

# Mathematical Statistics 2 – Midterm Exam

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1. Describe and prove the Central Limit Theorem (CLT).
2. Let  $X_1, \dots, X_n$  be a random sample from the normal distribution,  $N(\mu, \sigma^2)$ . We know that the sample mean  $\bar{X}$  and the sample variance  $S^2$  are independent.

- (1) Find the distribution of the sample mean  $\bar{X}$  using the mgf of  $X_i$ .
- (2) Find the distribution of

$$V = \frac{(n-1)S^2}{\sigma^2}.$$

- (3) Find the distribution of

$$W = \frac{\bar{X} - \mu}{s/\sqrt{n}}.$$

- (4) Compute the mean and variance of  $W$ .

3. Let  $X_1, \dots, X_n$  ( $n \geq 2$ ) be a random sample following an exponential distribution with mean  $\theta$ . We have order statistics  $X_{(1)}, \dots, X_{(n)}$  from  $X_1, \dots, X_n$ .

- (1) Find the pdf of  $X_{(i)}$ ,  $i = 1, \dots, n$ .
- (2) Find the joint pdf of  $X_{(i)}$  and  $X_{(j)}$ , where  $1 \leq i < j \leq n$ .
- (3) Find a S.S for  $\theta$ .
- (4) Find the MLE of  $\theta$  using a S.S for  $\theta$ .
- (5) Find the UE of  $\theta$  using the pdf of MLE of  $\theta$  and its mean.
- (6) Find the MME of  $\theta$ .

4. Let  $X_1, \dots, X_n$  be a random sample from the distribution with a pdf

$$f(x|\theta) = \frac{2x}{\theta^2} I(0 \leq x \leq \theta), \quad \theta > 0.$$

where  $I(\cdot)$  is the indicator function.

- (1) Find the MME of  $\theta$ .
  - (2) Find the MLE of  $\theta$ .
  - (3) Find the UE (unbiased estimator) of  $\theta$  using the pdf of MLE for  $\theta$ .
5. We have a random sample,  $X_1, \dots, X_n$  with the following distribution.
- (1)  $N(0, \theta_1 = \sigma^2)$ . Find the MLE of  $\theta_1$  using a S.S for  $\theta_1$ .
  - (2) We have a pdf of  $X_i$ ,

$$f(x|\theta_2) = \frac{1}{2} I(\theta_2 - 1 \leq x \leq \theta_2 + 1).$$

Find the MLE of  $\theta_2$ , if it exists.