미분기하학개론 1 - HW4

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2-3.3

$z = x^2+y^2 \cong$ plane.

Let $\mathbb{X} : \mathbb{R}^2 \to \mathbb{R}^3 \quad (x,y) \mapsto (x, y, h(x,y) = x^2+y^2)$.

$S := \{(x,y,z) \in \mathbb{R}^3 \mid z = x^2+y^2\}$ is grap of $h(x,y)$.

Thus S is regular surface.

$\mathbb{X}$ differentiable. $d\mathbb{X}_q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2x & 2y \end{pmatrix}$ injective $\left(\because \frac{\partial(x,y)}{\partial(x,y)} = 1 \neq 0\right)$.

and $\mathbb{X}$ injective.

So $\mathbb{X}$ is homeomorphism and parametrization.

To show that $S \cong \mathbb{R}^2$, we need to show $\mathbb{X}^{-1}$ is differentiable.

$\mathbb{X}^{-1} \circ \mathbb{X} = id$ is differentiable. By prop 1, $\mathbb{X}^{-1}$ is differentiable.

Hence $S \cong \mathbb{R}^2$

2-3.5

S : reg. surface.

$d : S \to \mathbb{R} \quad p \mapsto |p-p_0| \quad p_0 \notin S$.

d is differentiable $\Leftrightarrow$ $\forall p \in S$, $d \circ \mathbb{X}$ is differentiable at p.

Let $F : \mathbb{R}^3 \to \mathbb{R}$, $p \mapsto |p-p_0|$

$\det(dF_{(x,y,z)}) \neq 0$. for $p \neq p_0$.

F is differentiable for $p \neq p_0$.

$\therefore$ $d \circ \mathbb{X} = F \circ \mathbb{X}|_U$ diff.

($\because F|_S$ is differentiable ($\because p_0 \notin S$).)

$\therefore$ d is differentiable.