

Mathematical Statistics 1 – Midterm Exam

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1. A test indicates the presence of a particular disease 90% of the time when the disease is present and the presence of the disease 2% of the time when the disease is not present. If 1% of the population has the disease, calculate the probability that a person selected at random has the disease if the test indicates the presence of the disease.
2. Let X be the number of total Bernoulli trials until the r th success occurs, where the probability of success is p .
 - (1) Provide the pmf of X .
 - (2) Compute the variance of X .
3. Let X have a Poisson distribution with $P(X = 0) = 0.5$, $X \sim \text{Pois}(\lambda)$. A random variable Y has a pmf

$$f_Y(y) = \begin{cases} p + (1-p)f_X(y), & y = 0 \\ (1-p)f_X(y), & y = 1, 2, 3, \dots \end{cases}$$

where $p \in (0, 1)$ is a parameter and is called as zero-inflated Poisson distribution. Also, a random variable Z has a pmf

$$f_Z(z) = \frac{P(X = z)}{P(X > 0)}, \quad z = 1, 2, \dots$$

and it is called as zero-truncated Poisson distribution.

- (1) Derive the moment generating function (mgf) of a Poisson random variable, X after obtaining the value of λ .
 - (2) Using the mgf of X , compute the variance of X .
 - (3) Derive the moment generating function (mgf) of Y .
 - (4) Using the mgf of Y , compute the mean and variance of Y .
 - (5) Find the pmf of Z .
 - (6) Compute the mean and variance of Z .
4. Let X be a random variable following the cumulative distribution

$$F_X(x) = \begin{cases} ce^x, & x \leq 0 \\ 1 - ce^{-x}, & x > 0 \end{cases}$$

where c is a constant.

- (1) Find the value of c and the pdf of X .
 - (2) Obtain the mgf of X .
 - (3) Using the mgf of X , compute the mean and variance of X .