수리통계학1 - HW4

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#4.1-3

(a) $f_X(x) = \sum_y f(x,y) = \sum_{y=1}^{4} \frac{x+y}{32} = \frac{4x+10}{32} = \frac{2x+5}{16} \quad x=1,2$

(b) $f_Y(y) = \sum_x f(x,y) = \sum_{x=1}^{2} \frac{x+y}{32} = \frac{2y+3}{32} \quad y=1,2,3,4$

(c) $P(X>Y) = P(x=2, y=1) = \frac{3}{32}$

(d) $P(Y=2X) = P(X=1, Y=2) + P(X=2, Y=4)$
$= \frac{3}{32} + \frac{6}{32} = \frac{9}{32}$

(e) $P(X+Y=3) = P(X=1, Y=2) + P(X=2, Y=1)$
$= \frac{3}{32} + \frac{3}{32} = \frac{6}{32} = \frac{3}{16}$

(f) $P(X \leq 3-Y) = P(X=1, Y=1) + P(X=2, Y=1) + P(X=1, Y=2)$
$= \frac{2+3+3}{32} = \frac{8}{32} = \frac{1}{4}$

(g) $f_{XY}(x,y) = f_X(x) f_Y(y)$ (independent).
$f_{XY}(X=1, Y=1) = \frac{2}{32}$
$f_X(1) = \frac{7}{16} \quad f_Y(1) = \frac{5}{32}$
$f_X(1) \cdot f_Y(1) = \frac{7 \cdot 5}{16 \cdot 32} \neq \frac{2}{32} = f_{XY}(1,1)$.
$\therefore$ X and Y are dependent.

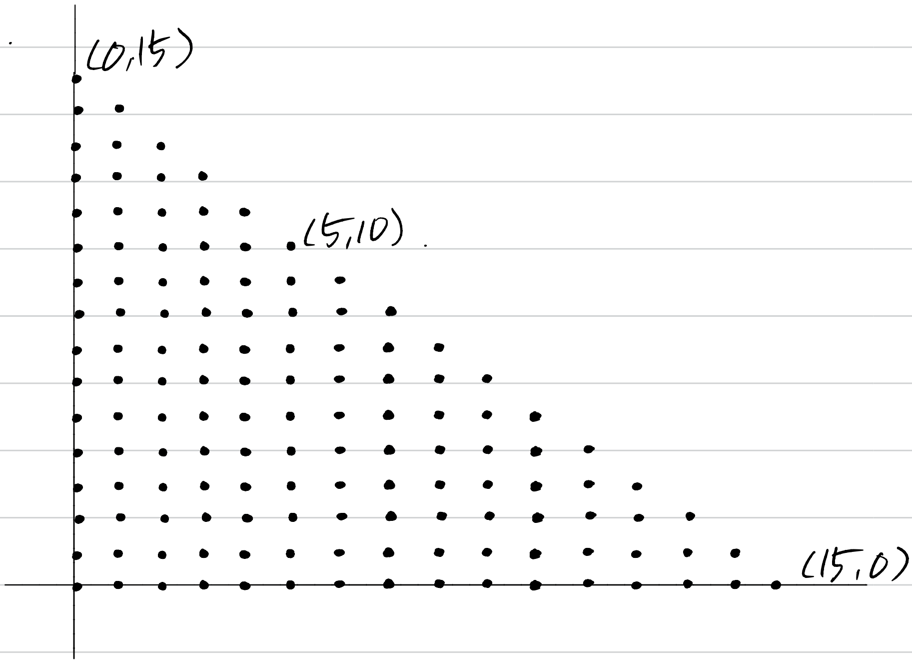
#4.1-9

(a) $f(x,y) = \frac{n!}{x!y!(n-x-y)!} P_X^x P_Y^y (1-P_X-P_Y)^{n-x-y}$

$= \frac{15!}{x!y!(15-x-y)!} \left(\frac{6}{10}\right)^x \left(\frac{3}{10}\right)^y \left(1-\frac{6}{10}-\frac{3}{10}\right)^{15-x-y}$

$= \frac{15!}{x!y!(15-x-y)!} \left(\frac{6}{10}\right)^x \left(\frac{3}{10}\right)^y \left(\frac{1}{10}\right)^{15-x-y}$

(b).



