

Weighted LR vs. Bayesian LR: Bankruptcy and Liquidation Prediction

Junwoo Yang^{*} Jaeseon Lee[†]

June 30, 2021

Abstract

In this study, we examine three methods of Logistic Regression (LR) to predict bankruptcy and liquidation of US companies: Maximum Likelihood LR, Weighted LR, and Bayesian LR. While most previous prior studies focused on predicting bankruptcy, this study predicts liquidation as well as bankruptcy. Since bankruptcy is an extremely rare event, including liquidation in the response variable could alleviate the problem caused by imbalanced data. Since there are about 400 financial variables, we take preselection steps by t-test and VIF before selecting variables by AIC. We predict companies that will go bankrupt or liquidate within three years based on a certain financial year.

Keywords: fundamental, bankruptcy, liquidation, imbalanced data, variable selection, maximum likelihood LR, weighted LR, Bayesian LR

1 Introduction

Business bankruptcy and liquidation forecasts are important. Creditors and entity investors must correctly assess the probability of an entity's default on its obligations for profitable decisions. For banks, accurate forecasts of corporate bankruptcy and liquidation enable safe lending operations for businesses and impose interest rates that adequately reflect risks. In addition, if an accounting firm makes inappropriate predictions about the possibility of bankruptcy of the audited company, it may be caught in a lawsuit. On the other hand, the choice of explanatory variables and the choice of functional form between these variables are key issues when constructing bankruptcy and liquidation prediction models.

Previous studies on bankruptcy prediction employ economic implication of variables. For example, [Laitinen and Laitinen \(2000\)](#) analyzed the ratio of cash, cash flow and equity to total assets as explanatory variables, based on the fact that the more cash flows, the more net cash flows, and the more flexible financing from outside. According to [Laitinen and Laitinen \(2000\)](#), only the ratio of cash to total assets and equity capital were statistically significant when the analysis was conducted based on financial position a year before bankruptcy. [Kuruppu, Laswad, and Oyeler \(2003\)](#) devised a model for forecasting liquidation for New Zealand entities, which included 63 variables in their financial statements. Of these, only 12 variables, in-

This paper is a project report of Math Capstone PBL (Data Analysis) taught at Hanyang University.

^{*}Department of Finance

[†]Department of Economics and Finance

cluding the ratio of equity to total assets, were statistically significant. Theoretical approaches to variable selection are rare.¹ In this study, approximately 900 financial variables are included in the candidates of variables in the final model.

Bankruptcy-themed studies typically postulate linear (linear discriminant analysis) or logistic (logistic regression) function relationships between variables. In particular, logistic regression analysis is effective in analyzing bipartisan data, such as whether an individual has cancer or whether a company has gone bankrupt. Indeed, [Laitinen and Laitinen \(2000\)](#) tried to devise a corporate bankruptcy prediction model using the logistic regression model and Taylor deployment. Meanwhile, [Hauser and Booth \(2011\)](#) derives a bankruptcy prediction model based on the robust logistic regression model. They analyzed two models after dividing the estimated direction of the coefficients by the Bianco-Yohai estimate and the maximum likelihood estimate. As a result, data from 2006 to 2007 showed that only the former model accurately predicted Lehman Brothers' bankruptcy. This work seeks to devise a corporate bankruptcy and liquidation prediction model using a logistic regression model. Based on the data, we would like to proceed with the fit and evaluate the performance of the fit model to determine whether it is useful as a predictive model.

This study seeks to model mid- to long-term bankruptcy and liquidation forecasts. In general, an entity's bankruptcy or liquidation is not determined in a short period of time, and signals about it are likely to be transmitted over a long period of time. That is, even if the entity's data for a particular year diagnoses that the entity is about to enter bankruptcy or liquidation, it may enter bankruptcy or liquidation two or three years later than the following year. On the other hand, setting the period too long will reduce the explanatory power of the model. Therefore, this study sought to cover most of the cases by presupposing an appropriate period of three years. In addition, because the financial position of the entity is likely to be very bad just before bankruptcy and liquidation, it would be very good in terms of the practicality of the forecast if a reasonable medium-term forecast model could be devised. For three years from 2011 to 2013, the company tried to analyze and explain whether it was bankrupt or liquidated based on 2010 data. Through this study, we would like to first identify which explanatory variables are significantly related to bankruptcy and liquidation, and predict which companies will go bankrupt and liquidated within the next three years based on the final model and the current data in 2020.

2 EDA

2.1 Data

The annual fundamental data used in this paper are from the Compustat Capital IQ database of WRDS. The observations are firms traded on the NYSE, AMEX, NASDAQ, TSX, and NYSE Arca from January 2000 through November 2020. This data set includes a total of 981 variables and 226,866 observations.

The location data are from the United States cities database of SimpleMaps². Meanwhile,

¹[Laitinen and Laitinen \(2000\)](#)

²<https://simplemaps.com/data/us-cities>

corporate fundamentals data only contained information about the states and cities in which individual companies existed. We wanted to see the distribution of companies by state, county, and city at a glance, and further analyze in detail whether the company's bankruptcy and liquidation are different depending on geographical requirements. Therefore, in addition to the enterprise data, additional data reflecting the geographical characteristics of the United States were built. These materials include the names of City, County and State, County FIPS, and the longitude and latitude of the city. We used maps to visually understand the county-specific data merged with the fundamental data based on City and State, and used longitude and latitude to show the company's coordinates to understand the distribution at a glance.

In the meantime, the headquarters wanted to exclude companies outside of the United States from the analysis. This provided convenience in the process of examining the relationship between geographical requirements and other variables. Increasing the proportion of bankruptcy and liquidation companies in the data also allowed for a slight reduction in the data's excessive imbalance. First of all, the headquarters excluded 3,058 companies that exist outside the United States from the analysis. It also excluded four entities that do not have information about the state from the analysis for entities existing in the United States. On the other hand, the analysis excluded 19 entities that disappeared before 2011 and seven in Puerto Rico and Guam for some reason.

The variables in the data are divided into the following eight categories: Identifying Information, company descriptor, balance sheet items, income statement items, cash flow items, miscellaneous items, supplemental data items, maps items.

The enterprise data utilized in this study encompasses 981 variables. Therefore, we found it difficult to explain all the meanings of individual variables, and it was meaningless when it came to analysis. Table 2 lists 414 of the 981 variables. Among them, we only want to elaborate on the variables included in the final model.

First, *aco* means an asset that is not included in the balance sheet in cash, cash equivalents, uncollected, or inventories. *aqpl1* represents an asset calculated at fair value. *Caps* are capital surplus and *csho* is the number of common shares in the market at the end of the year. The *cstk* represents the total face value of all general equity. *Glcea* is the after-tax amount of sales increases and decreases not included in the calculation of S&P. *idbflag* is a categorical variable that represents the source of the data, divided into three cases: within the United States (*idbflag*=D), outside the United States (*idbflag*=I), and both (*idbflag*=B). Since the source of the data has never been outside the United States (*idbflag*=I), it has virtually two categories. On the other hand, companies (B), whose sources are domestic and foreign, can be interpreted as multinational companies. *Optfvgr* represents a weighted average of the fair value of the option granted over the year. *Spced* refers to diluted earnings per share on an S&P basis. On the other hand, *stalt* is a categorical variable indicating that the entity is at risk of bankruptcy or is in the process of acquiring the entity (*stalt*=1). *stkcpa* refers to the amount of share-based compensation that has been treated as an expense in the income statement.

This study defines BL variables that represent 1, if and when an entity goes bankrupt and liquidates, as dependent variables and proceeds with the analysis. Within the data built from WRDS, the BL variable could be redefined by classifying only observations corresponding to bankruptcy and liquidation in DLRSN variables that separated the entity from 14 causes of dis-

appearance. Table 6 summarizes the number of companies that went bankrupt and liquidated during the 2011-2020 period in the DLRSN variable in the raw material.

