Bankruptcy and Liquidation Prediction with Weighted Likelihood and Bayesian Approach

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Abstract

In this study, we examine three methods of Logistic Regression (LR) to predict bankruptcy and liquidation of North America companies: Maximum Likelihood LR, Weighted LR, and Bayesian LR. While most previous prior studies focused on predicting bankruptcy, this study predicts liquidation as well as bankruptcy. Since bankruptcy is an extremely rare event, including liquidation into the response variable could alleviate the problem caused by imbalanced data. Since there are more than 400 variables, we take preselection steps by t-test and VIF before stepwise selection by AIC and BIC. We forecast companies that will go bankrupt or liquidate within three years based on the certain fiscal year.

Keywords: bankruptcy, liquidation, variable selection, imbalanced data, logistic regression, weighted likelihood, Bayesian approach, ROC curve, forecasting

1 Introduction

The main problems in constructing bankruptcy prediction models are the choice of the independent variables and the functional form between these variables. There are some major bankruptcy theories: the gambler's ruin model, perfect-access model (Scott, 1981). However theoretical approaches to variable selection are rare and too simplified. In most previous bankruptcy prediction studies, the independent variables are empirically chosen.

Most bankruptcy prediction research use linear discriminant analysis or logistic regression. Kuruppu, Laswad, and Oyelere (2003) used multiple discriminant analysis to develop the liquidation prediction model on New Zealand sample. Of the 63 ratios, 12 were selected through stepwise methodology. Linear model, however, requires an assumption that independent variables are normally distributed. Laitinen and Laitinen (2000) tested whether Taylor's series expansion can be used to solve the problems associated with the functional form of bankruptcy prediction models, and showed improvement of classification accuracy in some of their logistic regression models. However, they used three financial ratio variables: cash to total assets, cash flow to total assets, and shareholder's equity to total assets, relying on simple theoretical analysis of the isolation concept for variable selection. Hauser and Booth (2011) use robust logistic regression with the Bianco and Yohai estimator which improved both the classification

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of bankrupt firms in the training set and the prediction of bankrupt firms in the testing set. In an out of sample test, the BY robust logistic regression correctly predicted bankruptcy for Lehman Brothers while maximum likelihood logistic regression never. However, this study also designated five financial ratios as variables without empirically selecting.[1]

In this study, 414 variables containing both financial and non-financial are included in the candidates of variables in the final model. We empirically select variables describing bankruptcy predictions through several steps. To avoid the normality assumption of independent variables, this study also uses logistic regression. First, we evaluate the basic maximum likelihood logistic regression. However, this method is subject to the problems caused by imbalanced data. As a solution, we use adjusted likelihood function, called the weighted likelihood. A total of four models are derived by considering both AIC and BIC as statistics scoring each model. For these four model specifications, we re-estimate the coefficients with the Bayesian approach. Therefore, a total of eight models are constructed. We measure the performance of these models and use the simplest model to out of sample test.

The remainder of this paper is organized as follows. Section 2 presents how to handle missing values and preselect continuous variables. Section 3 derives mathematical notation for three logistic regression models and fits training data to eight final models. In the process, we apply two variable selection methods: AIC and BIC. Section 4 measures performance and forecasts future bankruptcies. Section 5 concludes by suggesting some directions for future research.

2 EDA

2.1 Data

The annual fundamental data used in this paper are from the Compustat Capital IQ database of WRDS. The data include a total of 981 variables and we merge these with US counties data in SimpleMaps[2] database by columns of state and city for Figure 1c. All variables are classified into following 7 categories: identifying information, company descriptor, balance sheet items, income statement items, cash flow items, miscellaneous items, supplemental data items. Prior to handling missing values, 26 variables with the same value for all observation and 552 variables with missing values greater than 80% were removed. There are left 414 variables including 349 continuous variables. These are listed in Table 1.

The observations are companies traded on the NYSE, AMEX, NASDAQ, TSX, and NYSE Arca from January 2000 through November 2020. In 2010, 7,978 of the 11,036 companies were headquartered in the United States and 1,887 in Canada. China, the United Kingdom, Israel, Bermuda and Hong Kong followed. However, none of the companies outside of the U.S. and

[1] Hauser and Booth (2011) used WCTA, RETA, EBITTA, MEDDEBT, and SALETA as independent variables where WCTA: working capital/total assets as a measure of the net liquid assets of the firm to total capitalization; RETA: retained earnings/total assets as a measure of cumulative profitability; EBITTA: earnings before interest and taxes/total assets as a measure of the true productivity of the firm's assets; MEDDEBT: market value of equity/book value of total debt as a measure of how much the firm's assets can decline in value before the liabilities exceed the assets and the firm becomes insolvent; SLAETA: sales/total assets as a measure of the sales generating ability of the firm's assets (for prediction capabilities of these ratios, see Altman, 1968; Boritz and Kennedy, 1995; Odom and Sharda, 1990; Zhang et al, 1999; Lee et al, 2005).

[2] https://simplemaps.com/data/us-cities

Canada went bankrupt, and only five Singapore-based companies were liquidated between 2011 and 2013. Therefore, we consider only 9,865 companies headquartered in the U.S. and Canada, removing the rest from the population. We define dependent variable y_i as follows:

$$y_i = \begin{cases} 1 & \text{if company } i \text{ was bankrupt or liquidated between 2011 and 2013,} \\ 0 & \text{otherwise. (solvent company)} \end{cases}$$

Table 4a shows the number of deleted companies due to bankruptcy and liquidation, respectively. Although bankruptcy and liquidation have been combined to define a response variable, the event is still rare. This is why 1,171 companies were deleted earlier.

2.2 Imputation

The North America Industry Classification System (NAICS) was developed jointly by the U.S., Canada, and Mexico to provide new comparability in statistics about business activity across North America. Each North America company is assigned a 6-digit NAICS code. A company's NAICS code classifies the company on a production and/or process-oriented basis at five different levels: Sector, Subsector, Industry Group, NAICS Industry, and National. For example, Microsoft Corporation's NAICS code 511210 breaks down as follows:

Sector	51	Information
Subsector	511	Publishing Industries (except Internet)
Industry Group	5112	Software Publishers
NAICS Industry	51121	Software Publishers
National	511210	Software Publishers

See Table 2 for a detailed structure of NAICS.

We replace each missing values of continuous variables with the average value of companies with the same 6-digit NAICS code.[3] If all companies with the same 6-digit have missing data, it expands to those with the same 5-digit code. Repeating this process to 2-digit code replaces most missing values.

There are other industry classification systems such as Standard Industry Classification (SIC) and Global Industry Classification Standard (GICS). However, SIC codes were not able to keep up with current industries, and as a result of the development of NAICS, more than 350 new industries were recognized.

2.3 Preselection by t-test and VIF

Table 3 shows the results of the preselection steps of variables. As a result of t-testing 349 continuous variables, 105 variables did not show significant differences in means. The remaining 244 variables were removed one by one until the largest VIF value was less than 10. 90 surviving continuous variables and all categorical variables are candidates for the variables to be used in the final model.

[3] White et al (2015) use the average 6-digit NAICS for imputation of inputs.

2.4 Summary statistics and correlation analysis

The frequentist method of MLLR-AIC contained the largest number of variables in Section 4. Correlation analysis was performed within these variables, leaving out explanations for all variables. Table 8a shows descriptive statistics of continuous variables in the MLLR model. Because the variables were standardized to compare the size of the estimated coefficients, the mean is equal to 0 and the variance equal to 1. Figure 1 shows a map of states, counties, and cities in the United States. By State, Figure 1a represents the proportion of bankruptcy and liquidation, and Figure 1b represents the number of samples. Figure 1d refers to the cities in which the corporate headquarters are located. Not only is the sample geographically identified through the map, but it is also included in the variable candidates.