The set of integers for which $f(x,y)>0$ is $\{(x,y) \mid x+y \le 15\}$

X and Y are dependent.

Because the shape of the graph of the set of integers is not a square and for independent variables the shape would be square.

(c) $P(X=10, Y=4) = \frac{15!}{10!\,4!\,(15-10-4)!} \left(\frac{6}{10}\right)^{10} \left(\frac{3}{10}\right)^{4} \left(\frac{1}{10}\right)^{15-10-4}$

≈ 0.07354

(d) $B(n, p_X) = B(15, \frac{6}{10})$.

$p = \frac{6}{10} = \frac{3}{5}$

$P(X=k) = {}_nC_k\, p^k (1-p)^{n-k} = \frac{n!}{k!\,(n-k)!} p^k (1-p)^{n-k}$

$P(X=k) = \frac{15!}{k!\,(15-k)!} 0.6^k \cdot 0.4^{15-k} = \frac{15!}{k!\,(15-k)!} 0.6^k\, 0.4^{15-k}$

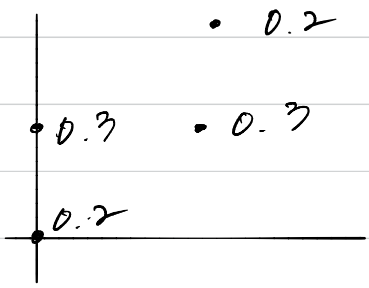
(e) $P(X=0) = \frac{15!}{0!\,15!} 0.6^0 \cdot 0.4^{15} \approx 0$.

$P(X=1) \approx 0$ $\qquad P(X=11) \approx 0.1268$.

$P(X \le 1) = 0.9095$.

4.2-2

(a)



$f(0,0) = f(1,2) = 0.2$

$f(0.1) = f(1.1) = 0.3$

(b) $f_X(0) = f(0,0) + f(0,1) = 0.2 + 0.3 = 0.5$

$f_X(1) = f(1,1) + f(1,2) = 0.3 + 0.2 = 0.5$

$f_Y(0) = f(0.0) = 0.2$

$f_Y(1) = f(0,1) + f(1.1) = 0.6$

$f_Y(2) = f(1,2) = 0.2$

(c) $\mu_X = \Sigma x f(x) = 0 \times 0.5 + 1 \times 0.5 = \frac{1}{2}$

$\mu_Y = \Sigma y f(y) = 0 \times 0.2 + 1 \times 0.6 + 2 \times 0.2 = 1$

$E(XY) = \Sigma xy f(x,y) = 0.7$

$\sigma_X^2 = \Sigma (x - \mu_X)^2 f(x,y) = 0.25$

$\sigma_Y^2 = \Sigma (y - \mu_Y)^2 f(x,y) = 0.4$

$Cov(X.Y) = E(XY) - \mu_X \mu_Y = 0.2$

$\rho = Cov / \sigma_X \sigma_Y \approx 0.6325$.

#42-7

$\mu_X = 1, \quad \mu_Y = 0.$

(a) $Cov(X,Y) = E(XY) - \mu_X\mu_Y$

$= 0\frac{1}{4} + 1\cdot 1\frac{1}{4} + 1\cdot(-1)\frac{1}{4} + 2\cdot 0\cdot\frac{1}{4} - 0 = 0$

$\therefore \rho = 0.$

$cov = 0 \nRightarrow$ independent.

($\Leftarrow$ is always true).

