

Financial Mathematics I – Midterm Exam 2

Junwoo Yang

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1. For a two-dimensional Brownian motion $(B_t^{(1)}, B_t^{(2)})_{t \geq 0}$, calculate

$$\mathbb{E}(e^{\int_0^T \sqrt{t} dB_t^{(1)} + B_T^{(2)}}).$$

2. Consider the progressive σ -algebra

$$\Sigma := \{A \subseteq [0, \infty) \times \Omega : A \text{ is progressive}\}.$$

Show that a stochastic process X is progressively measurable if and only if X is Σ -measurable.

3. Let $\theta \in \mathcal{H}_{\text{loc}}^2$. Show that a process

$$M_t := e^{\int_0^t \theta_s dB_s - \frac{1}{2} \int_0^t \theta_s^2 ds}, \quad 0 \leq t \leq T$$

is a local martingale.

4. Show that a left-continuous adapted process is progressively measurable.
5. Let $(B_t)_{t \geq 0}$ be a Brownian motion. Show that the process

$$\left(\cos B_t + \frac{1}{2} \int_0^t \cos B_s ds \right)_{t \geq 0}$$

is a martingale.

6. Let $(B_t^{(1)}, B_t^{(2)}, B_t^{(3)})_{t \geq 0}$ be a three-dimensional Brownian motion. Define

$$X_t = te^{B_t^{(2)}} + \int_0^t s B_s^{(1)} dB_s^{(3)}.$$

Find the quadratic variation $\langle X \rangle_t$ for $t \geq 0$.