미분기하학개론 I - HW3

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2020-91659
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2-2. 3

$S := \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2-z^2 = 0\}$.

$F := \mathbb{R}^3 \to \mathbb{R} \quad (x,y,z) \mapsto x^2+y^2-z^2$

$dF_p = (F_x, F_y, F_z) = (2x, 2y, -2z)$

$dF_{(0,0,0)} = (0,0,0) \Rightarrow$ vertex $(0,0,0)$: critical point.

$(0,0,0) \in S, \Rightarrow S$ is not regular surface.

2-2. 7.

(a) $f : \mathbb{R}^3 \to \mathbb{R}$

$(x,y,z) \mapsto (x+y+z-1)^2$

$df_p = (2x+2y+2z-2,\ 2x+2y+2z-2,\ 2x+2y+2z-2)$.

p is critical point $\Leftrightarrow df_p = 0 \Leftrightarrow 2x+2y+2z-2 = 0$.

$\therefore$ critical points of f : $\{t = (x,y,z) \in \mathbb{R}^3 \mid x+y+z = 1\}$

critical value of f : $f(t) = 0$.

(b) By prop 2, for any regular point r of f,

$f^{-1}(r)$ is regular surface.

0 is only critical value.

$\therefore \forall p \neq 0$, $f^{-1}(p)$ is regular surface.

(c) $f : \mathbb{R}^3 \to \mathbb{R}$

$(x,y,z) \mapsto xyz^2$

$dF_p = (yz^2, xz^2, 2xyz)$.

p is critical point $\Leftrightarrow yz^2 = xz^2 = 2xyz = 0 \Leftrightarrow z = 0$ or $x = y = 0$

critical point : $\{(x,y,0) \in \mathbb{R}^3 \mid x,y \in \mathbb{R}\} \cup \{(0,0,z) \in \mathbb{R}^3 \mid z \in \mathbb{R}\}$

critical value : $f(x,y,0) = 0$. $f(0,0,z) = 0$.

$\therefore$ The only critical value is 0.

$f^{-1}(p)$ is regular surface for $\forall p \neq 0$.