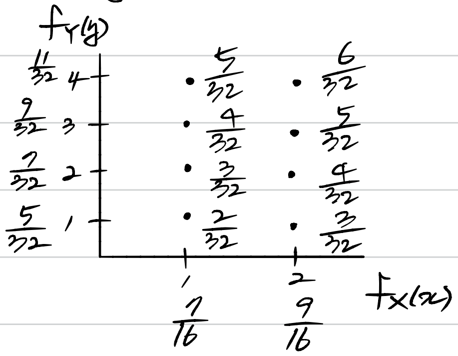
$\therefore$ not must to be independent.

(b) $Cov = 0, \quad \rho = 0.$

#4.3-1

$f_{XY}(x,y) = \frac{x+y}{32} \quad x = 1, 2, \quad y = 1, 2, 3, 4.$

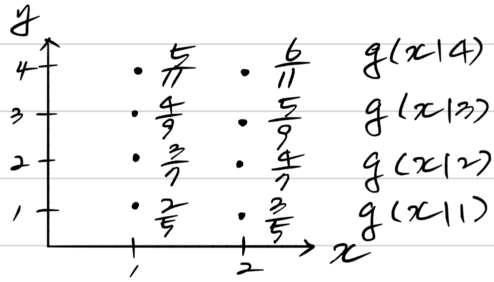
(a)



$f_X(x) = \frac{4x+10}{32} = \frac{2x+5}{16}$

$f_Y(y) = \frac{2y+3}{32}$

(b).

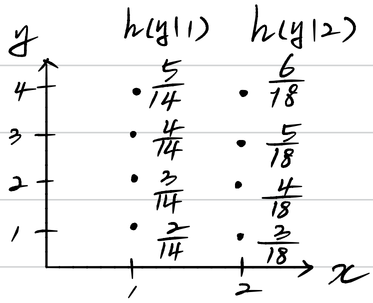


$g(x|y) = \frac{(x+y)/32}{(2y+3)/32}$

$= \frac{x+y}{2y+3}$

$x = 1, 2$ when $y = 1, 2, 3, 4$

(c).



$h(y|x) = \frac{(x+y)/32}{(2x+5)/16}$

$= \frac{x+y}{4x+10}$

$y = 1, 2, 3, 4$ when $x = 1, 2.$

(d). $P(1 \le Y \le 3 \mid X=1) = h(1|1) + h(2|1) + h(3|1)$

$= \frac{2+3+4}{14} = \frac{9}{14}$

$P(Y \le 2 \mid X=2) = h(1|2) + h(2|2)$

$= \frac{3}{18} + \frac{4}{18} = \frac{7}{18}$

$P(X=2 \mid Y=3) = g(2|3) = \frac{5}{9}$

(e) $E(Y \mid X=1) = \sum_{y=1}^{4} y \cdot h(y|1)$

$= \sum_{y=1}^{4} y\left(\frac{y+1}{14}\right) = 1 \cdot \frac{2}{14} + 2 \cdot \frac{3}{14} + \frac{3 \cdot 4}{14} + 4 \cdot \frac{5}{14}$

$= \frac{1}{14}(2+6+12+20) = \frac{40}{14} = \frac{20}{7}$

$\sigma^2_{Y|1} = E\left[\left(Y-\frac{20}{7}\right)^2 \middle| X=1\right] = \sum_{y=1}^{4}\left(y-\frac{20}{7}\right)^2 \frac{y+1}{14}$

$= \left(-\frac{13}{7}\right)^2 \frac{2}{14} + \left(-\frac{6}{7}\right)^2 \frac{3}{14} + \left(\frac{1}{7}\right)^2 \frac{4}{14} + \left(\frac{8}{7}\right)^2 \frac{5}{14}$

$= \frac{770}{49 \times 14} = \frac{55}{49}$

4 3 - 6

X : deductible on the homeowners' insurance.