There are three categorical variables in total, including dependent variables. Table 5 shows the results of cross-tabulation analysis among them. While stalt and idbflag are associated with dependent variables, stalt and idbflag are independent of each other. In terms of multicollinearity, variable selection is considered appropriate. It can also be visually identified by box plots in Figure 2. Since BL is t-tested, continuous variables differ visibly with BL, but idbflag and stalt differ relatively less significant with respect to continuous variables. The multicollinearity of the selected explanatory variables and their association with the response variables cannot be fully reflected, but they are at a satisfactory.

3 Logistic Regressions

This section provides model and notation of Logistic Regression (LR). We begin by estimating parameters of Maximum Likelihood LR (MLLR) and Weighted LR (WLR), and then determine the distribution of parameters of Bayesian LR (BLR) by using R packages and WinBUGS.

3.1 Subsampling imbalanced data

In 2010, there were 7,380 U.S. companies, 76 of which went bankrupt or liquidated between 2011 and 2013. The proportion of events in the population is 1.03% which is rare and has been established in the literature that these variables are difficult to predict and explain. For example, the problem arises that the fitted model boasts high accuracy even if all X_i are classified into negative. Thus, we manipulate the proportion by constructing training set with 60 $Y_i = 1$ and 600 $Y_i = 0$ at random. It gives more weight to group of $Y_i = 1$.

However, this increases not only the rate at which the actual positive is correctly diagnosed as positive, but also the rate at which the actual negative is diagnosed as positive. That is, type 1 error increases instead of reducing type 2 error. In this case, it was considered more tremendous to predict that the company that would actually go bankrupt would not go bankrupt than to the contrary. Therefore, we try to solve the problem caused by imbalanced data through subsampling despite the increase in type I error. The test set consists of 16 bankrupt companies and 160 non-bankrupt companies.

3.2 Maximum likelihood estimation

Let $X \in \mathbb{R}^{n(k+1)}$ be a data matrix where n is the number of observation and k is the number of variables, and $Y \in \mathbb{R}^n$ be a binary outcomes vector. For all observation $X_i \in \mathbb{R}^{k+1}$ (a row vector in X), the outcome is either $y_i = 1$ (positive) or $y_i = 0$ (negative). The goal is to classify X_i as positive or negative. It can be treated as a Bernoulli trial with an expected value $E(y_i)$ or probability p_i. The logistic function commonly used to model each X_i with its expected outcome is given by the following formula:

$$E[y_i|X_i, \beta] = p_i = \frac{e^{X_i\beta}}{1 + e^{X_i\beta}} = \frac{1}{1 + e^{-X_i\beta}}$$

where $y_i \sim \text{Bernoulli}(p_i)$ and $\beta = (\beta_0, \beta_1, \cdots, \beta_m)^\top$. Let η be the logit link which is the logarithm of the odds ratio. It is defined as

$$\eta_i = \text{logit}(p_i) = \log \frac{p_i}{1 - p_i} = X_i\beta,$$

and can be written as $\eta = X\beta$ in matrix form. Now, assuming that the observations are independent, the likelihood function is

$$L(\beta|X, Y) = \prod_{i=1}^{n} (p_i)^{y_i}(1 - p_i)^{1-y_i} = \prod_{i=1}^{n} \left(\frac{e^{X_i\beta}}{1 + e^{X_i\beta}}\right)^{y_i} \left(\frac{1}{1 + e^{X_i\beta}}\right)^{1-y_i} = \prod_{i=1}^{n} \frac{e^{X_i\beta y_i}}{1 + e^{X_i\beta}}.$$

and the regularized log likelihood function is defined as

$$\log L(\beta|X, Y) = \sum_{i=1}^{n} \log \frac{e^{y_i X_i\beta}}{1 + e^{X_i\beta}} - \frac{\lambda}{2}\|\beta\|^2 \tag{1}$$

where the regularization (penalty) term was added to obtain better generalization. Since the log likelihood function is strictly concave, $\hat{\beta}_{\text{MLE}}$ which maximizes the log likelihood can be found.

3.3 Weighted likelihood

In addition to subsampling, we can use adjusted likelihood function, called weighted likelihood (King and Zeng, 2001; Maalouf and Siddiqi, 2014). Let $\overline{y}$ and τ be proportion of events in the sample and in the population, respectively. The weighted likelihood is defined as

$$L_W(\beta|X, Y) = \prod_{i=1}^{n} (p_i)^{w_1 y_i}(1 - p_i)^{w_0(1-y_i)},$$

where $w_1 = \tau/\overline{y}$, and $w_0 = (1-\tau)/(1-\overline{y})$. Now, instead of maximizing Eq.(1), we can maximize the weighted log likelihood

$$\log L_W(\beta|X, Y) = w_1 \sum_{y_i=1} \log(p_i) + w_0 \sum_{y_i=0} \log(1 - p_i) = -\sum_{i=1}^{n} w_i \log(1 + e^{(1-2y_i)X_i\beta})$$

where $w_i = w_1 y_i + w_0(1 - y_i)$.

3.4 Bayesian approach

Bayesian inference is a method of estimating parameters using probability models for the observed data and the parameters of interest. First, we determine the prior distribution of parameters, and estimate the parameters using a posterior distribution combining the sampling distribution and prior distribution. Let β be the set of parameters for model, and then the likelihood function for the observed data Y is

$$L(Y|\beta) = \prod_{i=1}^{n} (p_i)^{y_i} (1 - p_i)^{1-y_i}.$$

The prior distribution for all parameters assumes a non-informative prior distribution. The prior distribution for regression coefficients assumes a standard normal distribution. We suppose all parameters are mutually independent, and therefore the prior distribution for β is defined as follow:

$$p(\beta) = \prod_{i=1}^{n} p_i(\beta_i).$$

The posterior distribution for the parameters is

$$p(\beta|Y) = L(Y|\beta)p(\beta).$$

The estimation of parameters uses the Markov Chain Monte Carlo (MCMC) method. The WinBUGS statistical package[4] was used. Two chains with different initial values were used to check the sample convergence. Every fifth sample was extracted as a posterior sample. After 1,000 burn-in, 2,800 samples for each chain, in total of 5,600 samples, were used for parameter estimation. We checked the convergence using trace plot, the Gelman-Rubin statistic, and autocorrelation plot.

3.5 Variable selection by AIC and BIC

Both Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are appropriate for models fit under the maximum likelihood estimation framework. Let $\hat{L}$ be the maximum value of the likelihood function for each model. Then the statistics of AIC and BIC are the following.

$$\text{AIC} = 2k - 2\log\hat{L}, \quad \text{BIC} = k\log n - 2\log\hat{L}$$

where k is the number of estimated parameters in the model and n is the number of observation. Compared to the BIC method, the AIC statistic penalizes complex models less, meaning that it may put more emphasis on model performance on the training set, and, in turn, select more complex models.

The selection of variables in the model utilizes the stepwise selection method. It is a compromise between Forward Selection and Backward Elimination, in which important variables

[4] http://www.mrc-bsu.cam.ac.uk/software/bugs

are found while repeating each step selection and elimination. The method reviews step by step whether the already selected variables can be eliminated, while selecting additional important variables one by one. In other words, variables included in the model in the early stages can also be erased in later stages. This work determines the model representing the lowest AIC as the final model through a stepwise selection method. In other words, we thought that the selection of variables was complete when the AIC could no longer be lowered even with the addition or exclusion of additional variables in the current stage. On the other hand, the statistical significance of individual variables was not taken into account because variable selection based on AIC was considered. Model fitting utilizes training data containing 44 variables and 660 observations that have gone through all of the preceding processes. The results of MLLR and WLR are in Table 6, and thoes of BLR are in Table 7. For example, the selected MLLR model by AIC method is

$$\begin{aligned}\eta_i = &-203.6 - 7.2x_{\text{aco},i} + 8.9x_{\text{aqppl1},i} + 2.1x_{\text{caps},i} - 7.6x_{\text{csho},i} - 13.5x_{\text{cstk},i} \\ &+ 1.6x_{\text{glcea},i} + 197.0x_{\text{idbflag},i} - 20.0x_{\text{optfvgr},i} - 2.4x_{\text{spced},i} + 2.7x_{\text{stalt},i} - 3.3x_{\text{stkcpa},i}\end{aligned}$$

which contains more variables than others. The likelihood ratio test for this model resulted in a very small p-value of 1.21×10^{-14}, which allowed the null hypothesis to be rejected. In other words, at least one explanatory variable could be determined to have explanatory power over the dependent variable.

