3.1.14.

(a)(b) Obvious

(c) For all n, one of the endpoints of an interval remains *fixed* when we pass to the next; i.e., either $a_n = a_{n-1}$ or $b_n = b_{n-1}$. Hence, we obatin $(a_n - a_{n-1})(b_n - b_{n-1}) = 0$ or $a_n b_n + a_{n-1}b_{n-1} = a_n b_{n-1} + a_{n-1}b_n$.

3.1.20.

Case 1. $c_{n+1} = \frac{a_n+c_n}{2}$ then $|c_n - c_{n+1}| = \frac{b_n-a_n}{4} = \frac{b_0-a_0}{2^{n+2}}$

Case 2. $c_{n+1} = \frac{c_n+b_n}{2}$ then $|c_{n+1} - c_n| = \frac{b_n-a_n}{4} = \frac{b_0-a_0}{2^{n+2}}$

3.2.4.

$g(x) = x^4 + 2x^3 - 7x^2 + 3$; $g'(x) = 4x^3 + 6x^2 - 14x$.

Since $g(0) = 3, g(1) = -1$, and $g(2) = 7$, there is a root in the interval $[0, 1]$ and in $[1, 2]$. For $x_0 = 1$, $x_1 = 0.75$, $x_2 = 0.791$, $x_3 = 0.791$, and for $x_0 = 2$, $x_1 = 1.75$, $x_2 = 1.64$, $x_3 = 1.62$, $x_4 = 1.62$. The positive roots are 0.79 and 1.62.

3.2.7.

By the same way as Prob. 1.2.30., we see that $y = \frac{1}{\sqrt{2}}(x - \frac{\pi}{4}) + \frac{1}{\sqrt{2}}$ is the best linear approximation for $f(x) = \sin x$ in the vicinity of $x = \frac{\pi}{4}$. When $y = 0$, $x = \frac{\pi}{4} - 1$ which is x_1 for Newton's method starting with $x_0 = \frac{\pi}{4}$.