수리통계학1 - HW1

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1.1-5

(a) $P(A) = \frac{1}{6}$

(b) B^c : 4 is observed at first.

$P(B) = 1 - P(B^c) = 1 - \frac{1}{6} = \frac{5}{6}$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

since $P(A \cap B) = 0$, $P(A \cup B) = P(A) + P(B) = 1$

1.1-9

(a) $P(A_1 \cup A_2 \cup A_3)$

$= P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_3 \cap A_1)$
$+ P(A_1 \cap A_2 \cap A_3)$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - (\frac{1}{3})^2 - (\frac{1}{3})^2 - (\frac{1}{3})^2 + (\frac{1}{3})^3$

$= 1 - \frac{1}{3} + \frac{1}{27}$

$= \frac{19}{27}$

(b) $(A_1 \cup A_2 \cup A_3)^c = A_1^c \cap A_2^c \cap A_3^c$

No 1,2 in first roll : $1 - \frac{1}{3}$

No 3,4 in first roll : $1 - \frac{1}{3}$

No 5,6 in first roll : $1 - \frac{1}{3}$

$\therefore P((A_1 \cup A_2 \cup A_3)^c) = (\frac{2}{3})^3 = \frac{8}{27}$

$\Rightarrow P(A_1 \cup A_2 \cup A_3) = 1 - \frac{8}{27} = \frac{19}{27}$

1.1-15. If A and B are mutually exclusive,
then $P(A \cup B) = P(A) + P(B)$

(a) $1 = P(S) = P(A_1 \cup A_2 \cup \cdots \cup A_m)$

$= P(A_1 \cup A_2 \cup \cdots \cup A_{m-1}) + P(A_m)$

$= P(A_1) + P(A_2) + \cdots + P(A_m)$

$P(A_1) = \cdots = P(A_m)$, thus $P(A_1) = \cdots = P(A_m) = \frac{1}{n}$

(b) $A = A_1 \cup A_2 \cup \cdots \cup A_h$

$$P(A) = P(A_1 \cup \cdots \cup A_h)$$
$$= P(A_1) + \cdots + P(A_h)$$
$$= \underbrace{\frac{1}{m} + \cdots + \frac{1}{m}}_{h} = \frac{h}{m} \qquad \blacksquare$$

1.3-3.

	B_1	B_2	Total
A_1	10	15	25
A_2	25	10	35
Total	35	25	60

(a) $P(A_2 \cap B_1) = \frac{25}{60} = \frac{5}{12}$

(b) $P(A_2 \cup B_1) = \frac{35+35-25}{60} = \frac{45}{60} = \frac{3}{4}$

(c) $P(A_2 | B_1) = \frac{P(A_2 \cap B_1)}{P(B_1)} = \frac{\frac{25}{60}}{\frac{35}{60}} = \frac{5}{7}$

(d) $P(B_2 | A_1) = \frac{P(B_2 \cap A_1)}{P(A_1)} = \frac{\frac{15}{60}}{\frac{25}{60}} = \frac{3}{5}$

(e) ① select a Right thumb on top

$$P(A_1 | B_1) = \frac{P(A_1 \cap B_1)}{P(B_1)} = \frac{\frac{10}{60}}{\frac{35}{60}} = \frac{2}{7}$$

② select a Left thumb on top

$$P(A_1 | B_2) = \frac{P(A_1 \cap B_2)}{P(B_2)} = \frac{\frac{15}{60}}{\frac{25}{60}} = \frac{3}{5}$$

I will select a left thumb on top

1.3-6. $A = \{$died from heart disease$\}$

$B = \{$at least one parent had heart disease$\}$

$$P(A | B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{N(A \cap B^c)}{N(B^c)} = \frac{110}{648} = \frac{55}{324}$$

1.4 − 2. $P(A) = 0.3$ $P(B) = 0.6$

(a) $P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.6 = 0.18$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.18 = 0.72$

(b) $P(A \cap B) = 0$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{0.6} = 0$

1.4 − 6.