In particular, this study seeks to devise a mid- to long-term forecast model for bankruptcy and liquidation, considering that the bankruptcy and liquidation process of an entity does not take place in a short period of time. Therefore, instead of defining an individual year as a period of the dependent variable, the entity postulates a sufficient period of three years. This study selected three years from 2011 to 2013. Thus, the response variable is defined as follows:

$$y_i = \begin{cases} 1 & \text{if firm } i \text{ was bankrupt or liquidated between 2011 and 2013,} \\ 0 & \text{otherwise. (solvent company)} \end{cases}$$

2.2 Handling missing value

Prior to data analysis, it is critical to properly process missing data. When studying social phenomena, missing values are often included in the data, which can cause serious errors if improper processing of missing values is done. First of all, missing values reduce the statistical power, which indicates the probability of correctly rejecting the null hypothesis when it is false. Furthermore, missing values can produce bias in parameter estimation. Thirdly, missing values reduce the representation of samples, and finally complicate data analysis.³ Therefore, missing-value processing is an important problem that is in line with the validity of data analysis, which must be done in an appropriate manner.

Prior to full-scale processing of missing values, 26 variables with the same value were removed from all observations. This is because variables that all entities have the same value do nothing to explain whether they are bankrupt or liquidated. We also exclude 552 variables with missing values above 80% from our analysis. This summarizes the data into 349 continuous variables, 65 categorical and other informational variables, a total of 414 variables and 7,461 observations.

Next, the analysis was conducted by estimating the missing values within the continuous variables as the industry sector-specific average. This is because for enterprise-related data, characteristics may vary depending on the industry sector in which the enterprise is engaged. By representing missing values as industry-specific average values rather than overall average values, we tried to maintain characteristics according to industry segments of the industry. This study postulates NAICS variables as a criterion for enterprise industry segment classification. The commonly used SIC code categorizes industries into four digits, with a limitation that has not changed since its revision in 1987. It was judged that this may be a somewhat inappropriate classification standard in modern society, where industrial flows are rapidly changing. Therefore, the NAICS code revised in 2017 was considered a more appropriate criterion, classifying the industry in more detail than the SIC with six digits.

The first two digits of the NAICS represent a general classification of the entity's economic activities, the third being its sub-sector and the fourth being its industrial group. The fifth represents the NAICS classification of industrial groups, and the last sixth represents the national industry. Table 3 represents the structure of NAICS. Column 1 represents the first two

³Kang (2013)

digits of NAICS, and column 2 represents the number of observations in the data. A detailed description of the first two digits of NAICS is described in column 3. For example, an entity with the first two digits of the NAICS code 31 is classified as a manufacturing industry, and if 52 is classified as a financial or insurance industry.

NAICS code for Monster Beverage Inc., known for its energy drinks, is 312111. An example of Monster Beverage Inc. in Table 4 shows that the classification becomes more granular as the number of digits increases from 31 to 312, 3121. Nike, meanwhile, is classified as a manufacturing company, starting with the same 31 as Monster Beverage Inc., but the detailed classification is different. Nike was classified as 316 for leather and related product manufacturing, indicating the final 316210. In the NAICS code, the industrial sector below 3162 is the same as the shoe manufacturing sector.

This work estimates missing values as NAICS-specific averages. Starting with the 6-digit average, we proceeded with the upper classification when it was impossible to classify the lower classification, such as the 5-digit mean and 4-digit mean. On the other hand, there were 560 missing NAICS variables, which could reasonably be estimated based on their relationship to other variables. The observation with missing NAICS values was either the entity's SIC code: 6722 or 6726. Therefore, they conducted the analysis by replacing the SIC-specific averages of the entity. The above processing of missing values allowed us to handle all the missing values present in 350 continuous variables.

2.3 Preselection by t-test and VIF

Before proceeding with the correlation analysis of 349 continuous variables and 65 categorical and other informational variables, we wanted to eliminate the analytical unnecessary variables. In particular, we try to first screen variables that do not help explain dependent variables and variables that have an overly high correlation between explanatory variables. First, after conducting a t-test to see if there is a difference in the mean between the continuous explanatory variables for each group of bankruptcy and liquidation, a continuous explanatory variable that is considered irrelevant to the dependent variable BL was primarily excluded from the analysis. Specifically, the null hypothesis of equal variance in groups within variables was tested using the F-test, and based on this result, a t-test (the null hypothesis: equal mean) was conducted for the difference in means. As a result of the F-test, we conducted a Welch's two sample test for variables that determined that the variance was different, and a variable that determined that the variance was the same proceeded with a two sample test. The t-test excluded 105 of the 349 continuous variables from the analysis.

Next, we investigate the existence of a multi-collinearity problem between explanatory variables and solve it. Multicollinearity, which means a complete or almost complete linear dependent relationship between explanatory variables, makes the variance of the estimated regression coefficients very large, which reduces reliability. It also poses problems with the interpretation of regression models. The interpretation of regression coefficients presupposes the fixed state of all other explanatory variables, where the existence of multicollinearity contradicts the assumption. Furthermore, the sign of the estimate of the regression coefficient contradicts empirical or theoretical expectations. This study conducted multicollinearity verification

using the Variance Inflation Factor (VIF). We investigate the VIFs of explanatory variables and try to solve the multicollinearity by excluding the largest variables sequentially one by one from the analysis at the same time that the corresponding figure has a value of 10 or higher. A total of 90 variables were included in the analysis, excluding 154 of the remaining 244 variables that were excluded through t-test throughout the process. Through the above two processes, we were able to organize the data into 90 continuous variables and 65 categories and other informational variables, a total of 155 variables. Table 5 is a summary of this.

Finally, the analysis also excludes missing values and variables with more than 6,000 zero values based on raw materials. This was aimed at screening as much meaningless variables as possible in the first-order variable selection process, as we judged that too many values with zero could not reasonably be processed, even if the missing values were not more than 80%. In other words, we found that variables with missing values of 60% and the remaining 30% have zero values are less convincing as they result in estimating missing values from observations of the remaining 10%. Through this process, the data was organized into 76 variables. In the meantime, the analysis excluded meaningless categorical variables, such as an entity's telephone number, and other information variables about the entity. As a result, the first-order variable selection process allowed us to select 44 variables and 7,380 observations to proceed with the analysis.

2.4 Summary statistics

It is important to adjust characteristics such as units and ranges of explanatory variables in the pre-processing process. If each variable has a different unit, the variable with the larger value may have a greater effect on the dependent variable. Furthermore, if the value of the data is too large or too small, the model can be problematic by converging to zero or diverting to infinity. The task of adjusting the properties of the data is called Feature Scaling, where scaling methods vary widely. In this work, we try to organize the data through standardization, which is useful in building classification models. Standardization is a scaling operation that makes the mean of individual variables zero and the variance one.

Table 7 summarizes the underlying statistics of continuous variables in the final model. For the mean and standard deviation, the standardization operation showed that the standardization was organized into 0 and 1, respectively. Figure 2 shows maps of the distribution of companies by states, counties, and cities in the United States. Many companies are distributed in California, where Silicon Valley is located, and in the northeastern United States.

3 Methods of logistic regression

This section provides model and notation of Logistic Regression (LR). We begin by estimating parameters of Maximum Likelihood LR (MLLR) and Weighted LR (WLR), and then determine the distribution of parameters of Bayesian LR (BLR) by using R packages and WinBUGS.

3.1 Subsampling imbalanced data

In 2010, there were 7,380 U.S. companies, 76 of which went bankrupt or liquidated between 2011 and 2013. The proportion of events in the population is 1.03% which is rare and has been established in the literature that these variables are difficult to predict and explain. For example, the problem arises that the fitted model boasts high accuracy even if all X_i are classified into negative. Thus, we manipulate the proportion by constructing training set with 60 $Y_i = 1$ and 600 $Y_i = 0$ at random. It gives more weight to group of $Y_i = 1$.

