\cos 2t + 8A\sin 2t$

$y_p^{(4)} = 16A\cos 2t + 16B\sin 2t$ 1

$y_p^{(4)} + 2y'' + y = 9A\cos 2t + 9B\sin 2t + C = 4 + \cos 2t \Rightarrow A = \frac{1}{9},\ B = 0,\ C = 4$

$\therefore y_p = \frac{1}{9}\cos 2t + 4 \Rightarrow y = \frac{1}{9}\cos 2t + 4 + c_1\cos t + c_2\sin t + c_3t\cos t + c_4t\sin t$

4.4-2

$y''' - y' = 4t$

$r^3 - r = r(r+1)(r-1) = 0. \quad r = -1, 0, 1 \Rightarrow y_c = c_1 e^t + c_2 e^{-t} + c_3$

$y_p = u_1 e^t + u_2 e^{-t} + u_3$

$y_p' = u_1 e^t - u_2 e^{-t} + \underline{u_1' e^t + u_2' e^{-t} + u_3'}$ Assume $= 0$.

$y_p'' = u_1 e^t + u_2 e^{-t} + \underline{u_1' e^t - u_2' e^{-t}}$ Assume $= 0$

$y_p''' = u_1 e^t - u_2 e^{-t} + \underline{u_1' e^t + u_2' e^{-t}} = 4t$

$$W = \begin{vmatrix} e^t & e^{-t} & 1 \\ e^t & -e^{-t} & 0 \\ e^t & e^{-t} & 0 \end{vmatrix} = 2$$

$$u_1' = \frac{1}{2} \cdot \begin{vmatrix} 0 & e^{-t} & 1 \\ 0 & -e^{-t} & 0 \\ 4t & e^{-t} & 0 \end{vmatrix} = 2te^{-t} \Rightarrow u_1 = -2te^{-t} - 2e^{-t} + c_1$$

$$u_2' = \frac{1}{2} \cdot \begin{vmatrix} e^t & 0 & 1 \\ e^t & 0 & 0 \\ e^t & 4t & 0 \end{vmatrix} = 2te^t \Rightarrow u_2 = 2te^t - 2e^t + c_2$$

$$u_3' = \frac{1}{2} \begin{vmatrix} e^t & e^{-t} & 0 \\ e^t & -e^{-t} & 0 \\ e^t & e^{-t} & 4t \end{vmatrix} = -4t \Rightarrow u_3 = -2t^2 + c_3$$

$y_p = (-2te^{-t} - 2e^{-t} + c_1)e^t + (2te^t - 2e^t + c_2)e^{-t} + (-2t^2 + c_3)$

$\Rightarrow -2t + 2t - 2t^2 = -2t^2$

$\therefore y = -2t^2 + c_1 e^t + c_2 e^{-t} + c_3$

4.4-7

$y''' + y' = \sec t, \quad y(0) = 2, \quad y'(0) = 1, \quad y''(0) = -2$

$r^3 + r = r(r^2+1) = 0. \quad r = 0, \pm i \Rightarrow y_c = c_1 \cos t + c_2 \sin t + c_3$

Let $y_p = u_1 \cos t + u_2 \sin t + u_3$,

$y_p' = u_2 \cos t - u_1 \sin t + \underline{u_1' \cos t + u_2' \sin t + u_3'}$ Assume $= 0$

$y_p'' = -u_1 \cos t - u_2 \sin t + \underline{u_2' \cos t - u_1' \sin t}$ Assume $= 0$

$y_p''' = -u_2 \cos t + u_1 \sin t - \underline{u_1' \cos t - u_2' \sin t} = \sec t$

$$W = \begin{vmatrix} \cos t & \sin t & 1 \\ -\sin t & \cos t & 0 \\ -\cos t & -\sin t & 0 \end{vmatrix} = 1$$

$$u_1' = \begin{vmatrix} 0 & \sin t & 1 \\ 0 & \cos t & 0 \\ \sec t & -\sin t & 0 \end{vmatrix} = -1 \Rightarrow u_1 = -t + c_1$$

$$u_2' = \begin{vmatrix} \cos t & 0 & 1 \\ -\sin t & 0 & 0 \\ -\cos t & \sec t & 0 \end{vmatrix} = -\tan t \Rightarrow u_2 = -\sec^2 t + c_2$$

$$u_3' = \begin{vmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ -\cos t & -\sin t & \sec t \end{vmatrix} = \sec t \Rightarrow u_3 = \sec t \cdot \tan t + c_3$$

$\therefore y_p = (-t + c_1)\cos t + (-\sec^2 t + c_2)\sin t + (\sec t \cdot \tan t + c_3)$

$\Rightarrow -t\cos t$

$\therefore y = -t\cos t + c_1\cos t + c_2\sin t + c_3 \Rightarrow y(0) = c_1 + c_3 = 2 \Rightarrow c_3 = 0.$

$y' = (c_2 - 1)\cos t + (t - c_1)\sin t \Rightarrow y'(0) = c_2 - 1 = 1 \Rightarrow c_2 = 2$

$y'' = (t - c_1)\cos t + (2 - c_2)\sin t \Rightarrow y''(0) = -c_1 = -2 \Rightarrow c_1 = 2$

$\therefore$ Solution of IVP : $y = -t\cos t + 2\cos t + 2\sin t$