① A and $(B \cap C)$

$P(A \cap (B \cap C)) = P[A \cap B \cap C]$

$= P(A) \cdot P(B) \cdot P(C)$

$= P(A) \cdot P(B \cap C)$

② A and $B \cup C$

$P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C))$

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$= P(A) \cdot P(B) + P(A) \cdot P(C) - P(A) \cdot P(B) \cdot P(C)$

$= P(A)\,[P(B) + P(C) - \underbrace{P(B) \cdot P(C)}_{= P(B \cap C)}]$

$= P(A) \cdot P(B \cup C)$

③ A' and $(B \cap C')$

$P[A' \cap (B \cap C')] = P(A' \cap C' \cap B)$

$= P(B) \cdot P(A' \cap C' | B)$

$= P(B)\,[1 - P(A \cup C) | B)$

$= P(B)(1 - P(A \cup C))$

$= P(B)(P(A \cup C)^c)$

$= P(B)\,P(A^c \cap C^c)$

$= P(B) \cdot P(A^c) \cdot P(C^c)$

$= P(A^c) \cdot P(B \cap C^c)$

④ A', B', C'

$$
\begin{aligned}
P(A' \cap B' \cap C') &= P[(A \cup B \cup C)'] \\
&= 1 - P(A \cup B \cup C) \\
&= 1 - P(A) - P(B) - P(C) + P(A)\cdot P(B) + P(B)\cdot P(C) \\
&\quad + P(C)\cdot P(A) - P(A)\cdot P(B)\cdot P(C) \\
&= (1-P(A))(1-P(B))(1-P(C)) \\
&= P(A')\cdot P(B')\cdot P(C')
\end{aligned}
$$

1.4 -7

(a)
$$
\begin{aligned}
P(A_1 \cup A_2 \cup A_3) &= 1 - P[(A_1 \cup A_2 \cup A_3)'] \\
&= 1 - P(A_1' \cap A_2' \cap A_3') \\
&= 1 - P(A_1')\cdot P(A_2')\cdot P(A_3') \\
&\quad (\text{By } 1.4-6 \text{ ④}) \\
&= 1 - 0.6 \cdot 0.57 \cdot 0.4 = 0.8632
\end{aligned}
$$

(b) $P(A_1 \cap A_2) = P(A_1)\cdot P(A_2) = 0.6 \cdot 0.57 = 0.342$

$P(A_2 \cap A_3) = P(A_2)\cdot P(A_3) = 0.57 \cdot 0.4 = 0.228$

$P(A_3 \cap A_1) = P(A_3)\cdot P(A_1) = 0.4 \cdot 0.6 = 0.24$

$P(A_1 \cap A_2) + P(A_2 \cap A_3) + P(A_3 \cap A_1) - 3 \times P(A_1 \cap A_2 \cap A_3)$

$= 0.342 + 0.228 + 0.24 - 3 \times 0.1368 = 0.3996$

1.5 -4

A : event that age of the driver is 16-25

B : denote on accidence

$$P(A|B) = \frac{0.1 \cdot 0.05}{0.1 \cdot 0.05 + 0.55 \cdot 0.02 + 0.2 \cdot 0.03 + 0.15 \cdot 0.04}$$

$$= \frac{50}{280} = 0.179$$

1.5 − 6.

A : policyholder dies.

B_1, B_2, B_3 : standards, preferred, ultra-preferred.

$$P(B_1|A) = \frac{0.6 \cdot 0.01}{0.6 \cdot 0.01 + 0.3 \cdot 0.008 + 0.1 + 0.007} = 0.659$$

$$P(B_2|A) = 0.264$$

$$P(B_3|A) = 0.077$$

1.5 − 10.

$P(D^-|A^+) = 0.08$ $\quad P(D^-|A^-) = 0.95$ $\quad P(A^+) = 0.02$

$P(D^+|A^-) = 0.05$ $\quad P(D^+|A^+) = 0.92$ $\quad P(A^-) = 0.98$

(a) $P(D^+) = (0.02) \cdot (0.92) + (0.98) \cdot (0.05) = 0.0674$

(b) $P(A^-|D^+) = \frac{0.0490}{0.0674} = 0.727$

$P(A^+|D^+) = \frac{0.0184}{0.0674} = 0.273$

(c) $$P(A^-|D^-) = \frac{0.98 \cdot 0.95}{0.02 \cdot 0.08 + 0.98 \cdot 0.95} = 0.998$$

$P(A^+|D^-) = 1 - P(A^-|D^-) = 0.002$

(d) Yes. They are alarming.