Additionally, Figure 3 shows the posterior distribution and convergence process of parameters estimated by the Bayesian method. If the 95% confidence interval contains zero, that is, if the colored part hits zero, the parameter is considered insignificant. Meanwhile, all parameters of the four Bayesian models converge.

4 Performance and forecasting

The performance is measured based on test data consists of 16 companies with $y_i = 1$ and 160 companies with $y_i = 0$. We give the same importance to sensitivity and specificity. Thus, the optimal cut-off is chosen to maximize the sum of sensitivity and specificity, which is called Youden's J statistic. This is where the ROC curve in Figure 4 meets a 45 degree line at one point.

We construct 8 models according to variable selection and coefficient estimation methods. The more variables used in the model, the more difficult it is to predict due to missing values. Thus, the simplest model, Bayesian WLR-BIC, was used to predict future bankruptcy or liquidation. The forecast results are summarized in Table 9.

5 Conclusion

We considered three methods of logistic regression and two methods for scoring and selecting a model. Considering that there are so many variables, we relied entirely on statistical tests, not theoretical approaches for variable selection. There are four cases of variable selection based on basic maximum likelihood function, weighted likelihood, AIC, and BIC. Adding to

the Bayesian approach, we conducted a total of eight models. The ROC curves were compared in each case as frequentist versus Bayesian.

The performance was similar, but the Frequentist-WLR-AIC model had the highest sum of 78.3% for sensitivity and 72.5% for specificity. The effect of weighting is considered to be somewhat significant.

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Table 1: Variable names

This table shows the names of the 414 variables in Compustat Capital IQ database of WRDS. Variables with missing values greater than 80% or all equal were removed. We derive columns of first 2- to 5-digit code from the 6-digit NAICS code, which represents a higher classification. A total of 414 variables became candidates for the variables used in the model. The variables in the colored cells are those included in the final model selected by the maximum likelihood logistic regression and AIC. Variables selected for other models do not deviate from the range of these variables. Because there are so many variables, variable descriptions and correlation analysis were performed within these variables.

acctstd	auop	costat	dlc	ebit	gind	ivch	naics4	pidom	reajo	tfvce	txtubbegin
acdo	auopic	county_fips	dlcch	ebitda	glcea	ivncf	naics5	pifo	recch	tfvl	txtubend
aco	bkvlps	county_name	dldte	ein	glced	ivst	naics6	pnca	recco	tic	txtubposdec
acodo	BL	cshfd	dlrsn	emp	glceeps	ivstch	naicsh	pncad	recd	tlcf	txtubposinc
acominc	busdesc	cshi	dltis	epsfi	glcep	lat	ni	pncaeps	rect	tstk	txtubpospdec
acox	caps	csho	dlto	epsfx	gp	lco	niadj	pncwia	recta	tstkc	txtubpospinc
act	capx	cshpri	dltp	epspi	gsector	lcox	nopi	pncwip	rectr	tstkn	txtubsettle
add1	capxv	cshr	dltr	epspx	gsubind	lcoxdr	nopio	pnrsho	reuna	tstkp	txtubsoflimit
addzip	census_region	cshtr_c	dltt	esopct	gvkey	lct	np	ppegt	revt	txach	txtubtxtr
adjex_c	ceoso	cshtr_f	dm	esopdlt	ib	lifr	oancf	ppent	sale	txbco	txtubxintbs
adjex_f	ceq	cstk	dn	esopnr	ibadj	lifrp	oiadp	ppeveb	scf	txbcof	txtubxintis
ajex	ceql	cstkcv	do	esopt	ibc	lno	oibdp	prca	seq	txc	txw
ajp	ceqt	cstke	donr	esub	ibcom	lo	opeps	prcad	seqo	txdb	upd
aldo	cfoso	curcd	dp	esubc	ibmii	lol2	oprepsx	prcaeps	sic	txdba	wcap
am	ch	curncd	dpact	exchg	icapt	long	optca	prcc_c	sich	txdbca	weburl
ano	che	currtr	dpc	exre	idbflag	loxdr	optdr	prcc_f	siv	txdbcl	xacc
ao	chech	cusip	dpvieb	fatb	idit	lqpl1	optex	prch_c	spce	txdc	xad
aocidergl	ci	datadate	drc	fatc	incorp	lse	optexd	prch_f	spced	txdfed	xi
aociother	cibegni	dc	drlt	fate	intan	lt	optfvgr	prcl_c	spceeps	txdfo	xido
aocipen	cicurr	dclo	ds	fatl	intano	lul3	optgr	prcl_f	spcindcd	txdi	xidoc
aocisecgl	cidergl	dcom	dt	fatn	intc	mib	optlife	priusa	spcseccd	txditc	xint
aodo	cik	dcpstk	dudd	fato	intpn	mibn	optosby	prsho	spcsrc	txds	xintopt
aol2	cimii	dcs	dv	fatp	invch	mibt	optosey	prstkc	spi	txfed	xopr
aoloch	ciother	dcvsr	dvc	fax	invfg	mii	optprcby	pstk	sppe	txfo	xpp
aox	cipen	dcvsub	dvp	fca	invo	mkvalt	optprcca	pstkc	sppiv	txndb	xpr
ap	cisecgl	dcvt	dvpa	fdate	invrm	mrc1	optprcex	pstkl	src	txndba	xrd
apalch	citotal	dd	dvpsp_c	fiao	invt	mrc2	optprcey	pstkn	sstk	txndbl	xrdp
apdedate	city	dd1	dvpsp_f	fic	invwip	mrc3	optprcgr	pstkr	stalt	txndbr	xrent
aqc	cld2	dd2	dvpsx_c	fincf	ipodate	mrc4	optprcwa	pstkrv	state	txo	xsga
aqi	cld3	dd3	dvpsx_f	fopo	ismod	mrc5	optrfr	rdip	state_name	txp	
aqpl1	cld4	dd4	dvt	fopox	itcb	mrct	optvol	rdipa	stkco	txpd	
aqs	cld5	dd5	dxd2	fyr	itci	mrcta	pdate	rdipd	stkcpa	txr	
at	cogs	dfs	dxd3	fyrc	ivaco	msa	pddur	rdipeps	stko	txs	
au	conm	diladj	dxd4	gdwl	ivaeq	naics2	phone	re	teq	txt	
aul3	conml	dilavx	dxd5	ggroup	ivao	naics3	pi	rea	tfva	txtubadjust	

Table 2: Structure of 2017 NAICS

This table shows the structure of NAICS. The first 2-digit of NAICS code represents a sector. (b) shows the NAICS code of companies belonging to the manufacturing sector. Monster Beverage has the same 6-digit code with Coca Cola. NAICS Industry and National sometimes have the same description like Kellogg and Nike. We replaced each missing value with the average value of companies with the same 6-digit NAICS code. If all companies in the same National are missing a value for a variable, they move on to a 5-digit step. This process is carried out to the sector average, which is a 2-digit code.