Y : deducitible on automobile insurance

(a) $P(X=500) = 0.1 + 0.2 + 0.1 = 0.4$

$P(Y=500) = 0.1 + 0.2 + 0.05 = 0.35$

$g(y|x) = \frac{f(x,y)}{f_X(x)}$

$P(Y=500 \mid X=500) = \frac{0.2}{0.4} = \frac{1}{2} = 0.5$

$P(Y=100 \mid X=500) = \frac{0.1}{0.4} = 0.25$

(b) $\mu_X = 100 \times 0.35 + 500 \times 0.4 + 1000 \times 0.25 = 485$

$\mu_Y = 1000 \times 0.3 + 500 \times 0.35 + 100 \times 0.35 = 510$

$\sigma_X^2 = E(X^2) - [E(X)]^2 = 353,500 - 485^2 = 118,275$

$\sigma_Y^2 = (Y-\mu_Y)^2 \cdot p_Y = 130,900$

(c) $E(X \mid Y=100)$

$= 100 \times \frac{0.2}{0.35} + 500 \times \frac{0.1}{0.35} + 1000 \times \frac{0.05}{0.35}$

$= \frac{120}{0.35} = \frac{12000}{35} = \frac{2400}{7}$

$E(Y \mid X=500) = 100 \times \frac{0.1}{0.4} + 500 \times \frac{0.2}{0.4} + 1000 \times \frac{0.1}{0.4} = \frac{210}{0.4} = 525$

(d) $Cov(X,Y) = E[XY] - \mu_X \mu_Y$

$E(XY) = 297{,}000$

$297{,}000 - 485 \times 510 = 49{,}650$

(e) $\rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = 49650 / 124427.48 = 0.399$

4.4 - 2

(a) $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$

$= \int_0^1 x + y \, dx = (xy + \frac{y^2}{2})\Big|_0^1 = x + \frac{1}{2}$

$f_Y(y) = \int_0^1 x + y \, dx = (\frac{x^2}{2} + xy)\Big|_0^1 = y + \frac{1}{2}$

$f_X f_Y = xy + \frac{1}{2}x + \frac{1}{2}y + \frac{1}{4} \neq f_{XY}(x,y).$

not indep.

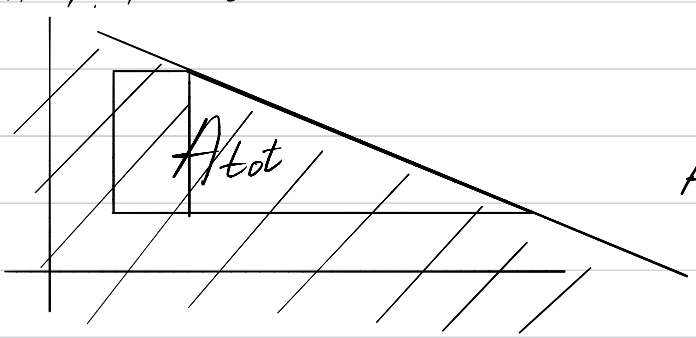
(b). $\mu_X = \int_0^1 x^2 + \frac{x}{2} \, dx = \frac{7}{12}$

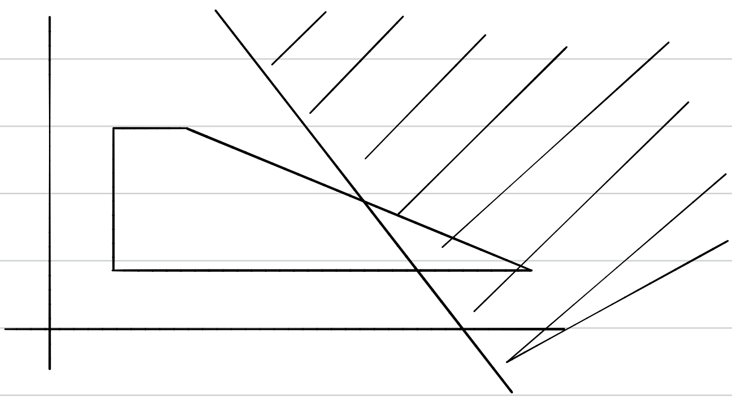
$\mu_Y = \int_0^1 y^2 + \frac{y}{2} \, dy = \frac{7}{12}$