However, this increases not only the rate at which the actual positive is correctly diagnosed as positive, but also the rate at which the actual negative is diagnosed as positive. That is, type 1 error increases instead of reducing type 2 error. In this case, it was considered more tremendous to predict that the company that would actually go bankrupt would not go bankrupt than to the contrary. Therefore, we try to solve the problem caused by imbalanced data through subsampling despite the increase in type I error. The test set consists of 16 bankrupt companies and 160 non-bankrupt companies.

3.2 Maximum Likelihood LR

Let $X \in \mathbb{R}^{n(k+1)}$ be a data matrix where n is the number of observation and k is the number of variables, and $Y \in \mathbb{R}^n$ be a binary outcomes vector. For all observation $X_i \in \mathbb{R}^{k+1}$ (a row vector in X), the outcome is either $y_i = 1$ (positive) or $y_i = 0$ (negative). The goal is to classify X_i as positive or negative. It can be treated as a Bernoulli trial with an expected value $E(y_i)$ or probability p_i . The logistic function commonly used to model each X_i with its expected outcome is given by the following formula:

$$E[y_i|X_i, \beta] = p_i = \frac{e^{X_i\beta}}{1 + e^{X_i\beta}} = \frac{1}{1 + e^{-X_i\beta}}$$

where $y_i \sim \text{Bernoulli}(p_i)$ and $\beta = (\beta_0, \beta_1, \dots, \beta_m)^\top$. Let η be the logit link which is the logarithm of the odds ratio. It is defined as

$$\eta_i = \text{logit}(p_i) = \log \frac{p_i}{1 - p_i} = X_i\beta,$$

and can be written as $\eta = X\beta$ in matrix form. Now, assuming that the observations are independent, the likelihood function is

$$L(\beta|X, Y) = \prod_{i=1}^n (p_i)^{y_i} (1 - p_i)^{1-y_i} = \prod_{i=1}^n \left(\frac{e^{X_i\beta}}{1 + e^{X_i\beta}} \right)^{y_i} \left(\frac{1}{1 + e^{X_i\beta}} \right)^{1-y_i} = \prod_{i=1}^n \frac{e^{X_i\beta y_i}}{1 + e^{X_i\beta}}.$$

and the regularized log likelihood function is defined as

$$\log L(\beta|X, Y) = \sum_{i=1}^n \log \frac{e^{y_i X_i \beta}}{1 + e^{X_i \beta}} - \frac{\lambda}{2} \|\beta\|^2 \quad (1)$$

where the regularization (penalty) term was added to obtain better generalization. Since the log likelihood function is strictly concave, $\hat{\beta}_{\text{MLE}}$ which maximizes the log likelihood can be

found.

3.3 Weighted LR

In addition to subsampling, we can use adjusted likelihood function, called weighted likelihood (King and Zeng, 2001; Maalouf and Siddiqi, 2014). Let \bar{y} and τ be proportion of events in the sample and in the population, respectively. The weighted likelihood is defined as

$$L_W(\beta|X, Y) = \prod_{i=1}^n (p_i)^{w_1 y_i} (1 - p_i)^{w_0 (1 - y_i)},$$

where $w_1 = \tau/\bar{y}$, and $w_0 = (1-\tau)/(1-\bar{y})$. Now, instead of maximizing Eq.(1), we can maximize the weighted log likelihood

$$\log L_W(\beta|X, Y) = w_1 \sum_{y_i=1} \log(p_i) + w_0 \sum_{y_i=0} \log(1 - p_i) = - \sum_{i=1}^n w_i \log(1 + e^{(1-2y_i)X_i\beta})$$

where $w_i = w_1 y_i + w_0 (1 - y_i)$.

3.4 Bayesian LR

4 Variable selection and final model

4.1 Stepwise backward elimination by AIC

Let \hat{L} be the maximum value of the likelihood function for the model. Then the AIC value of the model is the following.

$$\begin{aligned} AIC &= 2k - 2 \log \hat{L} \\ BIC &= k \log n - 2 \log \hat{L} \end{aligned}$$

where k is the number of estimated parameters in the model and n is the number of observation.

Compared to the BIC method (below), the AIC statistic penalizes complex models less, meaning that it may put more emphasis on model performance on the training dataset, and, in turn, select more complex models.

We fit the model based on training data. Meanwhile, model selection was based on the Akaike information critic (AIC) statistic. AIC is a measure that helps maintain a balance between fit and simplicity in the process of model selection. The smaller the AIC, the better the model is, and as the expression suggests, the higher the number of variables, the higher the AIC. In particular, AIC has the advantage of enabling comparisons between non-nested models. For example, a model with explanatory variables X_1, X_2, X_3 and a model with explanatory variables X_4, X_5 cannot be compared with an F-test, but can be based on an AIC. From a conformational point of view, we determine that there is no significant difference in the model

where AIC differs by less than 2, and furthermore, the smaller AIC model is the better model.⁴

The selection of variables in the model utilizes the Stepwise Selection Method. The stepwise selection method is a compromise between Forward Selection and Backward Elimination, in which important variables are found while repeating each step selection and elimination. The method reviews step by step whether the already selected variables can be eliminated, while selecting additional important variables one by one. In other words, variables included in the model in the early stages can also be erased in later stages. This work determines the model representing the lowest AIC as the final model through a stepwise selection method. In other words, we thought that the selection of variables was complete when the AIC could no longer be lowered even with the addition or exclusion of additional variables in the current stage. On the other hand, the statistical significance of individual variables was not taken into account because variable selection based on AIC was considered. Model fitting utilizes training data containing 44 variables and 660 observations that have gone through all of the preceding processes. Table 10 is a clean-up of the variables used to fit the model.

The fitted results included nine continuous explanatory variables and two categorical explanatory variables, with a total of 11 explanatory variables in the final model. Seven explanatory variables, $aqpl1$, $caps$, $csho$, $glcea$, $optfvgr$, $stalt$, and $stkcpa$, were shown to be statistically significant. Information on the coefficients of the final model is briefly described in Table 11. The selected model by AIC method is

$$\eta_i = -203.6 - 7.19x_{aco,i} + 8.9x_{aqpl1,i} + 2.12x_{caps,i} - 7.58x_{csho,i} - 13.51x_{cstk,i} \\ + 1.56x_{glcea,i} + 197.01x_{idbflag,i} - 19.96x_{optfvgr,i} - 2.42x_{spced,i} + 2.66x_{stalt,i} - 3.31x_{stkcpa,i}.$$

The likelihood ratio test for the final model resulted in a very small p-value of 1.21×10^{-14} , which allowed the null hypothesis to be rejected. In other words, at least one explanatory variable could be determined to have explanatory power over the dependent variable.

The selected model by BIC method is a less complex:

$$\eta_i = -142.9 - 7.3x_{aco} + 6.9x_{aqpl1} - 7.1x_{csho} + 137.6x_{idbflag} - 21.8x_{optfvgr} - 3.3x_{stkcpa}.$$

The selected weighted LR model by AIC method is

$$\eta_i = -189.460 - 5.595x_{aco} + 8.451x_{aqpl1} - 4.672x_{csho} - 15.164x_{cstk} + 185.161 - 22.422x_{idbflag} - 2.900x_{stkcpa}.$$

The selected weighted LR model by BIC method is

$$\eta_i = -3.235 - 15.430x_{aco} - 25.100x_{optfvgr}.$$

5 Performance

The performance evaluation of the model is based on test data. The test data consists of 16 companies with $BL = 1$ and 160 companies with $BL = 0$. We determine the optimal threshold

⁴Chatterjee and Hadi (2015)

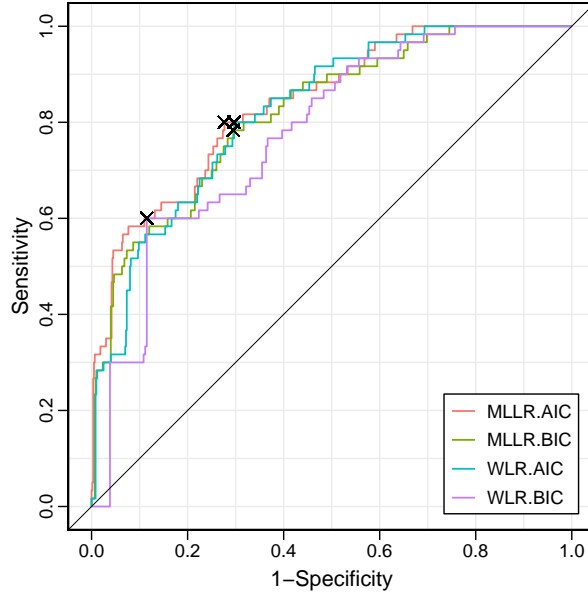


Figure 1: ROC curves

by drawing ROC curves, and determine the superiority of the model through AUC. We also evaluate the performance of the model by understanding classification accuracy, etc., based on the confusion matrix.

