

# Mathematical Statistics 1 – Midterm Exam

Junwoo Yang

April 25, 2019

1. A test indicates the presence of a particular disease 90% of the time when the disease is present and the presence of the disease 2% of the time when the disease is not present. If 1% of the population has the disease, calculate the probability that a person selected at random has the disease if the test indicates the presence of the disease.
2. Let  $X$  be the number of total Bernoulli trials until the  $r$ th success occurs, where the probability of success is  $p$ .
  - (1) Provide the pmf of  $X$ .
  - (2) Compute the variance of  $X$ .
3. Let  $X$  have a Poisson distribution with  $P(X = 0) = 0.5$ ,  $X \sim \text{Pois}(\lambda)$ . A random variable  $Y$  has a pmf

$$f_Y(y) = \begin{cases} p + (1-p)f_X(y) & y = 0 \\ (1-p)f_X(y) & y = 1, 2, 3, \dots \end{cases}$$

where  $p \in (0, 1)$  is a parameter and is called as zero-inflated Poisson distribution. Also, a random variable  $Z$  has a pmf

$$f_Z(z) = \frac{P(X = z)}{P(X > 0)}, \quad z = 1, 2, \dots$$

and it is called as zero-truncated Poisson distribution.

- (1) Derive the moment generating function (mgf) of a Poisson random variable,  $X$  after obtaining the value of  $\lambda$ .
  - (2) Using the mgf of  $X$ , compute the variance of  $X$ .
  - (3) Derive the moment generating function (mgf) of  $Y$ .
  - (4) Using the mgf of  $Y$ , compute the mean and variance of  $Y$ .
  - (5) Find the pmf of  $Z$ .
  - (6) Compute the mean and variance of  $Z$ .
4. Let  $X$  be a random variable following the cumulative distribution

$$F_X(x) = \begin{cases} ce^x, & x \leq 0 \\ 1 - ce^{-x}, & x > 0 \end{cases}$$

where  $c$  is a constant.

- (1) Find the value of  $c$  and the pdf of  $X$ .
  - (2) Obtain the mgf of  $X$ .
  - (3) Using the mgf of  $X$ , compute the mean and variance of  $X$ .