미분기하학개론 1 - HW2

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2-2.11

$S = \{(x,y,z) \in \mathbb{R}^3 \mid z = x^2 - y^2\}$

$f : \mathbb{R}^2 \to \mathbb{R}$ $\qquad$ $\mathbb{X} : \mathbb{R}^2 \to \mathbb{X}(\mathbb{R}^2) \subset \mathbb{R}^3$

$(x,y) \mapsto (x^2 - y^2).$ $\qquad$ $(x,y) \mapsto (x, y, x^2 - y^2).$

S is graph f.

condition 1: $(x,y) \mapsto x$, $(x,y) \mapsto y$, $(x,y) \mapsto (x^2-y^2)$ all diff.

$\therefore \mathbb{X}$ is differentiable.

condition 2: $\mathbb{X}$ is obviously injective

$\exists\, \mathbb{X}^{-1} : \mathbb{X}(\mathbb{R}^2) \to \mathbb{R}^2$ continuous. $(x,y,z) \mapsto (x,y).$

$\mathbb{X}$ is homeomorphism.

condition 3: $\forall (x_0, y_0) \in \mathbb{R}^2,$

$$d\mathbb{X}_{(x_0,y_0)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2x_0 & -2y_0 \end{pmatrix} \quad \text{injective.}$$

$\therefore S^2$ is regular surface.

a. $\mathbb{X}(u,v) = (u+v, u-v, 4uv)$ $\quad$ $(u,v) \in \mathbb{R}^2$

$u + v = x$, $\quad u - v = y$.

$2u = x + y$. $\quad u = \frac{1}{2}(x+y)$.

$2v = x - y$. $\quad v = \frac{1}{2}(x-y)$.

$4uv = x^2 - y^2$.

$\therefore \{\mathbb{X}(u,v) \mid (u,v) \in \mathbb{R}^2\} = S$.

$\therefore \mathbb{X}$ is parametrization for S.

↳ covers the whole surface S.

b. $\mathbb{X}(u, v) = (u\cosh v, u\sinh v, u^2)$, $(u, v) \in \mathbb{R}^2$ $u \neq 0$.

$u\cosh v = x$, $u\sinh v = y$.

$\frac{u}{2}(e^v + e^{-v}) = x$, $\frac{u}{2}(e^v - e^{-v}) = y$.

$\frac{u}{2} \cdot 2e^v = ue^v = x + y$.

$\frac{u}{2} \cdot 2e^{-v} = ue^{-v} = x - y$.

$ue^v \cdot ue^{-v} = u^2 = (x+y)(x-y) = x^2 - y^2 > 0$ $(\because u \neq 0)$.

Note. $\cosh v \geq 1$. $u^2 > 0$.

$\Rightarrow u \neq 0$, $x \neq 0$, $z > 0$

$\therefore \mathbb{X}$ is parametrization for upper half of S without curve $\alpha(t) = (0, -t^2, t)$.