The ROC curve represents an aspect of sensitivity and $1 - \text{specificity}$ with threshold changes. Specifically, the $1 - \text{singularity}$ represents a false positive rate. The closer sensitivity and specificity are to 1, the better the model is, and thus the optimal critical point is determined closest to the ROC curve and the coordinates with $(1 - \text{specificity}, \text{sensitivity})$ values (0,1). Meanwhile, the ROC curve derived from the logistic model evaluates the overall performance of the model. The area under the ROC curve is called AUC, and the larger the AUC, the better the model is. If the AUC is 0.5, it is a model of the same level as random estimation, and if it is 1, it is a perfect model. Generally, if the AUC is 0.8 or higher, we judge it as an excellent model.

Figure 1 shows the ROC curve of the final model. The optimal critical point on the ROC curve is shown to be 0.1267. This means that if the predicted probability of bankruptcy and liquidation exceeds 0.1267, the sensitivity and fit of the entity is best when classifying it as bankruptcy and liquidation. Therefore, on an optimal critical point basis, an entity is classified as a bankruptcy and liquidation entity if the estimated probability of bankruptcy and liquidation exceeds 0.1267. Conversely, if the estimated probability is less than 0.1267, the entity is not expected to go bankrupt and liquidate. Meanwhile, the AUC was 0.857. Based on this, the final model could be judged to be relatively good.

Before we begin, I want to summarize the terms. In particular, Positive, Negative within a confusion matrix can be interpreted positively and negatively, which can cause confusion in analysis. Consequently, an entity sought to minimize possible confusion in subsequent processes by reinterpreting the term in terms of bankruptcy and liquidation of an entity. The classification results of the prediction model can be summarized in four ways: TP refers to when an entity classified as bankruptcy and liquidation in the forecast model has actually gone bankrupt and liquidated, and TN refers to when an entity classified as insolvent and

liquidated in the model has not actually gone bankrupt and liquidated. On the other hand, FP predicts that the model will bankrupt and liquidate a particular entity, but in reality it is bankrupt and not liquidated, meaning type 1 error. FN predicted that the model would not go bankrupt and liquidate, but in reality it would go bankrupt and liquidate, which is class II error. In particular, the case of classifying bankruptcy and liquidation entities as not likely to be the case when forecasting is a very serious error. For example, if business decisions, such as investment, are made based on forecasting models, the error can result in massive losses. As mentioned earlier, this study focused on this and proceeded with the analysis.

Confusion matrices allow us to determine sensitivity and specificity. Sensitivity refers to the rate at which positive predictions are made correctly (TPR), and specificity refers to the rate at which negative predictions are made correctly (TNR). We determine that the closer the sensitivity and specificity at a particular threshold to 1, the better the classification performance of the model.

Table 1 shows a confusion matrix based on the optimal cut-off 0.105. The final model classified 14 of the 16 actual bankrupt or liquidated companies (BL) as bankrupt or liquidated companies. In addition, 117 out of 160 companies that are neither bankrupt nor liquidated (NBL) were correctly classified. That is, the sensitivity of the model was 87.5%, and the specificity was 73.125%. The closer both sensitivity and specificity are to 1, the better the model is judged, both of which are relatively superior. On the other hand, the accuracy representing the correct classification ratio of the model was 74.43%, and the misclassification rate was 25.57%. Among the companies classified as bankruptcy and liquidation, precision, which means the ratio of actual bankruptcy and liquidation companies, was 24.56%. The ratio of bankruptcy and non-liquidation companies to bankruptcy and liquidation was 26.875%. In other words, there were more cases in which the final model was judged to be bankrupt and liquidated than in the case. However, as noted earlier, it can be interpreted that the threshold is the result of determining that it is more dangerous to predict otherwise the entity to go bankrupt and liquidate.

| | | Predicted | | |
|--------|-----|-----------|-----|-----|
| | | BL | NBL | |
| Actual | BL | 14 | 2 | 16 |
| | NBL | 43 | 117 | 160 |
| | | 57 | 119 | 176 |

Table 1: Confusion matrix with cut-off 0.105

6 Conclusion

Nine continuous variables and two categorical variables were used in the final model. The net amount of cash, cash equivalent, or assets calculated at fair value (aqpl1), and the total amount of ordinary shares distributed at the end of the year (glcea). The explanatory variables

are aco, csho, cstk, optfvgr, spced, and stkcpa variables, while aqpl1, caps, and glcea are shown to fluctuate in the same direction as the probability of bankruptcy and liquidation of an entity. On the other hand, when other conditions are fixed, business activities are found to be more likely to go bankrupt and liquidate than those operating outside the United States (idbflag=D), and those currently at risk of bankruptcy or undergoing acquisitions (stalt=1).

The performance of the model is evaluated by leveraging test data to represent the confusion matrix with the ROC curve. We derive optimal thresholds and AUCs through the ROC curve. The optimal threshold was 0.1267, which means that the sensitivity and goodness of fit are the best when the criteria for classification as bankruptcy and liquidation are set at 0.1267. On the other hand, the AUC, which means the area under the curve, was 0.857, which makes it possible to judge that the fitted model is excellent. Meanwhile, we construct a confusion matrix based on optimal thresholds. The sensitivity was 0.875, fit was 0.731, and accuracy was 0.744. On the other hand, the false positive ratio was 0.269, which was relatively high. This is because the analysis was conducted in the direction of lowering class II errors, judging that the prediction of bankruptcy would be worse than the opposite case, i.e., false positive case, to predict that an entity would not go bankrupt. Based on heresy extraction, this study lowered type 2 errors, which unfortunately increased type 1 errors.

Variables with opposite common sense and signs also existed in the model. This is a common phenomenon due to multicollinearity in regression analysis, but in this work, we diagnose the problem of multicollinearity. The limitations were determined to be attributed to the implementation of heresy extraction in the process of fitting the model and the selection of variables based on AIC. Therefore, we expect to produce results consistent with common sense by supplementing the limits of unbalanced data in other ways instead of heresy extraction, or proceeding with variable selection by setting a criterion other than AIC.

It was judged that diagnosing that a company that would go bankrupt would not go bankrupt would be a more serious error than the opposite. Therefore, this work has been able to reduce the corresponding error by arbitrarily adjusting the number of training data. However, the error of predicting that a company that would not actually go bankrupt, or type 1 error, would go bankrupt, had to be tolerated. Depending on the purpose of the study, sensitive errors will vary, and threshold adjustments will also be made differently. The proper balance between the two errors requires careful judgement by the researcher.

Based on current data and appropriate models, this study sought to predict which companies will go bankrupt and liquidate in the next three years. However, for the 2019 and 2020 data, observations of both variables included in the final model were often missing. Therefore, it was not possible to deal with the missing values in an appropriate way, and the analysis was not carried out to the stage of predicting bankruptcy and liquidation entities. If we proceed with the processing of missing values based on raw materials from 2000 to 2020, we expect to be able to predict bankruptcy and liquidation companies with current data.