(a) Sectors of NAICS

Sector	Obs	Description
11	18	agriculture, forestry, fishing and hunting
21	426	mining, quarrying, and oil and gas extraction
22	248	utilities
23	78	construction
31–33	2193	manufacturing
42	169	wholesale trade
44–45	235	retail trade
48–49	148	transportation and warehousing
51	652	information
52	2122	finance and insurance
53	341	real estate and rental and leasing
54	233	professional, scientific, and technical services
55	0	management of companies and enterprises
56	111	administrative and support and waste management and remediation services
61	26	educational services
62	117	health care and social assistance
71	43	arts, entertainment, and recreation
72	106	accommodation and food services
81	17	other services (except public administration)
92	0	public administration
99	105	unclassifiable

(b) Examples of NAICS

Monster Beverage Corp		Kellogg Co	
31	manufacturing	31	manufacturing
312	beverage and tobacco product manufacturing	311	food manufacturing
3121	beverage manufacturing	3112	grain and oilseed milling
31211	soft drink and ice manufacturing	31123	breakfast cereal manufacturing
312111	soft drink manufacturing	311230	breakfast cereal manufacturing
Coca Cola Consolidated Inc		**Nike Inc**	
31	manufacturing	31	manufacturing
312	beverage and tobacco product manufacturing	316	leather and allied product manufacturing
3121	beverage manufacturing	3162	footwear manufacturing
31211	soft drink and ice manufacturing	31621	footwear manufacturing
312111	soft drink manufacturing	316210	footwear manufacturing

Table 3: Preselection by t-test and VIF

This table shows the results of the preselection steps of variables. As a result of t-testing 349 continuous variables, 105 variables did not show significant differences in means. The remaining 244 variables were removed one by one until the largest VIF value was less than 10. 90 surviving continuous variables and all categorical variables are candidates for the variables to be used in the final model.

Removed variables by t-test (105/349)											
adjex_c	aoloch	currtr	do	epspi	glceeps	lno	oprepsx	prcaeps	pstkr	spi	txo
adjex_f	apalch	dcom	donr	epspx	invch	lol2	optca	prcc_c	pstkrv	sstk	txtubadjust
ajex	aqi	dcpstk	dvp	esopdlt	invt	long	optlife	prcc_f	rdip	tfva	txtubxintis
ajp	che	dcvsr	dvpsp_c	esopnr	ivaco	lqpl1	optrfr	prch_c	rdipa	tfvl	xi
ano	cicurr	dcvt	dvpsp_f	exre	ivao	mib	pncwia	prch_f	rea	tstkp	xido
aocidergl	cidergl	diladj	dvpsx_c	fatl	ivch	msa	pncwip	prcl_c	recco	txach	xintopt
aociother	ciother	dlc	dvpsx_f	fca	ivncf	nopi	pnrsho	prcl_f	seqo	txdfed	
aocisecgl	cshr	dltis	epsfi	fiao	ivst	np	prca	pstkc	siv	txdi	
aol2	cstkcv	dltr	epsfx	glced	lat	opeps	prcad	pstkl	spce	txndbr	
Removed variables by VIF (154/244)											
acodo	ceq	cshfd	dlcch	dxd3	ibc	loxdr	ni	pifo	reuna	txdba	txtubpospinc
acominc	ceql	cshi	dlto	dxd4	ibcom	lse	niadj	pncaeps	revt	txditc	txtubtxtr
acox	ceqt	cshpri	dltt	dxd5	ibmii	lt	oancf	ppegt	sale	txds	txtubxintbs
act	ch	cshtr_f	dn	ebit	icapt	lul3	oiadp	ppent	seq	txfo	xacc
am	ci	cstke	dp	ebitda	intan	mibn	oibdp	ppeveb	spceeps	txndb	xint
ao	cibegni	dd	dpact	esub	intpn	mibt	optosby	pstk	sppiv	txndba	xopr
aodo	cimii	dd1	dpc	fopo	lco	mkvalt	optosey	pstkn	stkco	txndbl	xpr
aox	cisecgl	dd2	dpvieb	fopox	lcox	mrc1	optprcby	rdipd	teq	txpd	xrd
ap	citotal	dd3	ds	gdwl	lcoxdr	mrc2	optprcca	re	tlcf	txt	xrdp
at	cld2	dd4	dt	glcep	lct	mrc3	optprcex	reajo	tstk	txtubbegin	xrent
aul3	cld4	dd5	dv	gp	lifr	mrc4	optprcey	recd	tstkc	txtubend	xsga
capx	cld5	dfs	dvt	ib	lifrp	mrc5	optprcgr	rect	txc	txtubposinc	
capxv	cogs	dilavx	dxd2	ibadj	lo	mrct	pi	rectr	txdb	txtubpospdec	
Preselected variables (90)											
acdo	caps	dclo	dvc	fate	intc	ivstch	optgr	rdipeps	txbco	txr	xidoc
aco	chech	dcs	dvpa	fatn	invfg	mii	optprcwa	recch	txbcof	txs	xpp
aldo	cipen	dcvsub	emp	fato	invo	mrcta	optvol	recta	txdbca	txtubposdec	
aocipen	cld3	dltp	esopct	fatp	invrm	nopio	pidom	spced	txdbcl	txtubsettle	
aqc	csho	dm	esopt	fincf	invwip	optdr	pnca	sppe	txdc	txtubsoflimit	
aqpl1	cshtr_c	drc	esubc	glcea	itcb	optex	pncad	stkcpa	txdfo	txw	
aqs	cstk	drlt	fatb	idit	itci	optexd	prsho	tfvce	txfed	wcap	
bkvlps	dc	dudd	fatc	intano	ivaeq	optfvgr	prstkc	tstkn	txp	xad	

Table 4: Variable discription and summary statistics

(a) motivated us to include liquidation in the response. Considered that population size of 7380, even if liquidation is included, the response variable is still a rare event. There were mergers and acquisitions for other reasons for the deletion, but it was not considered as a response variable because it did not necessarily mean a dangerous financial state. (b) shows the description of variables which was selected by MLLR and AIC. Because the variables were standardized to compare the size of the estimated coefficients, the mean is equal to 0 and the variance equal to 1 in (c).

(a) Number of deleted companies

year	All deletion	Bankruptcy	Liquidation	BL
2011	241	1	16	17
2012	363	6	29	35
2013	348	8	38	46
2014	369	3	47	50
2015	348	8	36	44
2016	356	10	31	41
2017	273	6	1	7
2018	243	8	1	9
2019	266	16	0	16
2020	99	4	0	4

(b) Variable description

aco	Current Assets Other Total
aqpl1	Assets Level1 (Quoted Prices)
caps	Capital Surplus/Share Premium Reserve
csho	Common Shares Outstanding
cstk	Common/Ordinary Stock (Capital)
glcea	Gain/Loss on Sale (Core Earnings Adjusted) After-tax
idbflag	International, Domestic, Both Indicator
optfvgr	Fair Value of Options Granted
spced	S&P Core Earnings EPS Diluted
stalt	Status Alert
stkcpa	After-tax stock compensation

(c) Summary statistics of continuous variables

	Obs	Mean	SD	Min	25%	75%	Max
aco	7,380	0.000	1.000	−0.150	−0.149	−0.107	42.624
aqpl1	7,380	0.000	1.000	−0.067	−0.067	−0.056	55.435
caps	7,380	0.000	1.000	−0.389	−0.209	−0.029	34.987
csho	7,380	0.000	1.000	−0.179	−0.163	−0.053	44.172
cstk	7,380	0.000	1.000	−0.138	−0.138	−0.119	35.533
glcea	7,380	0.000	1.000	−6.029	−0.134	−0.043	71.202
optfvgr	7,380	0.000	1.000	−0.080	−0.065	−0.008	66.953
spced	7,380	0.000	1.000	−83.980	−0.016	−0.006	5.143
stkcpa	7,380	0.000	1.000	−1.930	−0.235	−0.005	42.503

Table 5: Correlation analysis among categorical variables

This table shows the results of correlation between categorical variables.

