ADVANCED CALCULUS 1
ASSIGNMENT # 2 : 2019 SPRING

§2.1. # 2. Let $S = \{(x, y) \in \mathbb{R}^2 \,|\, xy > 1\,\}$. Show that S is open.

§2.3. # 5. Let $S = \{x \in \mathbb{R} \,|\, x \text{ is irrational}\,\}$. Is S closed?

§2.4. # 3. Find the accumulation points of the following sets in $\mathbb{R}^2$:
a. $\{(m, n) \,|\, m, n \text{ integers}\,\}$
b. $\{(p, q) \,|\, p, q \text{ rational}\,\}$
c. $\{(\frac{m}{n}, \frac{1}{n}) \,|\, m, n \text{ integers},\ n \neq 0\,\}$
d. $\{(\frac{1}{n} + \frac{1}{m}, 0) \,|\, n, m \text{ integers},\ n \neq 0,\ m \neq 0\}$

§2.6. # 5. Let $A \subset \mathbb{R}$ be bounded and nonempty and let $x = \sup(A)$. Is $x \in \mathrm{bd}(A)$?

§2.8. # 2. Let (M, d) be a metric space with the property that every bounded sequence has a convergent subsequence. Prove that M is complete

(Exercises for Chapter 2)

18. If $x, y \in M$ and $x \neq y$, then prove that there exist open sets U and V such that $x \in U, y \in V$, and $U \cap V = \varnothing$

29. Let $A, B \subset \mathbb{R}^n$ and x be an accumulation point of $A \cup B$. Must x be an accumulation point of either A or B?