Data for the year immediately before the bankruptcy and liquidation of the entity contains more signals than in any other year. In other words, if a company goes bankrupt or liquidated, the warning will remain intact in the previous year's data. Furthermore, [Laitinen and Laitinen \(2000\)](#) found that the model had the best explanatory power when the analysis was conducted based on data from the year just before bankruptcy. In addition, many prior studies have

devised predictive models based on data from the previous year of bankruptcy. However, in this work, we tried to derive a mid- to long-term prediction model over three years, meaning to try a new direction rather than to follow the existing direction. As has already been proven, it would be helpful to use data from the year just before bankruptcy when trying to create a predictive model.

References

- Erkki K. Laitinen and Teija Laitinen. Bankruptcy prediction: Application of the Taylor's expansion in logistic regression. *International review of financial analysis*, 9(4), 327–349, 2000.
- Nirosh Kuruppu, Fawzi Laswad, and Peter Oyelere. The efficacy of liquidation and bankruptcy prediction models for assessing going concern. *Managerial auditing journal*, 2003.
- Richard P. Hauser and David Booth. Predicting bankruptcy with robust logistic regression. *Journal of Data Science*, 9(4), 565–584, 2011.
- Michelle J. White. The corporate bankruptcy decision. *Journal of Economic Perspectives*, 3(2), 129–151, 1989.
- Gary King and Langche Zeng. Logistic regression in rare events data. *Political analysis*, 9(2), 137–163, 2001.
- Maher Maalouf and Mohammad Siddiqi. Weighted logistic regression for large-scale imbalanced and rare events data. *Knowledge-Based Systems*, 59, 142–148, 2014.
- Hyun Kang. The prevention and handling of the missing data. *Korean journal of anesthesiology*, 64(5), 402–406, 2013.
- Samprit Chatterjee and Ali S. Hadi. *Regression analysis by example*. John Wiley & Sons, 2015.
- James H. Stock and Mark W. Watson. *Introduction to Econometrics*. Pearson, 2015.
- David G. Kleinbaum and Mitchel Klein. *Logistic regression: A self-learning text*. Springer, 2010.

| | | | | | | | | | | | |
|-----------|---------------|-------------|---------|---------|---------|--------|----------|---------|------------|-------------|---------------|
| acctstd | auop | costat | dlc | ebit | gind | ivch | naics4 | pidom | reajo | tfvce | txtubbegin |
| acdo | auopic | county_fips | dlcch | ebitda | glcea | ivncf | naics5 | pifo | recch | tfvl | txtubend |
| aco | bkvlp | county_name | dldte | ein | glced | ivst | naics6 | pnca | recco | tic | txtubposdec |
| acodo | BL | csbfd | dlnsn | emp | glceeps | ivstch | naics8 | pncad | recd | tlcf | txtubposinc |
| acominc | busdesc | csbi | dltis | epsfi | glcep | lat | ni | pnaeaps | rect | tstk | txtubpospdec |
| acox | caps | csbo | dlto | epsfx | gp | lco | niadj | pncwia | recta | tstkc | txtubpospinc |
| act | capx | csbpri | dltp | epsfi | gsector | lcox | nopi | pncwip | rectr | tstkn | txtubsettle |
| add1 | capxv | csbr | dltr | epspx | gsubind | lcoxdr | nopio | pnrsho | reuna | tstkp | txtubsoflimit |
| addzip | census_region | csbtr_c | dltt | esopct | gvkey | lct | np | ppeg | revt | txach | txtubtxtr |
| adjex_c | ceoso | csbtr_f | dm | esopdlt | ib | lifr | oancf | ppent | sale | txbco | txtubxintbs |
| adjex_f | ceq | cstk | dn | esopnr | ibadj | lifrp | oiadp | ppeveb | scf | txbcof | txtubxintis |
| ajex | ceql | cstkcv | do | esopt | ibc | lno | oibdp | prca | seq | txc | txw |
| ajp | ceqt | cstke | donr | esub | ibcom | lo | opeps | prcad | seqo | txdb | upd |
| aldo | cfoso | curcd | dp | esubc | ibmii | lol2 | oprepsx | prcaeps | sic | txdba | wcap |
| am | ch | curncd | dpact | exchg | icapt | long | optca | prcc_c | sich | txdbca | weburl |
| ano | che | currtr | dpc | exre | idbflag | loxdr | optdr | prcc_f | siv | txdbcl | xacc |
| ao | chech | cusip | dpvieb | fatb | idit | lqpl1 | optex | prch_c | spce | txdc | xad |
| aocidergl | ci | datadate | drc | fatc | incorp | lse | optexd | prch_f | spced | txdfed | xi |
| aociother | cibegni | dc | drlt | fate | intan | lt | optfvgr | prcl_c | spceeps | txdfo | xido |
| aocipen | cicurr | dclo | ds | fatl | intano | lul3 | optgr | prcl_f | spcindcd | txdi | xidoc |
| aocisecgl | cidergl | dcom | dt | fatn | intc | mib | optlife | priusa | spcseccd | txditc | xint |
| aodo | cik | dcpstk | dudd | fato | intpn | mibn | optosby | prsho | spcsrc | txds | xinto |
| aol2 | cimii | dc | dv | fatp | invch | mibt | optosey | prstk | spi | txfed | xopr |
| aoloch | ciother | dcvsr | dvc | fax | invfg | mii | optprby | pstk | sppe | txfo | xpp |
| aox | cipen | dcvsub | dvp | fca | invo | mkvalt | optprcca | pstk | sppiv | txndb | xpr |
| ap | cisecgl | dcvt | dvp | fdate | invrm | mrc1 | optprcex | pstkl | src | txndba | xrd |
| apalch | citotal | dd | dvpsp_c | fiao | inv | mrc2 | optprcey | pstkn | sstk | txndbl | xrdp |
| apdate | city | dd1 | dvpsp_f | fic | invwip | mrc3 | optprcgr | pstkr | stalt | txndbr | xrent |
| aqc | cld2 | dd2 | dvpsx_c | finf | ipodate | mrc4 | optprcwa | pstkrv | state | txo | xsga |
| aqi | cld3 | dd3 | dvpsx_f | fopo | ismod | mrc5 | optprfr | rdip | state_name | txp | |
| aqpl1 | cld4 | dd4 | dvt | fopox | itcb | mrc | optvol | rdipa | stkco | txpd | |
| aq | cld5 | dd5 | dxd2 | fyr | itci | mrc | pdate | rdipd | stkcpa | txr | |
| at | cogs | dfs | dxd3 | fyr | ivaco | msa | pddur | rdipeps | stko | txs | |
| au | conm | diladj | dxd4 | gdwl | ivaeq | naics2 | phone | re | teq | txt | |
| aul3 | conml | dilavx | dxd5 | ggroup | ivao | naics3 | pi | rea | tfva | txtubadjust | |

Table 2: Variable names

| Sector | N | Description |
|--------|------|--|
| 11 | 18 | Agriculture, Forestry, Fishing and Hunting |
| 21 | 426 | Mining, Quarrying, and Oil and Gas Extraction |
| 22 | 248 | Utilities |
| 23 | 78 | Construction |
| 31–33 | 2193 | Manufacturing |
| 42 | 169 | Wholesale Trade |
| 44–45 | 235 | Retail Trade |
| 48–49 | 148 | Transportation and Warehousing |
| 51 | 652 | Information |
| 52 | 2122 | Finance and Insurance |
| 53 | 341 | Real Estate and Rental and Leasing |
| 54 | 233 | Professional, Scientific, and Technical Services |
| 55 | 0 | Management of Companies and Enterprises |
| 56 | 111 | Administrative and Support and Waste Management and Remediation Services |
| 61 | 26 | Educational Services |
| 62 | 117 | Health Care and Social Assistance |
| 71 | 43 | Arts, Entertainment, and Recreation |
| 72 | 106 | Accommodation and Food Services |
| 81 | 17 | Other Services (except Public Administration) |
| 92 | 0 | Public Administration |
| 99 | 105 | Nonclassifiable |

