Homework 2

Due date: 2019. 3.20.

In this homework, let G be a group.

1. Find the order of the following group and the order of each element in the group.

(1) $\mathbb{Z}_{12}$

(2) $U(10)$

(3) D_4

2. In $< \mathbb{Q}, + >$, find the elements in $\left\langle \frac{1}{2} \right\rangle$.

3. In $< \mathbb{Q}^*, \cdot >$, find the elements in $\left\langle \frac{1}{2} \right\rangle$.

4. Prove that the D_3 does not have a subgroup of order 4.

5. Must the centralize of an element of a group be Abelian?

6. Must the center of a group be Abelian?

Prove or disprove: (7-8)

7. If $H \leq G$ and $K \leq G$, then $H \cap K \leq G$.

8. If $H \leq G$ and $K \leq G$, then $H \cup K \leq G$.

현대대수1 - HW2

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1. (1) $\mathbb{Z}_{12} = \{0, 1, 2, 3, \cdots, 10, 11\}$

Order of $\mathbb{Z}_{12}$, $|\mathbb{Z}_{12}| = 12$ $\quad e = 0$

$|0| = 1$

$|1| = 12$ $(1^{12} = 1+1+\cdots+1 = 12 \equiv 0)$

$|2| = 6$ $(2^6 = 2+2+\cdots+2 = 12 \equiv 0)$

$|3| = 4$, $|4| = 3$, $|5| = 12$, $|6| = 2$

$|7| = 12$, $|8| = 3$, $|9| = 4$, $|10| = 6$, $|11| = 12$

(2) $U(10) = \{1, 3, 7, 9\}$

$|U(10)| = 4$, $\quad e = 1$

$|1| = 1$, $|3| = 4$, $|7| = 4$, $|9| = 2$

(3) $D_4 = \{R_0, R_{90}, R_{180}, R_{270}, H, V, D, D'\}$

$|D_4| = 8$, $\quad e = R_0$

$|R_0| = 1$, $|R_{90}| = 4$, $|R_{180}| = 2$, $|R_{270}| = 4$

$|H| = 2$, $|V| = 2$, $|D| = 2$, $|D'| = 2$

2. $\langle \mathbb{Q}, + \rangle$

$\langle \frac{1}{2} \rangle = \{\cdots, -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \cdots\}$

3. $\langle \mathbb{Q}^*, \cdot \rangle$

$\langle \frac{1}{2} \rangle = \{\cdots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \cdots\}$

4. $D_3 = \{R_0, R_{120}, R_{240}, L_1, L_2, L_3\}$

R_0 is identity,
Thus Subgroups of D_3 have to contain R_0

$R_{120} \cdot R_{240} = e$, $L_1 \cdot L_1 = e$, $L_2 \cdot L_2 = e$, $L_3 \cdot L_3 = e$

i) subgroup of order 1.
$\{R_0\}$

ii) subgroup of order 2.
$\{R_0, L_1\}$, $\{R_0, L_2\}$, $\{R_0, L_3\}$

iii) subgroup of order 3.
$\{R_0, R_{120}, R_{240}\}$

iv) subgroup of order 6
D_3

	R_0	R_{120}	R_{240}	L_1	L_2	L_3
R_0	R_0	R_{120}	R_{240}	L_1	L_2	L_3
R_{120}	R_{120}	R_{240}	R_0	L_3	L_1	L_2
R_{240}	R_{240}	R_0	R_{120}	L_2	L_3	L_1
L_1	L_1	L_2	L_3	R_0	R_{120}	R_{240}
L_2	L_2	L_3	L_1	R_{240}	R_0	R_{120}
L_3	L_3	L_1	L_2	R_{120}	R_{240}	R_0

When ii), iii) added one more other element,
It become D_3 to be subgroup of G

$\therefore$ D_3 does not have a subgroup of order 4.

5. Let G is not Abelian group.
$\forall g \in G$, $ge = eg = g$
$C(e) = G$
$\therefore$ $C(e)$ is not Abelian

6. $Z(G) = \{a \in G \mid ax = xa, \forall x \in G\}$
$\forall a, b \in Z(G)$, $ab = ba$ ($\because a, b \in G$)
$\therefore$ $Z(G)$ is Abelian

7. $H \leq G,\ K \leq G \rightarrow H \cap K \leq G$

prove).

i) existance of e

Since H, K are subgroup of G,

e is in H, K.

$\therefore\ e \in H \cap K$ (nonempty set)

ii) existance of inverse

$\forall x, y \in H \cap K,\ x, y \in H,\ x, y \in K$

Since H, K are group, $\exists\ x^{-1}, y^{-1}, xy$ in H, K

$\therefore\ x^{-1}, y^{-1}, xy \in H \cap K$

$\therefore$ $H \cap K$ is subgroup of G

8. $H \leq G,\ K \leq G \longrightarrow H \cup K \leq G$

disprove).

Let $H \not\subseteq K,\ K \not\subseteq H,\ h \in H\,(h \notin K),\ k \in K\,(k \notin H)$

Then we know that $h, k, h^{-1}, k^{-1} \in H \cup K$

WTS $hk \notin H \cup K$

Suppose $hk \in H$, then, $h^{-1}, hk \in H$

$h^{-1} \cdot hk = (h^{-1}h) \cdot k = e \cdot k = k \notin H$ ($\Rightarrow\Leftarrow$)

$\therefore\ hk \notin H$

Similarly, also $hk \notin K$ (same argument)

Thus $hk \notin H \cup K$

$\therefore$ $H \cup K$ is not subgroup of G