$\sigma_X^2 = \int_0^1 x^3 - \frac{2x^2}{3} - \frac{35x}{144} + \frac{49}{288} \, dx = \frac{11}{144}$

$\sigma_Y^2 = \int_0^1 (y - \frac{7}{12})^2 (y + \frac{1}{2}) \, dy = \frac{11}{144}$

4.4 - 10





$1 < t_1 < 10, \quad 2 < t_2 < 6$

$t_1 + 2t_2 < 14.$

$A_{tot} = 4 \times 1 + \frac{8+4}{2} = 4 + 16 = 20$

$\underline{T_1 + T_2 > 10}$

$A_{T_1+T_2>10} = \frac{2 \times 2}{2} = 2$

$P(T_1 + T_2 > 10)$

$= \frac{A_{T_1+T_2>10}}{A_{tot}} = \frac{2}{20} = 10\%$

4.4 − 11

(a) $\int_0^1 \int_0^{1-y} cx(1-y)\,dx\,dy$

$= \int_0^1 \frac{c}{2}(1-y)^3\,dy = \frac{c}{2}\int_0^1 (1-y)^3\,dy$

$= \frac{c}{2}\left[-\frac{1}{4}(1-y)^4\right]_0^1 = \frac{c}{2}\cdot\frac{1}{4} = \frac{c}{8} = 1.$ $\quad \therefore c = 8.$

(b) $P(Y < X \mid X \le \frac{1}{4}) = \dfrac{P(Y < X \le \frac{1}{4})}{P(X \le \frac{1}{4})}$

$P(X \le \frac{1}{4}) = \int_0^{\frac{1}{4}} f_X(x)\,dx$

$f_X(x) = \int_0^{1-x} 8x(1-y)\,dy = \left[8xy - 4xy^2\right]_0^{1-x}$

$= 8x(1-x) - 4x(1-x)^2$

$= 8x - 8x^2 - 4x + 8x^2 - 4x^3$

$= -4x^3 + 4x$

$P(X \le \frac{1}{4}) = \int_0^{\frac{1}{4}} (-4x^3 + 4x)\,dx$

$= \left[-x^4 + 2x^2\right]_0^{\frac{1}{4}} = -(\frac{1}{4})^4 + 2\cdot(\frac{1}{4})^2$

$= -\frac{1}{256} + \frac{1}{8} = \frac{31}{256}$

$P(0 < Y < X \le \frac{1}{4}) = \int_0^{\frac{1}{4}}\int_0^{x} 8x(1-y)\,dy\,dx$

$= \int_0^{\frac{1}{4}} (8x^2 - 4x^3)\,dx$

$= \left[\frac{8}{3}x^3 - x^4\right]_0^{\frac{1}{4}} = \frac{8}{3}(\frac{1}{4})^3 - (\frac{1}{4})^4$

$= \frac{32 - 3}{4^4 \times 3} = \frac{29}{768}$

$P(Y < X \mid X \le \frac{1}{4}) = \frac{29}{768} \Big/ \frac{31}{256} = \frac{29}{93}$ //.

4.5 − 4

(a) $E(Y \mid X = 76) = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(76 - \mu_X)$

$= 5 + 0.6\cdot\frac{15}{12} = 5.75$

(b) $Var(Y \mid X = 76) = \sigma_Y^2(1 - \rho^2)$

$= 225 \times 0.64 = 144$

(c) $P(Y \le 90 \mid X = 76)$

$P(Z \le \frac{90 - 5.75}{12}) \doteqdot 1.$