Table 3: Structure of 2017 NAICS

| Monster Beverage Corp | | Kellogg Co | |
|----------------------------|--|------------|--|
| 31 | Manufacturing | 31 | Manufacturing |
| 312 | Beverage and Tobacco Product Manufacturing | 311 | Food Manufacturing |
| 3121 | Beverage Manufacturing | 3112 | Grain and Oilseed Milling |
| 31211 | Soft Drink and Ice Manufacturing | 31123 | Breakfast Cereal Manufacturing |
| 312111 | Soft Drink Manufacturing | 311230 | Breakfast Cereal Manufacturing |
| Coca Cola Consolidated Inc | | Nike Inc | |
| 31 | Manufacturing | 31 | Manufacturing |
| 312 | Beverage and Tobacco Product Manufacturing | 316 | Leather and Allied Product Manufacturing |
| 3121 | Beverage Manufacturing | 3162 | Footwear Manufacturing |
| 31211 | Soft Drink and Ice Manufacturing | 31621 | Footwear Manufacturing |
| 312111 | Soft Drink Manufacturing | 316210 | Footwear Manufacturing |

Table 4: Examples of NAICS

| Removed variables by t-test (105/349) | | | | | | | | | | | |
|---------------------------------------|---------|---------|---------|---------|---------|---------|----------|---------|---------|---------------|--------------|
| adjex_c | aoloch | currtr | do | epsi | glceeps | lno | oprepsx | prcaeps | pstkr | spi | txo |
| adjex_f | apalch | dcom | donr | epspx | invch | lol2 | optca | prcc_c | pstkrv | sstk | txtubadjust |
| ajex | aqi | dcpstk | dvp | esopdlt | inv | long | optlife | prcc_f | rdip | tfva | txtubxintis |
| ajp | che | dcvsr | dvpsp_c | esopnr | ivaco | lqpl1 | optrfr | prch_c | rdipa | tfvl | xi |
| ano | cicurr | dcvt | dvpsp_f | exre | ivao | mib | pncwia | prch_f | rea | tstkp | xido |
| aocidergl | cidergl | diladj | dvpsx_c | fatl | ivch | msa | pncwip | prcl_c | recco | txach | xintopt |
| aociother | ciother | dlc | dvpsx_f | fca | ivncf | nopi | pnrsho | prcl_f | seqo | txdfed | |
| aocisecgl | cshr | dltis | epsfi | fiao | ivst | np | prca | pstk_c | siv | txdi | |
| aol2 | cstkcv | dltr | epsfx | glced | lat | opeps | prcad | pstkl | spce | txndbr | |
| Removed variables by VIF (154/244) | | | | | | | | | | | |
| acodo | ceq | cshfd | dlch | dxd3 | ibc | loxdr | ni | pifo | reuna | txdba | txtubpospinc |
| acominc | ceql | cshi | dlto | dxd4 | ibcom | lse | niadj | pncaeps | revt | txditc | txtubtxtr |
| acox | ceqt | cshpri | dltt | dxd5 | ibmii | lt | oancf | ppeg | sale | txds | txtubxintbs |
| act | ch | cshtr_f | dn | ebit | icapt | lul3 | oiadp | ppent | seq | txfo | xacc |
| am | ci | cstke | dp | ebitda | intan | mibn | oibdp | ppeveb | spceeps | txndb | xint |
| ao | cibegni | dd | dpact | esub | intpn | mibt | optosby | pstk | sppiv | txndba | xopr |
| aodo | cimii | dd1 | dpc | fopo | lco | mkvalt | optosey | pstk_n | stkco | txndbl | xpr |
| aox | cisecgl | dd2 | dpvieb | fopox | lcox | mrc1 | optprcby | rdipd | teq | txpd | xrd |
| ap | citotal | dd3 | ds | gdwl | lcoxdr | mrc2 | optprcca | re | tlcf | txt | xrdp |
| at | cld2 | dd4 | dt | glcep | lct | mrc3 | optprcex | reajo | tstk | txtubbegin | xrent |
| aul3 | cld4 | dd5 | dv | gp | lifr | mrc4 | optprcey | recd | tstk_c | txtubend | xsga |
| capx | cld5 | dfs | dvt | ib | lifrp | mrc5 | optprcgr | rect | txc | txtubposinc | |
| capxv | cogs | dilavx | dxd2 | ibadj | lo | mrct | pi | rectr | txdb | txtubpospdec | |
| Preselected variables (90) | | | | | | | | | | | |
| acdo | caps | dclo | dvc | fate | intc | ivstch | optgr | rdipeps | txbco | txr | xidoc |
| aco | chch | dcs | dvpa | fatn | invfg | mii | optprcwa | rech | txbcof | txs | xpp |
| aldo | cipen | dcvsub | emp | fato | invo | mrcta | optvol | recta | txdbca | txtubposdec | |
| aocipen | cld3 | dltp | esopct | fatp | invrm | no pio | pidom | spced | txdbcl | txtubsettle | |
| aqc | csho | dm | esopt | fincf | invwip | optdr | pnca | sppe | txdc | txtubsoflimit | |
| aqpl1 | cshtr_c | drc | esubc | glcea | itcb | optex | pncad | stkcpa | txdfo | txw | |
| aqs | cstk | drlt | fatb | idit | itci | optexd | prsho | tfvce | txfed | wcap | |
| bkvlp | dc | dudd | fatc | intano | ivaeq | optfvgr | prstkc | tstkn | txp | xad | |

Table 5: Preselected variables by t-test and VIF

| year | All deletion | Bankruptcy | Liquidation | B + L |
|------|--------------|------------|-------------|-------|
| 2011 | 241 | 1 | 16 | 17 |
| 2012 | 363 | 6 | 29 | 35 |
| 2013 | 348 | 8 | 38 | 46 |
| 2014 | 369 | 3 | 47 | 50 |
| 2015 | 348 | 8 | 36 | 44 |
| 2016 | 356 | 10 | 31 | 41 |
| 2017 | 273 | 6 | 1 | 7 |
| 2018 | 243 | 8 | 1 | 9 |
| 2019 | 266 | 16 | 0 | 16 |
| 2020 | 99 | 4 | 0 | 4 |

Table 6: Number of deleted companies

| | Obs | Mean | Std.Dev | Min | 25% | 75% | Max |
|---------|-------|-------|---------|---------|--------|--------|--------|
| aco | 7,380 | 0.000 | 1.000 | −0.150 | −0.149 | −0.107 | 42.624 |
| aqpl1 | 7,380 | 0.000 | 1.000 | −0.067 | −0.067 | −0.056 | 55.435 |
| caps | 7,380 | 0.000 | 1.000 | −0.389 | −0.209 | −0.029 | 34.987 |
| csho | 7,380 | 0.000 | 1.000 | −0.179 | −0.163 | −0.053 | 44.172 |
| cstk | 7,380 | 0.000 | 1.000 | −0.138 | −0.138 | −0.119 | 35.533 |
| glcea | 7,380 | 0.000 | 1.000 | −6.029 | −0.134 | −0.043 | 71.202 |
| optfvgr | 7,380 | 0.000 | 1.000 | −0.080 | −0.065 | −0.008 | 66.953 |
| spced | 7,380 | 0.000 | 1.000 | −83.980 | −0.016 | −0.006 | 5.143 |
| stkcpa | 7,380 | 0.000 | 1.000 | −1.930 | −0.235 | −0.005 | 42.503 |

Table 7: Summary statistics of continuous variables

| | | stalt | | |
|----|---|-------|------|------|
| | | 1 | 0 | |
| BL | 1 | 4 | 72 | 76 |
| | 0 | 16 | 7288 | 7304 |
| | | 20 | 7360 | 7380 |

| | | idbflag | | |
|----|---|---------|------|------|
| | | B | D | |
| BL | 1 | 0 | 76 | 76 |
| | 0 | 445 | 6859 | 7304 |
| | | 445 | 6935 | 7380 |

| | | idbflag | | |
|-------|---|---------|------|------|
| | | B | D | |
| stalt | 1 | 1 | 19 | 20 |
| | 0 | 444 | 6916 | 7360 |
| | | 445 | 6935 | 7380 |

