

# Financial Mathematics 1 – Exam 2

Junwoo Yang

June 4, 2020

1. For a two-dimensional Brownian motion  $(B_t^{(1)}, B_t^{(2)})_{t \geq 0}$ , calculate

$$\mathbb{E}(e^{\int_0^T \sqrt{t} dB_t^{(1)} + B_T^{(2)}}).$$

2. Consider the progressive  $\sigma$ -algebra

$$\Sigma := \{A \subseteq [0, \infty) \times \Omega : A \text{ is progressive}\}.$$

Show that a stochastic process  $X$  is progressively measurable if and only if  $X$  is  $\Sigma$ -measurable.

3. Let  $\theta \in \mathcal{H}_{\text{loc}}^2$ . Show that a process

$$M_t := e^{\int_0^t \theta_s dB_s - \frac{1}{2} \int_0^t \theta_s^2 ds}, \quad 0 \leq t \leq T$$

is a local martingale.

4. Show that a left-continuous adapted process is progressively measurable.

5. Let  $(B_t)_{t \geq 0}$  be a Brownian motion. Show that the process

$$\left( \cos B_t + \frac{1}{2} \int_0^t \cos B_s ds \right)_{t \geq 0}$$

is a martingale.

6. Let  $(B_t^{(1)}, B_t^{(2)}, B_t^{(3)})_{t \geq 0}$  be a three-dimensional Brownian motion. Define

$$X_t = te^{B_t^{(2)}} + \int_0^t s B_s^{(1)} dB_s^{(3)}.$$

Find the quadratic variation  $\langle X \rangle_t$  for  $t \geq 0$ .