Mathematical Statistics II – Midterm Exam

October 29, 2019

1. Describe and prove the Central Limit Theorem (CLT).
2. Let $X_1, \ldots, X_n$ be a random sample from the normal distribution, $N(\mu, \sigma^2)$. We know that the sample mean $\overline{X}$ and the sample variance S^2 are independent.
   (1) Find the distribution of the sample mean $\overline{X}$ using the mgf of X_i.
   (2) Find the distribution of
   $$V = \frac{(n-1)S^2}{\sigma^2}.$$
   (3) Find the distribution of
   $$W = \frac{\overline{X} - \mu}{s/\sqrt{n}}.$$
   (4) Compute the mean and variance of W.
3. Let $X_1, \ldots, X_n$ $(n \geq 2)$ be a random sample following an exponential distribution with mean θ. We have order statistics $X_{(1)}, \ldots, X_{(n)}$ from $X_1, \ldots, X_n$.
   (1) Find the pdf of $X_{(i)}$, $i = 1, \ldots, n$.
   (2) Find the joint pdf of $X_{(i)}$ and $X_{(j)}$, where $1 \leq i < j \leq n$.
   (3) Find a S.S for θ.
   (4) Find the MLE of θ using a S.S for θ.
   (5) Find the UE of θ using the pdf of MLE of θ an its mean.
   (6) Find the MME of θ.
4. Let $X_1, \ldots, X_n$ be a random sample from the distribution with a pdf
   $$f(x|\theta) = \frac{2x}{\theta^2} I(0 \leq x \leq \theta), \quad \theta > 0.$$
   where $I(\cdot)$ is the indicator function.
   (1) Find the MME of θ.
   (2) Find the MLE of θ.
   (3) Find the UE (unbiased estimator) of θ using the pdf of MLE for θ.
5. We have a random sample, $X_1, \ldots, X_n$ with the following distribution.
   (1) $N(0, \theta_1 = \sigma^2)$. Find the MLE of θ_1 using a S.S for θ_1.
   (2) We have a pdf of X_i,
   $$f(x|\theta_2) = \frac{1}{2} I(\theta_2 - 1 \leq x \leq \theta_2 + 1).$$
   Find the MLE of θ_2, if it exists.