Table 8: Contingency tables

| | Pearson's chi-squared test | | | Fisher's exact test | | | |
|-----------------|----------------------------|----|-------------------------|---------------------|--------|----------|------------------------|
| | χ^2 | df | p -value | odds ratio | 95% CI | | p -value |
| BL – stalt | 53.376 | 1 | 2.755×10^{-13} | 25.238 | 5.994 | 80.874 | 4.441×10^{-5} |
| BL – idbflag | 3.911 | 1 | 0.048 | ∞ | 1.299 | ∞ | 0.014 |
| stalt – idbflag | 2.648×10^{-29} | 1 | 1 | 1.219 | 0.193 | 50.795 | 1 |

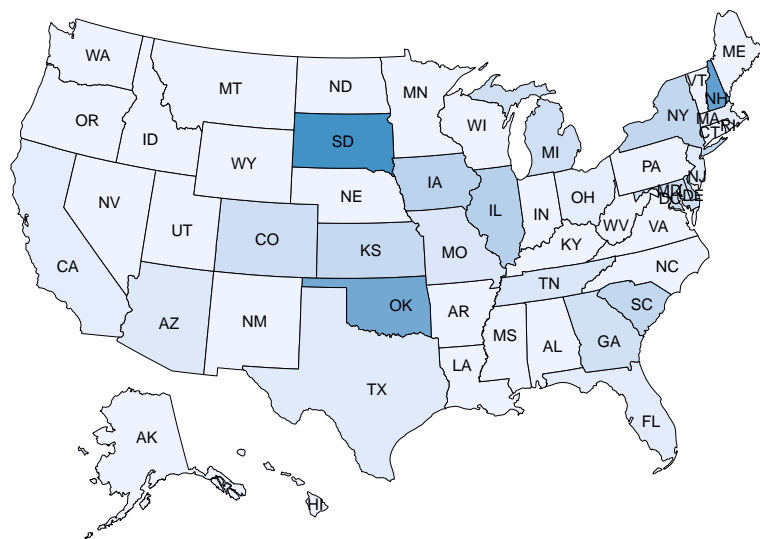
Table 9: Result of tests

| variable | type | variable | type | variable | type | variable | type |
|---------------|---------|----------|---------|----------|---------|----------|---------|
| aco | numeric | dvc | numeric | naics2 | factor | spced | numeric |
| aqpl1 | numeric | emp | numeric | nopio | numeric | stalt | factor |
| bkvlp | numeric | exchg | factor | optex | numeric | state | factor |
| BL | factor | fate | numeric | optexd | numeric | stkcpa | numeric |
| caps | numeric | fic | factor | optfvgr | numeric | tstkn | numeric |
| census_region | factor | finch | numeric | optgr | numeric | txdbca | numeric |
| chech | numeric | glcea | numeric | optprcwa | numeric | txdc | numeric |
| csho | numeric | idbflag | factor | optvol | numeric | txfed | numeric |
| cshtr_c | numeric | idit | numeric | prstkc | numeric | txs | numeric |
| cstk | numeric | intano | numeric | recch | numeric | wcap | numeric |
| dm | numeric | mrcta | numeric | recta | numeric | xad | numeric |

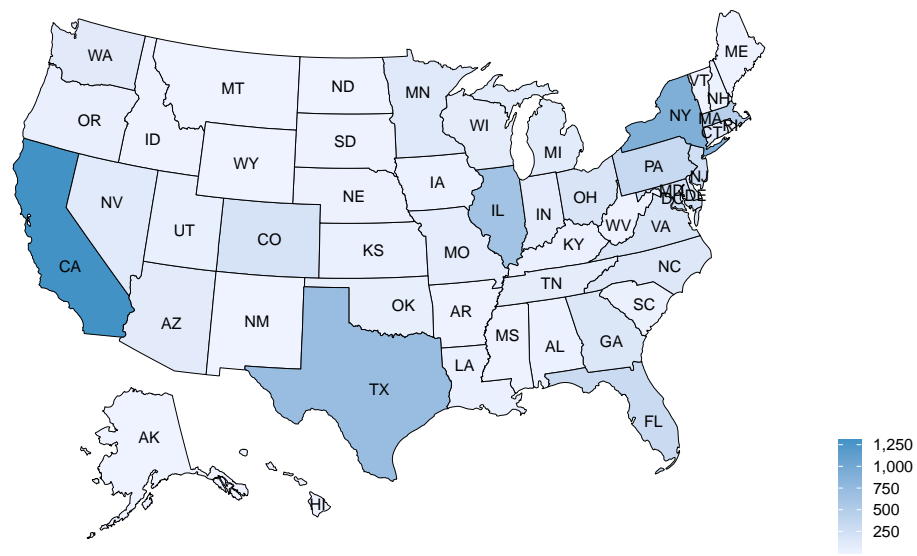
Table 10: 44 variables before stepwise selection

| | Estimate | Std. Error | z value | $P(> z)$ |
|----------------------|----------|------------|-----------|------------|
| (Intercept) | -203.596 | 970.743 | -0.210 | 0.834 |
| aco | -7.190 | 6.036 | -1.191 | 0.234 |
| aqpl1 | 8.901 | 3.520 | 2.529 | 0.011 |
| caps | 2.117 | 0.961 | 2.204 | 0.028 |
| csho | -7.575 | 3.088 | -2.453 | 0.014 |
| cstk | -13.510 | 11.405 | -1.185 | 0.236 |
| glcea | 1.561 | 0.675 | 2.312 | 0.021 |
| idbflag _D | 197.005 | 970.673 | 0.203 | 0.839 |
| optfvgr | -19.957 | 6.620 | -3.015 | 0.003 |
| spced | -2.420 | 1.488 | -1.627 | 0.104 |
| stalt ₁ | 2.663 | 1.242 | 2.144 | 0.032 |
| stkcpa | -3.308 | 1.449 | -2.284 | 0.022 |

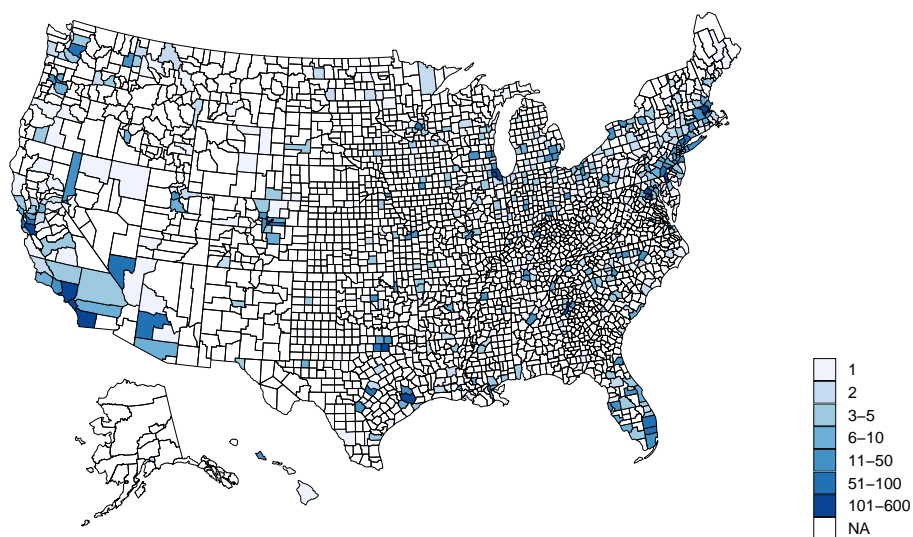
Table 11: Coefficients



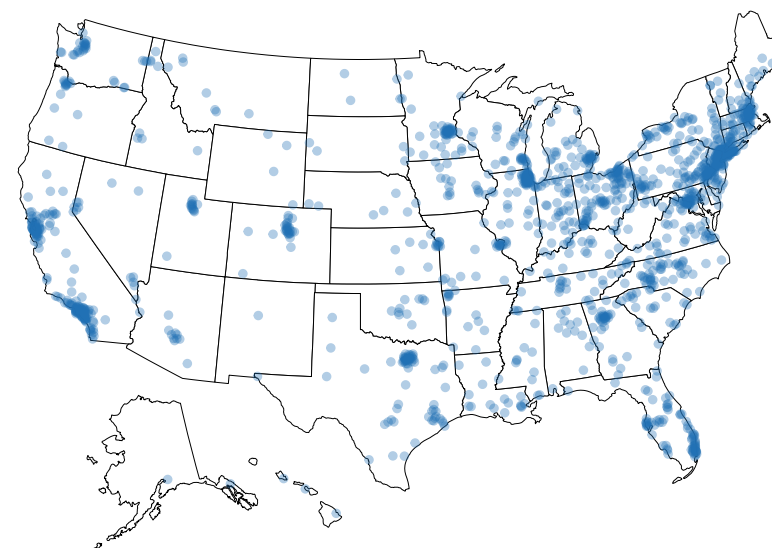
(a) Proportion of bankruptcy and liquidation by states



