Mathematical Statistics 1 – Final Exam

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- 1. Let X_1, X_2 be independent random variables, each of which follows an exponential distribution with a mean 1. Let us consider $Y_1 = X_1 X_2$, $Y_2 = X_1 + X_2$, and $Z = X_1/(X_1 + X_2)$.
 - (1) Find the joint pdfs of Y_1 and Y_2 .
 - (2) Find the marginal pdfs of Y_1 and Y_2 .
 - (3) Compute $P(Y_2 < 5|Y_1 = -2)$.
 - (4) Find the marginal pdf of Z.
- 2. Solve each question.
 - (1) If two random variables X and Y have the correlation coefficient ρ , show that ρ satisfies the inequality $-1 \le \rho \le 1$.
 - (2) If X have continuous strictly increasing cdf $F_X(x)$ on the range a < x < b, show that $Y = F_X(x)$ has a uniform distribution on (0, 1).
 - (3) If (X, Y) has the bivariate normal pdf

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho^2)}(x^2 - 2\rho xy + y^2)\right\}$$

where $\operatorname{Corr}(X,Y) = \rho$, then find the conditional distribution of X given Y = y and write down its distribution notation (e.g. $X \sim \operatorname{Poi}(\theta)$). Here, you don't need to derive the marginal distribution of Y.

- 3. Let $X \sim N(\mu, \sigma^2)$.
 - (1) Derive the moment generating function (mgf) of X.
 - (2) Compute the variance of X.
 - (3) Find the pdf of $W=(X-\mu)^2/\sigma^2=V^2$ after showing the pdf of $V=(X-\mu)/\sigma$.
 - (4) Derive the mgf of W.
 - (5) Compute the mean and variance of W.
- 4. Let X_1 and X_2 be nonnegative integer random variables such that $X_1 + X_2 \le n$ with joint pmf

$$f(x_1, x_2) = \frac{n!}{x_1! x_2! (n - x_1 - x_2)!} p_1^{x_1} p_2^{x_2} (1 - p_1 - p_2)^{n - x_1 - x_2},$$

where $0 < p_1, p_2 < 1$ and $p_1 + p_2 \le 1$.

- (1) Derive the marginal pmf of X_1 .
- (2) Find the conditional pmf of X_2 , given $X_1 = x_1$.
- (3) Compute the covariance of X_1 and X_2 .