(a) Contingency tables

stalt

BL	1	0	
1	4	72	76
0	16	7288	7304
	20	7360	7380

idbflag

BL	B	D	
1	0	76	76
0	445	6859	7304
	445	6935	7380

idbflag

stalt	B	D	
1	1	19	20
0	444	6916	7360
	445	6935	7380

(b) The significance of the difference between the two proportions

	Pearson's chi-squared test			Fisher's exact test			
	χ^2	df	p-value	odds ratio	95% CI		p-value
BL – stalt	53.376	1	2.755×10^{-13}	25.238	5.994	80.874	4.441×10^{-5}
BL – idbflag	3.911	1	0.048	∞	1.299	∞	0.014
stalt – idbflag	2.648×10^{-29}	1	1	1.219	0.193	50.795	1

Table 6: Results of frequentist methods of logistic regressions

This table reports the results of frequentist methods of logistic regressions.

	Maximum Likelihood LR		Weighted LR	
	AIC	BIC	AIC	BIC
aco	−7.190	−7.303	−5.595	−15.430*
	(6.036)	(7.196)	(6.785)	(9.297)
aqpl1	8.901**	6.869*	8.451	
	(3.520)	(3.762)	(5.564)	
caps	2.117**			
	(0.961)			
csho	−7.575**	−7.104**	−4.672	
	(3.088)	(3.109)	(2.876)	
cstk	−13.510		−15.164	
	(11.405)		(12.998)	
glcea	1.561**			
	(0.675)			
idbflagD	197.005	137.557	185.161	
	(970.673)	(658.688)	(1,554.751)	
optfvgr	−19.957***	−21.830***	−22.422***	−25.100***
	(6.620)	(6.349)	(7.920)	(7.745)
spced	−2.420			
	(1.488)			
stalt1	2.663**			
	(1.242)			
stkcpa	−3.308**	−3.291**	−2.900*	
	(1.449)	(1.420)	(1.559)	
intercept	−203.596	−142.864	−189.460	−3.235**
	(970.743)	(658.668)	(1,554.795)	(1.260)
Observations	660	660	660	660
Log Likelihood	−155.707	−163.898	−30.190	−33.437
Akaike Inf. Crit.	335.414	341.796	76.379	72.873
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 7: Posterior distributions of parameters

This table reports the results of Bayesian logistic regression.

	Mean	SD	2.5%	25%	50%	75%	97.5%	Rhat	N.eff
Panel A: The same variables as MLLR-AIC.									
aco	−1.066	0.814	−2.713	−1.603	−1.040	−0.504	0.470	1.001	5,600
aqpl1	0.010	0.518	−1.228	−0.274	0.097	0.383	0.784	1.001	5,600
caps	−0.008	0.598	−1.283	−0.386	0.033	0.420	1.050	1.001	4,400
csho	−1.391	0.798	−3.048	−1.926	−1.360	−0.849	0.130	1.001	5,600
cstk	−0.988	0.625	−2.381	−1.381	−0.935	−0.537	0.073	1.001	5,600
glcea	1.046	0.418	0.184	0.780	1.055	1.329	1.838	1.002	1,900
idbflagD	−0.947	0.760	−2.511	−1.454	−0.900	−0.417	0.491	1.001	5,600
optfvgr	−1.146	0.923	−3.008	−1.743	−1.139	−0.514	0.648	1.003	820
spced	0.568	0.781	−1.015	0.039	0.577	1.096	2.073	1.001	2,600
stalt1	1.122	0.802	−0.475	0.591	1.128	1.666	2.677	1.001	4,000
stkcpa	−1.717	0.658	−3.090	−2.146	−1.697	−1.257	−0.493	1.001	5,600
intercept	−2.890	0.212	−3.329	−3.031	−2.884	−2.745	−2.493	1.001	2,800
deviance	359.627	5.755	349.697	355.600	359.200	363.200	372	1.002	1,700
Panel B: The same variables as MLLR-BIC.									
aco	−1.038	0.792	−2.659	−1.561	−0.995	−0.486	0.403	1.002	2,300
aqpl1	0.120	0.538	−1.173	−0.185	0.220	0.508	0.896	1.001	5,600
csho	−1.401	0.813	−3.029	−1.952	−1.386	−0.845	0.150	1.001	5,600
idbflagD	−1.003	0.746	−2.547	−1.500	−0.976	−0.484	0.339	1.001	2,900
optfvgr	−1.327	0.894	−3.125	−1.930	−1.304	−0.721	0.396	1.001	5,100
stkcpa	−1.821	0.656	−3.156	−2.252	−1.796	−1.360	−0.606	1.001	5,600
intercept	−2.756	0.199	−3.150	−2.889	−2.752	−2.620	−2.386	1.002	2,000
deviance	371.046	5.244	361.797	367.300	370.700	374.325	382.200	1.001	5,600
Panel C: The same variables as WLR-AIC.									
aco	−0.985	0.791	−2.629	−1.502	−0.952	−0.424	0.432	1.001	5,600
aqpl1	0.120	0.537	−1.137	−0.185	0.225	0.512	0.904	1.005	1,600
csho	−1.339	0.800	−2.973	−1.868	−1.310	−0.786	0.155	1.001	5,600
cstk	−0.829	0.674	−2.302	−1.257	−0.756	−0.337	0.296	1.001	5,600
idbflagD	−0.998	0.755	−2.565	−1.488	−0.961	−0.472	0.401	1.001	5,600
optfvgr	−1.269	0.898	−3.060	−1.870	−1.257	−0.657	0.460	1.001	5,600
stkcpa	−1.811	0.657	−3.128	−2.242	−1.792	−1.355	−0.560	1.001	5,600
intercept	−2.826	0.207	−3.246	−2.960	−2.820	−2.682	−2.438	1.001	5,600
deviance	369.900	5.192	360.900	366.200	369.600	373.100	381.100	1.001	5,600
Panel D: The same variables as WLR-BIC.									
aco	−1.611	0.750	−3.193	−2.103	−1.581	−1.079	−0.258	1.001	5,600
optfvgr	−1.510	0.894	−3.296	−2.102	−1.496	−0.903	0.214	1.001	5,600
intercept	−2.467	0.155	−2.777	−2.571	−2.463	−2.362	−2.166	1.001	5,600
deviance	389.530	4.149	382.100	386.700	389.300	392.200	398.302	1.001	4,400

Table 8: Performance

(a) reports the performance of 8 models. AUC is area under the roc curve. (b) shows confusion matrix with optimal cut-off which maximizes Youden's J statistic. The optimal cut-off value on frequentist MLLR-AIC is 0.105. This means that if the predicted probability of bankruptcy and liquidation exceeds 0.105, the sensitivity and fit of the entity is best when classifying it as bankruptcy and liquidation. Therefore, on an optimal cut-off basis, an entity is classified as a bankruptcy and liquidation entity if the estimated probability of bankruptcy and liquidation exceeds 0.105. Conversely, if the estimated probability is less than 0.105, the entity is not expected to go bankrupt and liquidate. Meanwhile, the AUC was 0.857. Based on this, the final model could be judged to be relatively good.