(b) Number of companies by states

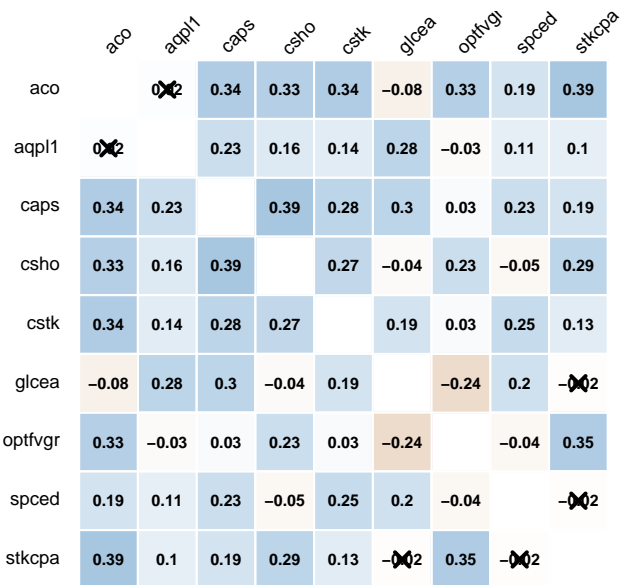


(c) Number of companies by counties

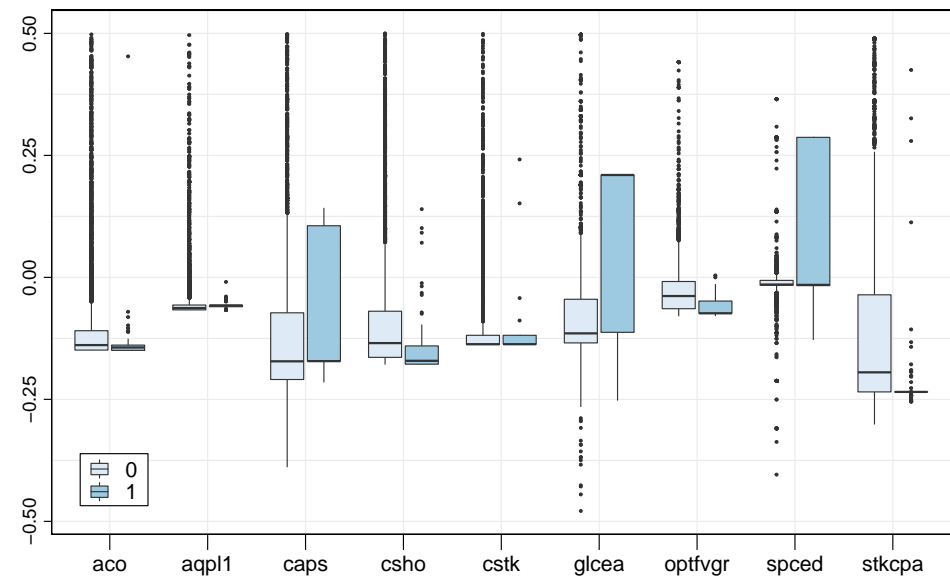


(d) Plotting cities where a company is located

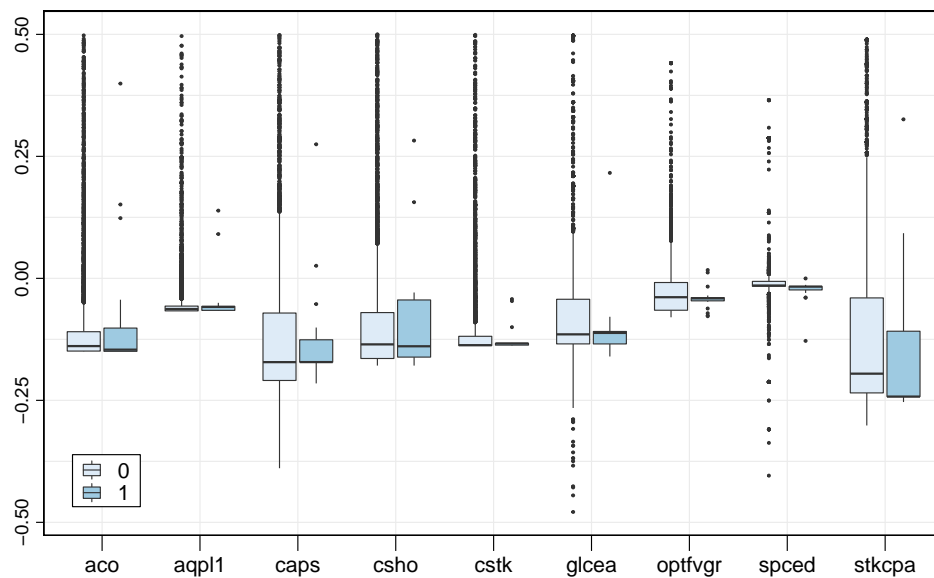
Figure 2: US map by states, counties, and cities



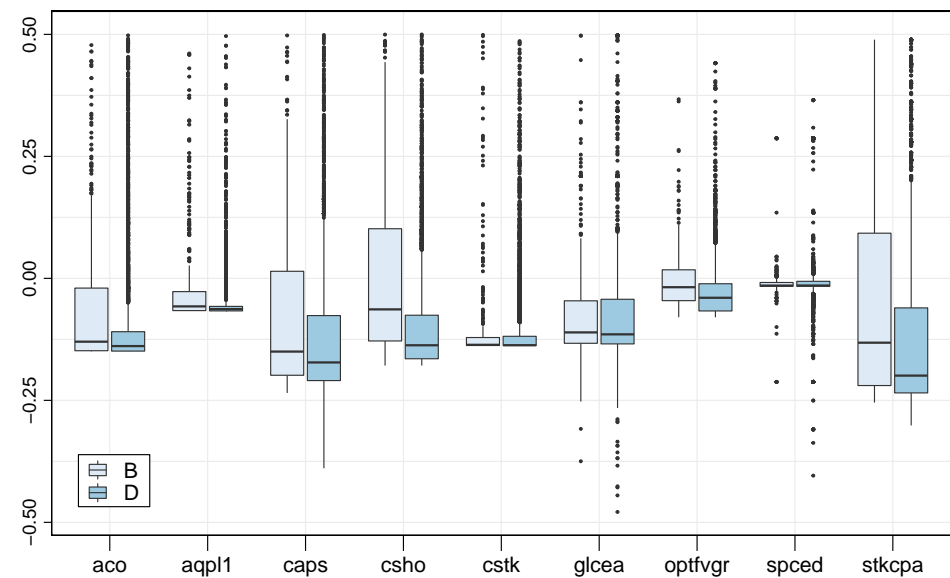
(a) Spearman correlogram with significance test



(b) Box plots by BL (truncated)



(c) Box plots by stalt (truncated)



(d) Box plots by idbflag (truncated)

Figure 3: Correlogram and box plots