Financial Mathematics I (SNU) – Exam 1

May 7, 2020

1. Consider a two-period binomial model with the time index $t = 0, 1, 2$. This model has two underlying assets; a bank account process $(G_t)_{t=0,1,2}$ given by

$$G_0 = G_1 = 1, \quad G_2 = 1.2$$

and stock with price process given by the following tree.

$$\begin{tikzcd}
 & & 240 \\
 & 160 \arrow[ur, "0.5"] \arrow[dr, "0.5"] & \\
140 \arrow[ur, "0.6"] \arrow[dr, "0.4"] & & 180 \\
 & 120 \arrow[ur, "0.5"] \arrow[dr, "0.5"] & \\
 & & 120
\end{tikzcd}$$

   (a) Construct a sample space Ω, a probability $\mathbb{P}$, a filtration $(\mathcal{F}_t)_{t=0,1,2}$, and stock price process $(S_t)_{t=0,1,2}$ representing this market.

   (b) Consider an option with payoff

$$X_2 = \begin{cases} 30 & \text{if } S_2 = 240 \\ 15 & \text{if } S_2 = 180 \\ 0 & \text{if } S_2 = 120 \end{cases}$$

   and maturity $t = 2$.

      i. Find the price process $(X_t)_{t=0,1,2}$ and the hedging portfolio of this option.

      ii. Find the risk-neutral measure $\mathbb{Q}$.

2. Let C_E and C_A be the prices of European call and American call options, respectively, for the same maturity and strike. Show that $C_E = C_A$ when the short rate $r \geq 0$.

3. Let B be a Brownian motion. Calculate $\mathbb{E}(B_1|B_3)$.

4. Solve the following problems.

   (a) Let $(X_t)_{t\geq 0}$ be a RCLL Gaussian process. Show that

$$\left(\int_0^t X_s\, ds\right)_{t\geq 0}$$

   is a continuous Gaussian process. You may use, without proof, the fact that the limit (in the sense of convergence in distribution) of a sequence of normal random variables is normal.

   (b) Calculate $\mathbb{E}(e^{\int_0^T B_u\, du})$.