(a) Performance

	Model	AUC	Sensitivity	Specificity	Youden's J
Frequentist	MLLR-AIC	0.8455278	0.7833333	0.76	0.6288611
	MLLR-BIC	0.8306944	0.8166667	0.69	0.5066667
	WLR-AIC	0.8309722	0.7833333	0.725	0.5083333
	WLR-BIC	0.7808056	0.6	0.8933333	0.4933333
Bayesian	MLLR-AIC	0.7969444	0.7	0.7883333	0.4883333
	MLLR-BIC	0.7873333	0.7333333	0.73	0.4633333
	WLR-AIC	0.7898889	0.7666667	0.7083333	0.475
	WLR-BIC	0.7783611	0.65	0.8533333	0.5033333

(b) Confusion matrix of frequentist MLLR-AIC model with cut-off 0.105

		Predicted		
		BL	NBL	
Actual	BL	14	2	16
	NBL	43	117	160
		57	119	176

Table 9: Forecasting

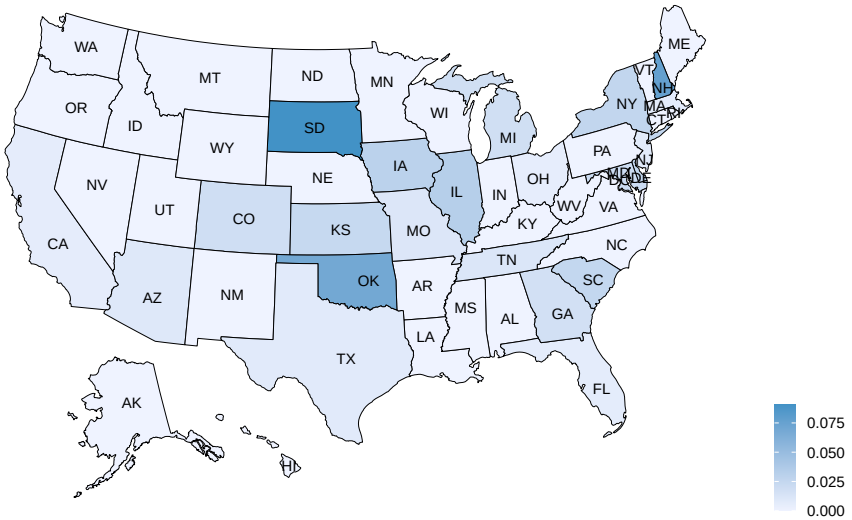
The table reports the top 50 companies with a high probability of bankruptcy and liquidation based on 2020 data. The results are predicted using the least complex Bayesian logistic regression model which simply has aco and optfvgr as explanatory variables. The gvkey is a global company key and loc is a current ISO country code where its headquarter locates.

rank	gvkey	company legal name	loc	aco	optfvgr	probability
1	18019	Brazil Minerals Inc	BRA	0.002	0.001	0.07786378
2	20612	Nuinsco Resources Ltd	CAN	0.000	0.010	0.07712260
3	187545	Lupaka Gold Corp	CAN	0.002	0.010	0.07689359
4	116614	Uravan Minerals Inc	CAN	0.000	0.020	0.07605470
5	164631	Sphinx Resources Ltd	CAN	0.006	0.015	0.07590614
6	175071	Enertopia Corp	CAN	0.014	0.010	0.07553256
7	174368	Solar Alliance Energy Inc	CAN	0.005	0.020	0.07549061
8	26761	Advanced Proteome Therapeutics Corp	CAN	0.010	0.020	0.07493035
9	190576	Angkor Gold Corp	CAN	0.013	0.020	0.07459604
10	175527	Petrolympic Ltd	CAN	0.006	0.030	0.07433256
11	21874	Prosper Marketplace Inc	USA	0.000	0.040	0.07395952
12	170692	Wellness Center USA Inc	USA	0.001	0.040	0.07384926
13	16611	Soperior Fertilizer Corp	CAN	0.013	0.030	0.07356033
14	135145	Candente Copper Corp	CAN	0.013	0.030	0.07356033
15	170573	Pershimex Resources Corp	CAN	0.017	0.030	0.07312238
16	184946	Focus Graphite Inc	CAN	0.029	0.020	0.07283608
17	65688	Sienna Resources Inc	CAN	0.011	0.040	0.07275495
18	30749	QYOU Media Inc	CAN	0.030	0.020	0.07272736
19	140669	Rusoro Mining Ltd	CAN	0.030	0.020	0.07272736
20	170814	Asante Gold Corp	CAN	0.044	0.010	0.07222627
21	142501	Maxtech Ventures Inc	CAN	0.013	0.044	0.07213258
22	106789	CardioComm Solutions Inc	CAN	0.008	0.050	0.07206535
23	186296	Transatlantic Mining Corp	CAN	0.020	0.040	0.07178286
24	141309	Fuse Cobalt Inc	CAN	0.011	0.050	0.07174282
25	106561	Engold Mines Ltd	CAN	0.049	0.010	0.07168837
26	174026	Mobi724 Global Solutions Inc	CAN	0.040	0.020	0.07164838
27	65703	Inhibitor Therapeutics Inc	USA	0.032	0.030	0.07150139
28	20629	Moneta Porcupine Mines Inc	CAN	0.023	0.040	0.07146150
29	107693	Euromax Resources Ltd	CAN	0.035	0.030	0.07118120
30	25106	NewOrigin Gold Corp	CAN	0.008	0.060	0.07106208
31	25502	Lincoln Ventures Ltd	CAN	0.000	0.070	0.07091620
32	106517	Kiplin Metals Inc	CAN	0.000	0.070	0.07091620
33	15481	Urbanimmersive Inc	CAN	0.019	0.050	0.07088926
34	108841	Canadian Spirit Resources Corp	CAN	0.031	0.040	0.07061103
35	122273	Timberline Resources Corp	USA	0.014	0.060	0.07042665
36	107217	Playfair Mining Ltd	CAN	0.025	0.050	0.07025525
37	130402	Slam Exploration Ltd	CAN	0.003	0.074	0.07020315
38	184045	Belgravia Hartford Capital Inc	CAN	0.046	0.030	0.07001845
39	170401	Independence Gold Corp	CAN	0.000	0.080	0.06992773
40	125996	VoIP-PAL.com	USA	0.066	0.010	0.06988703
41	13825	Red Moon Resources Inc	CAN	0.007	0.075	0.06968572
42	178094	Xtierra Inc	CAN	0.055	0.030	0.06908020
43	176180	iCo Therapeutics Inc	CAN	0.029	0.060	0.06886097
44	26203	Forum Energy Metals Corp	CAN	0.011	0.080	0.06878394
45	18342	IOU Financial Inc	CAN	0.000	0.100	0.06798892
46	160541	Hercules Capital Inc	USA	0.000	0.100	0.06798892
47	22727	PureBase Corp	USA	0.005	0.100	0.06748027
48	184811	Bravada Gold Corp	CAN	0.024	0.080	0.06745454
49	17384	Banyan Gold Corp	CAN	0.054	0.050	0.06726419
50	179948	Rockhaven Resources Ltd	CAN	0.040	0.065	0.06725817

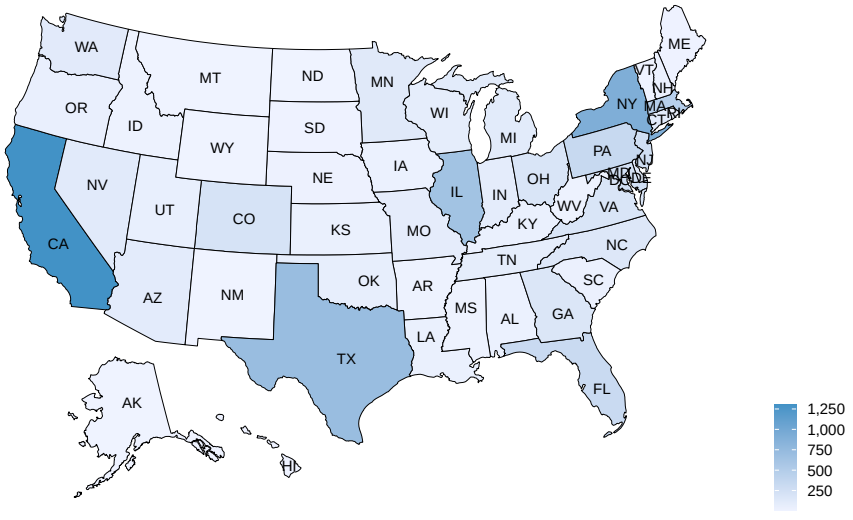
Figure 1: United States maps

These figures are the maps of states, counties, and cities in United States. (a) represents the proportion of event (bankruptcy or liquidation) by state. (b) and (c) show distribution of companies by states and counties. (d) refers to cities in which a company's headquarter is located. Not only is the sample geographically identified through the map, but it is also included in the variable candidates.

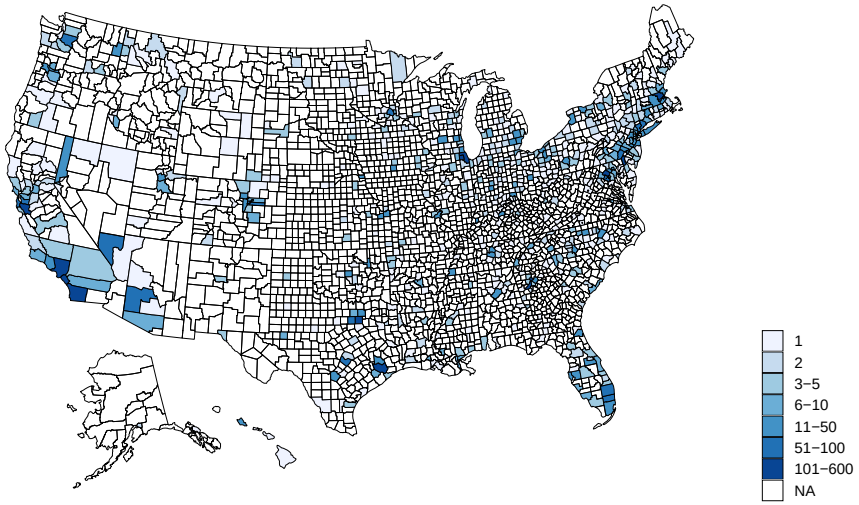
(a) Proportion of bankruptcy and liquidation by states



(b) Number of companies by states



(c) Number of companies by counties



(d) Plotting cities where a company is located

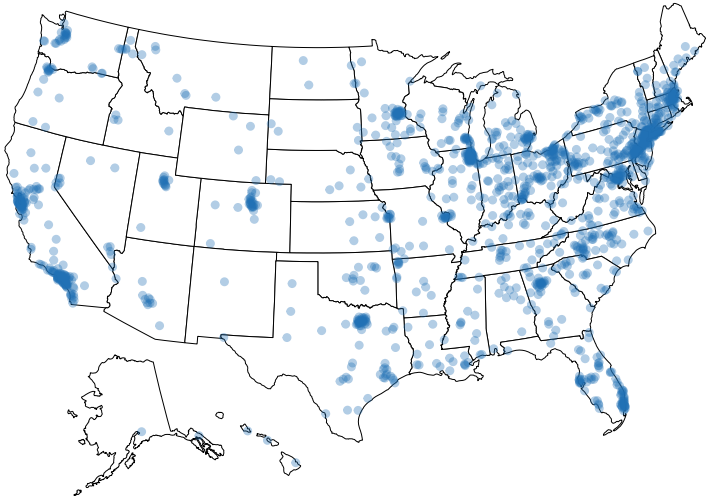
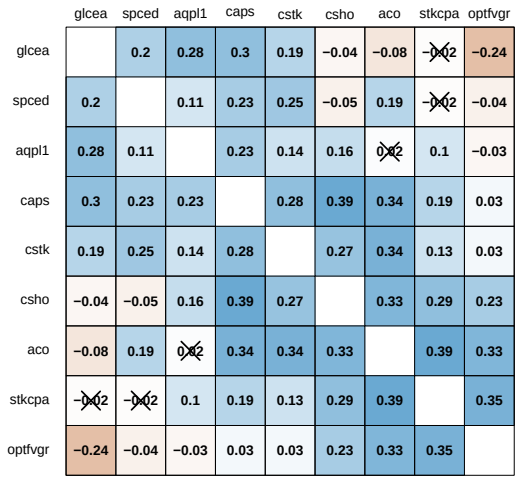


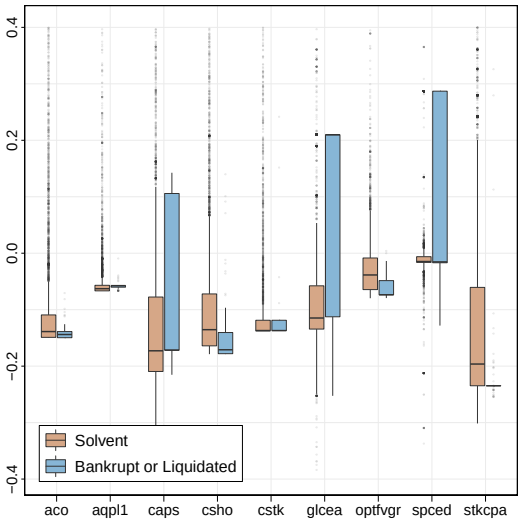
Figure 2: Correlation analysis with continuous variables

These figures show correlation analysis with continuous variables. Since all continuous variables are t-tested by BL, they differ visibly with BL. But idbflag and stalt differ relatively less significant with respect to continuous variables. The multicollinearity between the selected explanatory variables and their association with the response variable are reasonable.

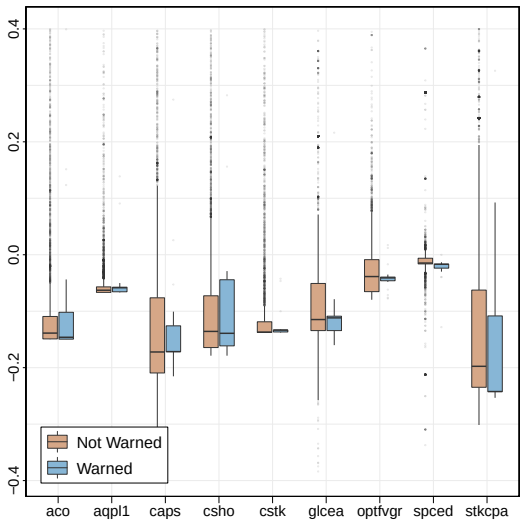
(a) Spearman correlogram with significance test



(b) Box plots by BL (truncated)



(c) Box plots by stalt (truncated)



(d) Box plots by idbflag (truncated)

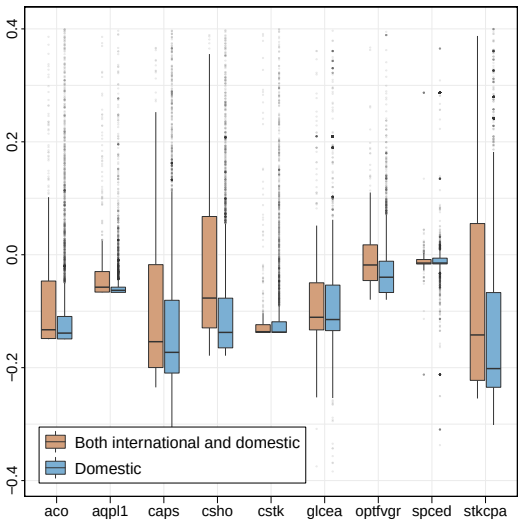
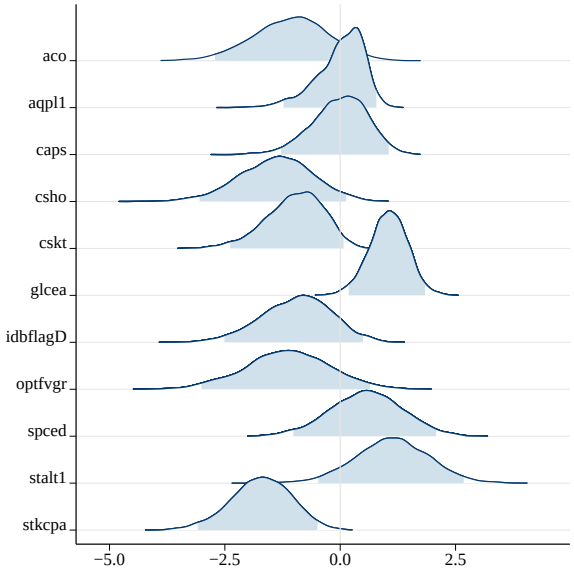


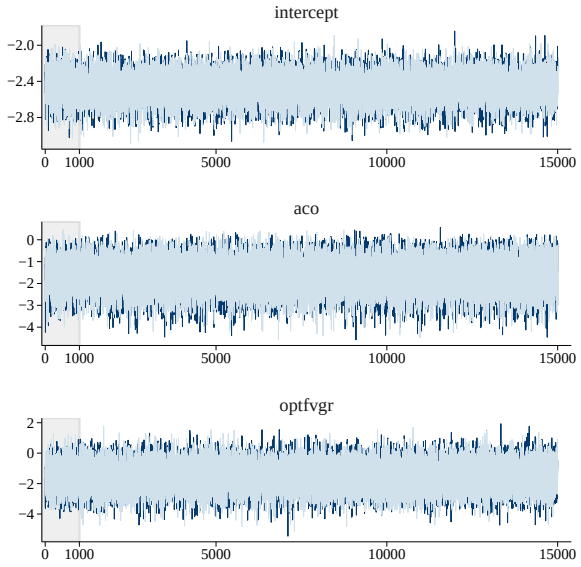
Figure 3: Bayesian logistic regression

(a) shows posterior distributions of parameters in model 1 (MLLR-AIC). The colored parts is the regions over 95% confidence intervals. If it contain zero, the parameter is not significant. (b) and (c) show trace and autocorrelation of parameters in model 4 (WLR-BIC) respectively. The distributions of parameters are driven along 15,000 iteration (1,000 warmup and 5 thin).

(a) Density plot with 95% interval



(b) Trace plot with 1,000 warmup



(c) Autocorrelation

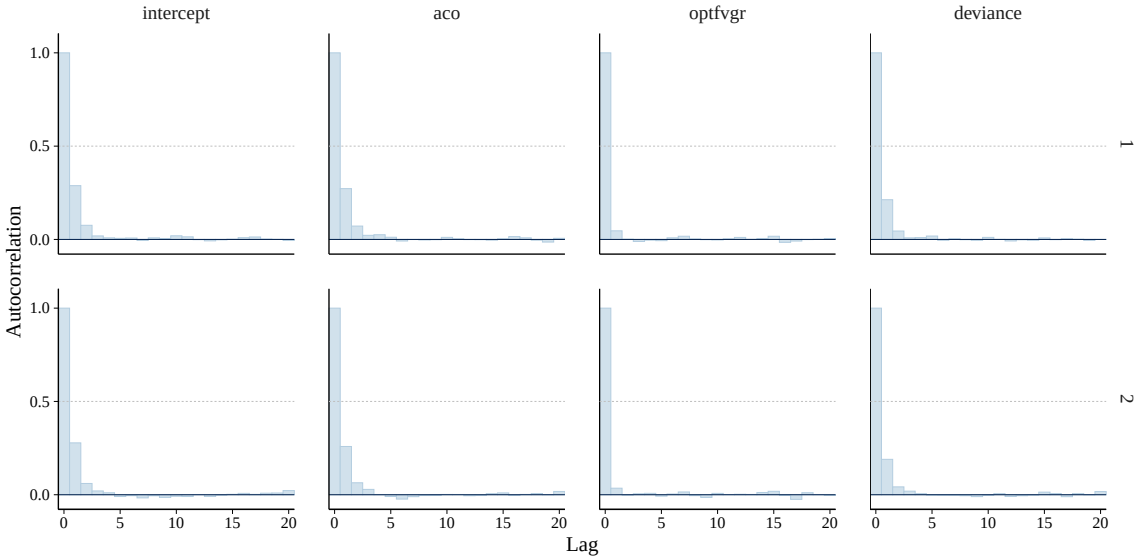
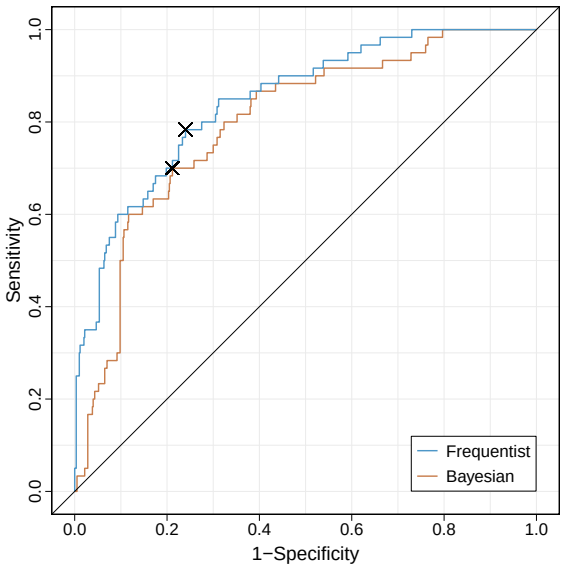


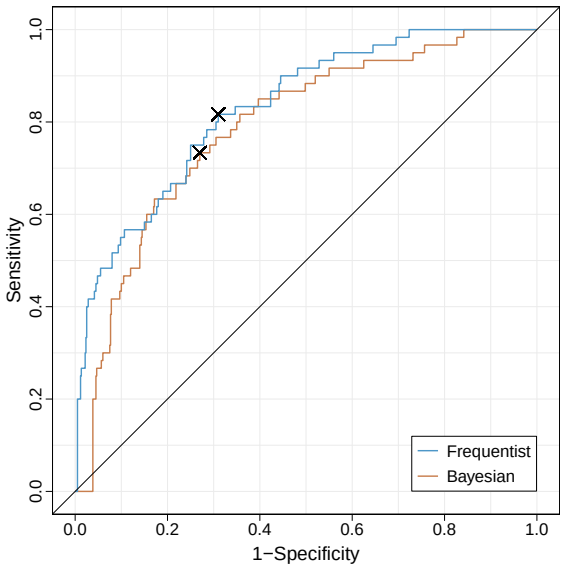
Figure 4: ROC curves

These figures show the ROC curves of final models. Subfigures are separated by variable selection methods and represent two path of frequentist and Bayesian. In other words, (a) includes aco, aqpl1, caps, csho, cstk, glcea, idbflagD, optfvgr, spced, stalt1, stkcpa, and (d) has aco, optfvgr as explanatory variables. The X-marked point is the position of the optimal cut-off given the same importance of sensitivity and specificity.

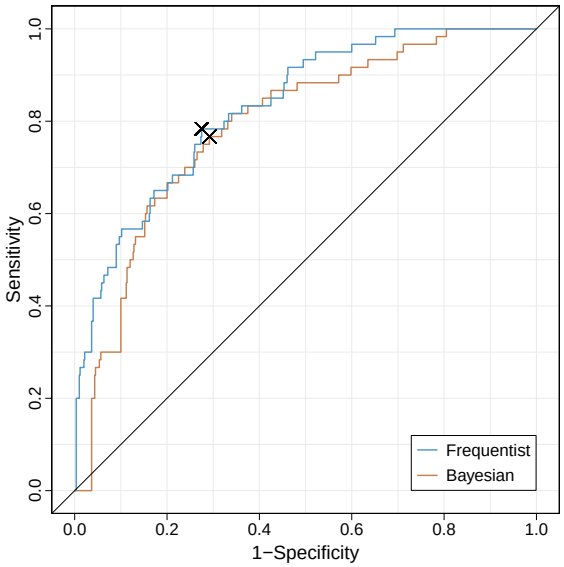
(a) MLLR-AIC



(b) MLLR-BIC



(c) WLR-AIC



(d) WLR-BIC